Real Analysis

Activity 1

July 11, 2021

1. Let $S \subseteq \mathbb{R}$. What is meant by a limit point of S? We shall denote the set of limit points of S by S'.

2. What is S' for the following sets S? No proof required.

   (a) $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \left\{\frac{-1}{n} : n \in \mathbb{N}\right\}$.

   (b) $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \left\{\frac{-1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$.

   (c) $S = [0, 1]$

   (d) $S = [0, 1)$

   (e) $S = (0, 1)$

   (f) $S = [0, 1] \setminus \left\{\frac{k}{10}\right\}_{k=1}^{9}$

3. Question will be given depending on how you define limit point in problem 1.

4. For a set $S \subseteq \mathbb{R}$ what is an open cover of S? Given an open cover of S, what is a finite subcover?

5. Consider $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$. Let $\varepsilon > 0$ be given. Find finitely many closed intervals $I_1 = [a_1, b_1], I_2 = [a_2, b_2], \cdots, I_k = [a_3, b_3]$ such that $S \subseteq \bigcup_{i=1}^{k} I_i, b_i > a_i \forall i$ and $\sum_{i=1}^{k} b_i - a_i \leq \varepsilon$. (Here k is not fixed and can depend on ε, but it must a be a natural number).