Image Compression

Jupyter notebook

[1]:
```
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#Python 3
```

[2]:
```
import numpy as np
from PIL import Image
import matplotlib.pyplot as plt
```

[3]:
```
image = Image.open('conway.jpg')
A = np.array(image, dtype=float) / 255.0
A = np.dot(A, [0.2989, 0.5870, 0.1140])
m,n = A.shape
```

[4]:
```
U, S, Vt = np.linalg.svd(A) #do the SVD
K = [40,80,120,160]         #storing values of k
l = len(K)
S = np.diag(S)              #original S is only a linear array, need to␣
 ↪convert to matrix
```

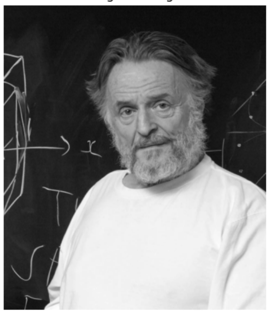
[5]:
```
approx = []
for i in range(l):
    approx.append(U[:, :K[i]] @ S[:K[i], :K[i]] @ Vt[:K[i], :])   #taking␣
 ↪approximations
```

[6]:
```
#obtaining and printing a table for Frobeius norm of differences
error = [0 for _ in range(l)]
print("k\t|\tFrobenius norm")
print("--------+-------------------------")
for i in range(len(K)):
    error[i] = np.linalg.norm(A-approx[i])
    print(str(K[i])+"\t|\t"+str(error[i]))
```

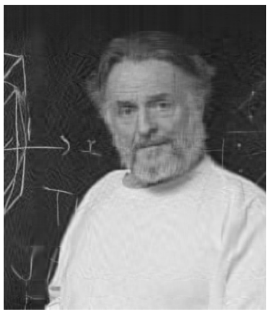
```
k       |       Frobenius norm
--------+-------------------------
40      |       31.733288485394482
80      |       19.016427727768185
120     |       13.055854194249326
160     |       9.511015173570117
```

[7]:
```
fig, ax = plt.subplots(len(K)+1, 1, figsize = (10, 30))

ax[0].set_title("Original Image")
ax[0].imshow(A, cmap = 'gray')
ax[0].axis('off')

for i in range(l):
    ax[i+1].imshow(approx[i], cmap = 'gray')
    ax[i+1].set_title("k = " + str(K[i]))
    ax[i+1].axis('off')

plt.show()
```

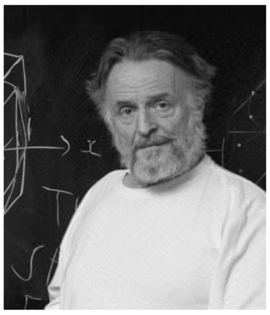
Original Image



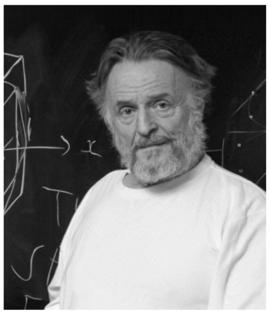
k = 40



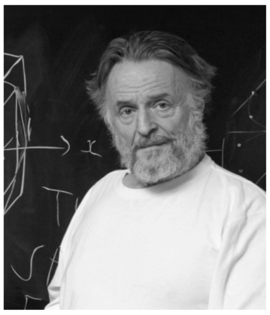
k = 80



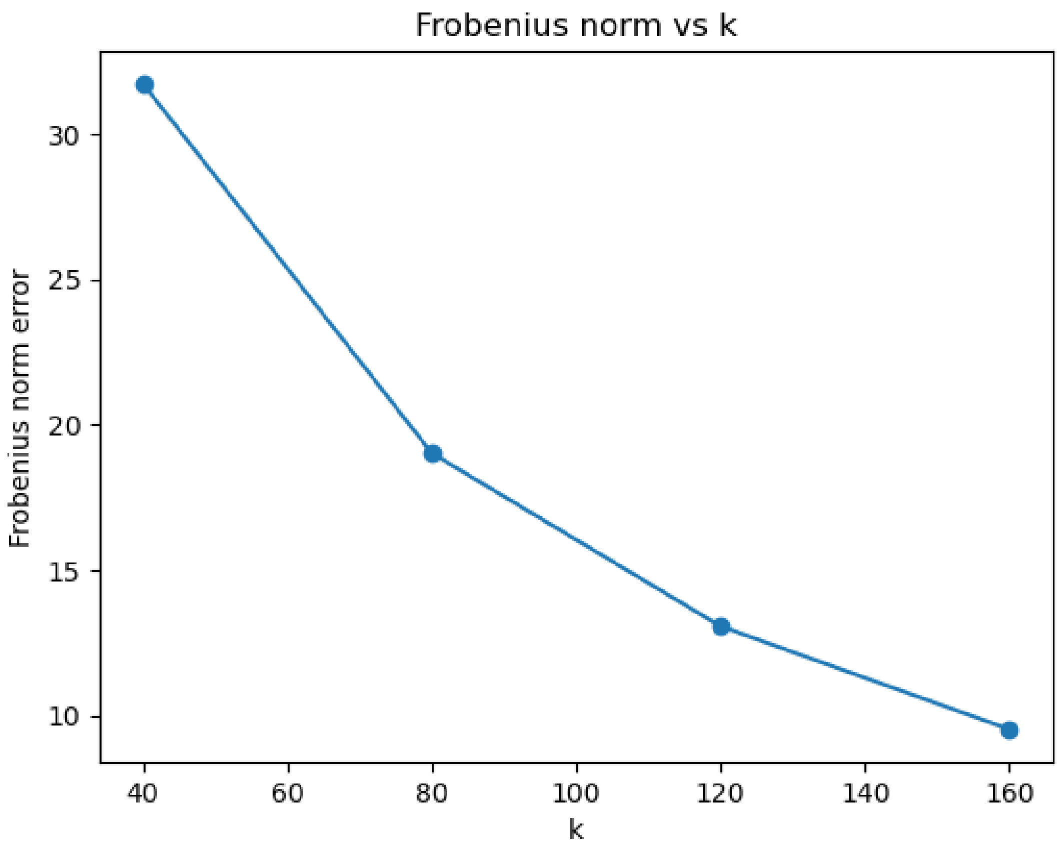
k = 120



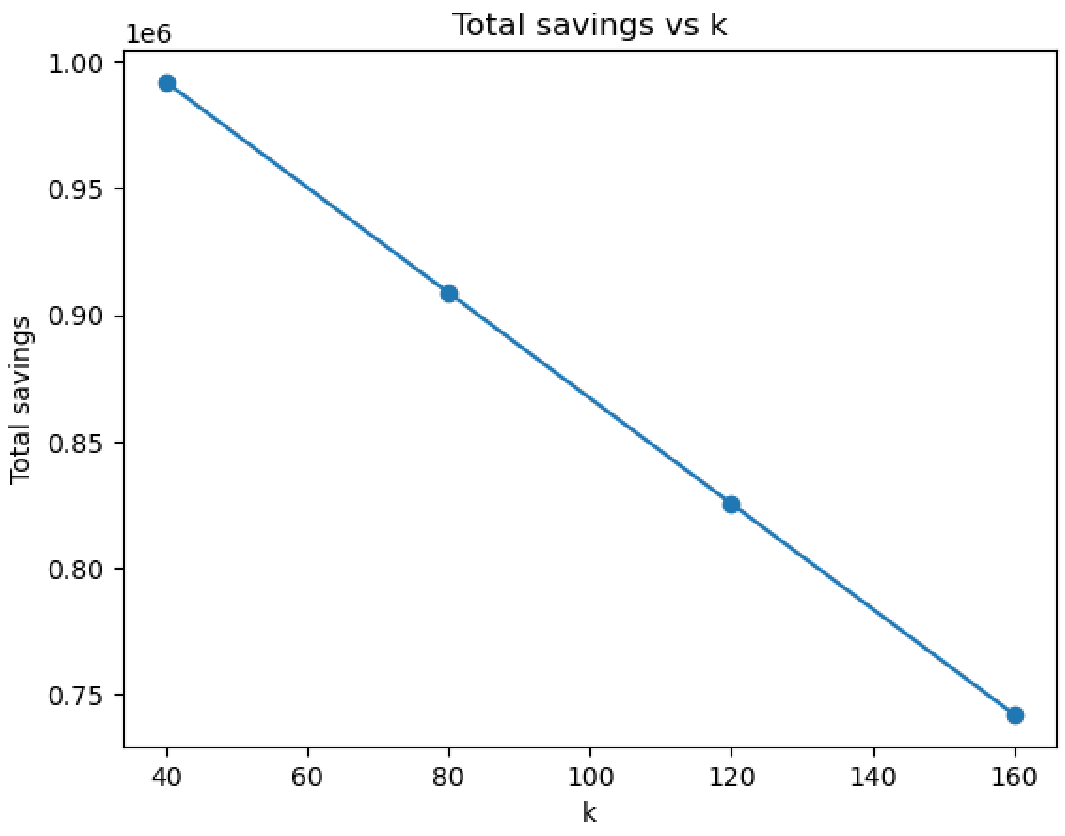
k = 160



[8]:
```
plt.plot(K, error, marker = 'o')
plt.xlabel('k')
plt.ylabel('Frobenius norm error')
plt.title("Frobenius norm vs k")
plt.show()
```



[9]:
```
plt.plot(K, [m*n-(m+n+1)*k for k in K], marker = 'o')
plt.xlabel('k')
plt.ylabel('Total savings')
plt.title("Total savings vs k")
plt.show()
```


Total savings vs k
1e6
Total savings
k