System of Linear Equation :-

$$a_{11}x_1 + \cdots + a_{1n}x_n = c_1$$

$$\vdots$$

$$a_{m1}x_1 + \cdots + a_{mn}x_n = c_m$$

$2(x+y) = 2$

$2x + 3y = 4$

$$Ax = C \qquad A = (a_{ij})$$

$$C = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

i) Multiplicⁿ by Scalars

ii) Adding c. (eqⁿ) to another (eqⁿ)

iii) Interchanging the eqⁿ among itself.

$R_i \to cR_i \to R_i$

* Rows operations are reversible $\quad A \rightleftarrows B$

$$E_1 = \operatorname{diag}(1, 1, \ldots, \underset{\substack{\downarrow \\ i}}{c}, 1, \ldots, 1)$$

$$E_1 A = R_i \longrightarrow cR_i$$

$$E_{ij} \quad \begin{array}{c|c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

$$E_{ij} A =$$

$$A \dashrightarrow A'$$

$$\begin{bmatrix} 1 & 0 & 5 & 7 & 0 & 3 \\ 0 & 1 & -1 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 4 & 8 & 0 & -1 \\ 0 & 1 & -1 & 1 & 0 & -4 \\ 0 & -2 & 2 & -2 & 1 & 14 \\ 0 & 3 & -3 & 3 & 0 & -12 \end{bmatrix}$$

$$M \longrightarrow E_1 M \longrightarrow E_2 E_1 M \longrightarrow \cdots \longrightarrow E_k E_{k-1} \cdots E_1 M$$

$= M'$ (row echelon)

HW

2) Suppose that the row echelon form of M are M' and M''. Then prove that $M' = M''$

3) Prove that if

$$M \longrightarrow M' \qquad m \times n \qquad 1 \times n \in \mathbb{R}^n$$

obtained by elementary row operation, then

$$\text{row rank}(M) = \text{row rank}(M')$$

$R_i \longrightarrow cR_i$

$R_i \longrightarrow R_i + cR_i$

$R_i \longleftrightarrow R_j$

k

$L \quad \{R_1, R_2, \dots, R_k\}$

$m-k$

$\{R_1, R_2, \dots, R_m\} \rightarrow k$

$\{R_2, R_1, \dots, R_m\}$

$\{cR_1, R_2, \dots, R_m\}$

L'

3) $\begin{cases} Ax = B \\ A'x = B' \end{cases}$

$A'x_0 = B'$

$$(A \mid B) = M$$
$$\downarrow$$
$$(A' \mid B') = M'$$

$M' = E_k E_{k-1} \cdots E_2 E_1 \cdot M$

$= PM$

$(A' \mid B') = P(A \mid B) = (PA \mid PB)$

$A' = PA, \quad B' = PB$

$Ax_0 = B$

$Ax = B$ $\qquad (A|B) \longrightarrow (A'|B')$

$A'x = B'$

$A \to m \times n$ $\qquad (A'|B')$

$0\ 0\ 0\ \cdots\ 0\ 1\ c$

A' $\qquad (A'|B') \to k$

r denote the # of non-zero rows of A' and $(A'|B')$

2) Prove that if $r = n$

$AX = B$ has only one solution

3) $n > r$ then there are infinitely sol.

$\mathbb{R}^m$, $\mathcal{B}$ $\quad L = \{v_1, \ldots, v_n\}$

$$A = \left([v_1]_{\mathcal{B}} \quad [v_2]_{\mathcal{B}} \quad \cdots \quad [v_n]_{\mathcal{B}} \right) \in M_{m \times n}(\mathbb{R})$$

$Ax = 0$ — L is linearly dependent if it has a nontrivial solution

$Bx = 0$ $\qquad A \xrightarrow[R.R]{} B$ $\qquad B = EA$

$c_1, c_2, \ldots, c_k, c_{k+1}, \ldots, c_n$

$A \longrightarrow k$

$B \longrightarrow k$ $\qquad c_1', c_2', \ldots, c_k', c_{k+1}', \ldots, c_n'$

$$Bx = 0 \qquad \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_k \\ 0 \\ \vdots \\ 0 \end{bmatrix} = x_0 \qquad \lambda_1 c_1' + \cdots + \lambda_k c_k' = 0$$

$$\begin{bmatrix} C_1 & C_2 & \cdots & C_l \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_l \end{bmatrix} = x_1 C_1 + x_2 C_2 + \cdots + x_l C_l$$

$$\lambda_1 C_1' + \cdots + \lambda_k C_k' + 0 + \cdots + 0 = 0$$

$$\lambda_1 C_1 + \cdots + \lambda_k C_k = 0$$

$A \longrightarrow A'$

1) The non zero rows of A' are L.I.

$$A' = \begin{bmatrix} \cdots R_1 \cdots \\ \cdots R_2 \cdots \\ \vdots \\ \cdots R_k \cdots \\ \cdots 0 \cdots \\ \cdots 0 \cdots \\ \vdots \\ \cdots 0 \cdots \end{bmatrix}$$

$$c_1 R_1 + \cdots + c_k R_k = 0$$

$$\longrightarrow \begin{bmatrix} c_1 R_1 \\ R_2 \\ \vdots \\ R_k \\ 0 \\ \vdots \end{bmatrix} \longrightarrow \begin{bmatrix} c_1 R_1 + c_2 R_2 \\ R_2 \\ R_3 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ R_2 \\ R_3 \\ \vdots \\ R_k \\ 0 \\ \vdots \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} R_2 \\ R_3 \\ \vdots \\ R_k \\ 0 \\ \vdots \\ 0 \end{bmatrix} = A''$$

$$\begin{array}{ccc} & A & \\ \swarrow & & \searrow \\ A' & = & A'' \end{array}$$

2)

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