

Lecture#4

Data Structures

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2D Array

	Col. 0	Col. 1	Col. 2	Col. 3
Row. 0	(0, 0)	(0, 1)	(0, 2)	(0, 3)
Row. 1	(1, 0)	(1, 1)	(1, 2)	(1, 3)
Row. 2	(2, 0)	(2, 1)	(2, 2)	(2, 3)
Row. 3	(3, 0)	(3, 1)	(3, 2)	(3, 3)



2D Array

The **syntax** for declaring a two-dimensional array is:

`DataType ArrayName [RowSize][ColumnSize]`

`int data[3][4]`

Number of elements = RowSize × ColumnSize

= 3 × 4

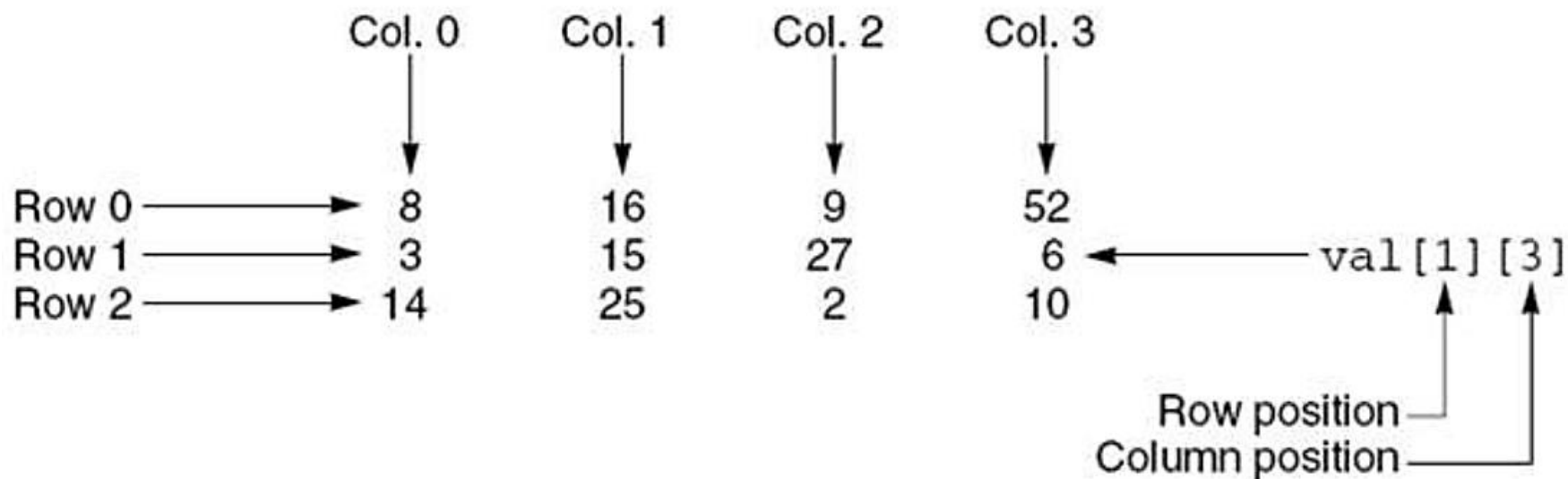
= 12





2D Array

Each array element is identified by its Row and Column position





2D Array

Assume 17 students had taken 4 class test.

The marks are stored in 17×4 array locations:

	CT0	CT1	CT2	CT3
Std 0	8	7	6	8
Std 1	0	7	8	9
Std 2	6	5	8	8
..
..
Std 16	7	8	8	7



Representation of 2D Array in Memory



	1	2	3	4
1	[1,1]	[1,2]	[1,3]	[1,4]
2	[2,1]	[2,2]	[2,3]	[2,4]
3	[3,1]	[3,2]	[3,3]	[3,4]

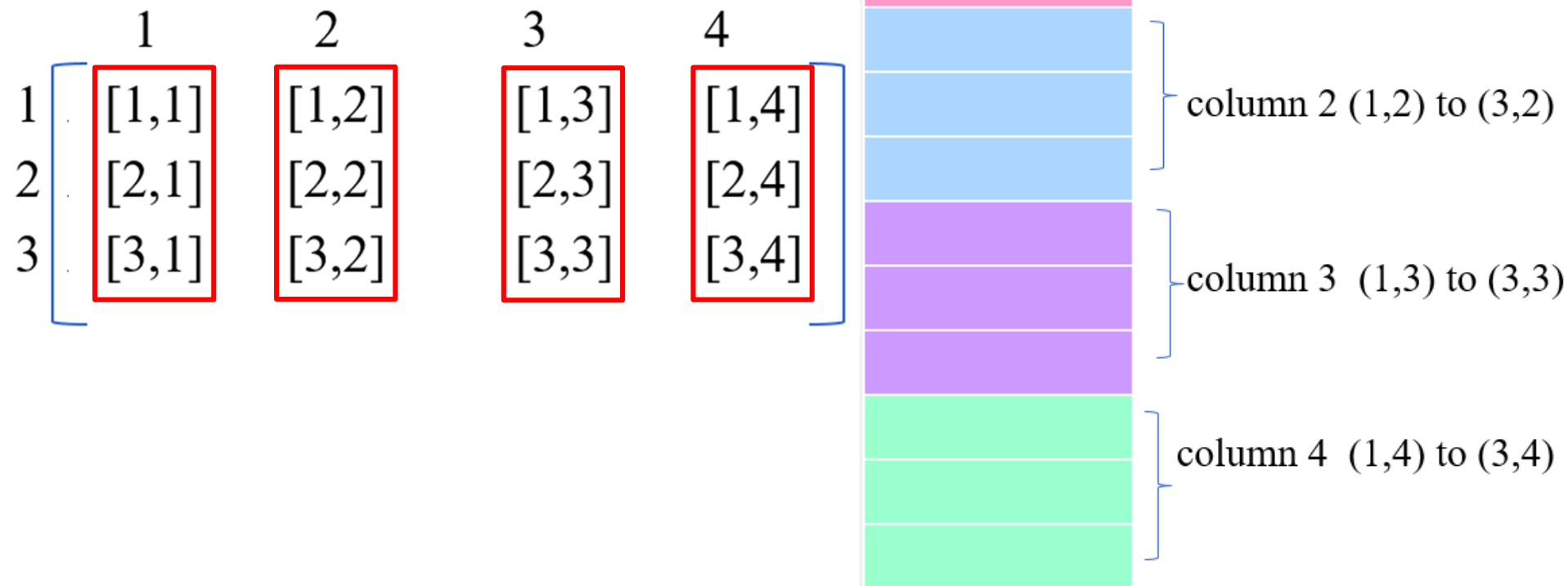
Two dimensional 3 x 4 array A



Representation of 2D Array in Memory



Column-major Order



Representation of 2D Array in Memory



Column-major Order

$$\text{LOC}(A[J, K]) = \text{Base}(LA) + w[n(J-1) + (K-1)]$$

where

$\text{Base}(A)$ = the address of the first element $A[1,1]$ of A

n = a number of rows in one column

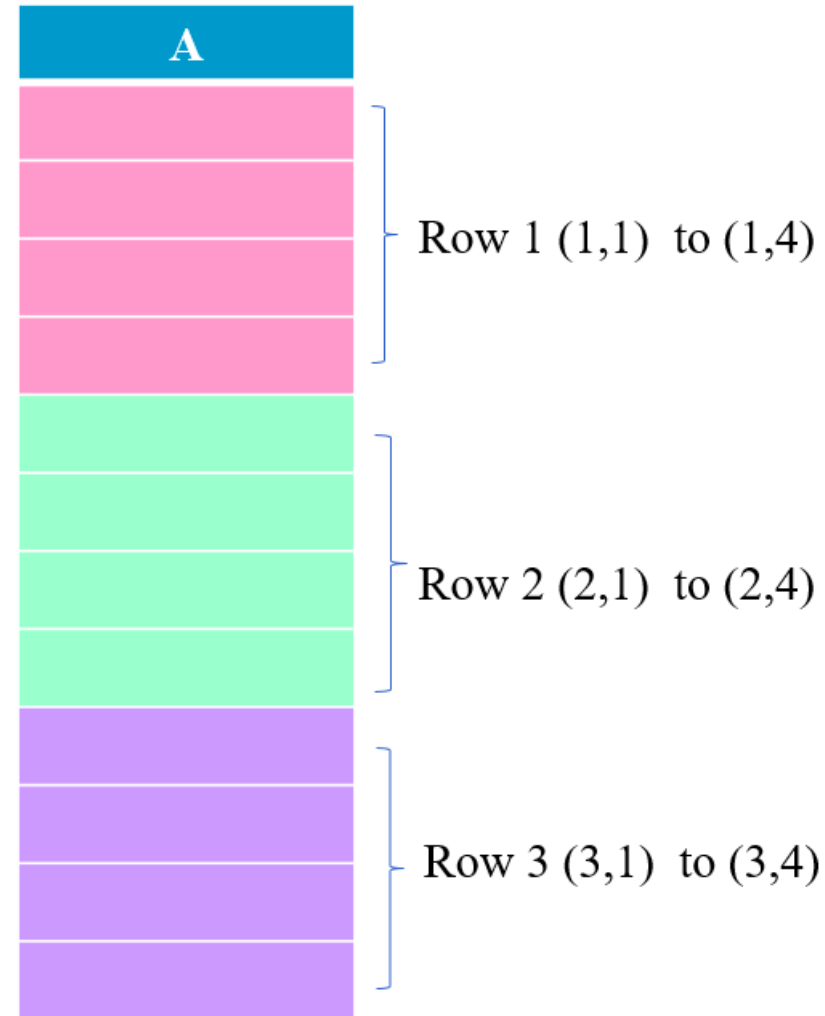


Representation of 2D Array in Memory



Row-major Order

	1	2	3	4
1	[1,1]	[1,2]	[1,3]	[1,4]
2	[2,1]	[2,2]	[2,3]	[2,4]
3	[3,1]	[3,2]	[3,3]	[3,4]



Representation of 2D Array in Memory



Row-major Order

$$\text{LOC}(A[J, K]) = \text{Base}(LA) + w[n(J-1) + (K-1)]$$

where

$\text{Base}(A)$ = the address of the first element $A[1,1]$ of A

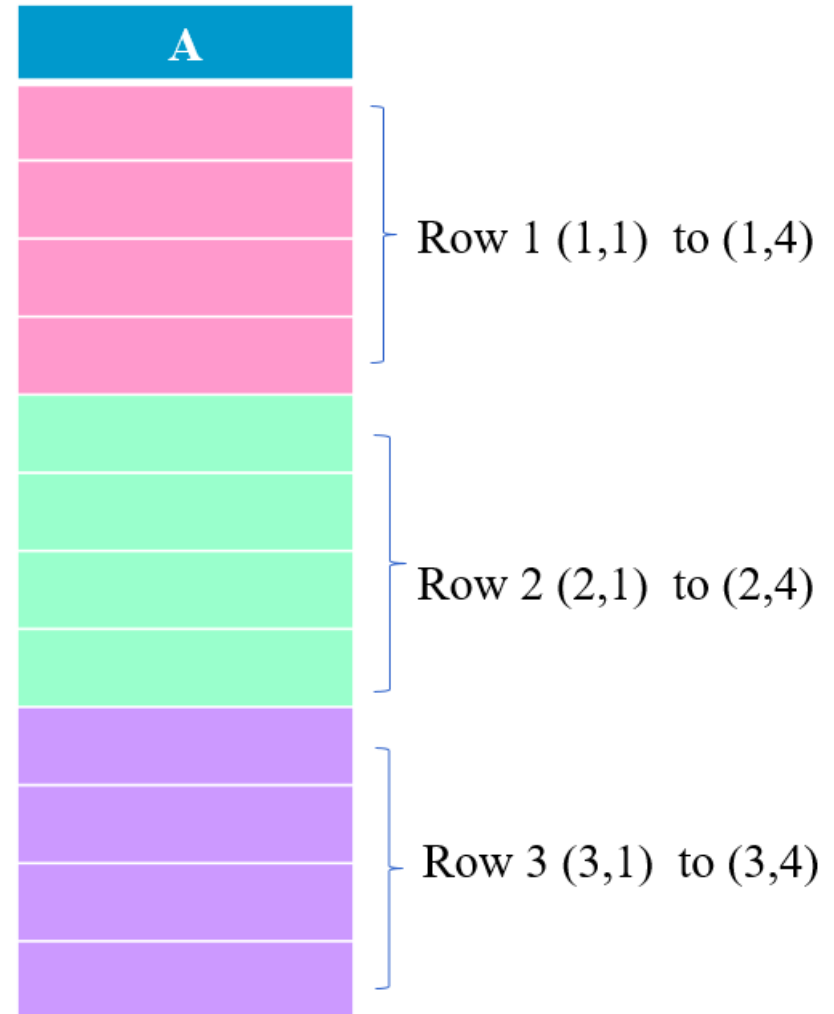
n = a number of columns in one row



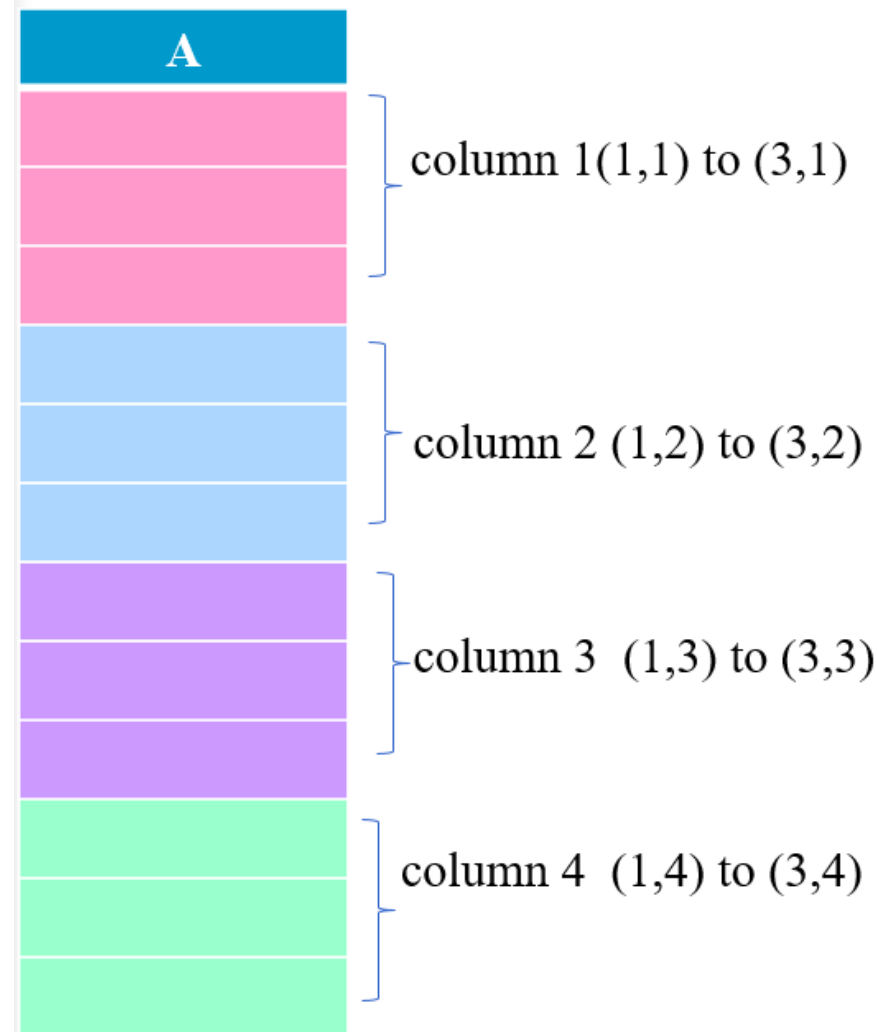
Representation of 2D Array in Memory



If $Base(\mathbf{LA}) = 200$ and $w = 4$
what is the address of
 $LOC(\mathbf{LA} [3, 2])$ in Row-major
Order representation?



Representation of 2D Array in Memory



If $Base(\text{LA}) = 200$ and $w = 4$
what is the address of
 $LOC(\text{LA}[3, 2])$ in Column-major
Order representation?





Longest Common Subsequence (LCS)





What is Subsequence?

A subsequence can be derived from the given sequence by deleting some or no elements **without changing the order** of the remaining elements.

The list of all subsequences for the word "apple" would be

"a", "ap", "al", "ae", "app", "apl", "ape", "ale", "appl",
"appe", "apple", "p", "pp", "pl", "pe", "ppl", "ppe",
"ple", "pple", "l", "le", "e", ""





Common Subsequences

Common subsequences are the subsequences that occur in both strings.

$S1 = abcde$

$S2 = cde$

Common Subsequences:

c, d, e, cd, de, cde





Longest Common Subsequence (LCS)

Given two sequences find the length of longest subsequence present in both items.

$S1 = a b c d e$

$S2 = c d e$

Common Subsequences:

c, d, e, cd, de, cde

Longest Common Subsequence (LCS): cde





Longest Common Subsequence (LCS)

Given two sequences find the length of longest subsequence present in both items.

$S1 = x \ y \ z \ a \ b \ c \ d \ e \ f$

$S2 = z \ a \ d \ f$

Longest Common Subsequences: $z \ a \ d \ f$





Longest Common Subsequence (LCS)

Given two sequences find the length of longest subsequence present in both items.

$S1 = x \ y \ z \ a \ b \ c \ d \ e \ f$

$S2 = z \ y \ d \ b \ f$

Longest Common Subsequences: $z \ d \ f$ / $y \ d \ f$





Longest Common Subsequence (LCS)

Given two sequences find the length of longest subsequence present in both items.

S1 = a b d a c e

S2 = b a b c e

Longest Common Subsequences?





Longest Common Subsequence Applications

- ❖ Similarity estimation of two strings or files
- ❖ Spoken word recognition
- ❖ Similarity of two biological sequences (DNA or protein)
- ❖ Sequence alignment
- ❖ In compressing genome resequencing data
- ❖ To authenticate users within their mobile phone through In-air signatures





Naïve /Brute Force Algorithm

For every subsequence of X , check whether it's a subsequence of Y .

- ❖ X has 2^m subsequences.
- ❖ Each subsequence takes $O(n)$ time to check:
scan Y for first letter, for second, and so on.

Time Complexity: $O(n2^m)$





Longest Common Subsequence (LCS)

We'll see how LCS algorithm works on the following example:

$X = A B C B$

$Y = B D C A B$

What is the **Longest Common Subsequence (LCS)** of X and Y ?





Longest Common Subsequence (LCS)



The following steps are followed for finding the longest common subsequence.

1. Create a table of dimension $m+1 * n+1$ where m and n are the lengths of X and Y respectively.





Longest Common Subsequence (LCS)

$X = \text{ABCB};$
 $m = |X| = 4$

$Y = \text{BDCAB};$
 $n = |Y| = 5$

		j	0	1	2	3	4	5
i			Y_j	B	D	C	A	B
0	X_i							
1	A							
2	B							
3	C							
4	B							





Longest Common Subsequence (LCS)

The following steps are followed for finding the longest common subsequence.

1. Create a table of dimension $m+1 * n+1$ where m and n are the lengths of X and Y respectively.
2. The first row and the first column are filled with zeros.





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
i		Yj	B	D	C	A	B	
0	Xi	0	0	0	0	0	0	
1	A	0						
2	B	0						
3	C	0						
4	B	0						

for $i = 1$ to m $c[i,0] = 0$
for $j = 1$ to n $c[0,j] = 0$





Longest Common Subsequence (LCS)

The following steps are followed for finding the longest common subsequence.

1. Create a table of dimension $m+1 * n+1$ where m and n are the lengths of X and Y respectively.
2. The first row and the first column are filled with zeros.
3. If the character corresponding to the current row and current column are matched, then fill the current cell by adding one to the diagonal element. Point an arrow to the diagonal cell.
4. Else take the maximum value from the previous column and previous row element for filling the current cell. Point an arrow to the cell with maximum value. If they are equal, point to any of them.





Longest Common Subsequence (LCS)

		j					
		0	1	2	3	4	5
i		Yj	B	D	C	A	B
0	Xi	0	0	0	0	0	0
1	A	0	0				
2	B	0					
3	C	0					
4	B	0					





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
i		Yj	B	D	C	A	B	
	0	Xi	0	0	0	0	0	0
	1	A	0	0	0			
	2	B	0					
	3	C	0					
	4	B	0					
	5							
	6							
	7							
	8							



Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i								
0	X _i	0	0	0	0	0	0	0
1	A	0	0	0	0	0		
2	B	0						
3	C	0						
4	B	0						





Longest Common Subsequence (LCS)

		j					
		0	1	2	3	4	5
i		Y _j	B	D	C	A	B
0	X _i	0	0	0	0	0	0
1	A	0	0	0	0	1	
2	B	0					
3	C	0					
4	B	0					





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i								
0	X _i	0	0	↑ 0	↑ 0	↑ 0	0	↑ 0
1	A	0	0	0	0	0	1	1
2	B	0						
3	C	0						
4	B	0						





Longest Common Subsequence (LCS)

		j					
		0	1	2	3	4	5
i		Y _j	B	D	C	A	B
0	X _i	0	0	0	0	0	0
1	A	0	0	0	0	1	1
2	B	0	1				
3	C	0					
4	B	0					





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
i		Yj						
		Xi		B	D	C	A	B
0	Xi		0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1			
3	C		0					
4	B		0					





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
i		Yj	B	D	C	A	B	
		Xi						
0	Xi	0	0	0	0	0	0	
1	A	0	0	0	0	1	1	
2	B	0	1	1	1			
3	C	0						
4	B	0						





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
i		Y _j		B	D	C	A	B
	X _i							
0			0	↑ 0	↑ 0	↑ 0	↖ 0	0
1	A	0	↖ 0	0	0	0	↖ 1	← 1
2	B	0	↖ 1	← 1	← 1	← 1	↖ 1	
3	C	0						
4	B	0						





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i								
0	X _i	0	0	0	0	0	0	0
1	A	0	0	0	0	0	1	1
2	B	0	1	1	1	1	1	2
3	C	0						
4	B	0						





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
i		Y _j	B	D	C	A	B	
0	X _i	0	0	0	0	0	0	
1	A	0	0	0	0	1	1	
2	B	0	1	1	1	1	2	
3	C	0	1	1	2	2	2	
4	B	0						





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
i		Yj						
		Yj	B	D	C	A	B	
0	Xi	0	0	0	0	0	0	
1	A	0	0	0	0	1	1	
2	B	0	1	1	1	1	2	
3	C	0	1	1	2	2	2	
4	B	0	1	1	2	2	3	





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	↑ 0	↑ 0	↑ 0	↖ 0	0
1	A		0	↖ 0	↑ 0	↑ 0	↖ 1	← 1
2	B		0	↖ 1	← 1	← 1	↑ 1	↖ 2
3	C		0	↑ 1	↑ 1	↖ 2	← 2	↑ 2
4	B		0	↖ 1	↑ 1	↑ 2	↑ 2	↖ 3





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i	X _i							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1	1	2	2	3

Arrows indicate the path for the LCS: A blue arrow points from (3,4) to (3,3), and a red arrow points from (4,5) to (3,4). Other black arrows show the backtracking path from (4,5) to (1,4) to (2,3) to (2,2) to (1,1) to (0,0).





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Yj	B	D	C	A	B
i	Xi							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1	1	2	2	3

Diagram illustrating the Longest Common Subsequence (LCS) problem. The table shows the dynamic programming table with indices i (rows) and j (columns). The sequences are X = A, B, C, B and Y = B, D, C, A, B. The table contains values representing the length of the LCS up to (i, j). Arrows indicate the path taken to compute the values, showing the backtracking path for the LCS. A red arrow points from (4, 5) to (3, 4), and a blue arrow points from (3, 4) to (3, 3). Other black arrows show the general flow of computation.





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Yj	B	D	C	A	B
i	Xi							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1	1	2	2	3

Diagram illustrating the Longest Common Subsequence (LCS) problem. The table shows the dynamic programming table with indices i (rows) and j (columns). The sequences are X = A, B, C, B and Y = B, D, C, A, B. The table contains values representing the length of the LCS up to (i, j). Arrows indicate the path of the LCS: black arrows for general transitions, blue arrows for horizontal transitions (indicating a match in Y), and red arrows for diagonal transitions (indicating a match in X).





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Yj	B	D	C	A	B
i	Xi							
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1	1	2	2	3

Diagram illustrating the Longest Common Subsequence (LCS) problem. The table shows the DP table with indices i (rows) and j (columns). The sequences are X = A B C B and Y = B D C A B. The table contains values for the LCS length at each position (i, j). Arrows indicate the path of the LCS: black arrows for the main sequence, red arrows for alternative paths, and blue arrows for the final path (A, B, C).





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
			Y _j	B	D	C	A	B
i								
0	X _i		0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1	1	2	2	3





Longest Common Subsequence (LCS)

		j	0	1	2	3	4	5
i		Y _j		B	D	C	A	B
		X _i						
0			0	0	0	0	0	0
1	A		0	0	0	0	1	1
2	B		0	1	1	1	1	2
3	C		0	1	1	2	2	2
4	B		0	1	1	2	2	3

Thus, the longest common subsequence is B C B





Longest Common Subsequence (LCS)

X



The first sequence

Y



Second Sequence

What is the **Longest Common Subsequence (LCS)** of X and Y?





Longest Common Subsequence (LCS)

		C	B	D	A
	0	0	0	0	0
A	0				
C	0				
A	0				
D	0				
B	0				





Longest Common Subsequence (LCS)

		C	B	D	A
	0	0	0	0	0
A	0	0	0	0	1
C	0				
A	0				
D	0				
B	0				





Longest Common Subsequence (LCS)

		C	B	D	A
A C A D B		0	0	0	0
	0	0	0	0	1
	0	1	1	1	1
	0	1	1	1	2
	0	1	1	2	2
	0	1	2	2	2





Longest Common Subsequence (LCS)

		C	B	D	A
		0	0	0	0
A		0	0	0	1
C		0	1	1	1
A		0	1	1	2
D		0	1	2	2
B		0	1	2	2





Longest Common Subsequence (LCS)

		C	B	D	A
A	0	0	0	0	0
C	0	1	1	1	1
A	0	1	1	1	2
D	0	1	1	2	2
B	0	1	2	2	2

Select the cells
with diagonal
arrows

		C	B	D	A
A	0	0	0	0	0
C	0	1	1	1	1
A	0	1	1	1	2
D	0	1	1	2	2
B	0	1	2	2	2

C

A





Longest Common Subsequence (LCS)

X = P R E S I D E N T

Y = P R O V I D E N C E

What is the Longest Common Subsequence of X and Y?





Longest Common Subsequence (LCS)

i \ j	0	1	2	3	4	5	6	7	8	9	10
	<i>o</i>	<i>p</i>	<i>r</i>	<i>o</i>	<i>v</i>	<i>i</i>	<i>d</i>	<i>e</i>	<i>n</i>	<i>c</i>	<i>e</i>
0	0	0	0	0	0	0	0	0	0	0	0
1 <i>p</i>	0	1	1	1	1	1	1	1	1	1	1
2 <i>r</i>	0	1	2	2	2	2	2	2	2	2	2
3 <i>e</i>	0	1	2	2	2	2	2	3	3	3	3
4 <i>s</i>	0	1	2	2	2	2	2	3	3	3	3
5 <i>i</i>	0	1	2	2	2	3	3	3	3	3	3
6 <i>d</i>	0	1	2	2	2	3	4	4	4	4	4
7 <i>e</i>	0	1	2	2	2	3	4	5	5	5	5
8 <i>n</i>	0	1	2	2	2	3	4	5	6	6	6
9 <i>t</i>	0	1	2	2	2	3	4	5	6	6	6

Output: PRIDEN





Longest Common Subsequence (LCS)

X = C G A T A A T T G A G A

Y = G T T C C T A A T A

What is the Longest Common Subsequence of X and Y?





Longest Common Subsequence (LCS)

<i>L</i>	-1	0	1	2	3	4	5	6	7	8	9	10	11
-1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	1
1	0	0	1	1	2	2	2	2	2	2	2	2	2
2	0	0	1	1	2	2	2	3	3	3	3	3	3
3	0	1	1	1	2	2	2	3	3	3	3	3	3
4	0	1	1	1	2	2	2	3	3	3	3	3	3
5	0	1	1	1	2	2	2	3	4	4	4	4	4
6	0	1	1	2	2	3	3	3	4	4	5	5	5
7	0	1	1	2	2	3	4	4	4	4	5	5	6
8	0	1	1	2	3	3	4	5	5	5	5	5	6
9	0	1	1	2	3	4	4	5	5	5	6	6	6

0 1 2 3 4 5 6 7 8 9 10 11
Y=CGATAATTGAGA
0 1 2 3 4 5 6 7 8 9
X=GTTCTAATA





Longest Common Subsequence (LCS)

INPUT: two strings

X = A C T G A A C T C T G T G C A C T

Y = T G A C T C A G C A C A A A A A C

OUTPUT: longest common subsequence

A C T G A A C T C T G T G C A C T

T G A C T C A G C A C A A A A A C





Longest Common Subsequence (LCS)

Algorithm 7.1 LCS

Input: Two strings A and B of lengths n and m , respectively, over an alphabet Σ .

Output: The length of the longest common subsequence of A and B .

```
1. for  $i \leftarrow 0$  to  $n$ 
2.    $L[i, 0] \leftarrow 0$ 
3. end for
4. for  $j \leftarrow 0$  to  $m$ 
5.    $L[0, j] \leftarrow 0$ 
6. end for
7. for  $i \leftarrow 1$  to  $n$ 
8.   for  $j \leftarrow 1$  to  $m$ 
9.     if  $a_i = b_j$  then  $L[i, j] \leftarrow L[i - 1, j - 1] + 1$ 
10.    else  $L[i, j] \leftarrow \max\{L[i, j - 1], L[i - 1, j]\}$ 
11.    end if
12.  end for
13. end for
14. return  $L[n, m]$ 
```

Time Complexity: $O(nm)$



