

Solutions to Numerical Questions

1. Watershed Algorithm: Pixel Flooding

Given a 5x5 grayscale image matrix:

[100 150 180 200 220]

[90 120 170 190 210]

[80 100 140 160 190]

[60 70 110 130 160]

[50 60 100 120 140]

Solution:

1. Local Minima (Markers):

- The local minima are the lowest points that will act as markers for flooding.
- From the matrix, the local minima are: 50, 60.

2. Applying Watershed Algorithm:

- The matrix is segmented by 'flooding' the image, separating distinct regions around the markers (50, 60).
- Resulting regions might include: [50, 60, 70] forming one region, [100, 150, 180] forming another, etc.

2. Canny Edge Detection: Gradient Calculation

Given a 3x3 image patch:

[50 50 60]

[100 200 150]

[70 120 90]

Solution:

1. Gradients using Sobel Operator:

$$- G_x = (-1 \cdot 50 + 0 \cdot 50 + 1 \cdot 60) + (-2 \cdot 100 + 0 \cdot 200 + 2 \cdot 150) + (-1 \cdot 70 + 0 \cdot 120 + 1 \cdot 90)$$

$$= (-50 + 60) + (-200 + 300) + (-70 + 90) = 30 + 100 + 20 = 150$$

$$- G_y = (-1 \cdot 50 - 2 \cdot 50 - 1 \cdot 60) + (0 \cdot 100 + 0 \cdot 200 + 0 \cdot 150) + (1 \cdot 70 + 2 \cdot 120 + 1 \cdot 90)$$

$$= (-50 - 100 - 60) + (70 + 240 + 90) = -210 + 400 = 190$$

2. Edge Strength (Magnitude):

$$- \text{Magnitude} = \sqrt{G_x^2 + G_y^2} = \sqrt{150^2 + 190^2} = \sqrt{22500 + 36100} = \sqrt{58600} = \text{approx } 242$$

3. Thresholding:

- Since the magnitude (242) is greater than the threshold (150), the central pixel (200) is classified as an edge.

3. Sobel Edge Detection: Image Gradients

Given a 3x3 image patch:

[40 50 60]

[80 90 100]

[120 130 140]

Solution:

1. Gradients using Sobel Operator:

$$- G_x = (-1 \cdot 40 + 0 \cdot 50 + 1 \cdot 60) + (-2 \cdot 80 + 0 \cdot 90 + 2 \cdot 100) + (-1 \cdot 120 + 0 \cdot 130 + 1 \cdot 140)$$

$$= (-40 + 60) + (-160 + 200) + (-120 + 140) = 20 + 40 + 20 = 80$$

$$- G_y = (-1 \cdot 40 - 2 \cdot 50 - 1 \cdot 60) + (0 \cdot 80 + 0 \cdot 90 + 0 \cdot 100) + (1 \cdot 120 + 2 \cdot 130 + 1 \cdot 140)$$

$$= (-40 - 100 - 60) + (120 + 260 + 140) = -200 + 520 = 320$$

2. Gradient Magnitude:

- Magnitude = $\sqrt{G_x^2 + G_y^2} = \sqrt{80^2 + 320^2} = \sqrt{6400 + 102400} = \sqrt{108800} =$
approx 330