Honours Project

(2nd Semester)

Determination of Control Parameters for Active Magnetic Bearings Using Genetic Algorithm

by

Nilanjan Saha

Roll: 001811201092; BME-IV

Under the supervision of

Prof. Anindya Sundar Das

Department of Mechanical Engineering

Jadavpur University

Kolkata-700032

Introduction

Magnetic bearing is a type of bearing which supports external loads and moving parts without any physical contact. Due to the absence of any contact between the bearing and the moving part(rotor), magnetic bearing has clear advantages over traditional contact bearings. They permit almost frictionless relative motion with the highest speeds among any type of bearing and consequently minimises the chances of wear. Magnetic bearings are being increasingly used in many areas such as machine tools, compressors, turbines, pumps, etc. They also have wide spread applications in various high precision medical devices such as blood pumps and artificial hearts. Magnetic bearings work on the principle of electromagnetic suspension caused by induction of eddy currents in a conductor moving in a magnetic field.

Due to its inherent instability, active control is necessary for proper functioning of a magnetic bearing system. The term active implies that the position of the rotor, when subjected to external disturbances, can be adjusted by controlling the bearing forces by means of feedback control loop. An active magnetic bearing (AMB) system can have two control modes: current control and voltage control. In current control, the control inputs are the electromagnet coil currents which are provided by a current amplifier according to the control law. And in voltage control, voltages across the electromagnet coils are the control inputs. The current amplifier used in current control mode is much more expensive than the voltage amplifier used in voltage control mode. Moreover, the effects of coil resistance and coil inductance on the system dynamics are not incorporated in the current control mode.

In this project, we have implemented the voltage control mode in a simple AMB system acted upon by a one-dimensional external disturbing force. For this purpose, we have designed a PID controller and applied the Genetic Algorithm for determining the optimal control parameters which will correspond to a set of optimal objective functions.

Magnetic bearing Model

In this project, we have considered an AMB system with two counteracting electromagnets as depicted in the schematic shown in fig.1. It consists of a rotor and two electromagnets. The coil currents in electromagnets 1 and 2 are i_1 an i_2 respectively. The voltages across the electromagnet coils are u_1 and u_2 respectively.

An external disturbance force f_d acts along the centre of mass of the rotor. The mass of the rotor is considered as M and in our analysis the effect of the rotor weight in the control strategy is neglected. Our analysis can be considered to be along the horizontal plane. The nominal air gap in between an electromagnet and the rotor is considered as X_o and the displacement of the rotor from its equilibrium position is considered as x. The flux linkages in the two electromagnets 1 and 2 are denoted by Φ_1 and Φ_2 respectively. The equation of motion of the rotor is –

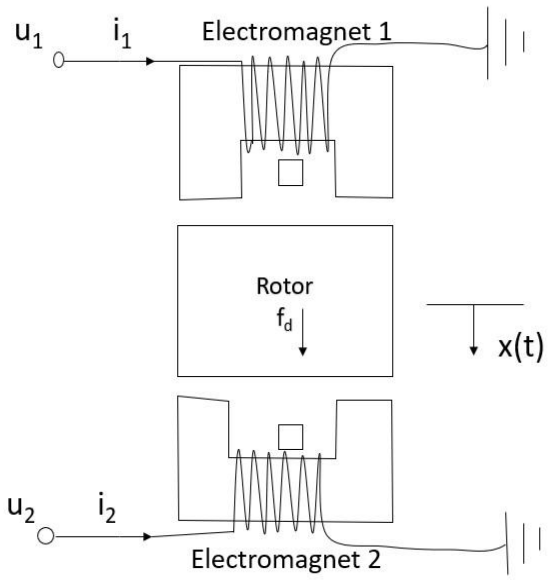

fig. 1

$$M\ddot{x} = \frac{1}{\mu_o A}(\phi_2^2 - \phi_1^2) + f_d \qquad (1)$$

$$\phi_1 = \frac{\mu_o A N i_1}{2(X_o + x)}; \quad \phi_2 = \frac{\mu_o A N i_2}{2(X_o - x)} \qquad (2)$$

In the above equations, μ_o, A and N represents the permeability, the pole surface area and the number of turns in each electromagnet respectively.

The force- current relationship i.e., equation (1) is nonlinear. To make computations easier, the nonlinear relation is approximated to its linear form by linearizing about the equilibrium point. In our case, the equilibrium point/operating point is characterised by the following points –

1. Both the air gaps between the two electromagnets and the rotor are equal to the value of the nominal air gap X_o.

2. The currents in both the electromagnet coils are equal to a bias current i_b which can be described as the current flowing through the coils when there are no external disturbances on the rotor.

The AMB model is linearized by the following substitutions –

$$i_1(t) = i_b - i_c(t)\,; \quad i_2(t) = i_b + i_c(t) \qquad (3)$$

In the above set of substitutions, i_c is referred to as control current which is the instantaneous current that must be subtracted from and added to the bias current i_b in coils 1 and 2 respectively, when the rotor is displaced from its equilibrium point as depicted in fig.2.

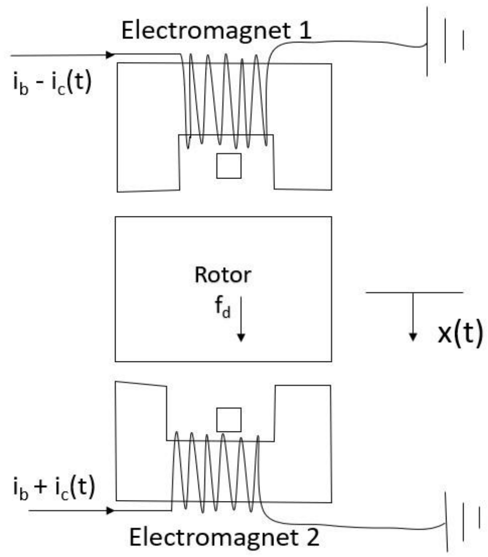

Fig.2

The objective of the control strategy is to ultimately obtain $i_c(t)$ in the AMB system subjected to any kinds of external disturbance, so that the rotor can be maintained in its equilibrium/operating point. Substituting (3) in the nonlinear force current relation (2), we get the following linear relation –

$$F = M\ddot{x} = -k_s x + k_i i_c + f_d \qquad (4)$$

$$k_s = -\frac{\mu_o N^2 A {i_b}^2}{X_o^3}\,; \quad k_i = \frac{\mu_o N^2 A i_b}{X_o^2} \qquad (5)$$

On assumption of the linear form is that the displacement of the rotor x is much less than the nominal air gap X_o. This means that the linear form is accurate only in the proximity of the equilibrium point.

In voltage control of the AMB system, the control inputs are the voltage across the electromagnet coils u and the external disturbance force on the rotor f_d. So, it becomes a multi-input control system. Generally state space description is useful for analysing multi-input systems.

For the AMB system, we consider the following –

State vector $X = [x \quad \dot{x} \quad i_c \quad \int x dt]^{\mathrm{T}}$

Control input vector $Y = [u \quad f_d]^{\mathrm{T}}$

Output $Y = x$

The state equations for the AMB system are –

$$\ddot{x} = \frac{1}{M}(-k_s x + k_i i_c + f_d)$$

$$u = Ri_c + L\frac{di_c}{dt} + k_i \dot{x}$$

Rearranging the above state equations and using the vectors mentioned above for the AMB system, we obtain the following state space mapping –

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{M} & 0 & \frac{k_i}{M} & 0 \\ 0 & \frac{-k_i}{L} & -\frac{R}{L} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{bmatrix} f_d$$

$$=> \dot{X} = AX + B_1 u + B_2 f_d$$

$$Y = [1 \quad 0 \quad 0 \quad 0]X$$

$$=> Y = C_1 X$$

The values of the system parameters are mentioned in the following table (1).

Parameters	Values
M	6 kg
A	5 cm^2
i_b	4 A
N	500
R	3.7 ohm
L	0.03926 H
k_i	157.08 N/A
k_s	-78.52 N/mm
f_d	25+12.5cos(15t) N

Table (1)

The set of equations (5) have been used to determine the values of K_i, K_s and L by putting the corresponding parameter values in them.

Voltage Control

In voltage control configuration, the controller sends a position-based voltage control signal as the power amplifier input from which it is converted into current in the coils. The voltage control mode, unlike current control, takes into consideration the effect of coil resistance and inductance in the overall system dynamics. Moreover, the amplifier used in current control is much more expensive than that used in the voltage control mode. For these reasons we will be considering voltage control mode for the AMB system

We will be implementing a PID controller for the voltage input. According to the PID control law –

$$u = P \times e + D \times \dot{e} + I \times \int e dt\ , \qquad (5)$$

where P, D, and I are referred to as the proportional, derivative and integral constants respectively and e represents the error of the instantaneous rotor position x with respect to a desired reference value r. We will be considering the reference position of the rotor to be zero.

$$\therefore e = r - x = -x$$

Substituting e in (5) –

$$u = [P \quad 0 \quad 0 \quad I]X - DC_1\dot{X}$$

$$=> u = C_2X - DC_1\dot{X} \qquad (6)$$

Substituting (6) in the state space description and rearranging, we get the following form –

$$\dot{X} = inv[I + DB_1C_1]([A - B_1C_1]X + B_2f_d \qquad I \text{ - 4 by 4 identity matrix}$$

$$=> \dot{X} = A_1X + B_3f_d \qquad (7)$$

The above equation (7) is the simplified form of the closed loop state space description of the AMB system. It represents a set of first order nonlinear ordinary differential equations. It is numerically solved in MATLAB using its ODE solver ODE45 with the following initial state vector X_i –

$$X_i = [0 \quad 0 \quad 0 \quad 0]^T$$

The initial zero state vector represents that initially the rotor is at the equilibrium/operating point.

Multi-objective optimisation problem

The presence of multiple objectives in an optimisation problem results in a set of optimal solutions known as pareto optimal solutions.

A general multi-objective optimisation problem consists of a number of objectives to be minimised and certain constraints to be satisfied at the same time. The objectives are functions of a set of variables also called as decision variables. And the constraints are imposed on those decision variables. In our case, the objective functions are –

i) the maximum magnitude of the transient displacement of the rotor
ii) the maximum magnitude of the control current.

And the decision variables are the control parameters P, D and I.

Thus, the multi-objective optimisation problem in our case is as follows-

$$\text{Minimise} = \begin{cases} f_1(P,D,I) = \max\left(|x|\right) \\ f_2(P,D,I) = \max\left(|i_c|\right) \end{cases}$$

$$\text{Subject to: } \begin{cases} P > 0 \\ D > 0 \\ I > 0 \end{cases}$$

We have attempted to solve the optimisation problem in MATLAB using its 'gamultiobj' function. 'gamultiobj' uses a controlled, elitist genetic algorithm (a variant of NSGA-II).

Genetic Algorithms

Genetic algorithms (GA) are a specific class of computation methods known as evolutionary optimisation (EO). An EO procedure is different from classical optimisation methods. An EO procedure does not use gradient information in its search process. Unlike most classical optimisation procedures, an EO procedure uses a set of solutions (population) in a single iteration. It uses stochastic operators unlike classical optimisation where deterministic operators are used. GA is a paradigm which uses gene recombination and the Darwinian survival of the fittest theory i.e., a group of individuals have a higher probability to survive if their fitness is high and offspring are created via a crossover operation as in gene recombination. GAs are stochastic search methos where an initial set of possible solutions (population) is modified in successive steps using the

Darwinian principle of natural selection, recombination and mutation to obtain an optimal solution.

GA begins with the initialisation process which results in a random creation of solutions (population). Then it enters into an iterative procedure of updating the current population to create a new population by the use of following operations-

a) Crossover: This operation generates a new solution from two existing solutions by exchanging some bits in those two solutions. The new solution thus formed inherit some characteristics of each of its parent solutions.
b) Mutation: Each child solution created by the crossover operation is now perturbed in its vicinity by a mutation operator. This operation generates a new solution by changing one or several bits in the existing solution. Thus, the new solution now possesses different characteristics from their parents. The mutation operator helps in creating random diversity in the population.
c) Selection: This operation choses to keep better solutions from the combined old and newly created population according to some predefined rules. This keeps the population size within a fixed constant and puts good solutions into the next generation with a high probability.

In a typical multi-objective optimisation problem, there exists family of equivalent solutions which are superior to the rest of the solutions and are considered equal from the perspective of the optimisation problem. These solutions are called non-inferior, non-dominated or pareto-optimal solutions. The nature of these solutions is such that no objective function can be improved without degrading one or more of the other objective functions. The goal of the multi-objective optimisation algorithms is to find the pareto front.

NSGA-II or Elitist Non-dominated sorting GA

NSGA-II differs from a simple genetic algorithm in the way the selection operator works. In this algorithm, multiple objectives are reduced to a single fitness measure by the creation of number of fronts sorted according to non-domination. It implements a selection method based on classes of domination of all the solutions. It introduces the concepts of rank and crowding distance for the solutions which are depicted in the following figure.

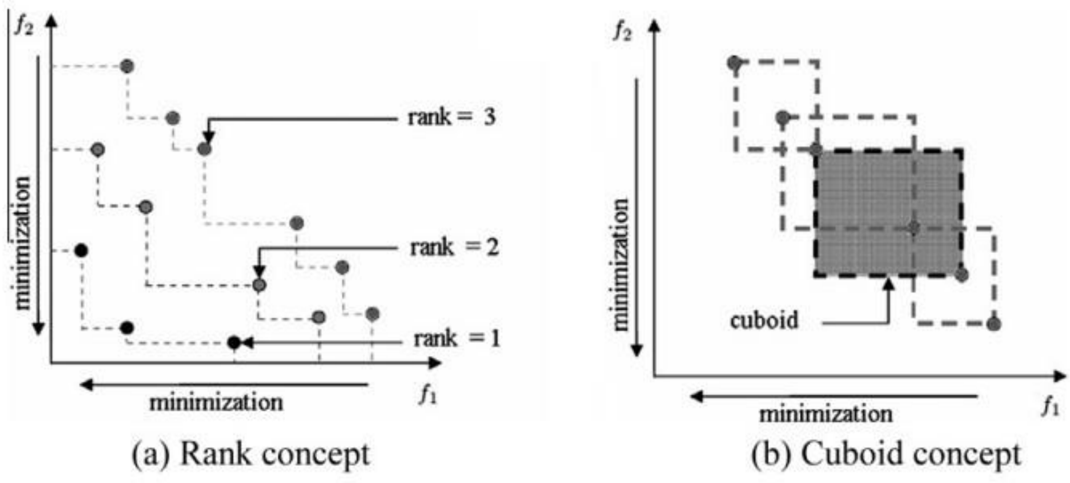

Fig. 3

Crowding distance is basically the perimeter of the cuboid formed between the two nearest neighbour solutions.

For two solutions belonging to the same rank, extreme solutions prevail over the non-extreme solutions. For two non-extreme solutions, the one with the bigger crowding distance prevails. Thus, NSGA-II encourages extreme and less crowded solutions.

The multi-objective genetic algorithm function of MATLAB 'gamultiobj' uses a variant of NSGA-II.

Results

The objective function vector is

$$f = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{Bmatrix} \max(|x|) \\ \max(|i_c|) \end{Bmatrix}$$

The decision variable vector is

$$X = \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} P \\ D \\ I \end{Bmatrix}$$

We have used the parameter values from table (1) for running the simulation in MATLAB.

The pareto-optimal solutions and the corresponding pareto front obtained from the simulation are depicted in the following figures.

Pareto front - function values and decision variables

Index	f1	f2	x1	x2	x3
1	0.007	0.241	26.305	19.477	27.315
2	0.006	0.242	27.611	19.302	29.334
3	0.007	0.242	27.03	19.565	27.88
4	0.006	0.242	27.743	19.518	29.07
5	0.006	0.242	27.517	19.375	29.097
6	0.007	0.241	26.305	19.477	27.315
7	0.006	0.242	27.482	19.653	29.015
8	0.006	0.242	27.605	19.804	29.036
9	0.007	0.241	26.305	19.477	27.315
10	0.007	0.241	26.305	19.477	27.315
11	0.006	0.242	28.674	18.302	30.334
12	0.006	0.242	28.82	20.783	29.996

Fig. 4

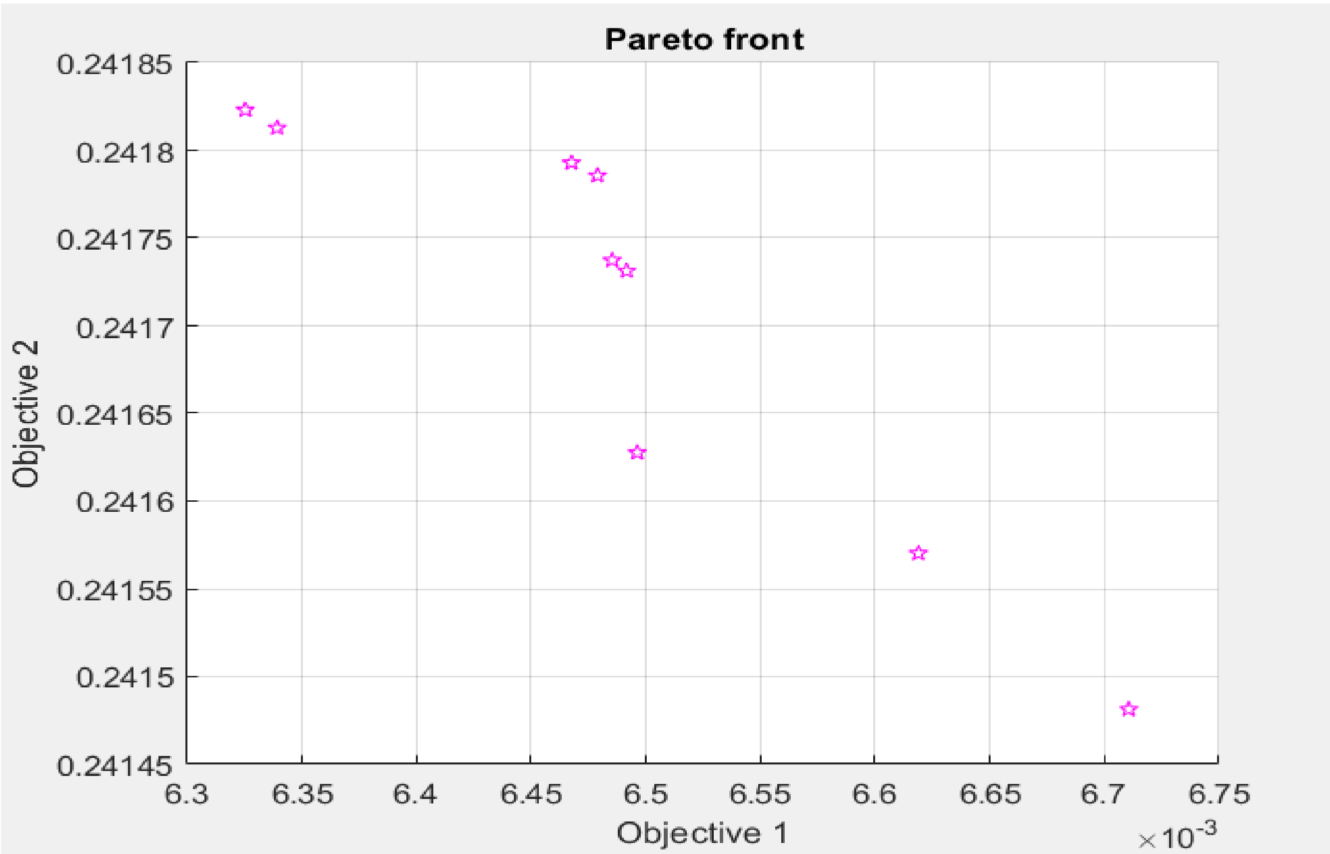

Fig. 5

The displacement, control current and voltage plots corresponding the pareto-optimal solution no. (7) i.e., P=27.48; D=19.65; I=29.01, are depicted in the following figure.

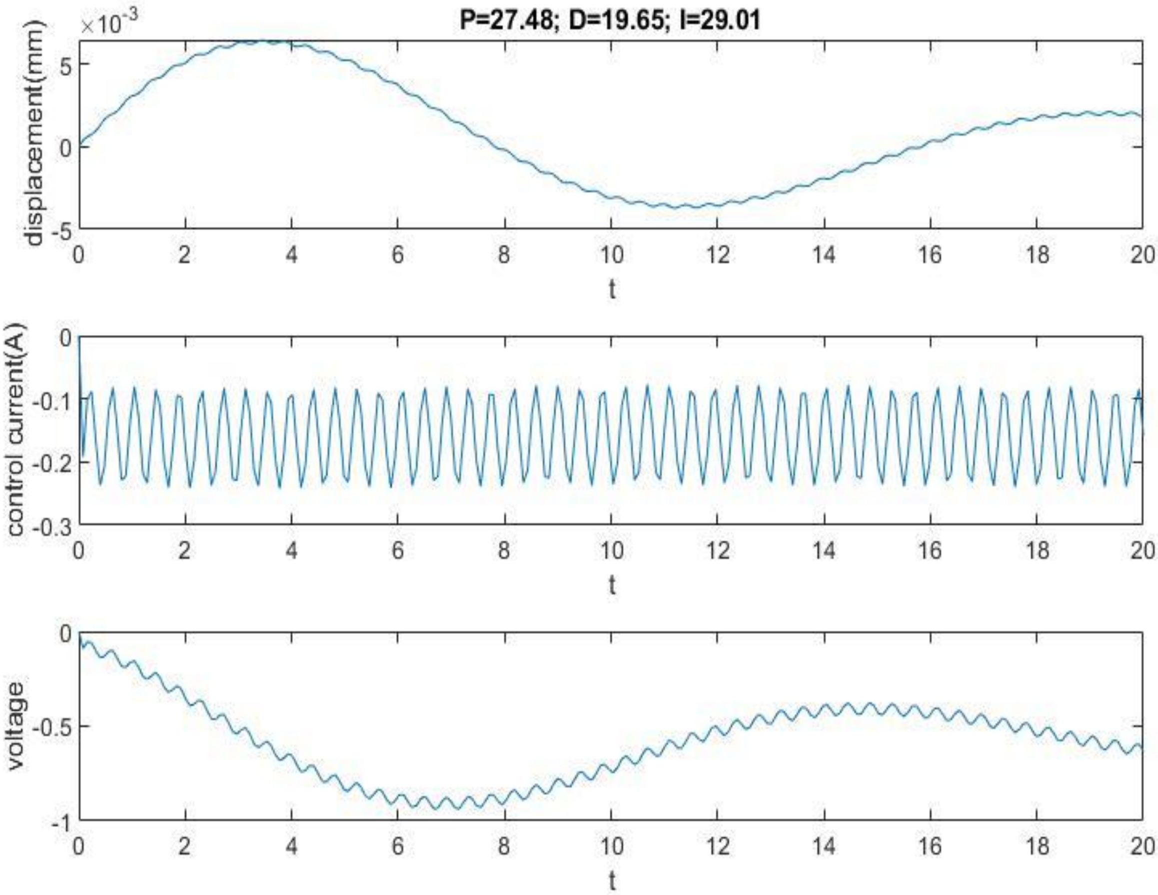

Fig. 6

Conclusion

In this project, we implemented voltage control in an active magnetic bearing by designing a PID controller for the same. We studied and presented the elementary concepts of multi-objective optimisation using genetic algorithms. And we implemented the same on our control problem to determine the optimal set of the control parameters with control current and rotor displacements as the objectives.

References

1) L. Li, T. Shinshi, A. Shimokohbe, “Asymptotically Exact Linearizations for Active Magnetic Bearing Actuators in Voltage Control Configuration”, 2003.
2) G. Schweitzer, E. Maslen, “Magnetic Bearings Theory, Design and Application to Rotating Machinery”, 2009.
3) K. Deb, “Multi-objective Optimization Using Evolutionary Algorithms: An Introduction”, 2011.
4) H. Ayala, L. Coelho, “Tuning of PID controller based on a multiobjective genetic algorithm applied to a robotic manipulator”.