Honours Project

Voltage Control of Active Magnetic Bearing

by

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Introduction

Magnetic bearing is a type of bearing which supports external loads and moving parts without any physical contact. Due to the absence of any contact between the bearing and the moving part(rotor), magnetic bearing has clear advantages over traditional contact bearings. They permit almost frictionless relative motion with the highest speeds among any type of bearing and consequently minimises the chances of wear. Magnetic bearings are being increasingly used in many areas such as machine tools, compressors, turbines, pumps, etc. They also have wide spread applications in various high precision medical devices such as blood pumps and artificial hearts. Magnetic bearings work on the principle of electromagnetic suspension caused by induction of eddy currents in a conductor moving in a magnetic field.

Due to its inherent instability, active control is necessary for proper functioning of a magnetic bearing system. The term active implies that the position of the rotor, when subjected to external disturbances, can be adjusted by controlling the bearing forces by means of feedback control loop. An active magnetic bearing (AMB) system can have two control modes: current control and voltage control. In current control, the control inputs are the electromagnet coil currents which are provided by a current amplifier according to the control law. And in voltage control, voltages across the electromagnet coils are the control inputs. The current amplifier used in current control mode is much more expensive than the voltage amplifier used in voltage control mode. Moreover, the effects of coil resistance and coil inductance on the system dynamics are not incorporated in the current control mode. Therefore, voltage control is preferred over current control, even though it is more complicated than current control due to added coil dynamics to the overall system dynamics.

The objective of this project is to implement voltage control in an AMB system by designing a PID controller for the voltage input. The effect of varying the parameters of the controller are also studied. The effects of the controller on the displacement of the rotor subjected to external loads are also simulated and presented.

Magnetic Bearing Model

Active magnetic bearing (AMB) is a combination of rotors, electromagnets, position sensors, power amplifiers and a control system. In this project, we have considered an AMB system with two counteracting electromagnets as depicted in the schematic shown in fig.1. It consists of a rotor and two electromagnets. The coil currents in electromagnets 1 and 2 are i_1 an i_2 respectively. The voltages across the electromagnet coils are u_1 and u_2 respectively.

The electromagnets exert attractive forces on the ferromagnetic rotor, the magnitudes of which can be controlled by the currents flowing through the electromagnet coils. An external disturbance force f_d acts along the centre of mass of the rotor. The mass of the rotor is considered as M and in our analysis the effect of the rotor weight in the control strategy is neglected. Our analysis can be considered to be along the horizontal plane. The nominal air gap in between an electromagnet and the rotor is considered as X_o and the displacement of the rotor from its equilibrium position is considered as x. The flux linkages in the two electromagnets 1 and 2 are denoted by Φ_1 and Φ_2 respectively. The equation of motion of the rotor is –

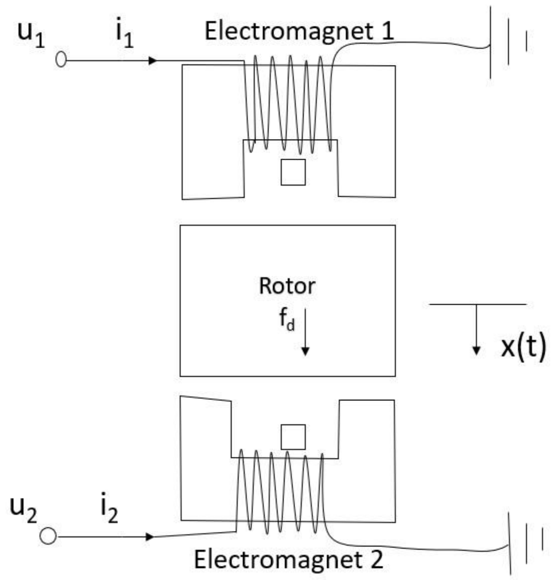


Fig.1

$$M\ddot{x} = \frac{1}{\mu_o A}(\phi_2^2 - \phi_1^2) + f_d \qquad (1)$$

$$\phi_1 = \frac{\mu_o A N i_1}{2(X_o + x)}; \quad \phi_2 = \frac{\mu_o A N i_2}{2(X_o - x)} \qquad (2)$$

In the above equations, μ_o, A and N represents the permeability, the pole surface area and the number of turns in each electromagnet respectively.

Equation (1) is referred to as the force-current relationship. It can be inferred from the above equations that the magnetic force exerted on the rotor is directly proportional to the square of the currents and inversely proportional to the square of the air gaps between the rotor and the electromagnets.

Linearization of the model

The force- current relationship is nonlinear. To make computations easier, the nonlinear relation is approximated to its linear form by linearizing about the equilibrium point. In our case, the equilibrium point/operating point is characterised by the following points –

1. Both the air gaps between the two electromagnets and the rotor are equal to the value of the nominal air gap X_o. This also means that at the equilibrium point, the displacement of the rotor x is equal to zero.
2. The currents in both the electromagnet coils are equal to a bias current i_b which can be described as the current flowing through the coils when there are no external disturbances on the rotor.

The AMB model is linearized by the following substitutions –

$$i_1(t) = i_b - i_c(t)\,; \quad i_2(t) = i_b + i_c(t) \qquad (3)$$

In the above set of substitutions, i_c is referred to as control current which is the instantaneous current that must be subtracted from and added to the bias current i_b in coils 1 and 2 respectively, when the rotor is displaced from its equilibrium point as depicted in fig.2.

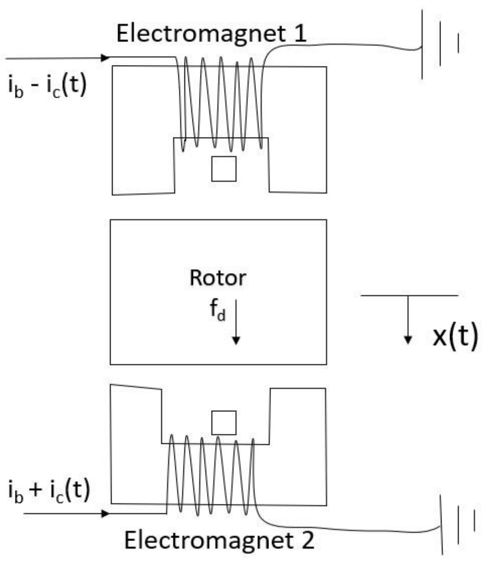


Fig.2

The objective of the control strategy is to ultimately obtain $i_c(t)$ in the AMB system subjected to any kinds of external disturbance, so that the rotor can be maintained in its equilibrium/operating point.

Substituting (3) in the nonlinear force current relation (2), we get the following linear relation –

$$F = M\ddot{x} = -k_s x + k_i i_c + f_d \qquad (4)$$

$$k_s = -\frac{\mu_o N^2 A i_b}{4X_o^3}\,; \quad k_i = \frac{\mu_o N^2 A i_b}{4X_o^2}$$

On assumption of the linear form is that the displacement of the rotor x is much less than the nominal air gap X_o. This means that the linear form is accurate only in the proximity of the equilibrium point.

Closed Control Loop

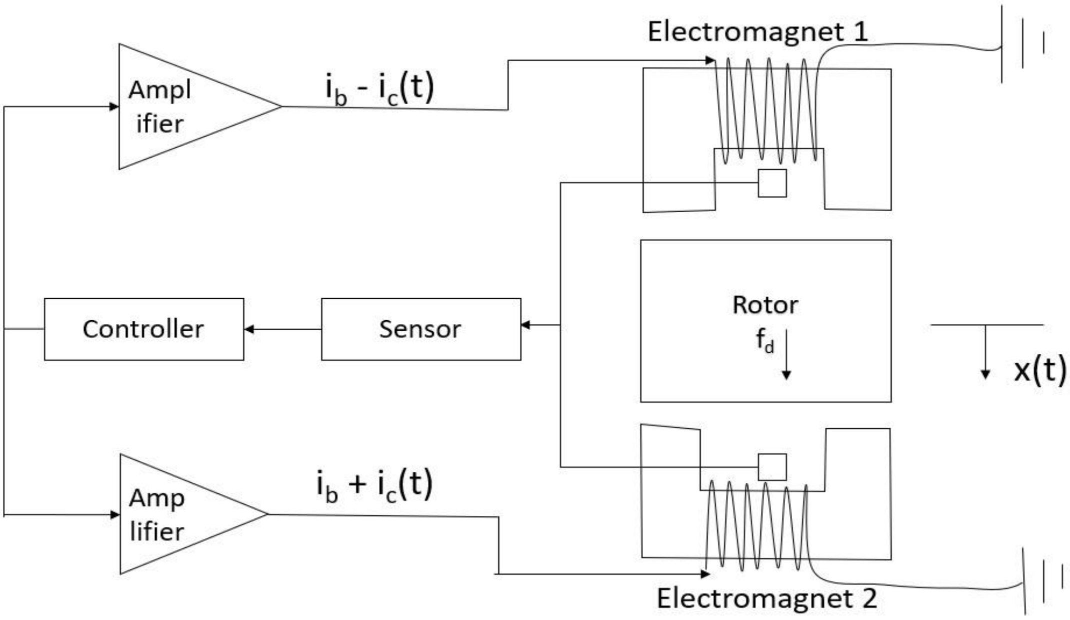


Fig.3

The above figure shows a simple magnetic bearing control loop with all the basic necessary components. The rotor which is subjected to an external disturbance force f_d, is to be levitated freely at a desired position from the electromagnets. A set of contactless position sensors measures the error of the rotor position with respect to the desired position and feeds the information to the controller. The objective of the controller is to stabilise the unless unstable open loop magnetic bearing system. The controller sends a position-based command signal to a power amplifier which transforms that signal into electric currents in the bearing electromagnet coils.

In voltage control configuration, the controller sends a voltage control signal as the power amplifier input from which it is converted into current in the coils. The voltage control mode, unlike current control, takes into consideration the effect of coil resistance and inductance in the overall system dynamics. Moreover, the amplifier used in current control is much more expensive than that used in the voltage control mode. For these reasons we will be considering voltage control mode for the AMB system.

State Space Description of the Model

In voltage control of the AMB system, the control inputs are the voltage across the electromagnet coils u and the external disturbance force on the rotor f_d. So, it becomes a multi-input control system. Generally state space description is useful for analysing multi-input systems.

General state space description of a control system is as follows –

$\dot{X} = AX + BU$ A- state matrix; B- input matrix

$Y = CX + DU$ C- output matrix; D- transmission matrix

where X, U, Y are the state vector, control input vector and output vector of the system respectively.

For the AMB system, we consider the following –

State vector $X = [x \quad \dot{x} \quad i_c \quad \int xdt]^{\mathrm{T}}$

Control input vector $Y = [u \quad f_d]^{\mathrm{T}}$

Output $Y = x$

The state equations for the AMB system are –

$$\ddot{x} = \frac{1}{M}(-k_s x + k_i i_c + f_d)$$

$$u = Ri_c + L\frac{di_c}{dt} + k_i\dot{x}$$

Rearranging the above state equations and using the vectors mentioned above for the AMB system, we obtain the following state space mapping –

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{M} & 0 & \frac{k_i}{M} & 0 \\ 0 & \frac{-k_i}{L} & -\frac{R}{L} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{bmatrix} f_d$$

$=> \dot{X} = AX + B_1u + B_2f_d$

$Y = [1 \quad 0 \quad 0 \quad 0]X$

$=> Y = C_1X$

The poles of this system before implementing any control (open loop system) are given by the eigen values of the state matrix A.

The values for the system parameters are given in the following table –

M	6 kg
k_s	-4.5 N/mm
k_i	3.51 N/A
R	3.7 ohm
L	5.94e-4 H
f_d	25+12.5cos(15t) N

Table 1

$$\text{Eigenvalues of A} = \begin{bmatrix} 0 \\ -6.23 \times 10^3 \\ -1.19 \\ 0.632 \end{bmatrix}$$

Thus, the open loop AMB system without any control is inherently unstable due to the presence of a pole/eigenvalue having a positive real part.

Voltage Control

We will be implementing a PID controller for the voltage input. According to the PID control law –

$$u = P \times e + D \times \dot{e} + I \times \int edt\ , \qquad (5)$$

where P, D, and I are referred to as the proportional, derivative and integral constants respectively and e represents the error of the instantaneous rotor position x with respect to a desired reference value r. We will be considering the reference position of the rotor to be zero.

$$\therefore e = r - x = -x$$

Substituting e in (5) –

$$u = [P \quad 0 \quad 0 \quad I]X - DC_1\dot{X}$$

$$=> u = C_2X - DC_1\dot{X} \qquad (6)$$

Substituting (6) in the state space description and rearranging, we get the following form –

$$\dot{X} = inv[I + DB_1C_1]([A - B_1C_1]X + B_2f_d \qquad I \text{ - 4 by 4 identity matrix}$$

$$=> \dot{X} = A_1X + B_3f_d \qquad (7)$$

The above equation (7) is the simplified form of the closed loop state space description of the AMB system. It represents a set of first order non-linear ordinary differential equations. It is numerically solved in MATLAB using its ODE solver ODE45 with the following initial state vector X_i –

$$X_i = [0 \quad 0 \quad 0 \quad 0]^T$$

The initial zero state vector represents that initially the rotor is at the equilibrium/operating point.

The values of the control parameters which have been considered, are enlisted in the table below –

P	500 V/mm
D	500 V.s/mm
I	500 V/mm.s
f_d	25+12.5cos(15t) N

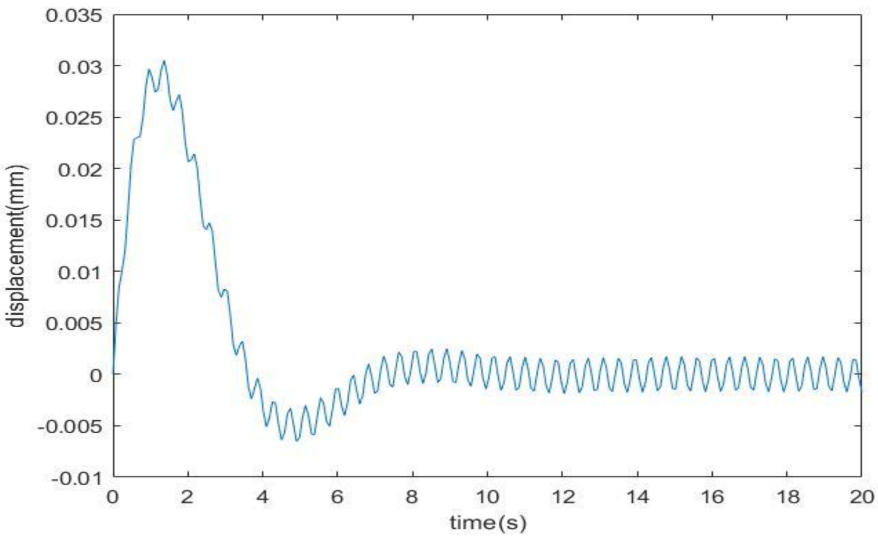
Results

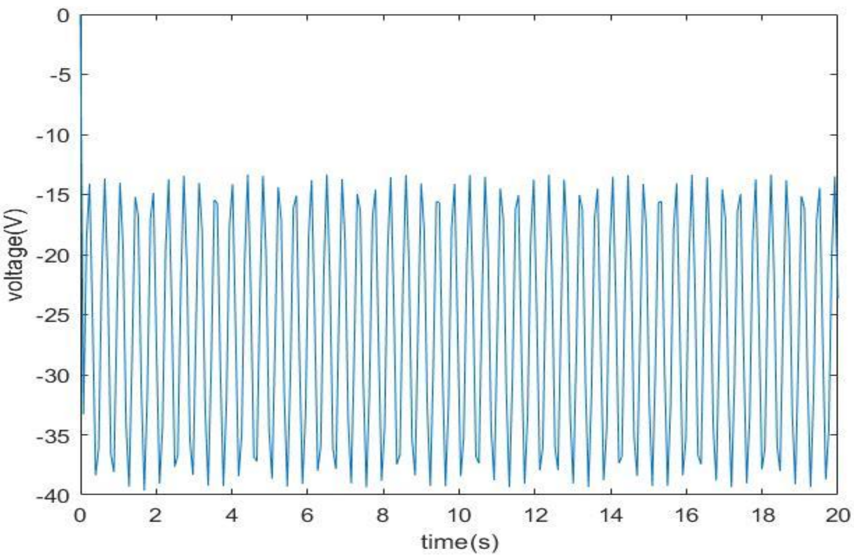
The poles of the closed loop AMB system with the above mentioned control parameters are –

$$\text{Eigenvalues of } A_1 = \begin{bmatrix} -6.15 \times 10^{3} \\ -4.92 \times 10^{-1} + 8.74 \times 10^{-1} i \\ -4.92 \times 10^{-1} - 8.74 \times 10^{-1} i \\ -7.97 \times 10^{1} \end{bmatrix}$$

Thus, the closed loop system is stable due to the absence of any unstable poles (poles with positive real part).

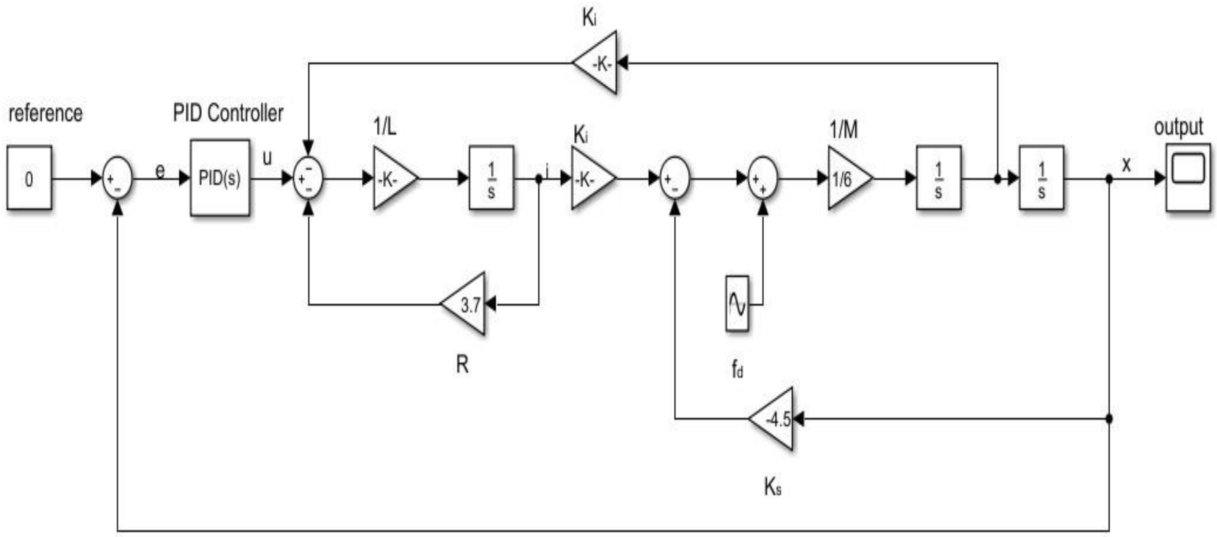
The displacement vs time and voltage vs time plots obtained from MATLAB are as follows –



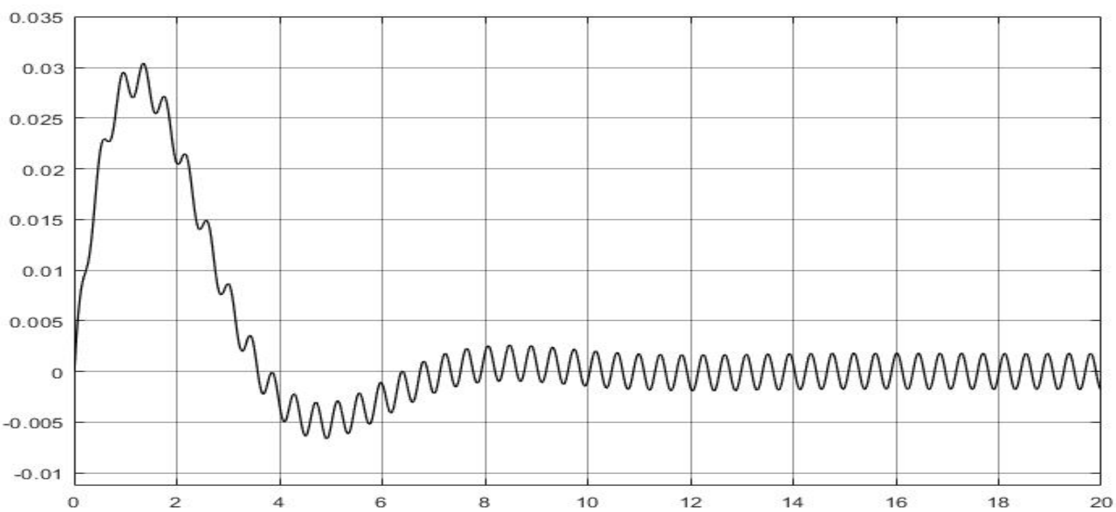


Simulink Model

The system is also solved with Simulink with the same system and control parameters to compare the results obtained through MATLAB. The following is the Simulink model –



The following is the displacement vs time plot obtained through Simulink –

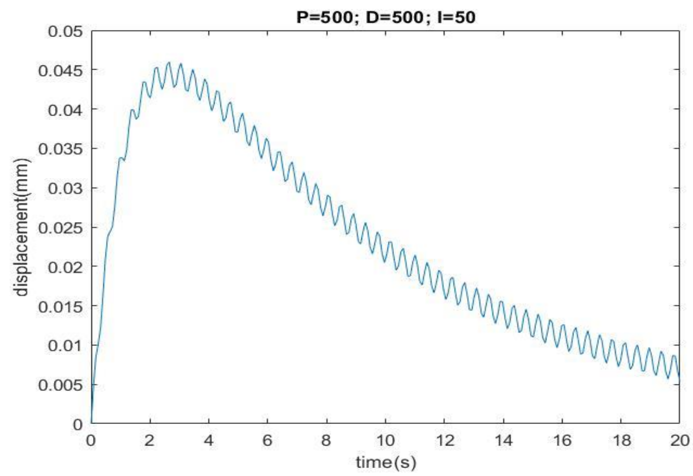


It is observed that the responses obtained through MATLAB and Simulink are similar.

Effect of Variation of Control Parameters

The following plots are the responses obtained by changing the integral constant while keeping the proportional and derivative constants both equal and constant.

Poles

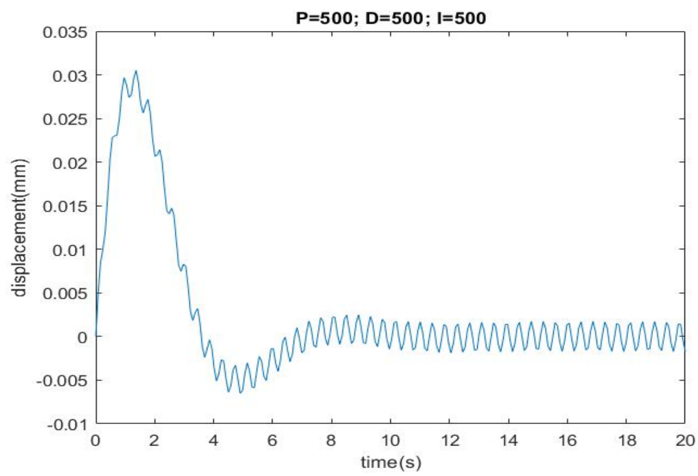


-6.15×10^3

-7.96×10^1

-8.81×10^{-1}

-1.14×10^{-1}

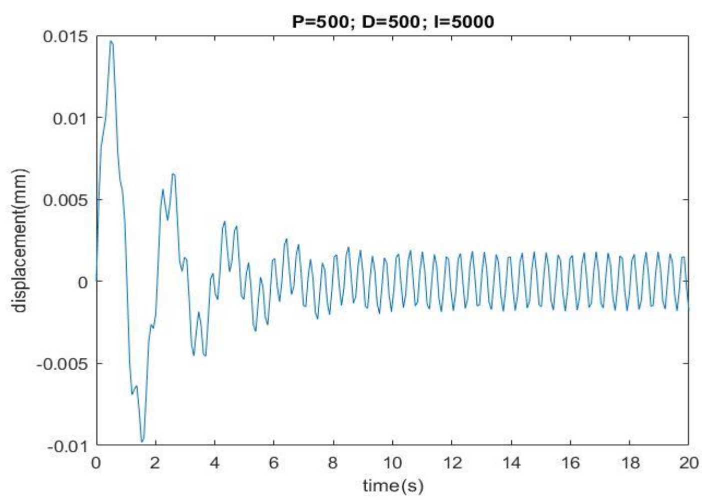


-6.15×10^3

$-0.492 + .874i$

$-0.492 - .874i$

-79.7

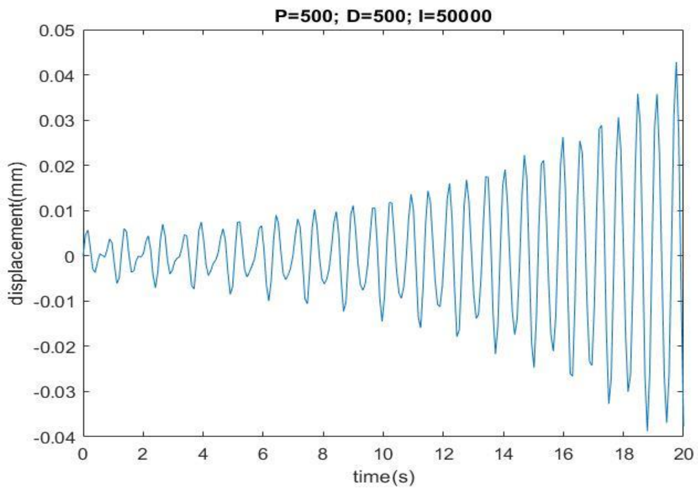


-6.15×10^3

$-0.433 + 3.14i$

$-0.433 - 3.14i$

-79.8



-6.15×10^3

$0.13 + 9.95i$

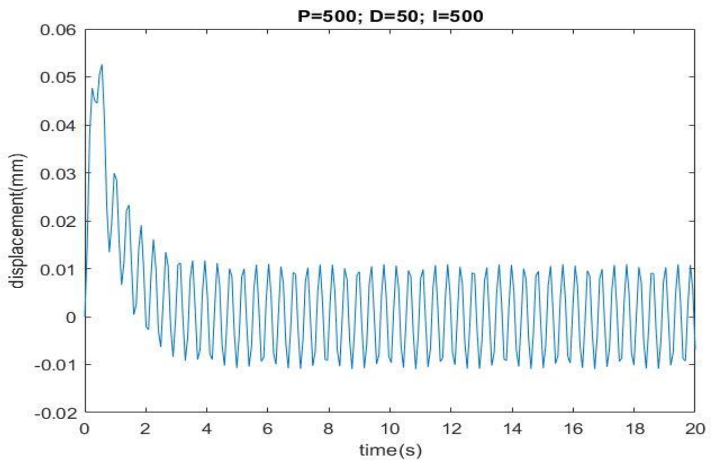
$0.13 - 9.95i$

-80.9

It is observed from the plots that as we increase the value of I, the rate of convergence of the response increases and maximum displacement of the rotor decreases. But after a certain point, as we have seen in the case of I=50000, the system becomes unstable.

The following plots are the responses obtained by changing the derivative constant while keeping the proportional and integral constants both equal and constant.

Poles

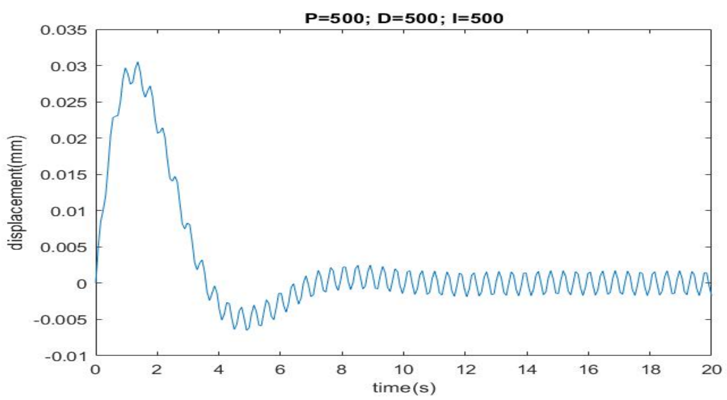


-6.22×10^3

$-3.67 + 7.53i$

$-3.67 - 7.53i$

-1.13

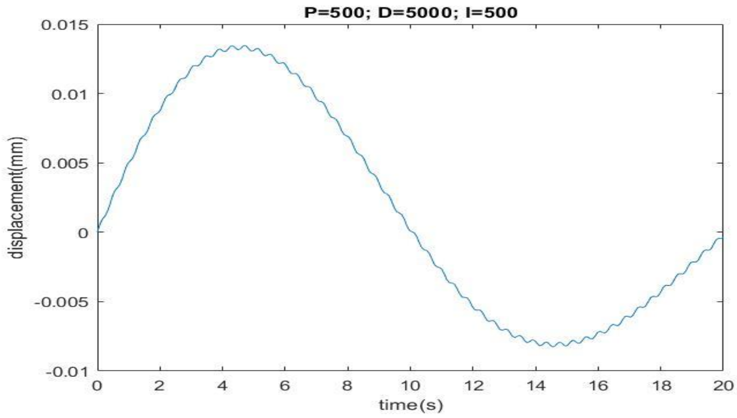


-6.15×10^3

$-0.492 + .874i$

$-0.492 - .874i$

-79.7

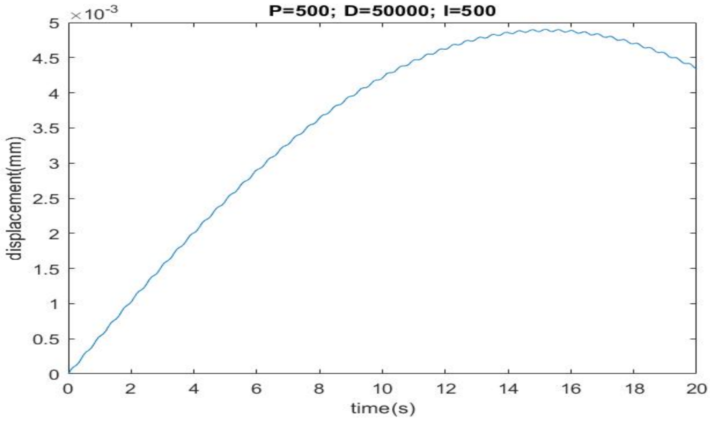


$$-5.3 \times 10^3$$

$$-9.3 \times 10^2$$

$$-0.0494 + 0.312i$$

$$-0.0494 - 0.312i$$



$$-3.1 \times 10^2 + 6.29 \times 10^3 i$$

$$-3.1 \times 10^2 - 6.29 \times 10^3 i$$

$$-4.9 \times 10^{-3} + 10^{-1} i$$

$$-4.9 \times 10^{-3} - 10^{-1} i$$

It is observed from the plots that as we increase the derivative constant D, the rate of convergence of the response decreases but the maximum displacement of the rotor decreases and the response gets smoother.

References

1) L. Li, T. Shinshi, A. Shimokohbe, “Asymptotically Exact Linearizations for Active Magnetic Bearing Actuators in Voltage Control Configuration”, 2003.
2) G. Schweitzer, E. Maslen, “ Magnetic Bearings Theory, Design and Application to Rotating Machinery”, 2009.