Algorithms & Data Structures	
Project 2	
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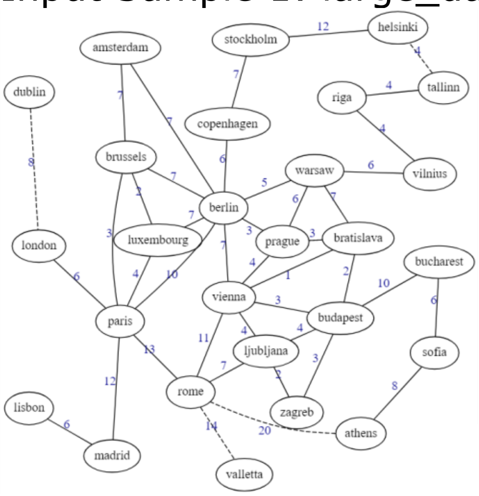
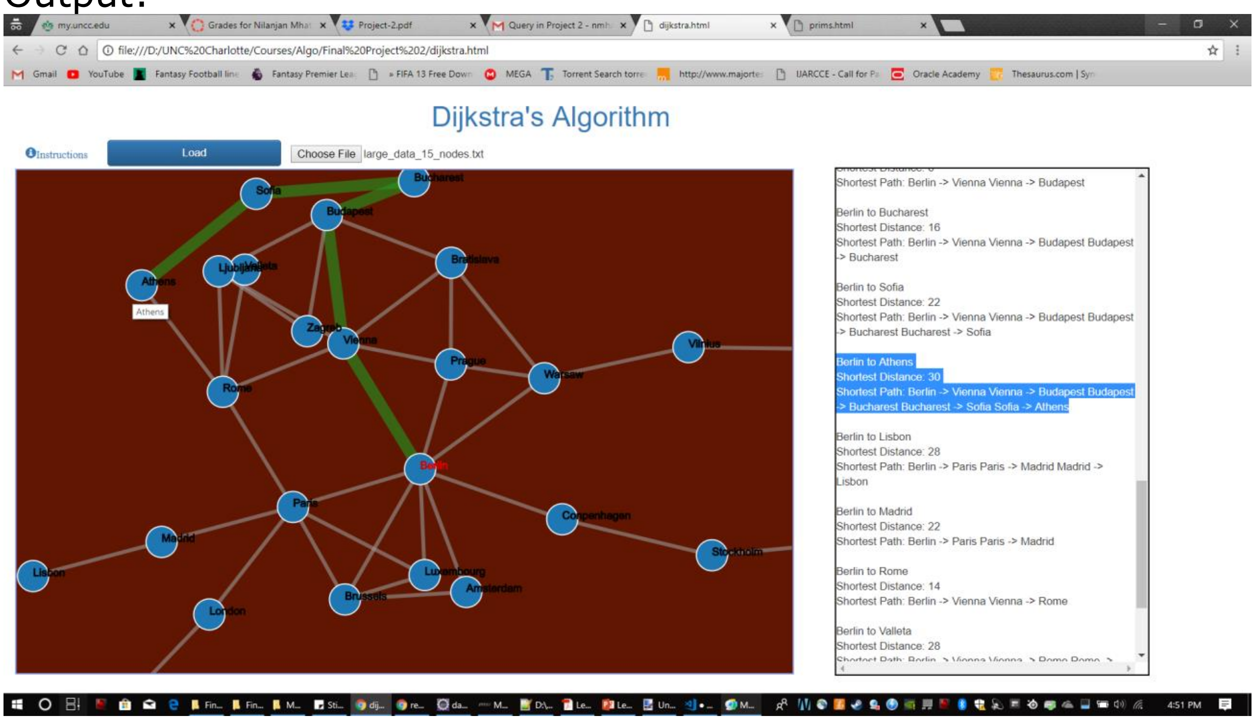
Problem 1: Find Shortest path in undirected graphs.

Dijkstra's Algorithm

Input Sample 1: large_data_27_nodes.txt



Output:



Dijkstra on Berlin

Berlin to Amsterdam
Shortest Distance: 7
Shortest Path: Berlin -> Amsterdam

Berlin to Brussels
Shortest Distance: 7
Shortest Path: Berlin -> Brussels

Berlin to Berlin
Shortest Distance: 0
Shortest Path:

Berlin to Dublin
Shortest Distance: 24
Shortest Path: Berlin -> Paris Paris -> London London -> Dublin

Berlin to London
Shortest Distance: 16
Shortest Path: Berlin -> Paris Paris -> London

Berlin to Stockholm
Shortest Distance: 13
Shortest Path: Berlin -> Conpenhagen Conpenhagen -> Stockholm

Berlin to Helsinki
Shortest Distance: 23
Shortest Path: Berlin -> Warsaw Warsaw -> Vilnius Vilnius -> Riga Riga -> Tallin Tallin -> Helsinki

Berlin to Conpenhagen
Shortest Distance: 6
Shortest Path: Berlin -> Conpenhagen

Berlin to Tallin
Shortest Distance: 19
Shortest Path: Berlin -> Warsaw Warsaw -> Vilnius Vilnius -> Riga Riga -> Tallin

Berlin to Riga
Shortest Distance: 15
Shortest Path: Berlin -> Warsaw Warsaw -> Vilnius Vilnius -> Riga

Berlin to Vilnius
Shortest Distance: 11
Shortest Path: Berlin -> Warsaw Warsaw -> Vilnius

Berlin to Luxembourg
Shortest Distance: 7
Shortest Path: Berlin -> Luxembourg

Berlin to Paris
Shortest Distance: 10
Shortest Path: Berlin -> Paris

Berlin to Warsaw
Shortest Distance: 5
Shortest Path: Berlin -> Warsaw

Berlin to Prague
Shortest Distance: 3
Shortest Path: Berlin -> Prague

Berlin to Vienna
Shortest Distance: 3
Shortest Path: Berlin -> Vienna

Berlin to Bratislava
Shortest Distance: 4
Shortest Path: Berlin -> Vienna Vienna -> Bratislava

Berlin to Budapest
Shortest Distance: 6
Shortest Path: Berlin -> Vienna Vienna -> Budapest

Berlin to Bucharest
Shortest Distance: 16
Shortest Path: Berlin -> Vienna Vienna -> Budapest Budapest -> Bucharest

Berlin to Sofia
Shortest Distance: 22
Shortest Path: Berlin -> Vienna Vienna -> Budapest Budapest -> Bucharest Bucharest -> Sofia

Berlin to Athens
Shortest Distance: 30
Shortest Path: Berlin -> Vienna Vienna -> Budapest Budapest -> Bucharest Bucharest -> Sofia Sofia -> Athens

Berlin to Lisbon
Shortest Distance: 28
Shortest Path: Berlin -> Paris Paris -> Madrid Madrid -> Lisbon

Berlin to Madrid
Shortest Distance: 22
Shortest Path: Berlin -> Paris Paris -> Madrid

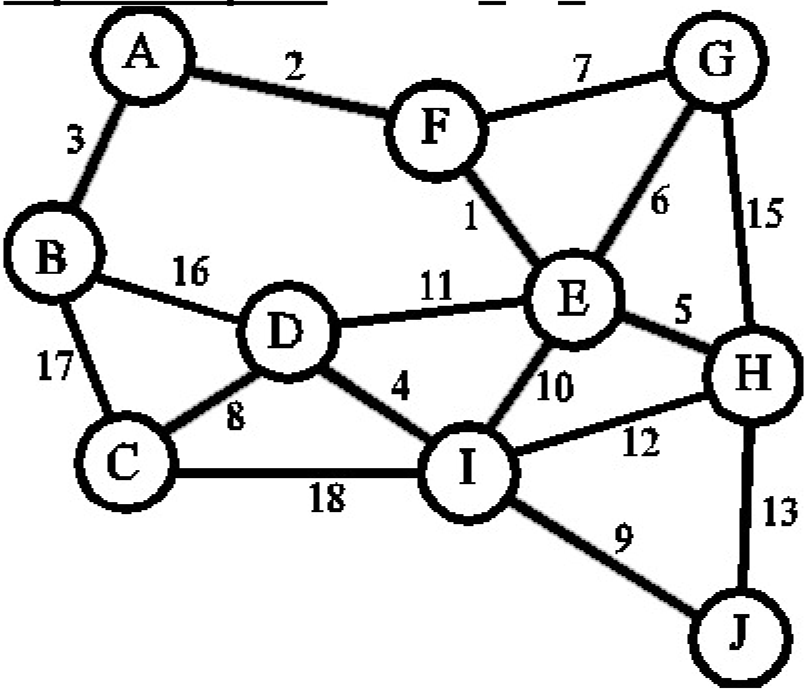
Berlin to Rome
Shortest Distance: 14
Shortest Path: Berlin -> Vienna Vienna -> Rome

Berlin to Valleta
Shortest Distance: 28
Shortest Path: Berlin -> Vienna Vienna -> Rome Rome -> Valleta

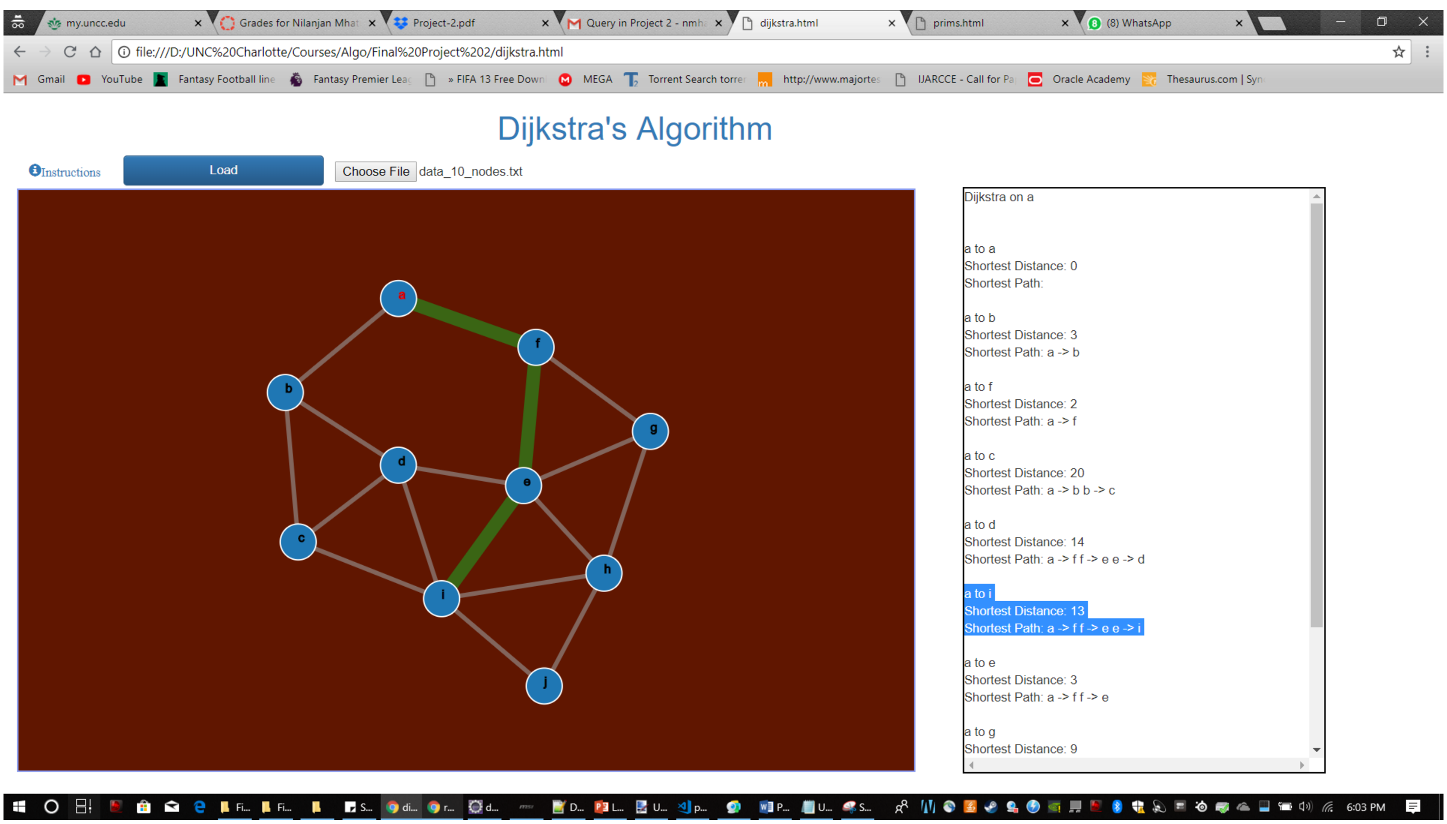
Berlin to Ljubljana
Shortest Distance: 7
Shortest Path: Berlin -> Vienna Vienna -> Ljubljana

Berlin to Zagreb
Shortest Distance: 9
Shortest Path: Berlin -> Vienna Vienna -> Budapest Budapest -> Zagreb

Input Sample 2: - data_10_nodes.txt



Output:



Dijkstra on a

a to a
Shortest Distance: 0
Shortest Path:

a to b
Shortest Distance: 3
Shortest Path: a -> b

a to f
Shortest Distance: 2
Shortest Path: a -> f

a to c
Shortest Distance: 20
Shortest Path: a -> b b -> c

a to d
Shortest Distance: 14
Shortest Path: a -> f f -> e e -> d

a to i
Shortest Distance: 13
Shortest Path: a -> f f -> e e -> i

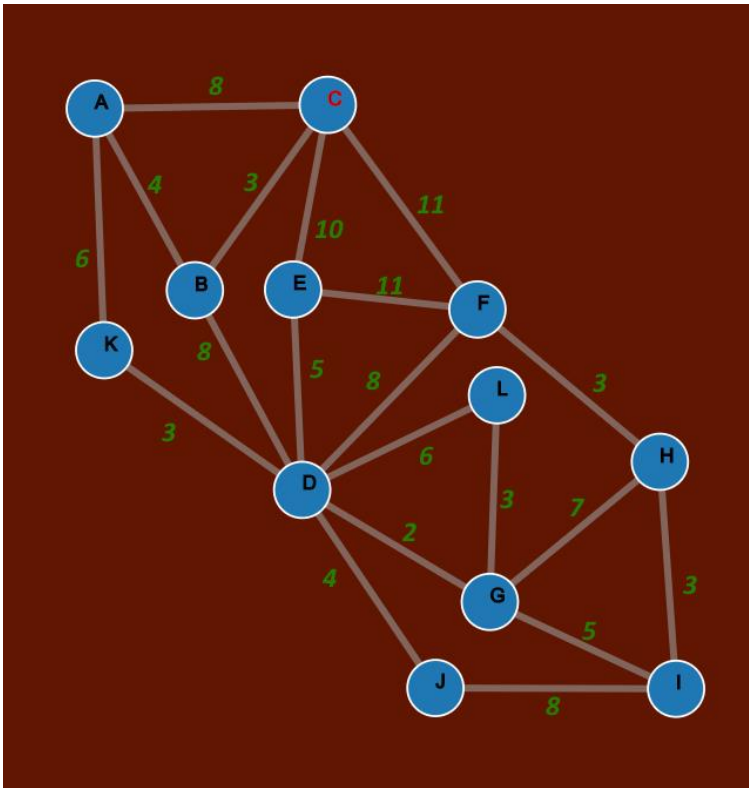
a to e
Shortest Distance: 3
Shortest Path: a -> f f -> e

a to g
Shortest Distance: 9
Shortest Path: a -> f f -> g

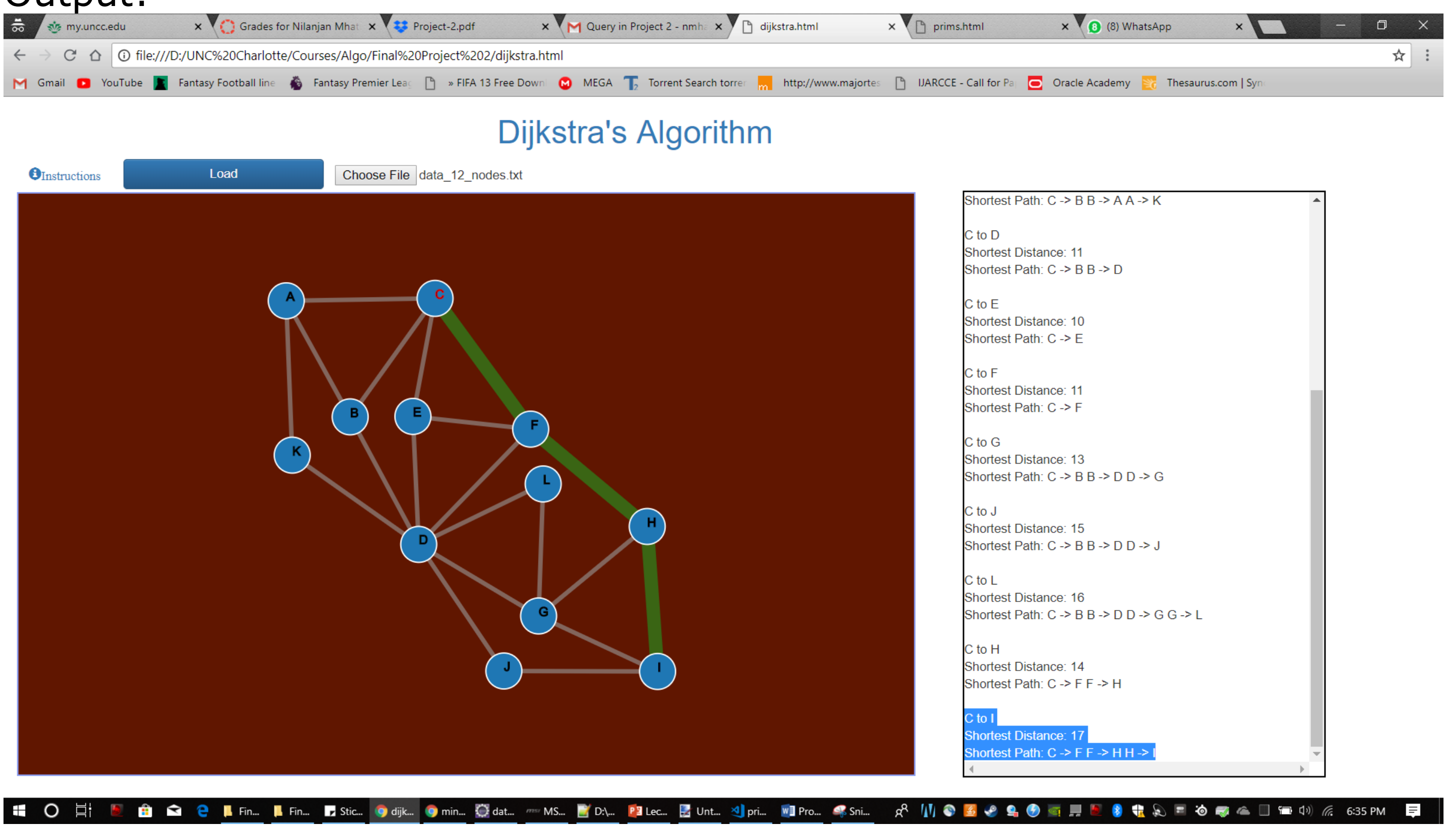
a to h
Shortest Distance: 8
Shortest Path: a -> f f -> e e -> h

a to j
Shortest Distance: 21
Shortest Path: a -> f f -> e e -> h h -> j

<u>Input Sample 3</u>: - data_12_nodes.txt



Output:-



Dijkstra on C

C to A
Shortest Distance: 7
Shortest Path: C -> B B -> A

C to B
Shortest Distance: 3
Shortest Path: C -> B

C to C
Shortest Distance: 0
Shortest Path:

C to K
Shortest Distance: 13
Shortest Path: C -> B B -> A A -> K

C to D
Shortest Distance: 11
Shortest Path: C -> B B -> D

C to E
Shortest Distance: 10
Shortest Path: C -> E

C to F
Shortest Distance: 11
Shortest Path: C -> F

C to G
Shortest Distance: 13
Shortest Path: C -> B B -> D D -> G

C to J
Shortest Distance: 15
Shortest Path: C -> B B -> D D -> J

C to L
Shortest Distance: 16
Shortest Path: C -> B B -> D D -> G G -> L

C to H
Shortest Distance: 14
Shortest Path: C -> F F -> H

C to I
Shortest Distance: 17
Shortest Path: C -> F F -> H H -> I

Runtime Analysis (Dijkstra Algorithm): -

Priority Queue is used that uses heap structure.
Each vertex insertion and the corresponding heapify operation takes 'O(log n)' time. 'Total = n log n'

Each vertex removal and the corresponding heapify operation takes 'O(log n)' time. 'Total = n log n'

Each vertex modified along with heapify operation takes 'O(log n)' time
The modification is done for each edge, for total 'm' edges.
Total = m log n

The total time = n log n + m log n + n log n
= O((m+n) log n)

```
for (i = 0; i < numberOfNodes; i++) {
    if(current == i) {
        queue.queue({"value": 0, "id": i});
    } else {
        queue.queue({"value": 9007199254740992, "id": i});
    }
    p[i] = -1;
}

//Start of Algorithm
d[current] = 0;
var dc = queue.dequeue().value;
while (true) {
    var adj = g[current];
    $.each(adj, function() {
        var eleArr = queue.findElement(this.vertex);
        if(eleArr.length > 0) {
            var ele = eleArr[0];
            if (this.weight != 0 && this.weight + dc < ele.value) {
                ele.value = this.weight + dc;
                d[this.vertex] = ele.value;
                p[this.vertex] = current;
                queue.priv._heapify();
            }
        }
    });
    if(queue.length == 0) {
        break;
    }
    var nextEle = queue.dequeue();
    dc = nextEle.value;
    current = nextEle.id;
}

//Display Paths
```

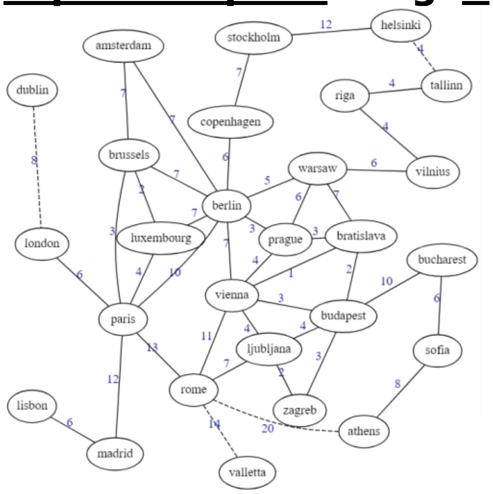
Insertion of each node takes 'log n' time.
-> Total time = n log n

The adjacency list is traversed for each node, for the number of times of its incident edges.
Hence, 'm' times in total.
The heapify operation will take 'log n' time.
-> Total time = m log n

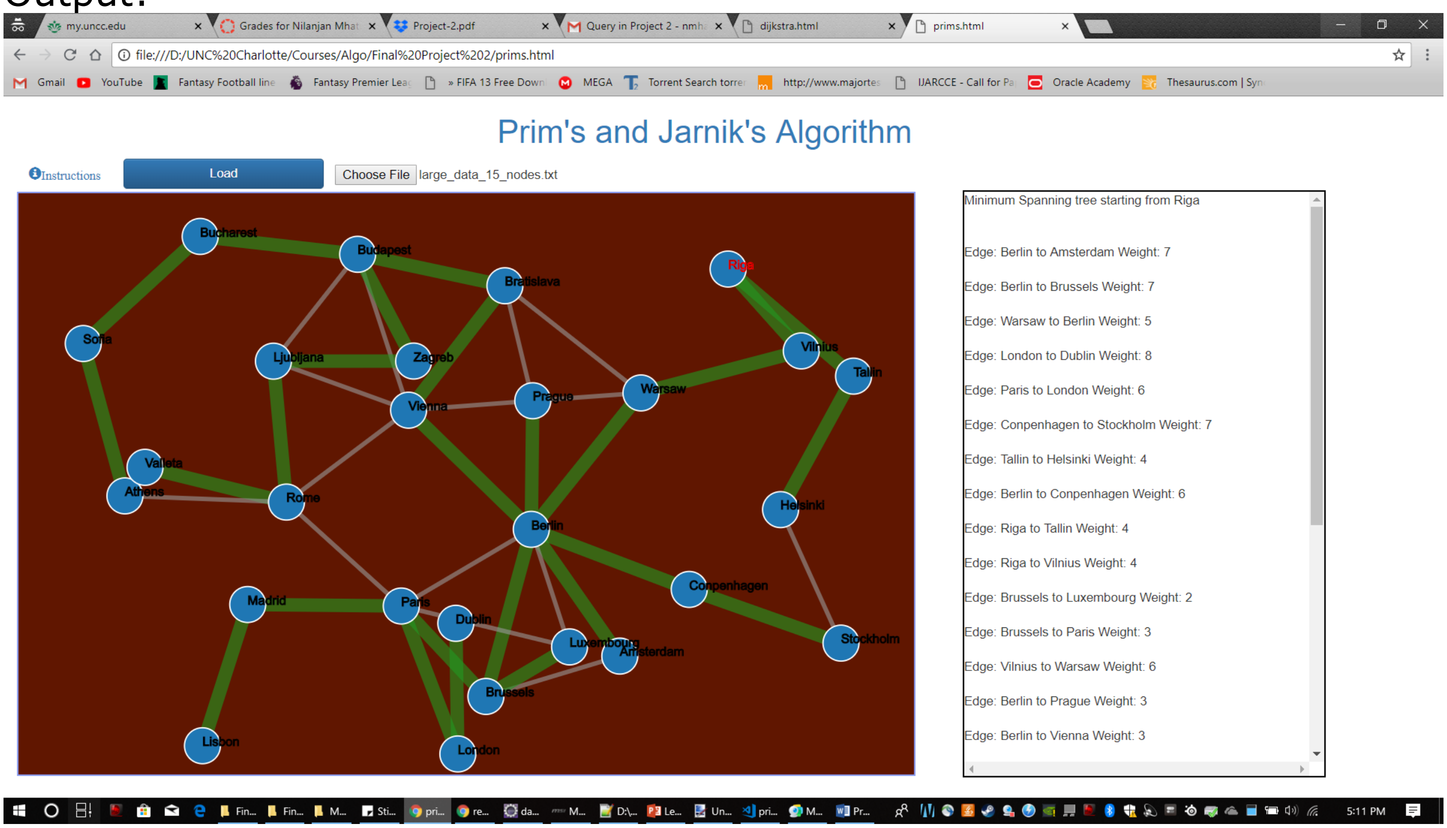
Dequeue operation will take place for each node
-> Total Time = n log n

Problem 2: Find the Minimum spanning tree (MST). Prim's and Jarnik's Algorithm

Input Sample 1: large_data_27_nodes.txt



Output:



Minimum Spanning tree starting from Riga

Edge: Berlin to Amsterdam Weight: 7

Edge: Berlin to Brussels Weight: 7

Edge: Warsaw to Berlin Weight: 5

Edge: London to Dublin Weight: 8

Edge: Paris to London Weight: 6

Edge: Conpenhagen to Stockholm Weight: 7

Edge: Tallin to Helsinki Weight: 4

Edge: Berlin to Conpenhagen Weight: 6

Edge: Riga to Tallin Weight: 4

Edge: Riga to Vilnius Weight: 4

Edge: Brussels to Luxembourg Weight: 2

Edge: Brussels to Paris Weight: 3

Edge: Vilnius to Warsaw Weight: 6

Edge: Berlin to Prague Weight: 3

Edge: Berlin to Vienna Weight: 3

Edge: Vienna to Bratislava Weight: 1

Edge: Bratislava to Budapest Weight: 2

Edge: Budapest to Bucharest Weight: 10

Edge: Bucharest to Sofia Weight: 6

Edge: Sofia to Athens Weight: 8

Edge: Madrid to Lisbon Weight: 6

Edge: Paris to Madrid Weight: 12

Edge: Ljubljana to Rome Weight: 7

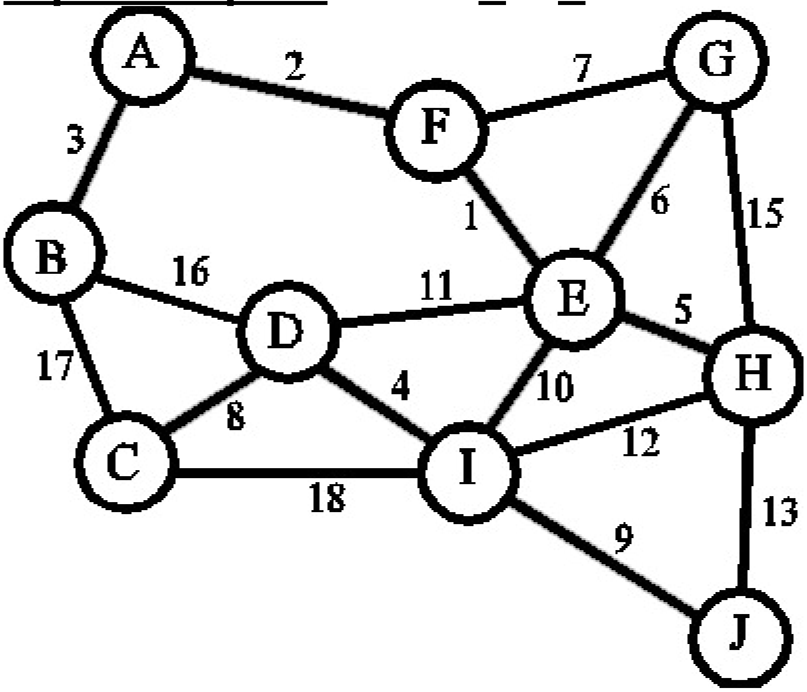
Edge: Rome to Valleta Weight: 14

Edge: Zagreb to Ljubljana Weight: 2

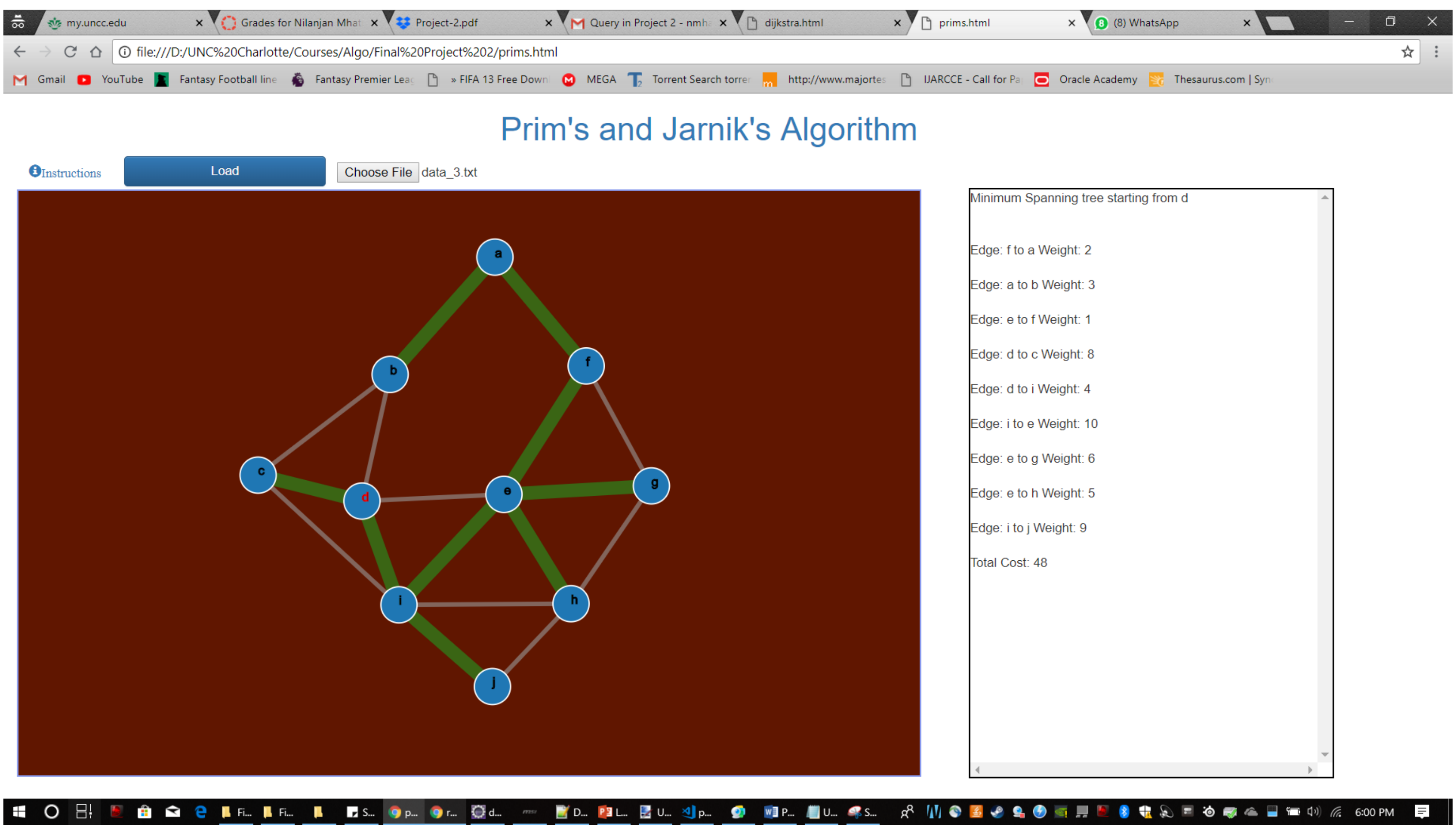
Edge: Budapest to Zagreb Weight: 3

Total Cost: 146

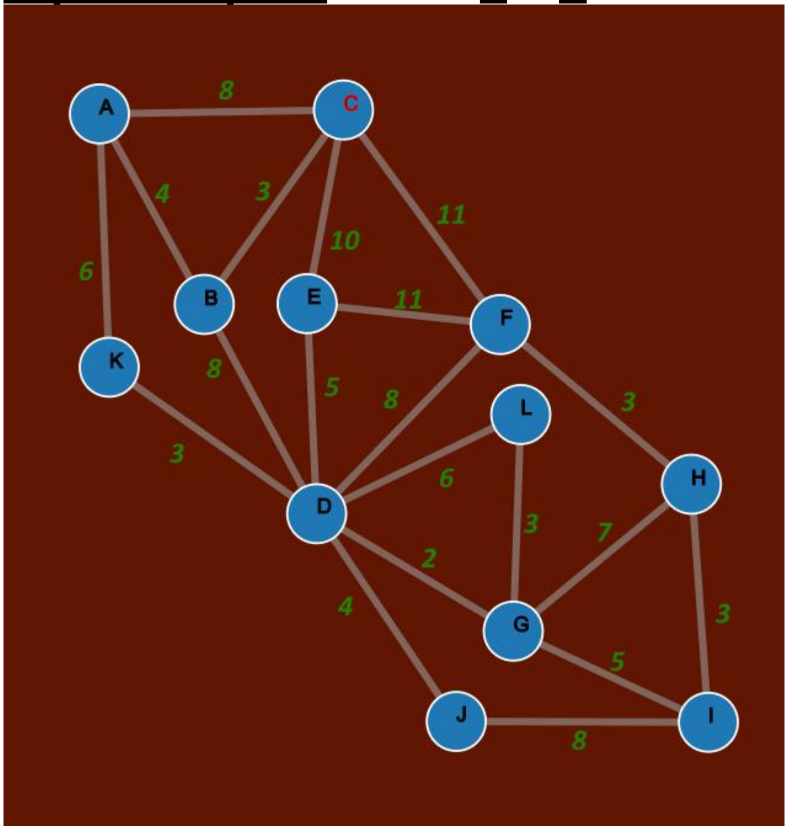
Input Sample 2: - data_10_nodes.txt



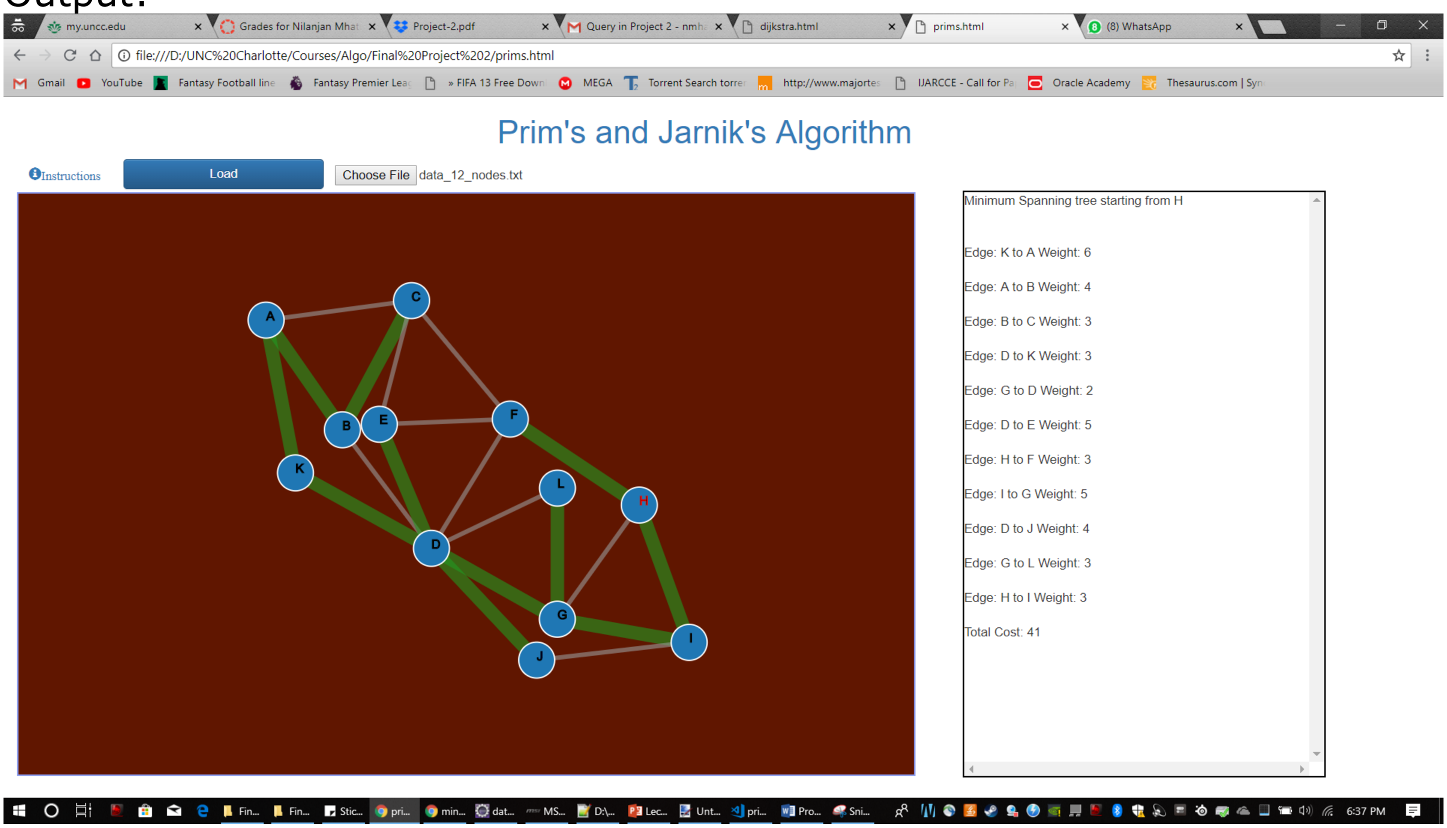
Output:



<u>Input Sample 3</u>: - data_12_nodes.txt



Output:-



Runtime Analysis (Prim's and Jarnik's Algorithm): -

Priority Queue is used that uses heap structure.
Each vertex insertion and the corresponding heapify operation takes 'O(log n)' time. 'Total = n log n'

Each vertex removal and the corresponding heapify operation takes 'O(log n)' time. 'Total = n log n'

Each vertex modified along with heapify operation takes 'O(log n)' time
Each vertex will be modified for maximum of its incident edges, i.e. for total 'm' edges.
Total = m log n

The total time = n log n + m log n + n log n
= O((m+n) log n)

```
for (i = 0; i < numberOfNodes; i++) {
    if(current == i) {
        queue.queue({"value": 0, "id": i});
    } else {
        queue.queue({"value": 9007199254740992, "id": i});
    }
    d[i] = 0;
    p[i] = -1;
}

//Start of Algorithm
d[current] = 0;
var c = 0;
var dc = queue.dequeue().value;
while (c != g.length-1) {
    var adj = g[current];
    $.each(adj, function() {
        var eleArr = queue.findElement(this.vertex);
        if(eleArr.length > 0) {
            var ele = eleArr[0];
            if (this.weight != 0 && this.weight < ele.value) {
                ele.value = this.weight;
                d[this.vertex] = ele.value;
                p[this.vertex] = current;
                queue.priv._heapify();
            }
        }
    });
    if(queue.length == 0) {
        break;
    }
    var nextEle = queue.dequeue();
    dc = nextEle.value;
    current = nextEle.id;
}

//Display the paths
```

Insertion of each node takes 'log n' time.
-> Total time = n log n

The adjacency list is traversed for each node,
and modified for the number of times of its incident edges
i.e. 'm' times in total.
Modification requires heapify operation.
Each node The heapify operation
will take 'log n' time.
-> Total time = m log n

Dequeue operation will take place
for each node
-> Total Time = n log n