Overcoming Braess's Paradox: A Modelling based Approach

Complex Social Systems: Modeling Agents, Learning, and Games Course Project HS 2022

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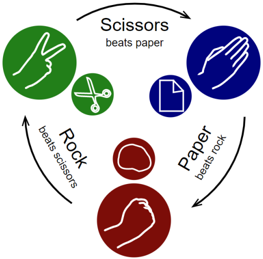
Outline

1. What is Braess's Paradox?
   - Refresher on Game Theory
   - A congestion game
2. Overcoming the paradox
   - The Centralized Scheduler
   - A Distributed Approach
3. Resolution
   - Conclusion and Outlook
   - Applications

Mathematically models interactions between agents $(i=1 \cdots n)$



https://commons.wikimedia.org/wiki/File:Rock-paper-scissors.svg

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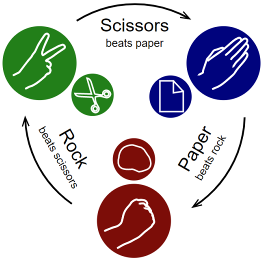

a_1 \ a_2	Rock	Paper	Scissors
Rock			
Paper			
Scissors			

Game Theory

A refresher

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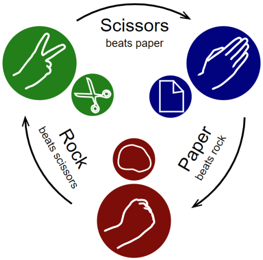
- Tries to maximize their reward $J_i(a_1, \cdots a_n)$



$\downarrow$

a_2 \ a_1	Rock	Paper	Scissors
Rock	0 / 0	1 / -1	-1 / 1
Paper	-1 / 1	0 / 0	1 / -1
Scissors	1 / -1	-1 / 1	0 / 0

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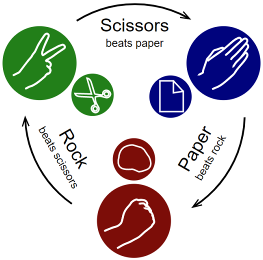
- If no agent can do better by unilaterally changing their action a_i - $\{a_1^*, \cdots a_n^*\}$ is a Nash Equilibrium



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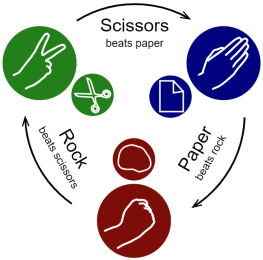
Social Cost

indicates the well-being of the society

$$\sum_{i=1}^{n} J_i(a_1, \cdots a_n)$$

Price of Anarchy

indicates if the NE cost is better or worse than the lowest achievable cost



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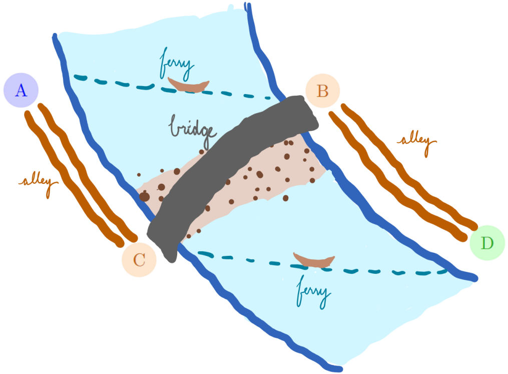
Crossing the Grand Canal in Venice

A congestion game



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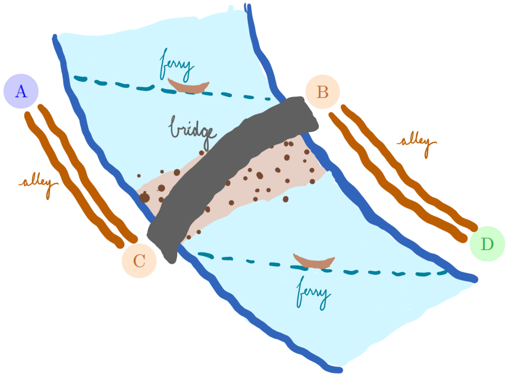

Narrow alleys

Crossing the Grand Canal in Venice

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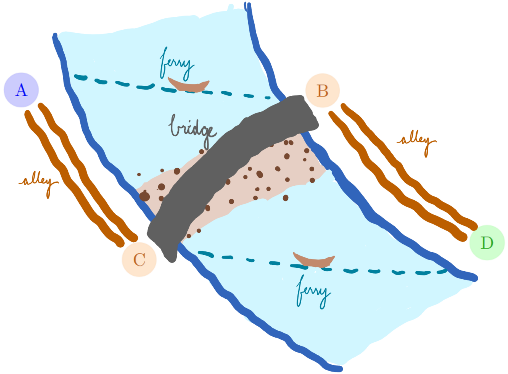
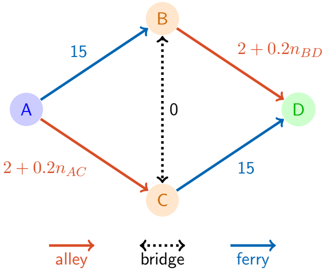




Modelling the Game

Why is it paradoxical?

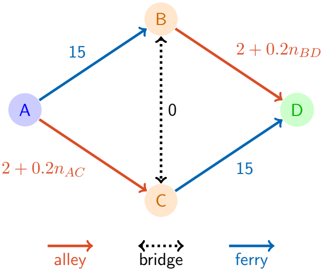
Time taken

Ferry: **15 min**

Alley: **2 min** + **12 sec** per person

Bridge: ϟ

and **60** agents.



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Why is it paradoxical?

No bridge

NE cost: $15 + 2 + 0.2 \times 30 =$ **23 min**

$n_{AC} = n_{BD} = 30$

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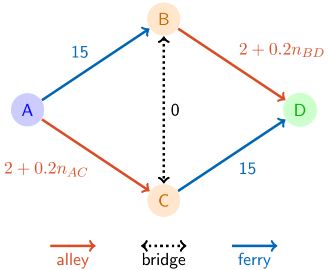

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With bridge

cost	ABD	ACD	ABCD	ACBD
Initial	23	23	30	**16**

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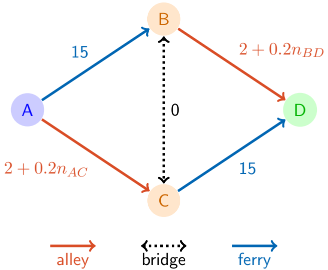

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cost	ABD	ACD	ABCD	ACBD
Initial	23	23	30	16
NE	29	29	30	28

5 min increase due to the bridge!

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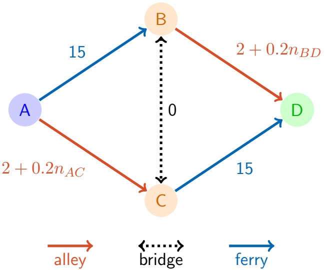

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cost	ABD	ACD	ABCD	ACBD
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5 min increase due to the bridge!

$$\text{PoA} = \frac{28}{22.96} = 121.95\%$$

Time taken

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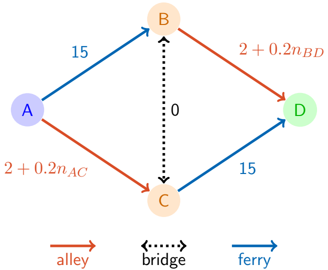
Bridge: ϟ

and **60** agents.



Modelling the Game

It is not just these numbers...

No bridge

NE cost: $40 + 15 + 0.1 \times 100 =$ **65 min**

$n_{AC} = n_{BD} = 100$

With bridge

cost	ABD	ACD	ABCD	ACBD
Initial	65	65	80	**50**
NE	75	75	80	70

5 min increase due to the bridge!

$$\text{PoA} = \frac{70}{64.375} = 108.74\%$$

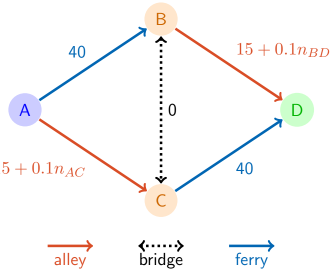
Time taken

Ferry: **40 min**

Alley: **15 min** + **6 sec** per person

Bridge: ⚡

and **200** agents.



Preliminary Agent Model

Introducing the parameters of each agent

Selfishness γ

$\gamma \in [0, 1]$ indicates how selfish the agent is.

$\gamma = 0$ completely selfish

$\gamma = 1$ completely benevolent

Connectivity $\mathcal{N}$

$|\mathcal{N}|$ indicates how well-connected an agent is.
$\mathcal{N}$ comprises of the "neighbours" of the agent.

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Cost

The cost that each player is trying to minimize

$$J_i(a_1, \cdots a_n) = (1-\gamma)C_i(a_1, \cdots a_n) + \gamma \sum_{l \in \mathcal{N}} C_l(a_1, \cdots a_n)$$

- Initially, we consider identical agents with fixed parameters
- The agents would be regulated by a centralized scheduler

The Centralized Scheduler

- An entity that would regulate the state of affairs

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- Its job is two-fold:
  - Decide the γ and $|\mathcal{N}|$ values
  - Schedule/prioritize the agents based on the cost they incur at every stage of the game

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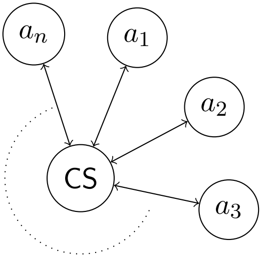

Figure: Centralized Scheduler Architecture

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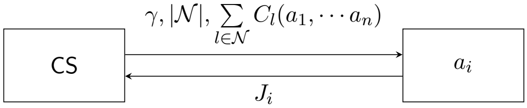
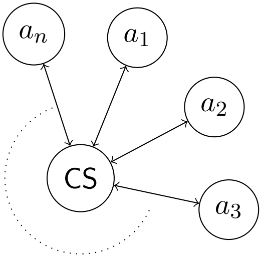


Figure: Information Exchange



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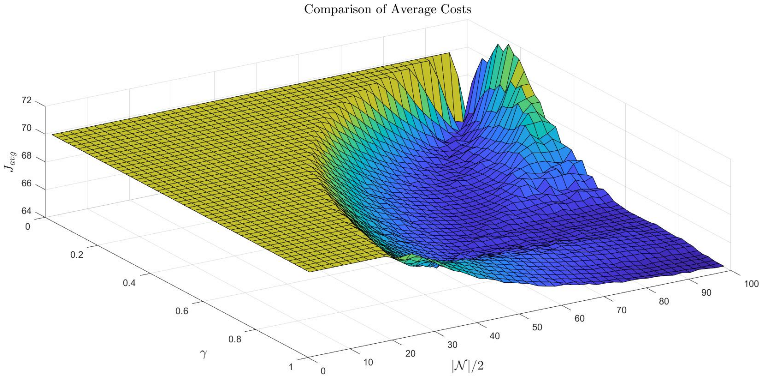
The Centralized Scheduler

Overall Result



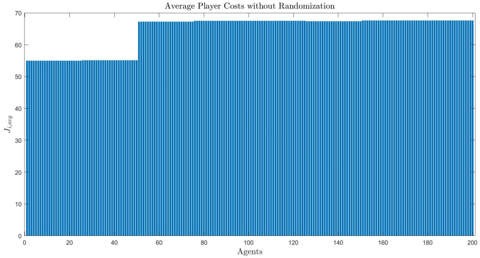
J_{avg} is the average cost incurred per agent per stage of the game. The lowest obtained value is $64.375 < 65$ for $\gamma = 1$ and $|\mathcal{N}| = 200$.

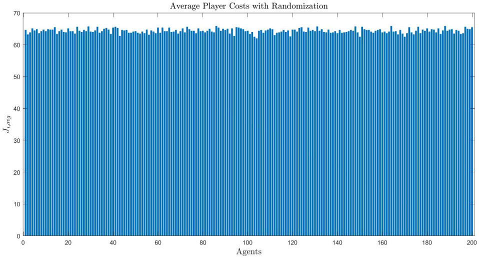
The Centralized Scheduler

Results for $\gamma = 1$ and $|\mathcal{N}| = 200$

Scheduling

The Centralized Scheduler sorts the costs of the different agents after each stage of the game and allows the agents with a high cost to choose their actions early in the next stage.





$J_{i,\text{avg}}$ is the average cost incurred by an agent i over the stages of the game

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Dynamic Agent Model

Introducing dynamics to the parameters of each agent

- In the absence of a centralized scheduler

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Cost Threshold τ

An "informed" decision of τ can be made by the agents based on the NE cost they used to incur prior to the construction of the bridge.

Dynamic Agent Model

Introducing dynamics to the parameters of each agent

- In the absence of a centralized scheduler
- What if the agents decide to update their parameters themselves?
- A "reasonable" agent, self-regulating model

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Dynamic Agent Model

A heuristic dynamic

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Selfishness $\gamma_i(t)$

If $J_i(t-1) > \tau$

$$\gamma_i(t) = \max(0, \gamma_i(t-1)/4)$$

Else

$$\gamma_i(t) = \min(1, 2\gamma_i(t-1))$$

Connectivity $\mathcal{N}_i(t)$

If $J_i(t-1) > \tau$

$$|\mathcal{N}_i(t)| = \max(0, |\mathcal{N}_i(t-1)|/4)$$

Else

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A Distributed Approach
Architecture

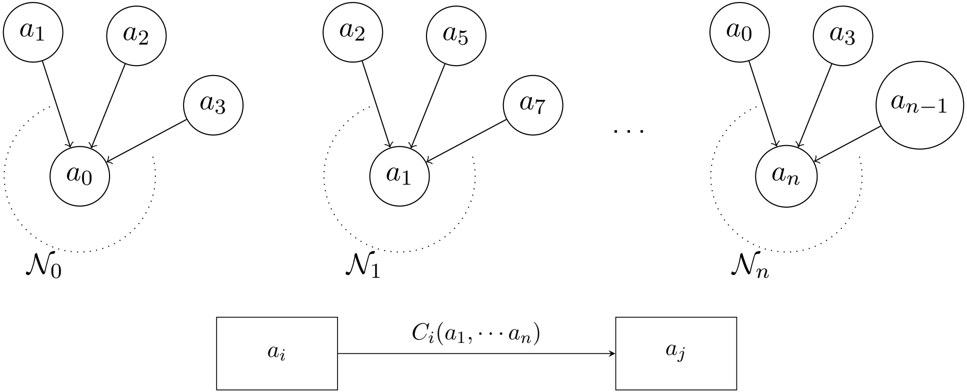

Figure: Information Exchange

A Distributed Approach

Does it mitigate the shortcomings of the centralized approach?

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Does it mitigate the shortcomings of the centralized approach?

- How to implement practically ? - More in Resolution
- ~~All decisions are controlled by one entity~~ Each agent has control over their decision
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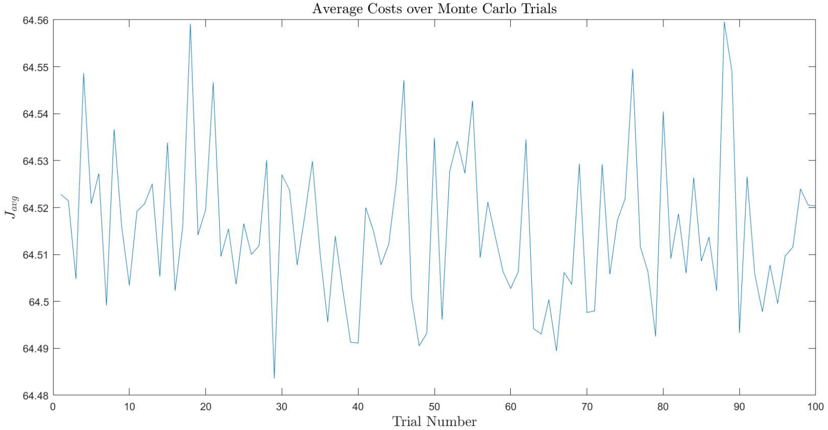
A Distributed Approach

Results of Monte Carlo Trials with $\gamma_i(0) \sim \mathcal{U}(0,1)$ and $|\mathcal{N}_i(0)| \sim \mathcal{U}(0,200)$



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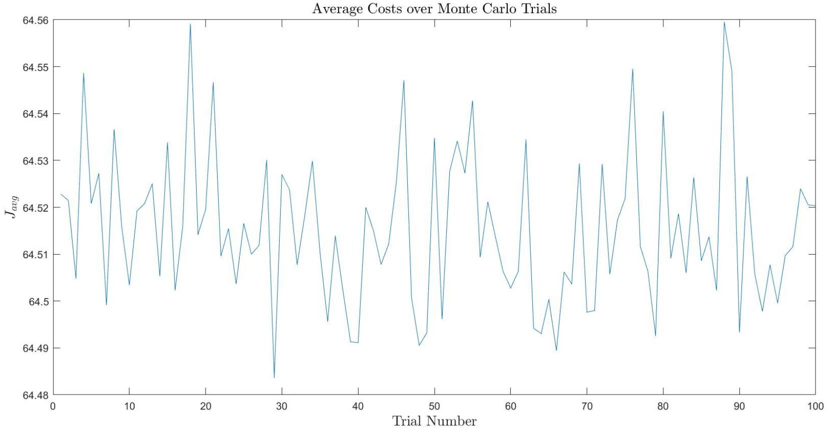

$$J_{avg} \sim \mathcal{N}(64.515, 0.016)$$

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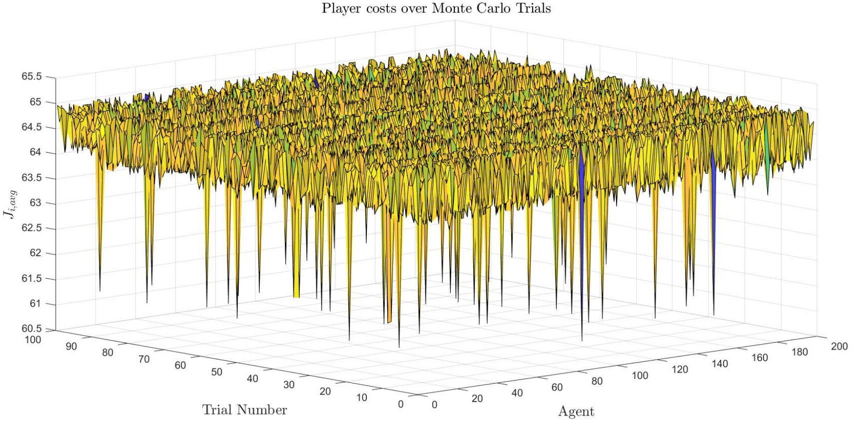

A Distributed Approach

Individual Player Parameter Behaviour with $\gamma_i(0) \sim \mathcal{U}(0,1)$ and $|\mathcal{N}_i(0)| \sim \mathcal{U}(0,200)$

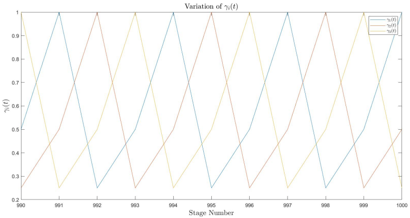

Figure: Variation of γ with stage

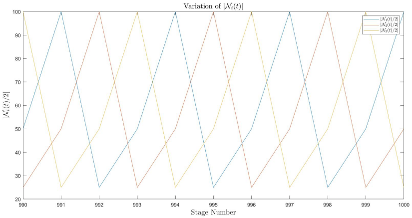

Figure: Variation of $|\mathcal{N}|$ with stage

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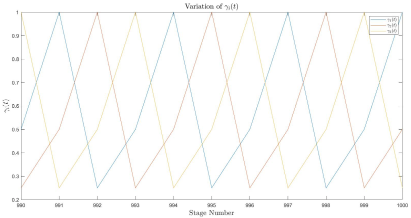

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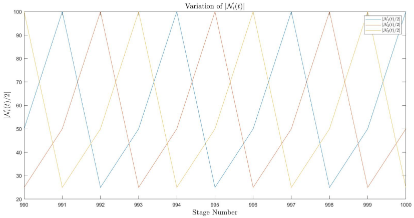

Figure: Variation of $|\mathcal{N}|$ with stage

- Only a few distinct types of agents, number varies based on their initial parameters

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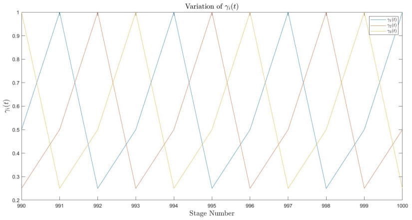

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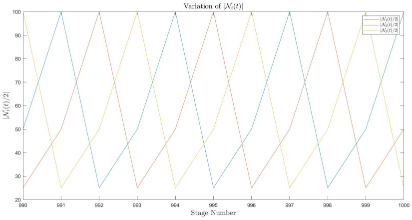

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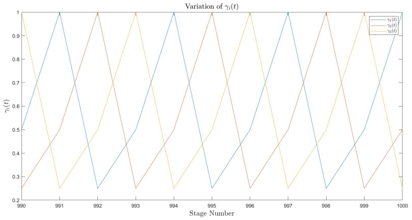

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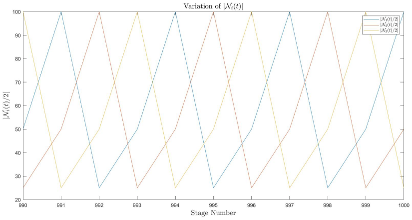
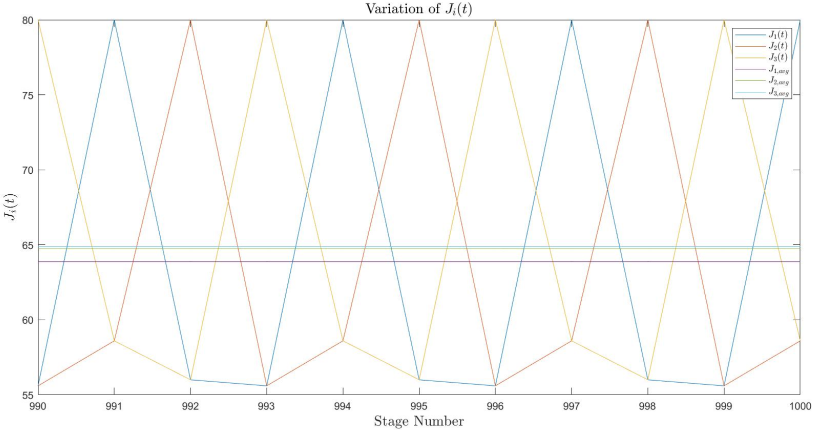


Figure: Variation of $|\mathcal{N}|$ with stage

- Only a few distinct types of agents, number varies based on their initial parameters
- A "Limit Cycle" convergence behaviour
- Value of convergence differs with the heuristic chosen to update the parameters of the agents

A Distributed Approach

Individual Player Cost Behaviour with $\gamma_i(0) \sim \mathcal{U}(0,1)$ and $|\mathcal{N}_i(0)| \sim \mathcal{U}(0,200)$



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- Does the bridge actually improve traffic?
  - Only if agents do not play selfishly
- What would be the practical considerations?
  - Dynamic agent model might not work with humans, but apps like Google Maps or Apple Maps can

Outlook
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- Can the heuristics to update the parameters of the agents be made better?
- The notion of "intelligent" agents - Can the agents be modelled as neural networks or ML-based entities such that they learn to be benevolent?

An example

… in real life


The New York Times

What if They Closed 42d Street and Nobody Noticed?

Give this article

By Gina Kolata

Dec. 25, 1990

"ON Earth Day this year, New York City's Transportation Commissioner <u>decided to close 42d Street</u>, which as every New Yorker knows is always congested. … But to everyone's surprise, Earth Day generated no historic traffic jam. Traffic flow actually improved when 42d Street was closed." [5]

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  Institute for Automatic Control, ETH Zürich, Spring 2022.

Thank you for your attention!

Images of Venice by authors, Nov 2022.