1) Determine primes p such that the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -2 & 0 & 2 \end{bmatrix}$$

is invertible in $M_3(\mathbb{F}_p)$

Soln → A is invertible iff $\det A \neq 0$

$AB = I_n$

$\det(AB) = 1 \Rightarrow \det(A) \cdot \det(B) = 1$

$A^{-1} = \frac{1}{\det A}\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \end{bmatrix}$

---

2) Let F be a finite field, $|F| = q$

W is a k-dimension F-v.s.

Prove that the number of basis for W is

$$(q^k - 1)(q^k - q) \cdots (q^k - q^{k-1})$$

$W \cong F^k$

$|W| = |F^k| = q^k$

Soln → $\{v_1, v_2, \ldots, v_k\}$

$\{\underset{q^k-1}{\_}, \underset{q^k-q}{\_}, \underset{q^k-q^2}{\_}, \ldots, \underset{q^k-q^{k-1}}{\_}\}$ $\{v, \omega, \_$

$\boxed{\lambda v + \mu \omega}$

$\{v, \quad , \ldots \}$

$\left| \{\lambda v + \mu \omega\}_{\substack{\lambda \in F \\ \mu \in F}} \right| = |F|^2 = q^2$

$\boxed{\{\lambda v\}_{\lambda \in F}}$

$(q^k - 1)(q^k - q) \cdots (q^k - q^{k-1})$

3) V be n dimensional F-vector space, $|F| = q$

Prove that the number of subspaces of V of dimension k $1 \leq k \leq n$ is

$$\frac{(q^n - 1)(q^n - q) \cdots (q^n - q^{k-1})}{(q^k - 1)(q^k - q) \cdots (q^k - q^{k-1})}$$

4) V be n dimensional F-vector space, $|F| = q$

$GL(V)$ denotes the set of $V \longrightarrow V$ isomorphism

$$|GL(V)| = (q^n - 1)(q^n - q) \cdots (q^n - q^{n-1})$$