6DL 2/16 Lecture 8 Group Actions and representations Def: X set, G group, Gacts on X if $q \times g \times or g \times or a(g, x)$ Such that 1) 1.x=x2) $g_1g_2x=\alpha(g_1,\alpha(g_2,x))$ = a(g, ogz, X) Write 6 C X: "Gacts on X", called a group action Ex: Dy C'R? Dy = < r, f | f2=1, r4=1, rf=fr3> $r.(\overset{\times}{y}) = (\overset{\times}{y}) \Rightarrow \text{define } r \text{ action}$ $f.(\overset{\times}{y}) = (\overset{\times}{y}) \Rightarrow \text{define } f \text{ action}$ $C_{5}(x) = L(L(x)) = L(x) = (-x)$ $r^{3}(x) = r(r^{2}(x)) = r(-x)$ Make sure to check that all relations are satisfied r=1 so r4x=1:x=x r''(x) = r(r''(x)) = r(-x) = (x)

Ex 2: Dy (Dy Group acts on itself what does a: 6x6 > 6 mean in this context?

Needs to satisfy oxioms of group action.

1) log=g and 2) (9,092)093 = 9,092093)

Turns out o satisfies this:

(9,092) ogz=9,0(92093) from Associativity of o

Ex 3: Dy P 213

Trivially g. 1=g so group action axioms are satisfied.

Ex 4: Sy (2 {1,2,3,43

a(o, K) = o(K), oesy

Ex 5: Sy C Sy

 $\alpha(g,h) = g \cdot h = g \cdot h g^{-1}$

Does this satisfy our group axioms?

Proof for Ex 5:

1) a(1,h)=h · 1.h= 1.h.1= h/

2) g. g2·h=(g.og2)()=g.(g2(h))

g,(g2(h))= g,(g2h,g2) = g, g2h,g2'g,-1

= (g,0g2)(h)(g2'0g,-1) (g,0g2)-1

 $= (g_1 \circ g_2)(h)(g_1 \circ g_2)^{-1} \longrightarrow g_2^{-1} \circ g_1^{-1} \circ g_2^{-1} \circ g_2^{-1} = 1$

So (9,092) = (92 09,-1) by uniqueness

{matrices}

gM >> gMg-1

Ex 7: 6 (R[6] = {f: 6 > R}

9.f = fog-1 h, g & G, f & R [G]

(g.f)(h)=f(g-1h) | Hw 3 check this is well-defined

EX8: GL, OR"

Def: If 6 PX then the orbit of xex is

 $O_{x} = Orbit_{g}(x) = G \times = \{g : x | g \in G\} \subseteq X$

Def: The stabilizer of X

 $Stab_G(x) = G_x = \{g \in G \mid gx = x\} \subseteq G$ subscript vs. $G \times above$

Prove Stab(X) is a subgroup for Hw 3

Thm: If 6 is finite then

|G| = | stab(x) | Orbit(x) |

Intuitively, a group element must either.

1) gx=x sogestab(x)

2) gx=Y so Ye orbit(x)

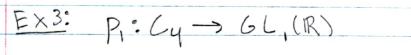
In class exercise: Find orbits and stabilizers
1) SO(2) OR2 a= roto(x)
2) SO(3) Q S= { X = R3 11×11=1}
3) 6L2 (C) (M2(C), g.M H) g Mg-1
11/ N 11 - 102 +1 M (-1 10) (-R ²
$(V,g)\cdot W = gw + V$
$(v, g) \cdot \omega - g\omega + v$
5) Sz (Sz by conjugation + Optional Hw)
23 24 Confidence
$\sigma \tau = \sigma \tau \sigma^{-1}$
Opt. Hw: Prove stabilizers of pairs in the same
orbit are conjugate.
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Det: A representation of 6 is a linear
Det: A representation of 6 is a linear group action on a vector space V.
VHgV is linear 4 geG
Since gacts linearly, it is represented by a matrix P(g).
From group action axioms:
Trom group action axioms: 1) $p(1) V = V \forall \forall \forall \forall \forall \forall \forall \forall \forall $
and $p(g) p(g^{-1}) = Iv$
and $p(g) p(g^{-1}) = Iv$ So $p(g)$ is an invertible matrix and $p(g^{-1}) = p(g)^{-1}$
Think of p : $p: G \rightarrow GL(V)$ Think of $p: G \rightarrow GL(V)$
$P(1) = I_V, P(g_1, g_2) = P(g_1) P(g_2)$
This means pis a homomorphism.
Linear Group Action +> p: 6 -> GL(V) G on V homomorphism there are identical

$$EX$$
 $C_{4} = \{1, r, r^{2}, r^{3} | r^{4} = 1\}$
 $C_{4} \cap \mathbb{R}^{4}$ Linear so

$$V \rightarrow D(I) = \begin{pmatrix} 0001 \\ 1000 \\ 0100 \end{pmatrix}$$

$$r \rightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, P_2(r)^4 = I_2$$



$$r \rightarrow (x)$$
, $x \neq c$

Hw: what are I $r \rightarrow (x)$, $x \neq 0$ dim reps of C_n, D_n ?