Problem Set 1
Linear Algebra

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This problem set contains some fairly easy problems and perhaps some not so easy problems.

1. Consider $\mathbb{R}^2$ as a $\mathbb{R}$ vector space. Are $(2,1)$ and $(0,1)$ Linear combination of the vectors $(1,1)$ and $(1,2)$? More generally which vectors are linear combination of $(1,1)$ and $(1,2)$?

2. In which of the following cases, $\mathbb{M}$ is a subspace of the given vector space?

(a) Consider $\mathbb{C}^3$ as a $\mathbb{C}$ vector space. Consider the **subsets** $\mathbb{M}$ consisted of vectors (α, β, γ) such that the following holds one by one

- $\alpha + \beta = 1$
- $\alpha + \beta$ is real and ≥ 0
- $\alpha \in \mathbb{R}$

(b) Consider the complex vector space $\mathbb{C}[x]$ and then consider the subsets $\mathbb{M}$ such that the following holds one by one

- $p(x) \in \mathbb{M}$ has degree 3.
- $p(x) \in \mathbb{M}$ such that $2p(0) = p(1)$.
- $p(x) \in \mathbb{M}$ such that $p(t) \geq 0$ whenever $0 \leq t \leq 1$.
- $p(x) \in \mathbb{M}$ such that $p(t) = p(1-t), \ \forall t$

3. Let V be a F- vector space. Is union of two subspaces is always a subspace of V? If yes give proof and if not give counter-example. Also if your answer is NO then under what condition is the union of two subspaces is a subspace? (There might be more than one condition but you can consider only one of them). How should you modify the condition (if possible) in case of any finite union/countable union/arbitrary union.

Answer the same question for intersection of two subspaces.

4.

(a) Consider $\mathbb{R}^3$ as a $\mathbb{R}$-vector space. Does $(1,4,9)$ belong to the span of the set $\{(1,1,1),(0,1,1),(0,0,1)\}$. Can you pose your solution using the language of matrix? This formulation is particularly important because when you will be asked the same question but with the set of larger size, it will help.

(b) Given a F- vector space V and a subset $E \subseteq V$, define $< E >$ to be the span of E. Then prove the following;

- $E \subseteq < E >$.
- Let $E \subseteq F$. Then $< E > \subseteq < F >$
- $<< E >> = < E >$