Lecture#10

Data Structures

Dr. Abu Nowshed Chy

Department of Computer Science and Engineering

University of Chittagong

February 24, 2025

Faculty Profile

Graphs

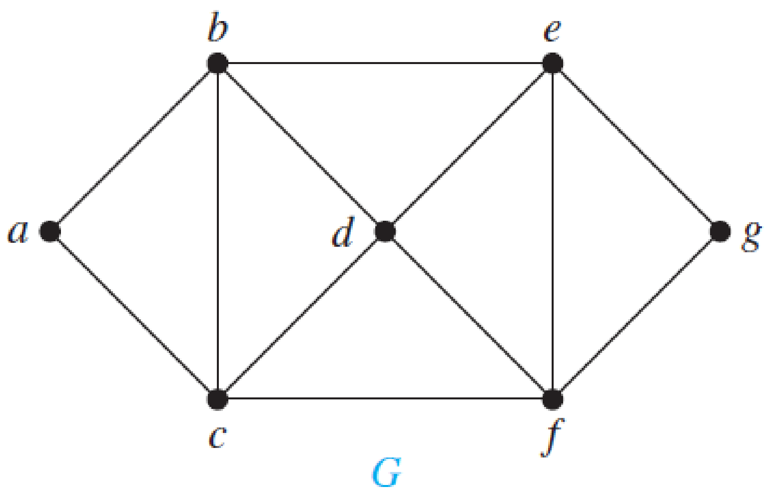
Graph Coloring

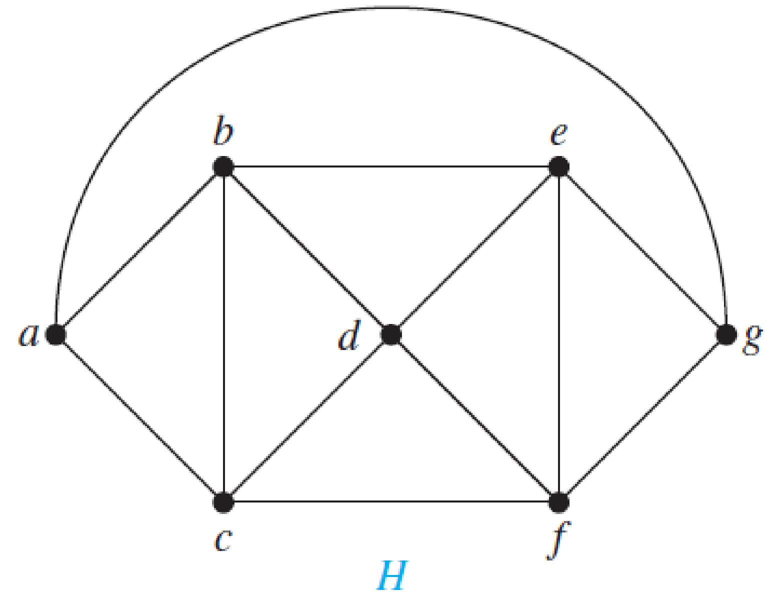
A *coloring* of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

The chromatic number of a graph is the least number of colors needed for a coloring of this graph.

Graph Coloring

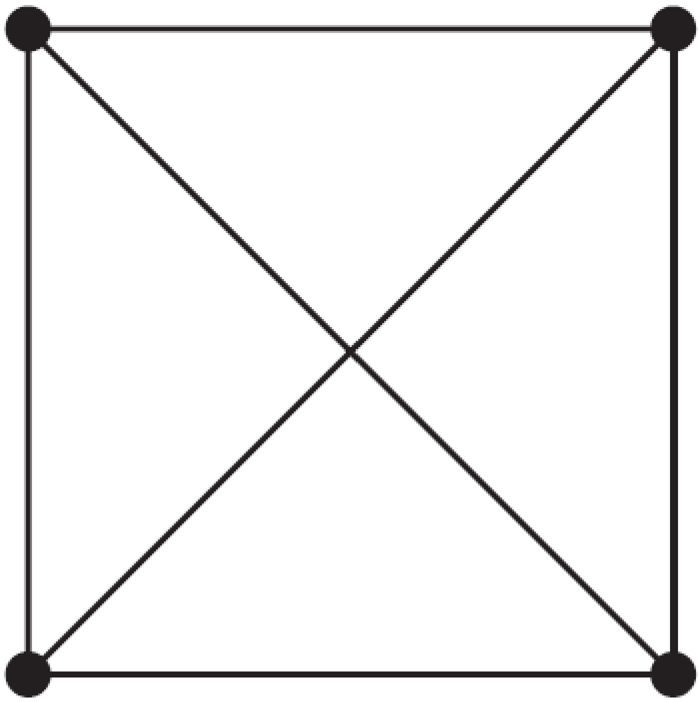
The Four Color Theorem:
The chromatic number of a planar graph is no greater than four.





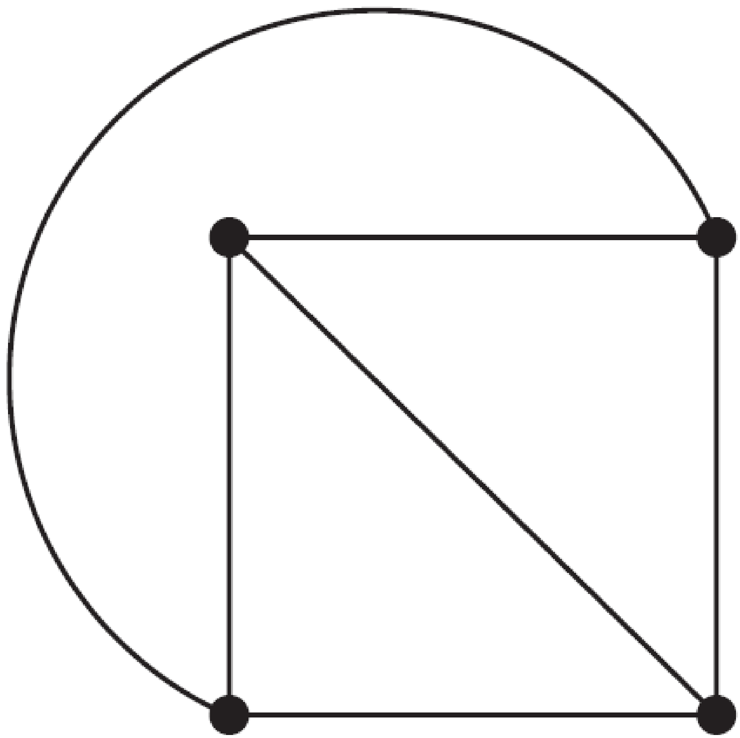
Planar Graph

A graph is called *planar* if it can be drawn in the plane without any edges crossing



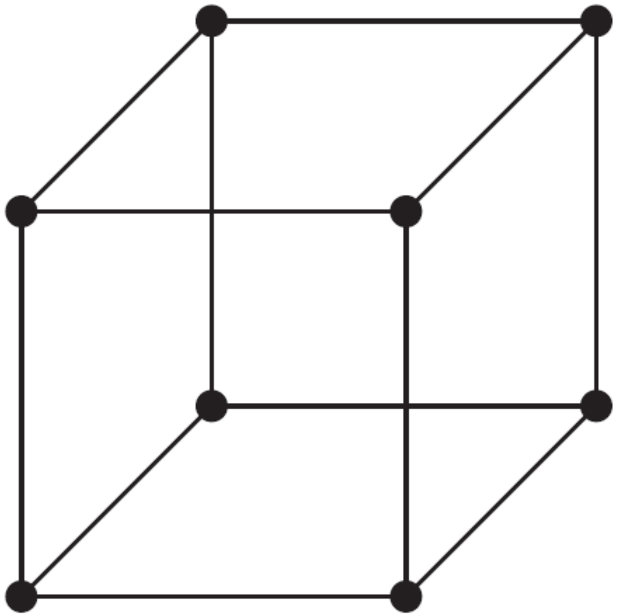
Planar Graph

A graph is called *planar* if it can be drawn in the plane without any edges crossing



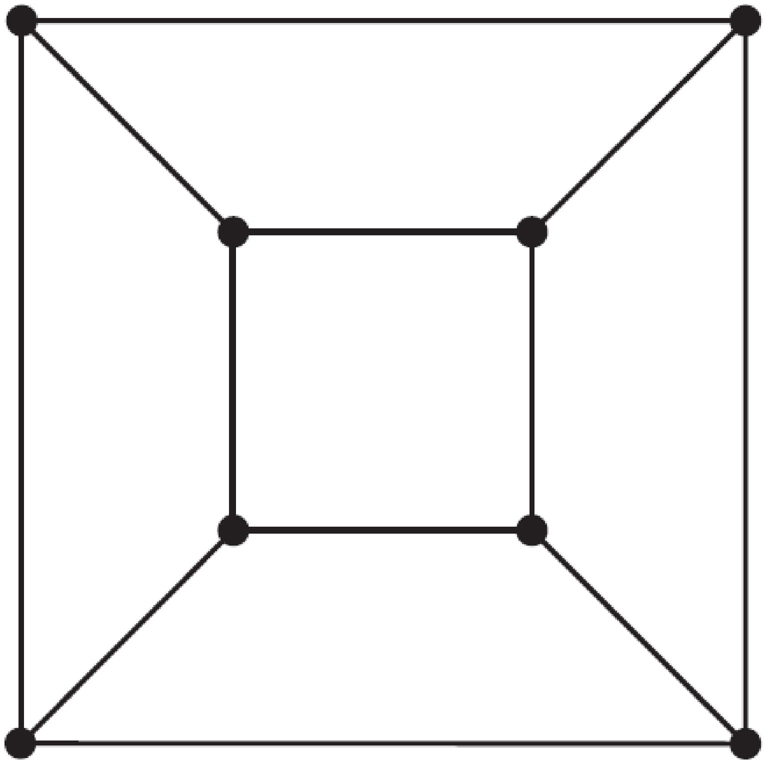
Planar Graph

A graph is called *planar* if it can be drawn in the plane without any edges crossing



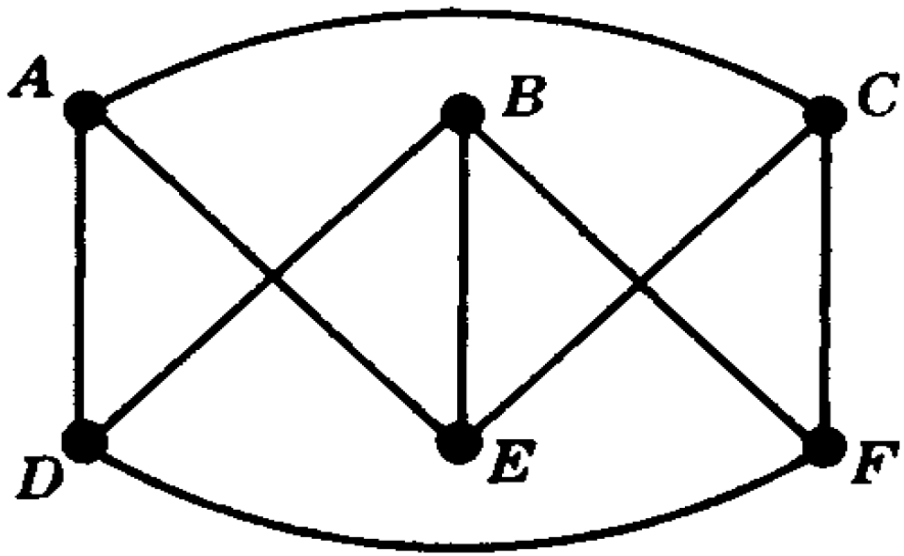
Planar Graph

A graph is called *planar* if it can be drawn in the plane without any edges crossing



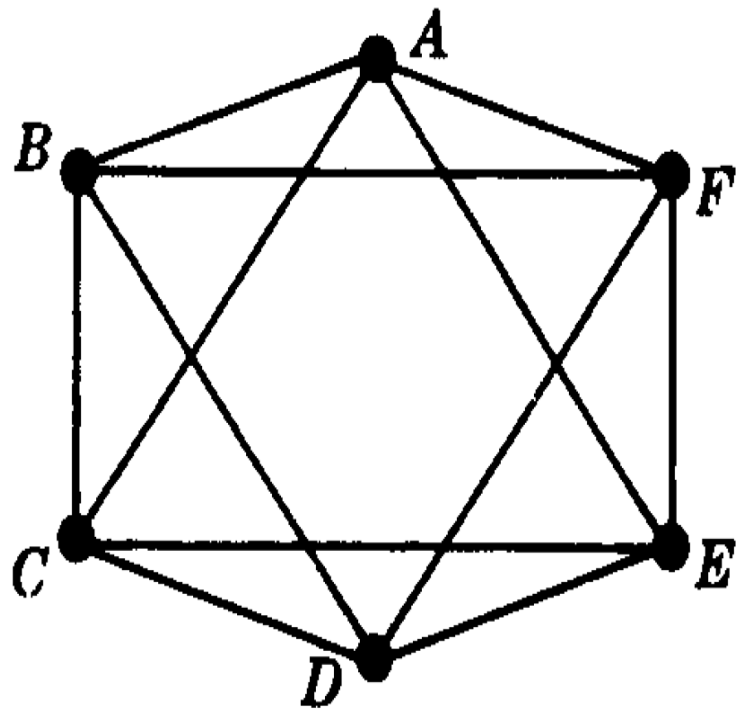
Planar Graph

A graph is called *planar* if it can be drawn in the plane without any edges crossing



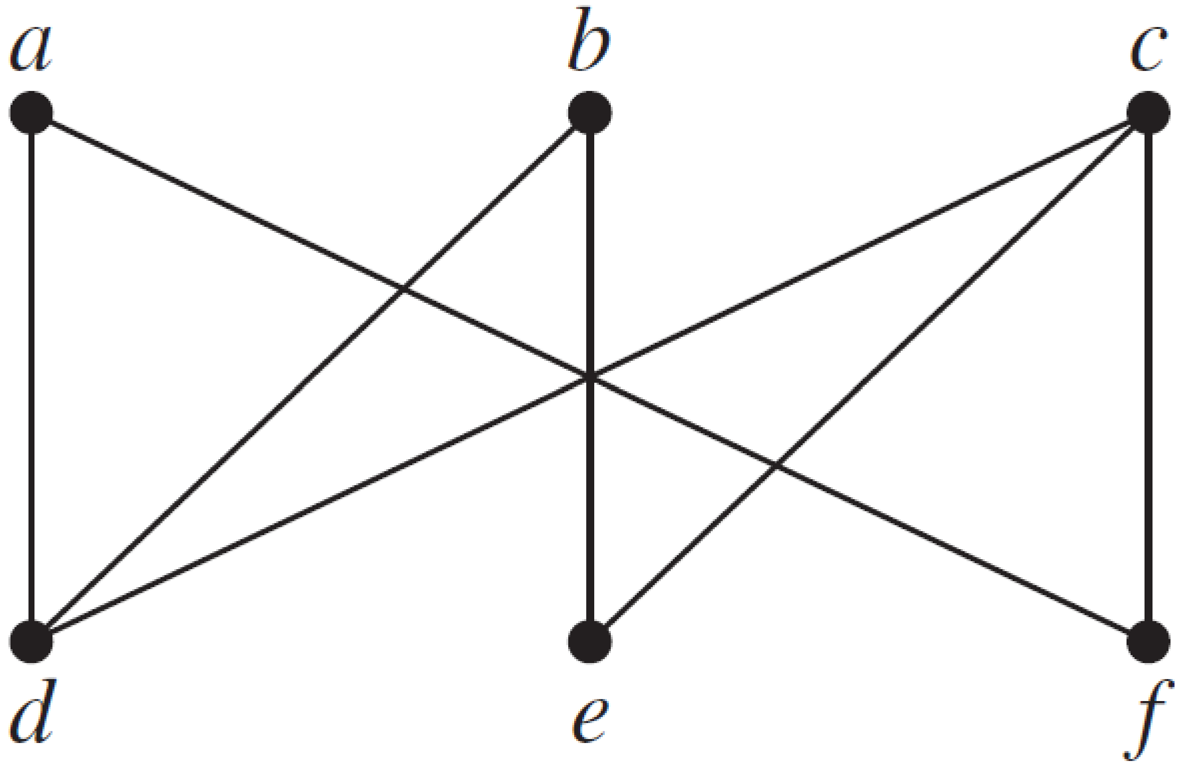
Planar Graph

A graph is called *planar* if it can be drawn in the plane without any edges crossing



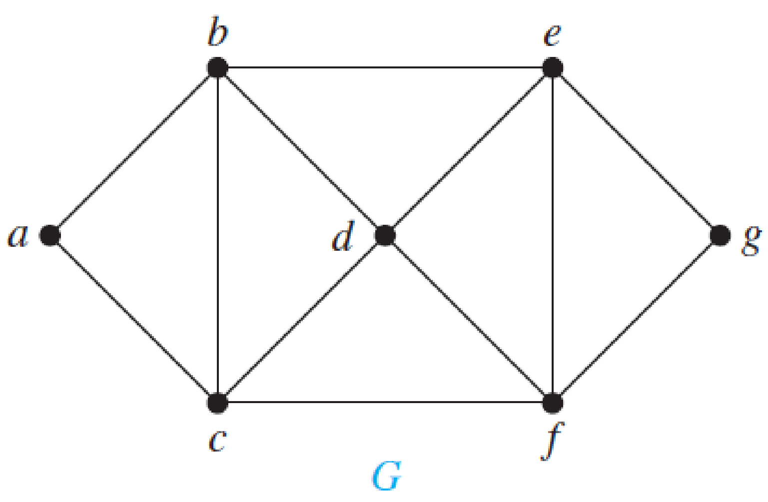
Planar Graph

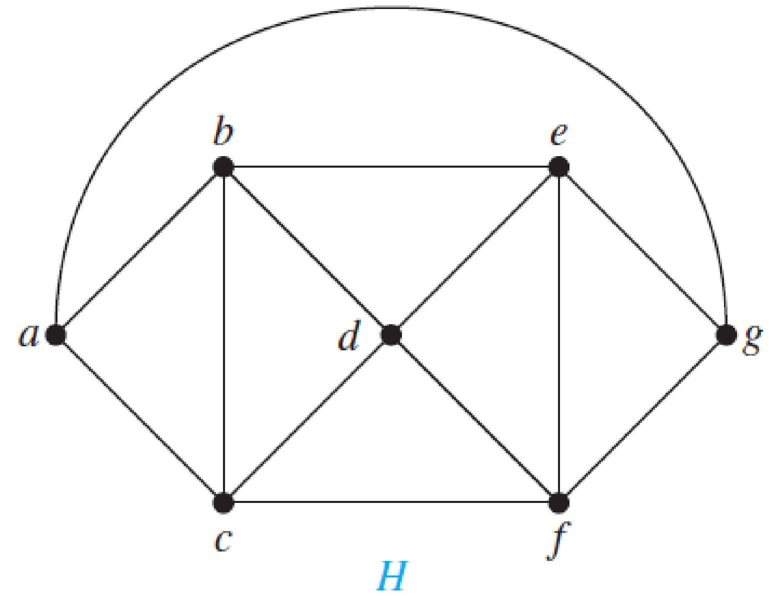
A graph is called *planar* if it can be drawn in the plane without any edges crossing



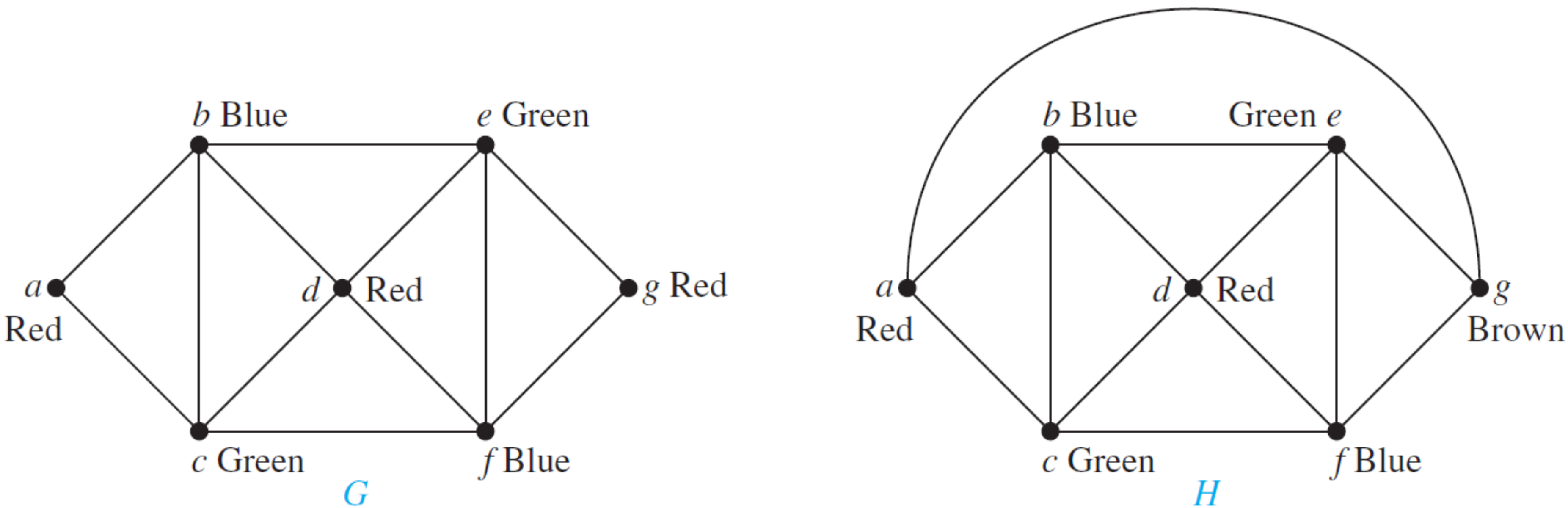
Graph Coloring

The Four Color Theorem:
The chromatic number of a planar graph is no greater than four.

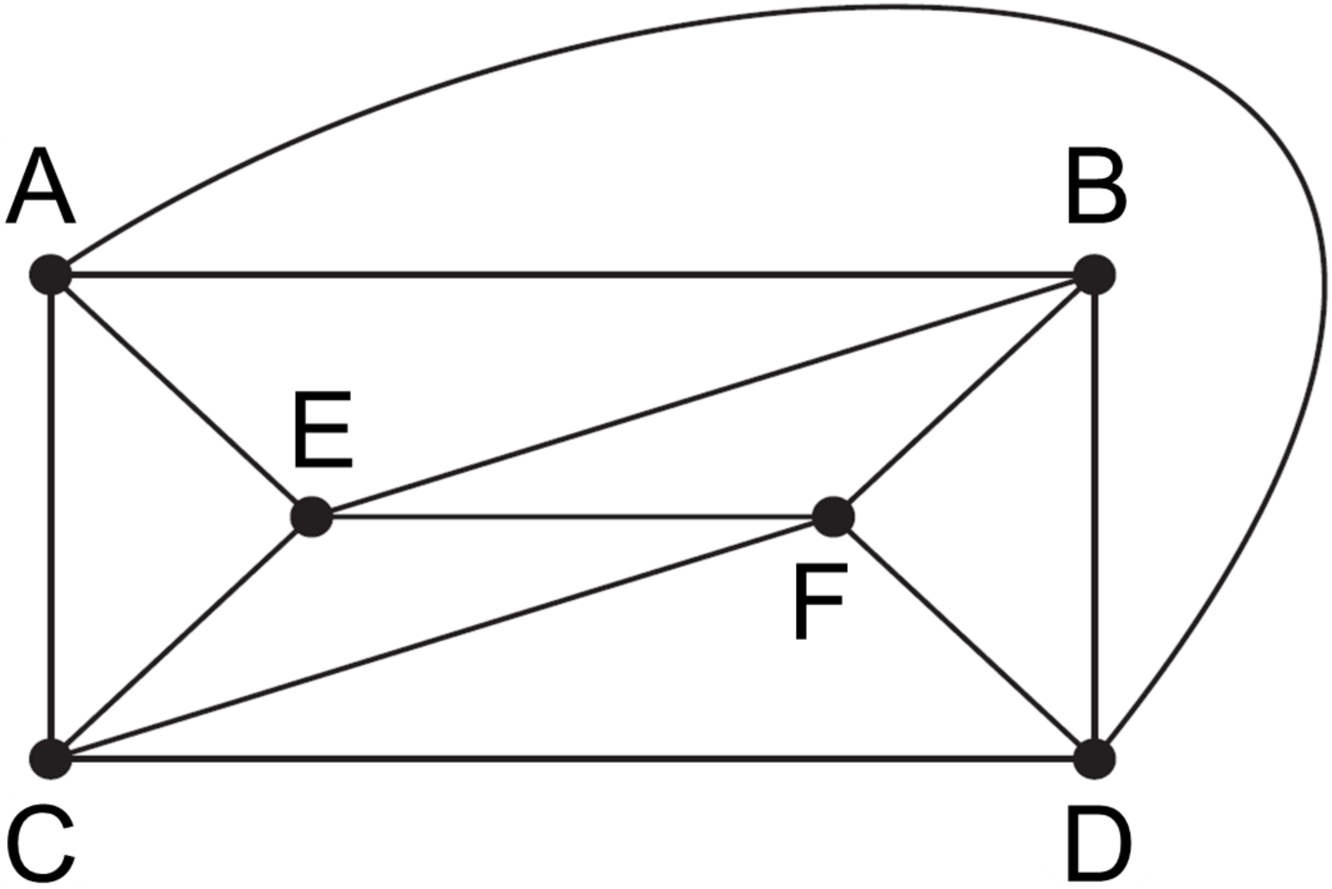




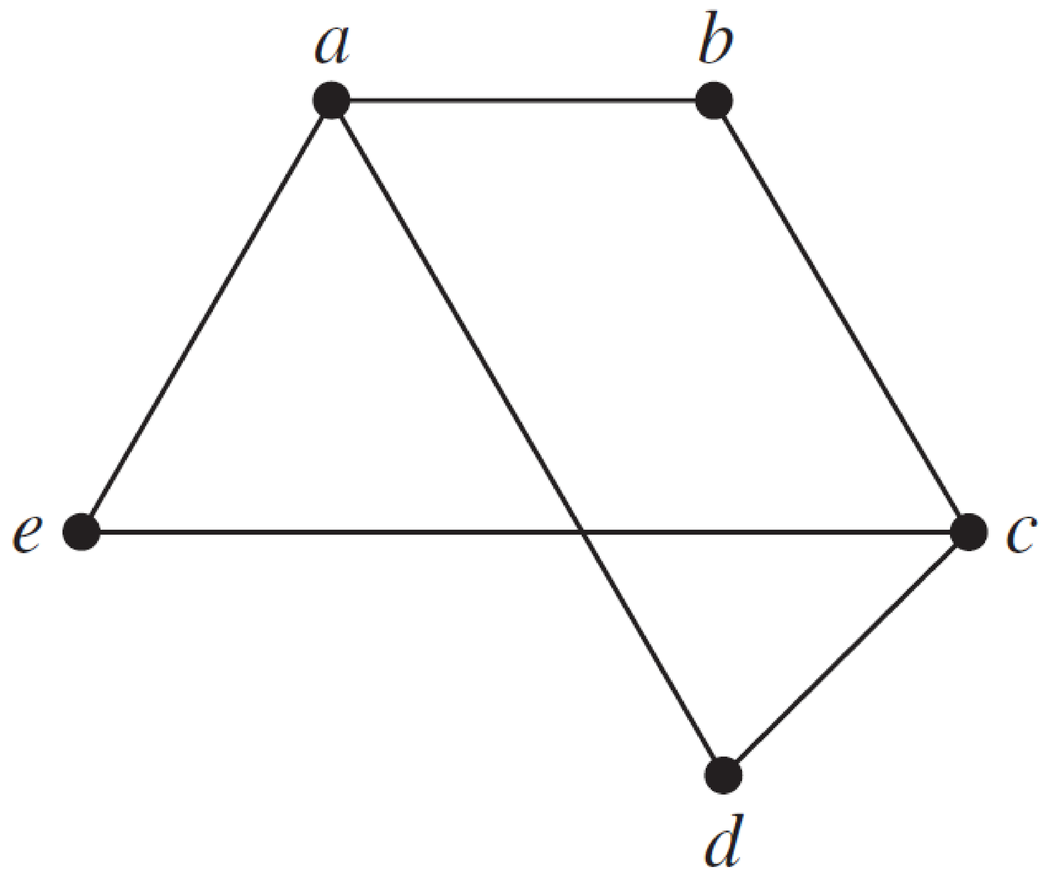
Graph Coloring



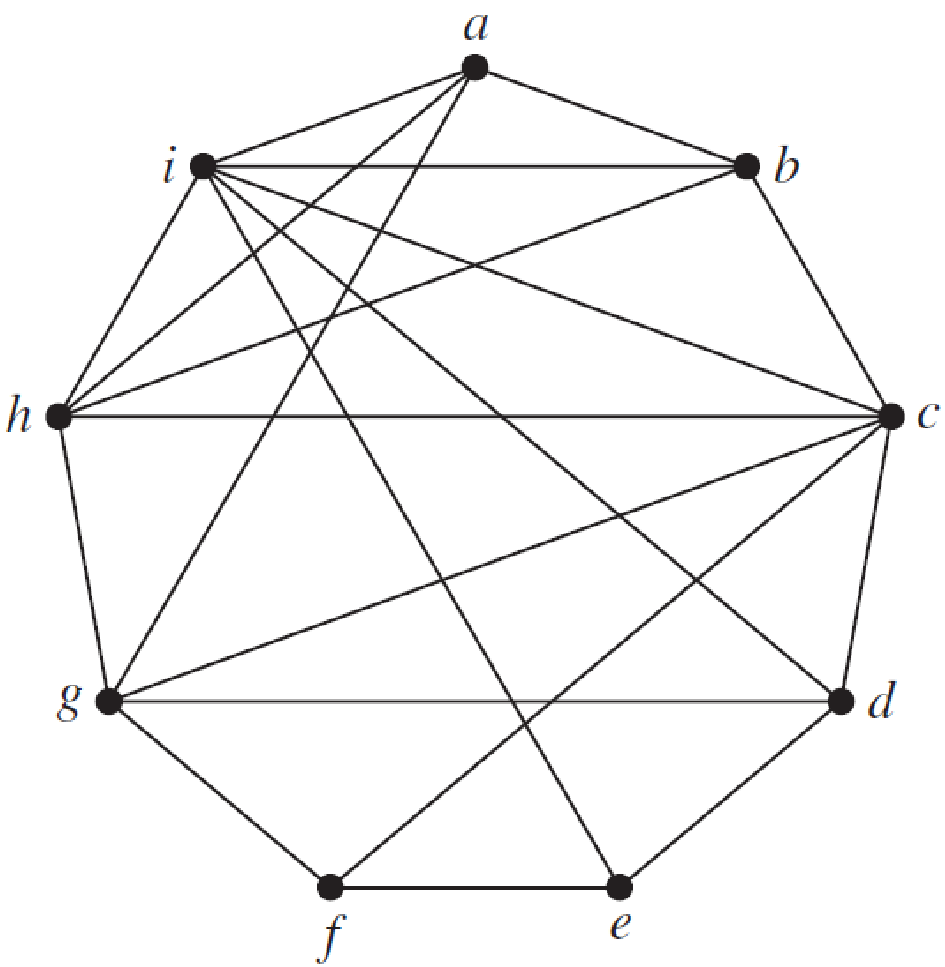
Graph Coloring



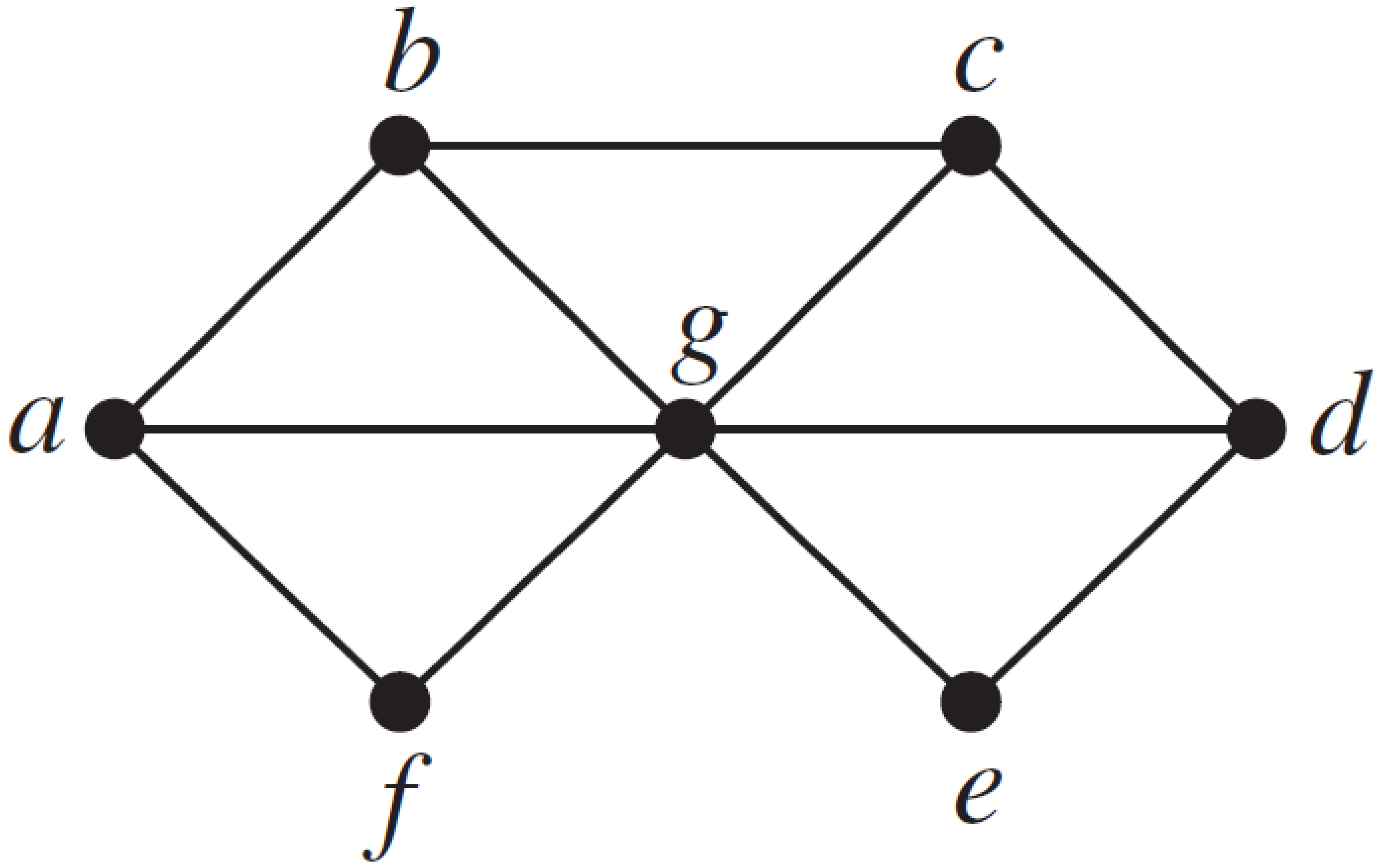
Graph Coloring



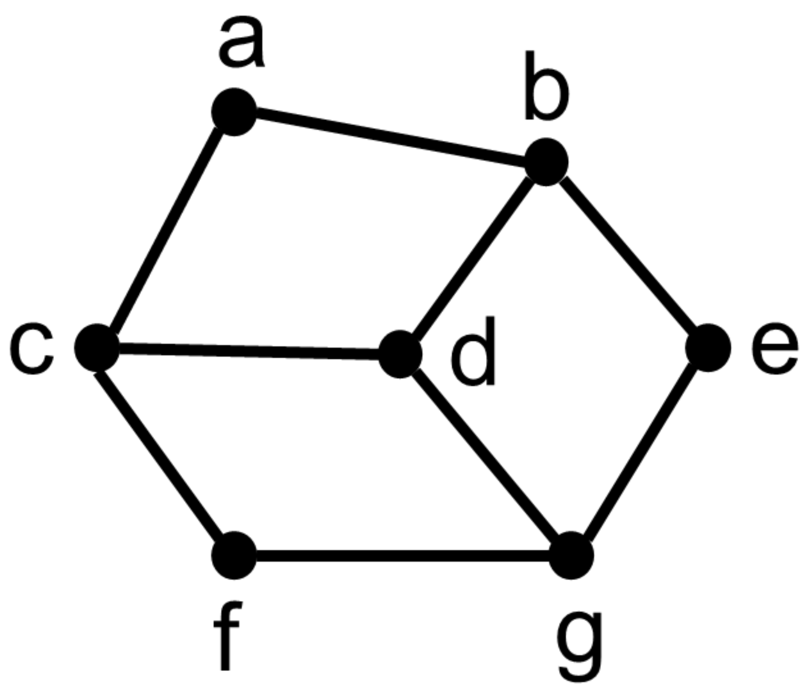
Graph Coloring



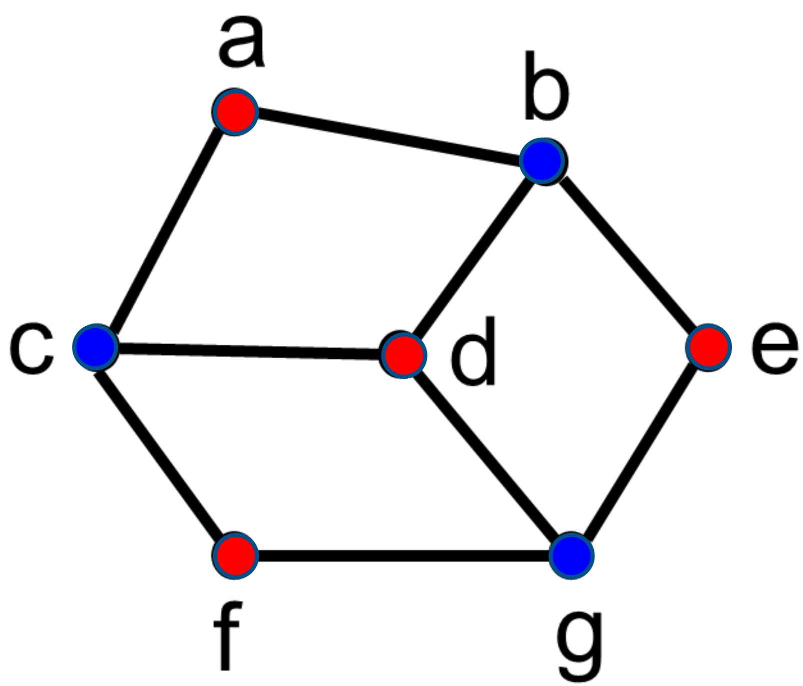
Graph Coloring



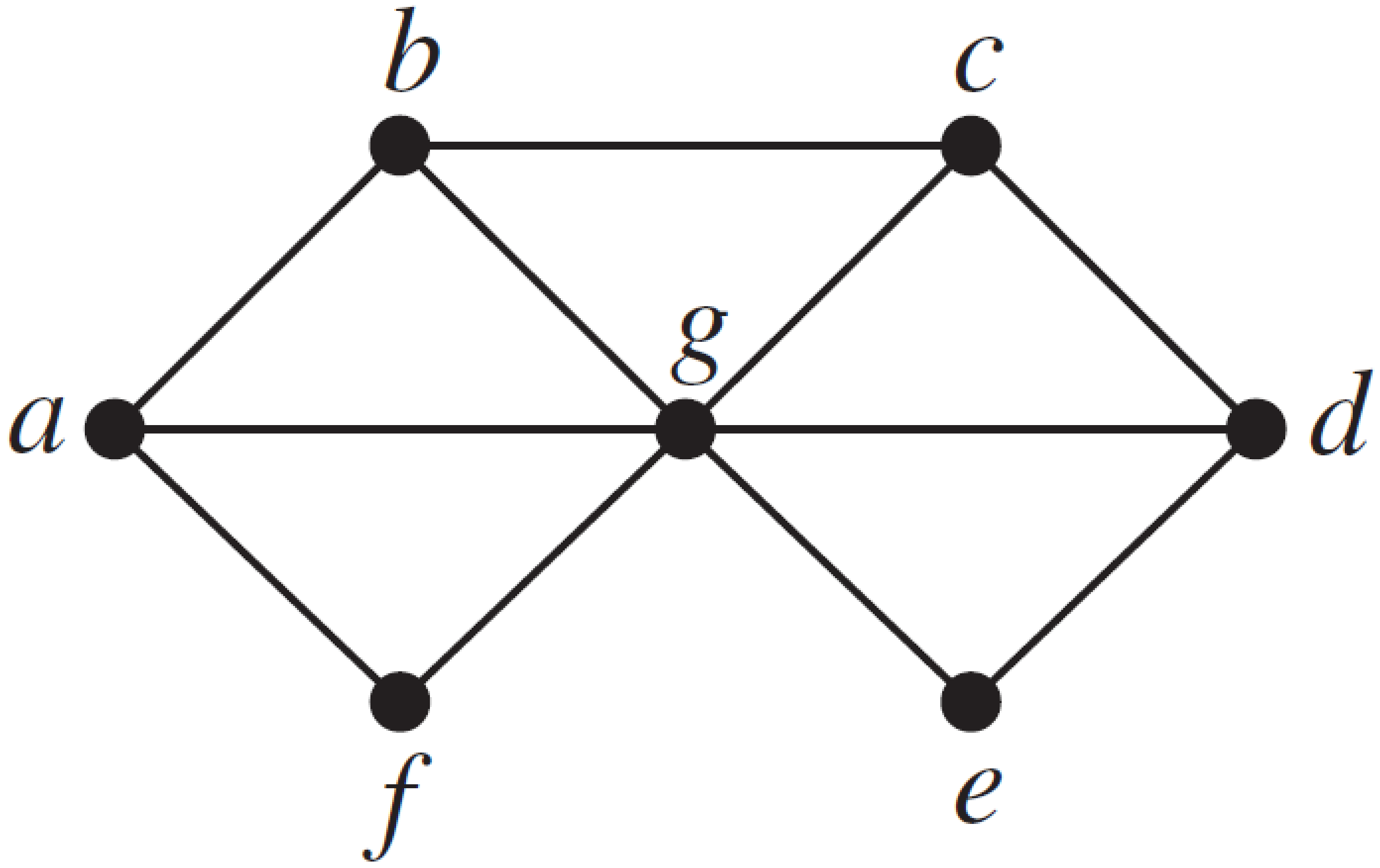
Bipartite



Bipartite



Bipartite



Graph Coloring Applications

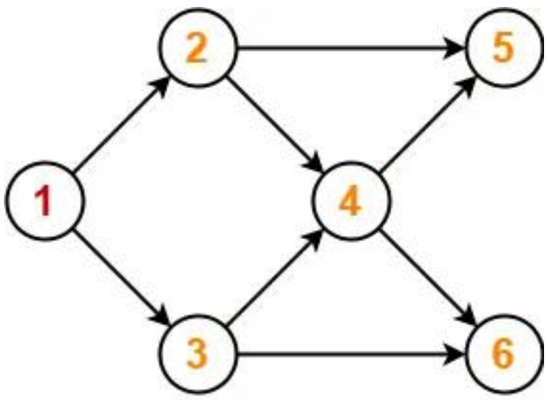
Scheduling Final Exams: How can the final exams at a university be scheduled so that no student has two exams at the same time?

Frequency Assignments

Topological Sorting

Topological Sorting

- ❑ Topological Sorting is possible if and only if the graph is a Directed Acyclic Graph (DAG)
- ❑ Topological Sort is a linear ordering of the vertices in such a way that if there is an edge in the DAG going from vertex ‘u’ to vertex ‘v’, then ‘u’ comes before ‘v’ in the ordering.
- ❑ There may exist multiple different topological orderings for a given directed acyclic graph



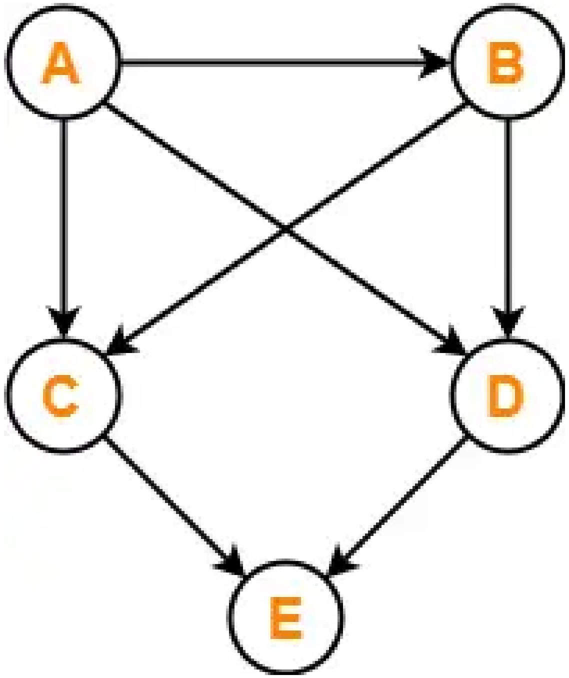
Topological Sort Example

For this graph, following 4 different topological orderings are possible-

- **1 2 3 4 5 6**
- **1 2 3 4 6 5**
- **1 3 2 4 5 6**
- **1 3 2 4 6 5**

Topological Sorting

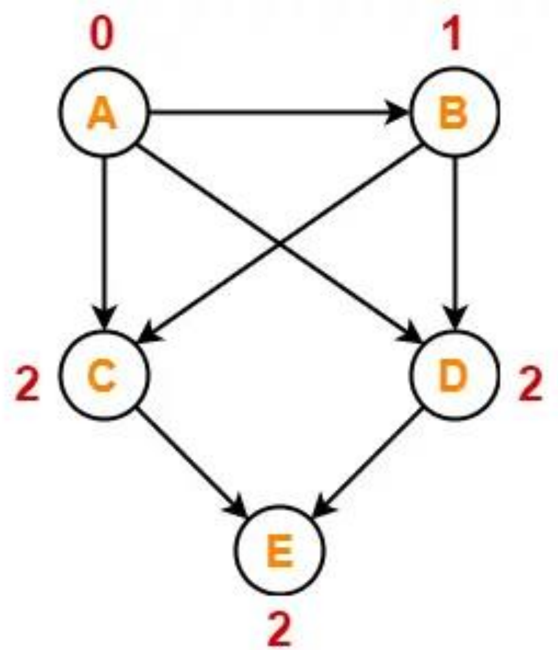
Find the number of different topological orderings possible for the given graph-



Topological Sorting

Step-01:

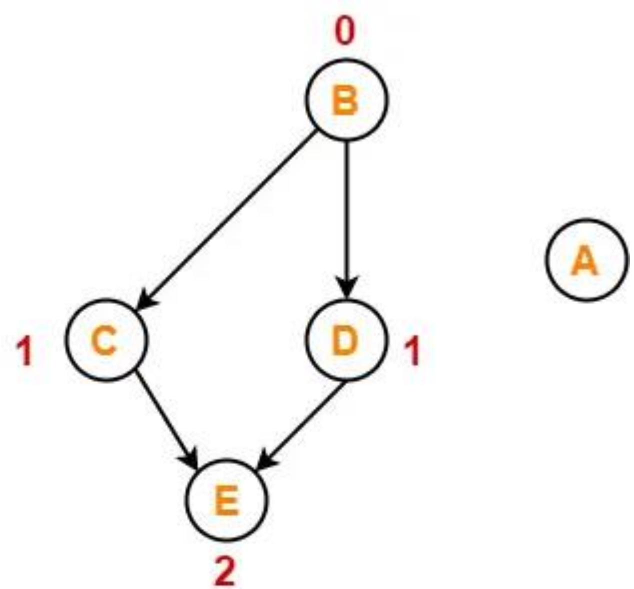
Write in-degree of each vertex-



Topological Sorting

Step-02:

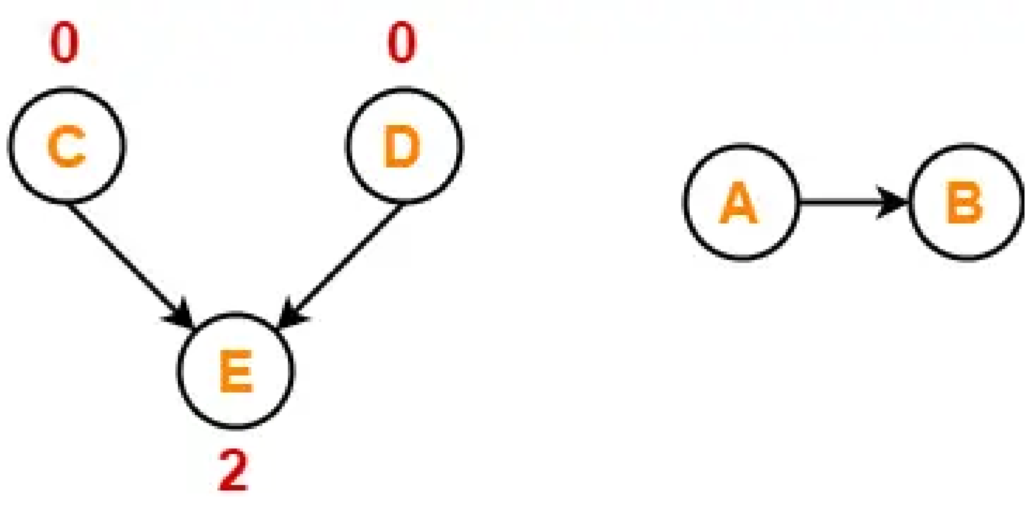
- Vertex-A has the least in-degree.
- So, remove vertex-A and its associated edges.
- Now, update the in-degree of other vertices.



Topological Sorting

Step-03:

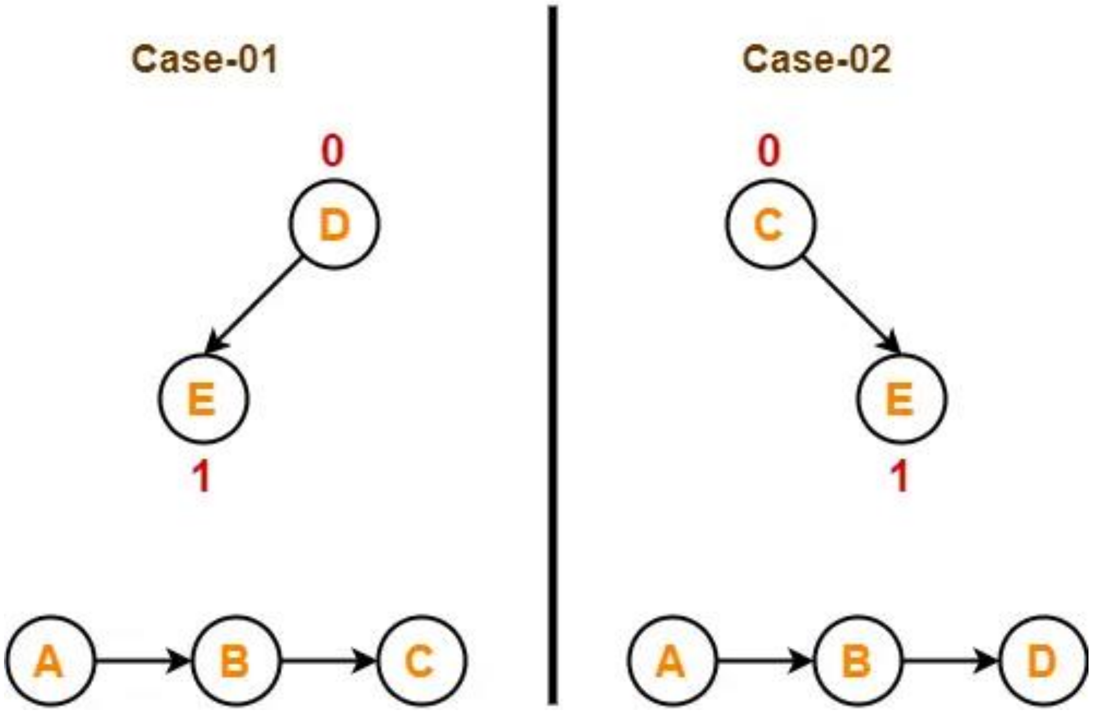
- Vertex-B has the least in-degree.
- So, remove vertex-B and its associated edges.
- Now, update the in-degree of other vertices.



Topological Sorting

Step-04:

There are two vertices with the least in-degree. So, following 2 cases are possible-



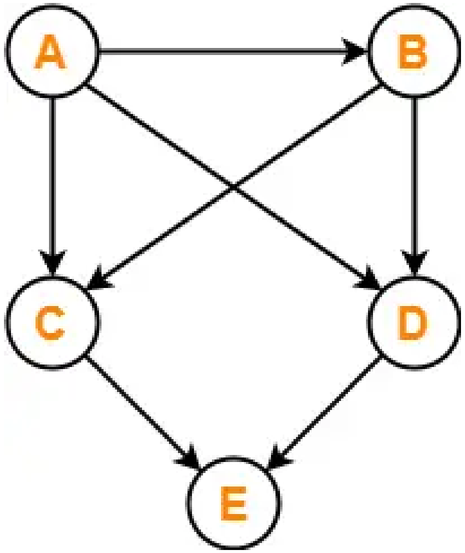
Topological Sorting

Step-05:

Now, the above two cases are continued separately in the similar manner.

Topological Sorting

Find the number of different topological orderings possible for the given graph-

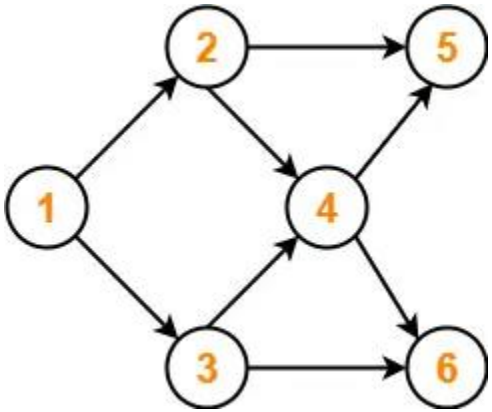


For the given graph, following 2 different topological orderings are possible-

- A B C D E
- A B D C E

Topological Sorting

Find the number of different topological orderings possible for the given graph-



For the given graph, following 4 different topological orderings are possible-

- 1 2 3 4 5 6
- 1 2 3 4 6 5
- 1 3 2 4 5 6
- 1 3 2 4 6 5


Thank You!