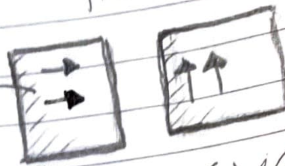
Eduardo Puerta CS7180: Geom Deep Learning

Lecture 9 continued: Steerable CNN

Last Class

How do vector fields rotate?

channels



$f(x)$ $\rho(g) f(g^{-1}x)$

for images we might have the trivial rep

example of maze, where a rotation can lead to permutting prob of moving north, south, east, west

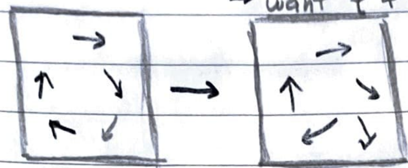
Tasks

Ex

Wind Prediction:

└ wind at time $t, \dots, t+k-1$

↳ want $t+k$



$H \subseteq O(2)$ $H = SO(2)$

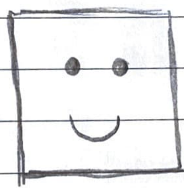
translation & rotational symmetries

$SO(2) \ltimes \mathbb{R}^2$-equivariant

ρ_1^k-field $\rightarrow \rho_1$-field

Ex

Image Classification



$\rightarrow$ {Happy, sad, ...}

$H = C_4$ we are not including reflection

ρ_0-field $\longrightarrow \mathbb{R}^2$

↳ trivial rep - pixels don't change

ρ_0-field $\rightarrow \rho_{reg}^{c_1} \rightarrow \dots \rho_0^2$-field $\rightarrow$ global pooling $\rightarrow \mathbb{R}^2$

↳ what does this mean? for model to determine

equivariant mappings between p-fields?

└ steerable convolution

Steerable Conv

Let's say — for ρ_{in}, ρ_{out} reps of $H \leq O(2)$

ρ_{in}-field $\longrightarrow$ ρ_{out}-field

$(\mathbb{R}^2, +) \rtimes H$ - equivariant linear

w/

$\rho_{in} : H \longrightarrow GL(\mathbb{R}^{c_{in}})$ $\quad V_{in} = \{ f : \mathbb{R}^2 \to \mathbb{R}^{c_{in}} \}$

$\rho_{out} : H \longrightarrow GL(\mathbb{R}^{c_{out}})$ $\quad V_{out} = \{ f : \mathbb{R}^2 \to \mathbb{R}^{c_{out}} \}$

so: $\left.\begin{array}{l} \varphi : V_{in} \longrightarrow V_{out} \\ f_{in} \longrightarrow f_{out} \end{array}\right]$ $\mathbb{R} \rtimes H$ - equivariant

V_{in}, V_{out} are fixed

Recall:

Convolution is already $\mathbb{R}^2$ (translation) equivariant

$k : \mathbb{R}^2 \longrightarrow \mathbb{R}^{c_{out} \times c_{in}}$

$$\underbrace{f_{out}(x)}_{\mathbb{R}^{c_{out}}} = \int_{y \in \mathbb{R}^2} \underbrace{k(x-y)}_{\mathbb{R}^{c_{out} \times c_{in}}} \underbrace{f_{in}(y)}_{\mathbb{R}^{c_{in}}} dy$$

how can I choose k s.t. it is H equivariant?

So we want that $\forall f_{in} \in V_{in}$, $h \in H$ s.t.

$$\boxed{h(k * f_{in}) = k * (h f_{in})}$$

LHS

$$h f_{out}(x)$$

$$= \rho_{out}(h) f_{out}(h^{-1}x)$$

$$= \rho_{out}(h) \int_z k(h^{-1}x - z) f_{in}(z) dz$$

- Let $z = h^{-1}y$
  $hz = y$

$$= \int_y \rho_{out}(h)\, k(h^{-1}x - h^{-1}y) f_{in}(h^{-1}y)\, dy$$

RHS

$$\int_y k(x-y)\rho_{in}(h) f_{in}(h^{-1}y)\, dy$$

$|\det(h)| = 1$

$O(2)$ preserves area

$\forall f_{in} \in V_{in}$

$$\implies \boxed{k(x-y)\rho_{in}(h) = \rho_{out}(h)\, k(h^{-1}x - h^{-1}y)}$$

$$k(x-y)\rho_{in}(h) = \rho_{out}(h)\, k(h^{-1}x - h^{-1}y)$$

$$\boxed{k(\omega) = \rho_{out}(h)\, k(h^{-1}\omega)\rho_{in}(h)^{-1}}$$

in terms of kernel

can also rewrite as

$$k(h\omega) = \rho_{out}(h)\, k(\omega)\rho(h)^{-1}$$

Theorem | Cohen et al.

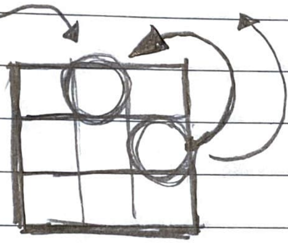
$f \longmapsto k * f$ is $\mathbb{R}^2 \rtimes H$ equivariant

iff $k(\omega) = \rho_{out}(h)\, k(h^{-1}\omega)\rho_{in}(h)^{-1}$

how do I solve this?!?

Easy way is to look in a book

$$k(h\omega) = \rho_{out}(h)\, k(\omega)\rho_{in}(h)^{-1}$$



$k(h\omega)$ is $k(\omega)$ after conjugation by some matrices

some constraint to solve for

$$\underline{k(\omega)} = \rho_{out}(h)\, \underline{k(\omega)}\rho_{in}(h)^{-1}$$

$$\boxed{k(\omega)\rho_{in}(h) = \rho_{out}(h)\, k(\omega)}$$