C4 action on Matrices

```
In[89]:= r = {{0, -1}, {1, 0}}
         M = {{a, b}, {c, d}}

Out[89]= {{0, -1}, {1, 0}}

Out[90]= {{a, b}, {c, d}}

In[92]:= r.M.Inverse[r]

Out[92]= {{d, -c}, {-b, a}}

In[93]:= r.r.M.Inverse[r.r]

Out[93]= {{a, b}, {c, d}}

In[94]:= r.r.r.M.Inverse[r.r.r]

Out[94]= {{d, -c}, {-b, a}}
```

SO(2) actions on ρ1 \[TensorProduct] ρ1

```
In[95]:= rθ = RotationMatrix[θ]

Out[95]= {{Cos[θ], -Sin[θ]}, {Sin[θ], Cos[θ]}}

In[99]:= rθ.M.Transpose[rθ] // FullSimplify // MatrixForm
```

Out[99]//MatrixForm=

$$\begin{pmatrix} a\,\mathrm{Cos}[\theta]^2 - (b+c)\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] + d\,\mathrm{Sin}[\theta]^2 & b\,\mathrm{Cos}[\theta]^2 + (a-d)\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] - c\,\mathrm{Sin}[\theta]^2 \\ c\,\mathrm{Cos}[\theta]^2 + (a-d)\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] - b\,\mathrm{Sin}[\theta]^2 & d\,\mathrm{Cos}[\theta]^2 + (b+c)\,\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] + a\,\mathrm{Sin}[\theta]^2 \end{pmatrix}$$

```
In[102]:= M1 = {{a, 0}, {0, a}}
          M2 = {{0, -b}, {b, 0}}

Out[102]= {{a, 0}, {0, a}}

Out[103]= {{0, -b}, {b, 0}}
```

In[104]:= `rθ.M1.Transpose[rθ] // FullSimplify // MatrixForm`

Out[104]//MatrixForm=

$$\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$$

In[105]:= `rθ.M2.Transpose[rθ] // FullSimplify // MatrixForm`

Out[105]//MatrixForm=

$$\begin{pmatrix} 0 & -b \\ b & 0 \end{pmatrix}$$

In[112]:=
```
RR = {Cos[θ]^2 -> (Cos[2 θ] + 1) / 2, Sin[θ] Cos[θ] -> Sin[2 θ] / 2, Sin[θ]^2 -> (Cos[2 θ] - 1) / -2}
```

Out[112]= $\left\{\mathrm{Cos}[\theta]^2 \to \frac{1}{2}(1+\mathrm{Cos}[2\theta]),\ \mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta] \to \frac{1}{2}\mathrm{Sin}[2\theta],\ \mathrm{Sin}[\theta]^2 \to \frac{1}{2}(1-\mathrm{Cos}[2\theta])\right\}$

In[116]:=
```
2 FullSimplify[rθ.M.Transpose[rθ]] /. RR // Simplify // MatrixForm
```

$$\begin{pmatrix} a+d+(a-d)\,\mathrm{Cos}[2\theta]-(b+c)\,\mathrm{Sin}[2\theta] & b-c+(b+c)\,\mathrm{Cos}[2\theta]+(a-d)\,\mathrm{Sin}[2\theta] \\ -b+c+(b+c)\,\mathrm{Cos}[2\theta]+(a-d)\,\mathrm{Sin}[2\theta] & a+d+(-a+d)\,\mathrm{Cos}[2\theta]+(b+c)\,\mathrm{Sin}[2\theta] \end{pmatrix}$$

In[118]:=
```
M3 = {{1, 0}, {0, -1}}
M4 = {{0, 1}, {1, 0}}
```

Out[118]= {{1, 0}, {0, -1}}

Out[119]= {{0, 1}, {1, 0}}

In[120]:=
```
FullSimplify[rθ.(α M3 + β M4).Transpose[rθ]]
```

Out[120]= $\{\{\alpha\,\mathrm{Cos}[2\theta]-\beta\,\mathrm{Sin}[2\theta],\ \beta\,\mathrm{Cos}[2\theta]+\alpha\,\mathrm{Sin}[2\theta]\},\ \{\beta\,\mathrm{Cos}[2\theta]+\alpha\,\mathrm{Sin}[2\theta],\ -\alpha\,\mathrm{Cos}[2\theta]+\beta\,\mathrm{Sin}[2\theta]\}\}$

In[123]:=
```
(α Cos[2 θ] - β Sin[2 θ]) M3 + (β Cos[2 θ] + α Sin[2 θ]) (M4)
```

Out[123]= $\{\{\alpha\,\mathrm{Cos}[2\theta]-\beta\,\mathrm{Sin}[2\theta],\ \beta\,\mathrm{Cos}[2\theta]+\alpha\,\mathrm{Sin}[2\theta]\},\ \{\beta\,\mathrm{Cos}[2\theta]+\alpha\,\mathrm{Sin}[2\theta],\ -\alpha\,\mathrm{Cos}[2\theta]+\beta\,\mathrm{Sin}[2\theta]\}\}$

In[125]:=
```
RotationMatrix[2 θ].{{α}, {β}}
```

Out[125]= $\{\{\alpha\,\mathrm{Cos}[2\theta]-\beta\,\mathrm{Sin}[2\theta]\},\ \{\beta\,\mathrm{Cos}[2\theta]+\alpha\,\mathrm{Sin}[2\theta]\}\}$

Rotation Matrix in SO(3)

In[6]:=
```
RotationMatrix[α, {0, 0, 1}].RotationMatrix[β, {1, 0, 0}].RotationMatrix[γ, {0, 0, 1}] // Simplify // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[\alpha]\,\mathrm{Cos}[\gamma]-\mathrm{Cos}[\beta]\,\mathrm{Sin}[\alpha]\,\mathrm{Sin}[\gamma] & -\mathrm{Cos}[\beta]\,\mathrm{Cos}[\gamma]\,\mathrm{Sin}[\alpha]-\mathrm{Cos}[\alpha]\,\mathrm{Sin}[\gamma] & \mathrm{Sin}[\alpha]\,\mathrm{Sin}[\beta] \\ \mathrm{Cos}[\gamma]\,\mathrm{Sin}[\alpha]+\mathrm{Cos}[\alpha]\,\mathrm{Cos}[\beta]\,\mathrm{Sin}[\gamma] & \mathrm{Cos}[\alpha]\,\mathrm{Cos}[\beta]\,\mathrm{Cos}[\gamma]-\mathrm{Sin}[\alpha]\,\mathrm{Sin}[\gamma] & -\mathrm{Cos}[\alpha]\,\mathrm{Sin}[\beta] \\ \mathrm{Sin}[\beta]\,\mathrm{Sin}[\gamma] & \mathrm{Cos}[\gamma]\,\mathrm{Sin}[\beta] & \mathrm{Cos}[\beta] \end{pmatrix}$$