Time : 15 minutes

1. Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in $\mathbb{R}^N$ and $\{x_{n_k}\}_{k\in\mathbb{N}}$ a subsequence. x_n converges to $x \in \mathbb{R}$. Show that x_{n_k} converges to x.

2. Say $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in $\mathbb{R}^N$ s.t. $\lim\limits_{n\to\infty} x_n^2 = 0$. Show $\lim\limits_{n\to\infty} x_n = 0$.

① Let $\varepsilon > 0$ given. $\exists\ N \in \mathbb{N}$ s.t. $|x_n - x| < \varepsilon$ $\forall n \geq N$.

$\therefore\ \forall\ k \geq N$ we have

$$n_k \geq k \geq N$$

$$\Rightarrow \quad |x_{n_k} - x| < \varepsilon \qquad \square$$

② Let $\varepsilon > 0$ given. $\exists\ N \in \mathbb{N}$ s.t. $|x_n^2 - 0| < \varepsilon^2$ $\forall\ n \geq N$

$[\because \lim x_n^2 = 0]$

$\Rightarrow \quad |x_n|^2 < \varepsilon^2$

$\Rightarrow \quad |x_n - 0| = |x_n| < \varepsilon$

$\Rightarrow \quad \lim x_n = 0$.