## Algebraic Structures :-

$\mathbb{Z}, \mathbb{Q} \cong \mathbb{R}[x]$, $\{f : [a,b] \to \mathbb{R}\}$

$f : [a,b] \to \mathbb{R}$

$g : [a,b] \to \mathbb{R}$

$(f+g) : [a,b] \to \mathbb{R}$

$(f+g)(x) = f(x) + g(x)$

Sets,

Groups, Rings / Fields, Module / Vector Space, Algebra

$(\mathbb{Z}, +)$ $(\mathbb{R}[x], +)$ $(\mathbb{Z}/n\mathbb{Z}, +)$ $V \cong F^n$

$(\mathbb{Q}^*, \cdot)$

$B \to X$

A

C

$X \cong Y$

## Linear Algebra :-

**Groups :-** $(G, +)$, $+ : G \times G \to G$ s.t

i) $+$ is associative, i.e.

$$(a+b)+c = a+(b+c) = a+b+c$$

ii) $\exists\, 0 \in G$ s.t $\forall\, g \in G$

$$g+0 = 0+g = g$$

iii) For every $g \in G$, $\exists\, h \in G$ s.t

$$g+h = h+g = 0$$

Since this $h$ is unique we define $h$ to be the "inverse of $g$", $(-g)$.

$a+b = b+a \quad \forall\, a,b \in G$

Abelian Group

$a+b = a+c \Rightarrow b = c$

$$b = 0+b = ((-a)+a)+b = (-a)+(a+b) = (-a)+(a+c) = ((-a)+a)+c = 0+c = c$$

b) Rings :- $(R, +, \cdot)$ such that

i) $(R, +)$ is a Abelian Group

$\cdot : R \times R \longrightarrow R$ s.t

ii $\cdot$ is associative, i.e.

$$(a \cdot b) \cdot c = a \cdot (b \cdot c) = abc$$

iii) $\exists \ 1 \in R$ s.t $a \cdot 1 = 1 \cdot a = a \quad \forall a \in R$

iv) $a(b+c) = ab + ac$

$(F, +, \cdot)$ be a commutative ring such that $1 \neq 0$ and

$(F \setminus \{0\}, \cdot)$ is a group

F is a field.