

# Package ‘SDNNtests’

October 8, 2020

**Title** SHAPE CONSTRAINED TESTS OF STOCHASTIC DOMINANCE

**Version** 0.0.0.9000

**Description** This package implements the methods discussed in Laha et al. (2020). In particular, this package performs nonparametric and shape-constrained (unimodality and log-concavity) test of one sided stochastic dominance against the null of non-dominance. This package also gives an estimator of the Hellinger distance between two densities under the shape restriction of unimodality or log-concavity.

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**Encoding** UTF-8

**LazyData** true

**Imports** logcondens,  
pracma

**RoxygenNote** 7.1.0

## R topics documented:

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calc_mode	<i>Unimodal density estimator when the mode is unknown</i>
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## Description

Estimates the density of a given sample under the assumption that the underlying density is unimodal. The mode is estimated from the data. The method is based on the unimodal regularization of Birge (1997).

## Usage

```
calc_mode(x, t)
```

## Arguments

- |   |   |
|---|---|
| x | Vector of independent and identically distributed random variables; must be sorted. |
| t | A positive real number. Default value is one.                                       |

## Details

Birge(1997)'s estimator gives a a piecewise constant unimodal density. The discontinuity points of the respective density are called the knots, which belong to the set of datapoints. The density estimator is constant between two consecutive knots.

t: The parameter  $t$  corresponds to the parameter  $\tau$  in Birge (1997). Higher values of  $t$  leads to more accurate estimation of the unimodal densities. This value ontrols the accuracy in unimodal density estimation upto the term  $n^{-t}$  where  $n$  is the sample size. We recommend a value greater than or equal to one. See Birge (1997) for more details.

## Value

- mode - The estimator of the mode
- x.knots - The vector of the knots of the estimated density.
- F.knots - A vector consisting the values of the estimated distribution function evaluated at the knots.
- f.knots - A vector whose i-th element gives the value of the estimated density on the segment joining the i-th and (i+1)-th knot. Recall that the estimated density is piecewise constant between two knots.

## Author(s)

**Nilanjana Laha** (maintainer), <nlaha@hsph.harvard.edu>, Alex Luedtke, <aluedtke@uw.edu>.

## References

Birge, L. (1997). *Estimation of unimodal densities without smoothness assumptions*, Ann. Statist., 25, 970–981.

## Examples

```
x <- sort(rnorm(100)); calc_mode(x, 1)
```

---

hd.lc

*Hellinger distance between log-concave densities*


---

## Description

Provides an estimate of the hellinger distance between two log-concave densities. This function uses the log-concave density estimator of Dumbgen and Rufibatch (2009), given by [logConDens](#) of logcondens package.

## Usage

```
hd.lc(x, y)
```

**Arguments**

- x** Vector of  $m$  independent and identically distributed random variables; corresponds to the first sample.
- y** Vector of  $n$  independent and identically distributed random variables; corresponds to the second sample.

**Value**

A point estimator of the Hellinger distance.

**Author(s)**

**Nilanjana Laha** (maintainer), <nlaha@hsph.harvard.edu>,  
 Alex Luedtke, <aluedtke@uw.edu>.

**References**

- Laha, N., Moodie, Z., Huang, Y., and Luedtke (2020), A. *Improved inference for vaccine-induced immune responses via shape-constrained methods*. Submitted.
- Dumbgen, L. and Rufibach, K. (2009). *Maximum likelihood estimation of a logconcave density and its distribution function: Basic properties and uniform consistency*, Bernoulli, 15, 40–68.

**See Also**

[hd.uni](#), [hd.lc.sm](#)

**Examples**

```
x <- sort(rnorm(100)); y <- sort(rgamma(50, shape=1));
hd.lc(x,y)
```

---

hd.lc.sm

*Hellinger distance between log-concave densities*


---

**Description**

Provides an estimate of the Hellinger distance between two log-concave densities. This function uses the smoothed log-concave density estimator of Dumbgen and Rufibach (2009), given by [log-ConDens](#) of logcondens package.

**Usage**

```
hd.lc.sm(x, y)
```

**Arguments**

- x** Vector of  $m$  independent and identically distributed random variables; corresponds to the first sample.
- y** Vector of  $n$  independent and identically distributed random variables; corresponds to the second sample.

**Value**

A point estimator of the Hellinger distance.

**Author(s)**

**Nilanjana Laha** (maintainer), <nlaha@hsph.harvard.edu>,  
Alex Luedtke, <aluedtke@uw.edu>.

**References**

Laha, N., Moodie, Z., Huang, Y., and Luedtke (2020), A. *Improved inference for vaccine-induced immune responses via shape-constrained methods*. Submitted.

Dumbgen, L. and Rufibach, K. (2009). *Maximum likelihood estimation of a logconcave density and its distribution function: Basic properties and uniform consistency*, Bernoulli, 15, 40–68.

**See Also**

[hd.lc](#), [hd.uni](#)

**Examples**

```
x <- sort(rnorm(100)); y <- sort(rgamma(50, shape=1));
hd.lc.sm(x,y)
```

---

hd.uni

*Hellinger distance between unimodal densities*


---

**Description**

Provides an estimate and a confidence interval for the hellinger distance between two unimodal densities. This function uses the density estimator of Birges (1997), given by [calc\\_mode](#). See Laha et al. (2020) for more details.

**Usage**

```
hd.uni(x, y, alpha)
```

**Arguments**

x	Vector of m independent and identically distributed random variables; corresponds to the first sample.
y	Vector of n independent and identically distributed random variables; corresponds to the second sample.
alpha	A number between 0 and 0.50. The function returns an (1-alpha)% confidence interval.

**Details**

This function calls `link{calc_mode}` where the parameter `t` is taken to be one.

**Value**

A vector of three elements. The first element is the point estimate of the Hellinger distance. The second and the third elements give the left and right endpoints of the confidence interval.

**Author(s)**

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**References**

Laha, N., Moodie, Z., Huang, Y., and Luedtke, A. (2020). *Improved inference for vaccine-induced immune responses via shape-constrained methods*. Submitted.  
Birge, L. (1997). *Estimation of unimodal densities without smoothness assumptions*, Ann. Statist., 25, 970–981.

**See Also**

[calc\\_mode](#), [hd.lc](#), [hd.lc.sm](#)

**Examples**

```
x <- sort(rnorm(100)); y <- sort(rgamma(50, shape=1));  
hd.uni(x, y, 0.05)
```

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SDNN	<i>One-sided test of stochastic dominance against the null of non-dominance</i>
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**Description**

Calculates the p-values of one-sided tests of restricted stochastic dominance against the null of non-dominance. The concerned tests are the minimum t-statistic test and the two sample empirical process (TSEP) test of *Laha et al. (2020)*. Each test can be either nonparametric, or semiparametric, ie. using unimodality or log-concavity assumption on the underlying densities.

**Usage**

```
SDNN(x, y, Method, t, p)
```

**Arguments**

x	Vector of m independent and identically distributed random variables; corresponds to the first sample.
y	Vector of n independent and identically distributed random variables; corresponds to the second sample.
Method	Must be one among "NP" (nonparametric), "UM" (unimodal), and "LC" (log-concave). See 'Details'.
t	A positive real number. Only required when Method="UM", default value is 1. See Details.
p	The proportion of combined data to be trimmed from each end prior to testing, should take value in [0, 0.50), the default is set to 0.05.

## Details

Suppose  $X_1, \dots, X_m$  and  $Y_1, \dots, Y_n$  are independent random variables with distribution  $F$  and  $G$ , respectively. Denote by  $D_{p,m,n}$  the set

$$[H_n^{-1}(p), H_n^{-1}(1-p)]$$

where  $p \in [0, 0.5)$  and  $H_n$  is the empirical distribution function of the combined sample

$$\{X_1, \dots, x_m, Y_1, \dots, Y_n\}.$$

The function SDNN tests  $H_0 : F(x) \geq G(x)$  for some  $x \in D_{p,m,n}$  vs  $H_1 : F(x) < G(x)$  for all  $x \in D_{p,m,n}$ . For more details, see Laha et al., 2020.

Method: "NP" corresponds to the nonparametric tests. "UM" corresponds to tests which use the function `calc_mode` to estimate the densities of  $X_i$ 's and  $Y_i$ 's. This function estimates the unimodal density estimator of Birge (1997). "LC" corresponds to tests which use the log-concave MLE (given by `logConDens` of R package "logcondens") of Dumbgen and Rufibach (2009) to estimate the latter densities. For more detail, see Laha et al. (2020).

t: The parameter  $t$  corresponds to the parameter  $\tau$  in Birge (1997). Higher values of  $t$  leads to more accurate estimation of the unimodal densities. This value controls the accuracy in unimodal density estimation upto the term  $(m+n)^{-t}$ . A value greater than or equal to one is recommended. See Birge (1997) for more details.

p: p corresponds to the set  $D_{p,m,n}$  in  $H_0$  and  $H_1$ . To overcome the difficulties arising from the tail region,  $100p$  percent of data is trimmed from both sides of the combined sample.

## Value

A list of two numbers.

- T1 - The p-value of the test based on minimum T-statistic of Laha et al., (2020)
- T2 - The p-value of the TSEP test of Laha et al., (2020).

## Author(s)

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## References

- Laha, N., Moodie, Z., Huang, Y., and Luedtke, A. (2020). *Improved inference for vaccine-induced immune responses via shape-constrained methods*. Submitted.
- Dumbgen, L. and Rufibach, K. (2009). *Maximum likelihood estimation of a logconcave density and its distribution function: Basic properties and uniform consistency*, Bernoulli, 15, 40–68.
- Birge, L. (1997). *Estimation of unimodal densities without smoothness assumptions*, Ann. Statist., 25, 970–981.

## See Also

`calc_mode`, `logConDens`

## Examples

```
x <- rnorm(100); y <- rgamma(50, shape=1);
SDNN(x, y, Method="UM", t=1, p=0.01)
```

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