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# Machine Learning (IS ZC464) Session 14: Support Vector Machines (SVM)

# Slides adapted from following internet recourses

- http://www.cs.cmu.edu/~awm/tutorials
- http://www-labs.iro.umontreal.ca/~pift6080/H09/documents/papers/svm_tutorial.ppt

# Classifiers

- Linear – perceptron model

$$w_1x_1+w_2x_2+\ldots+w_nx_n > T \text{ for class C1}$$

And $$w_1x_1+w_2x_2+\ldots+w_nx_n < T \text{ for class C2}$$

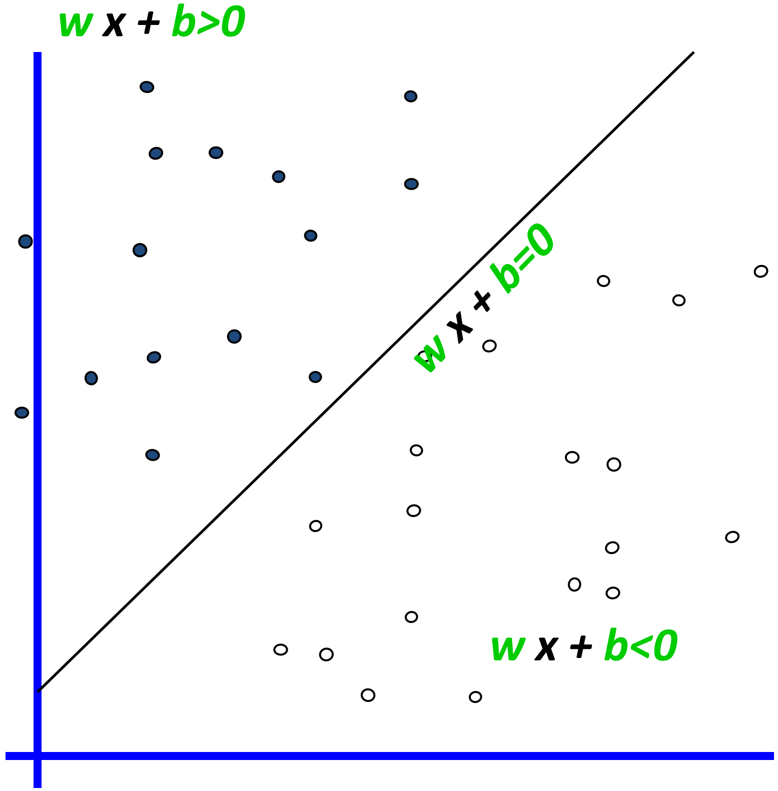
- Non linear – Multi-layer perceptron (MLP) neural network, radial basis function neural network

$$w_1\phi(x_1)+w_2\phi(x_2)+\ldots+w_n\phi(x_n) > T \text{ for class C1}$$

And $$w_1\phi(x_1)+w_2\phi(x_2)+\ldots+w_n\phi(x_n) < T \text{ for class C2}$$

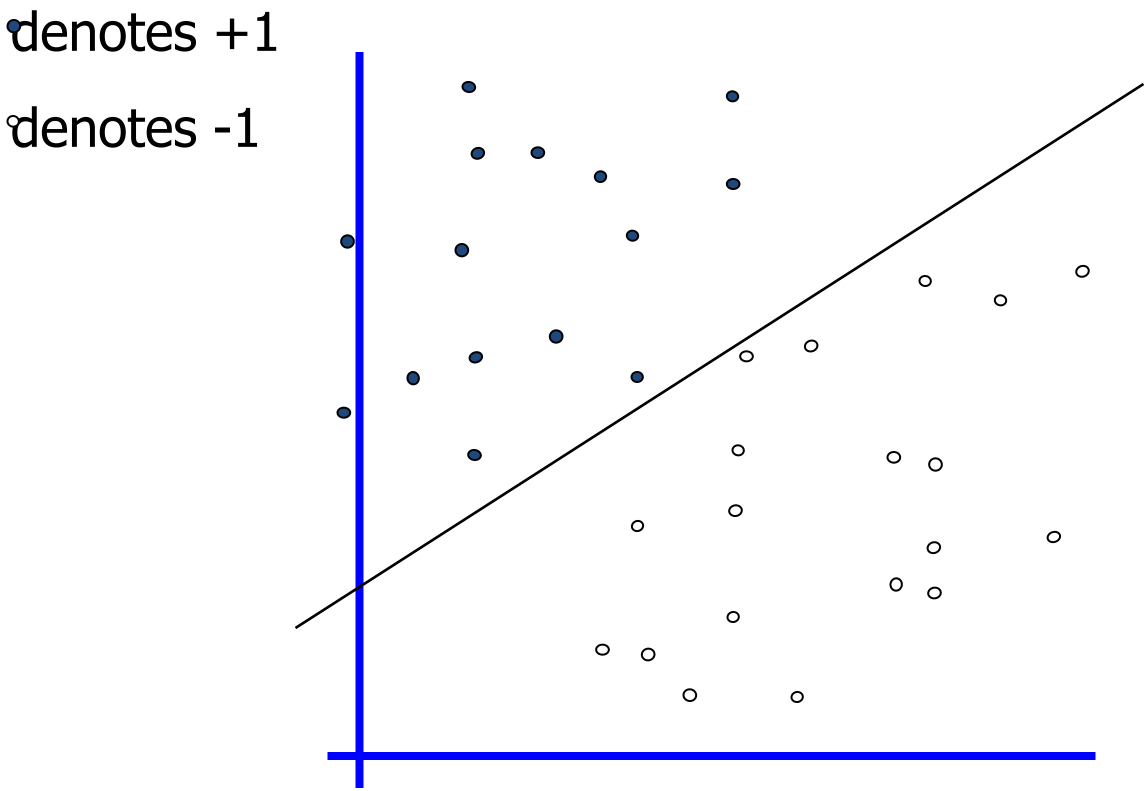
# Linear Classifiers

- denotes +1
- denotes -1




How would you classify this data?

# Linear Classifiers




How would you classify this data?

# Linear Classifiers

- denotes +1
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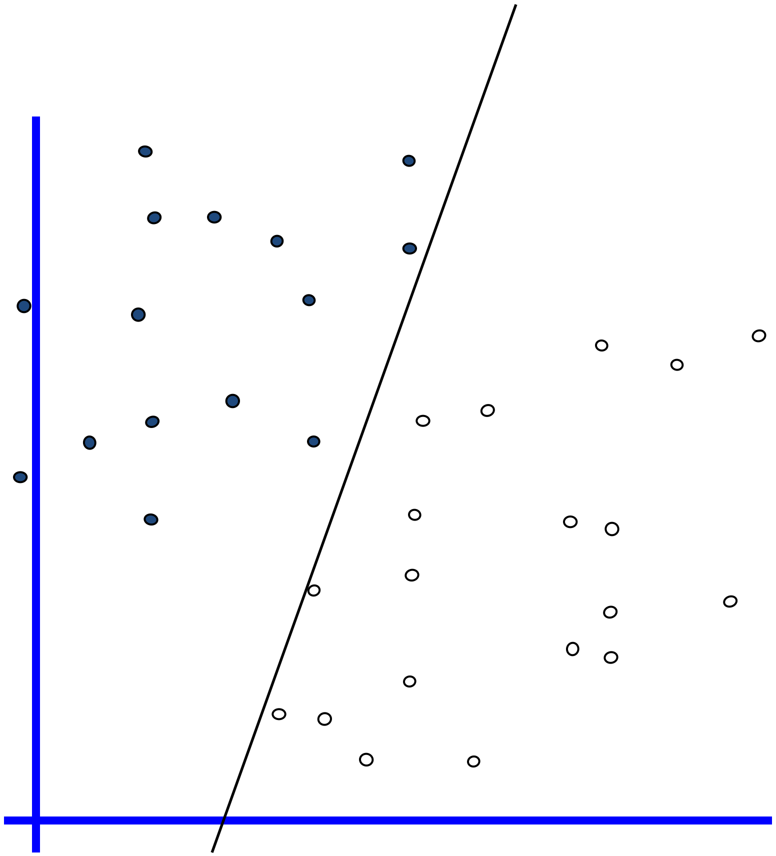
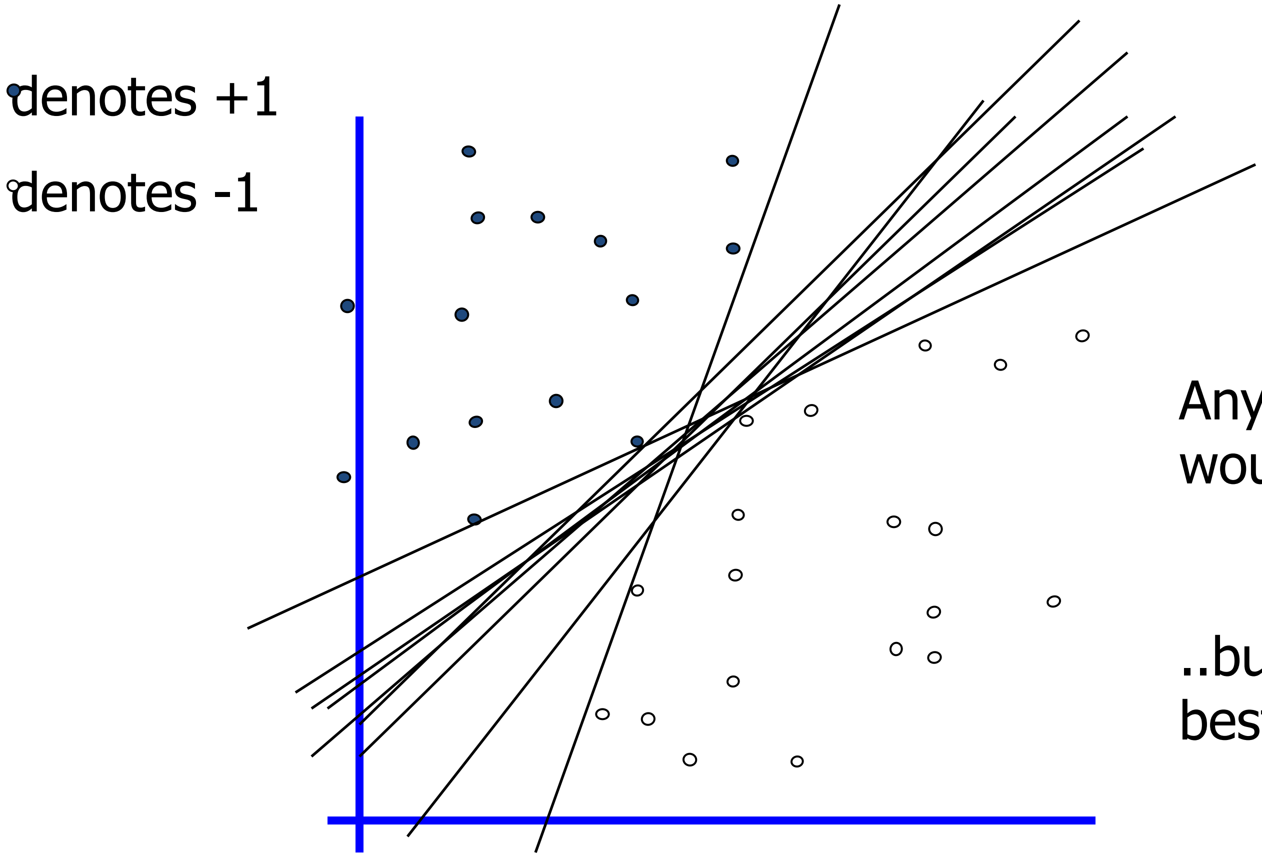


How would you classify this data?

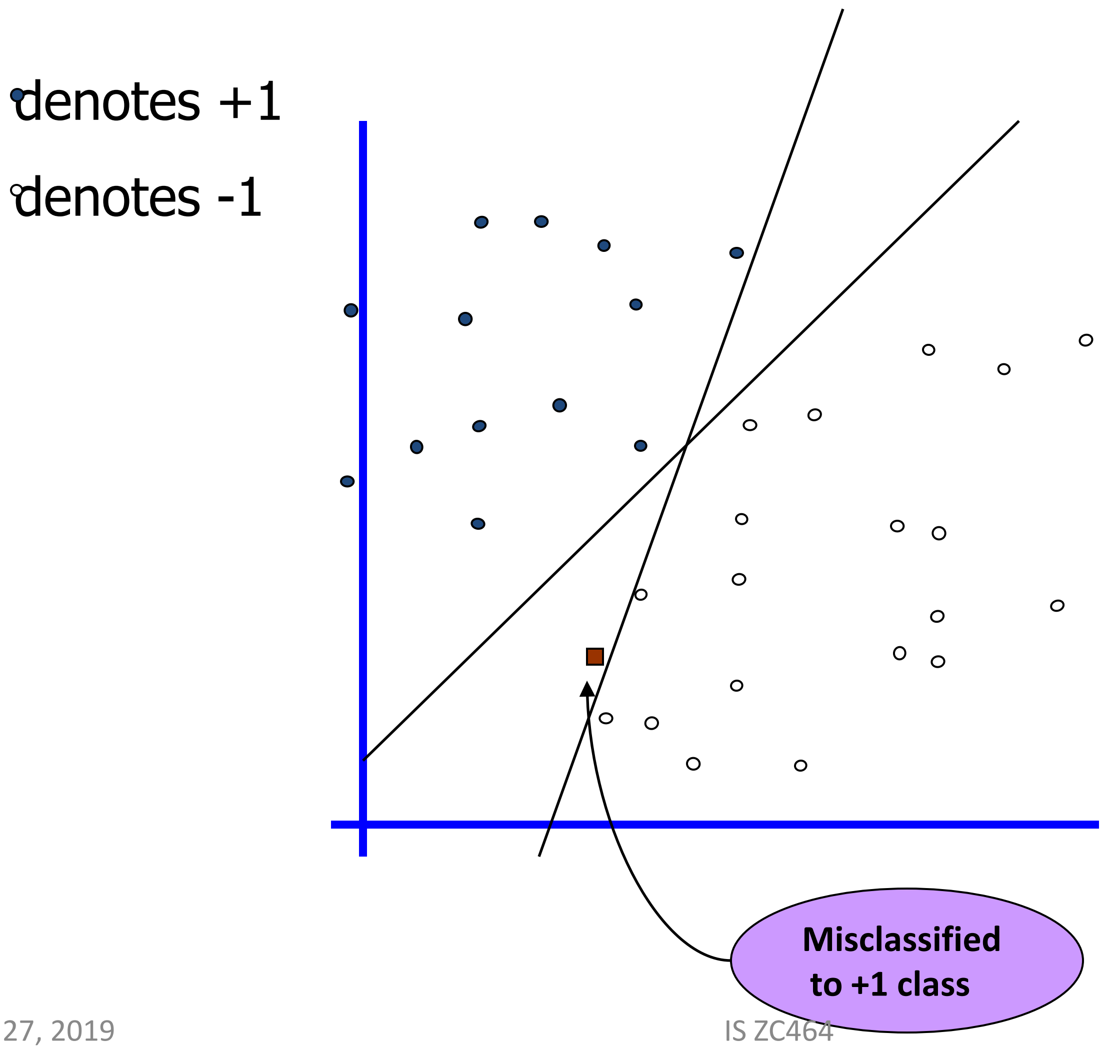
# Linear Classifiers




Any of these would be fine..

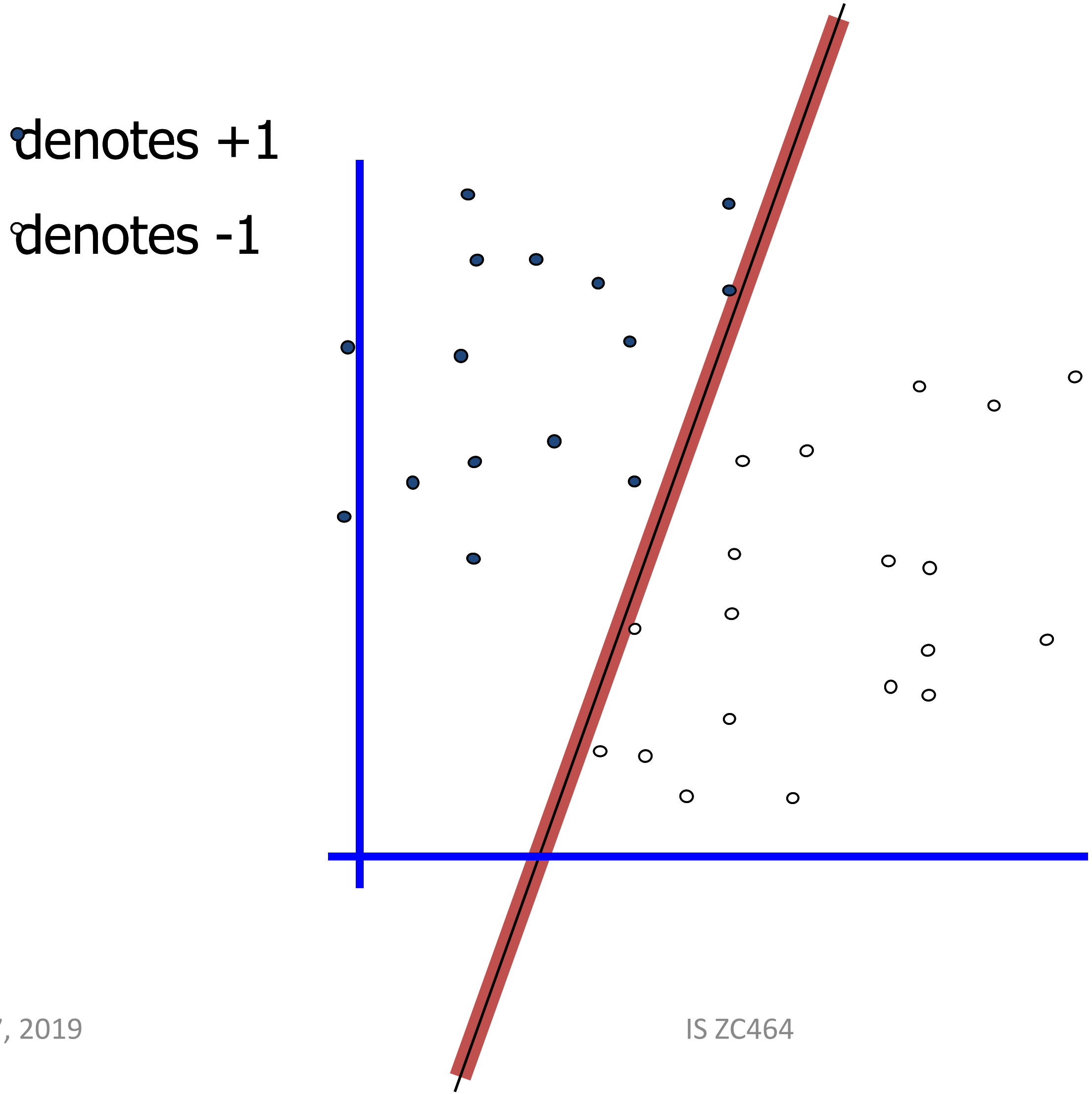
..but which is best?

# Linear Classifiers




How would you classify this data?

# Classifier Margin



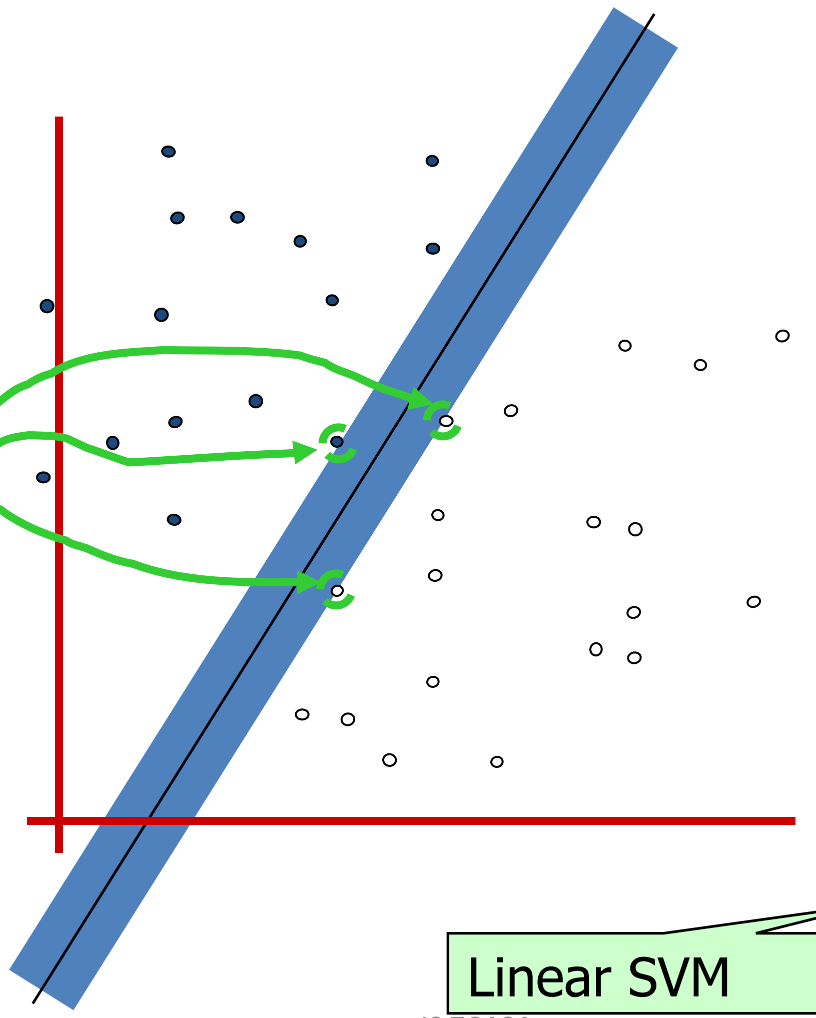

Define the margin of a linear classifier as the width that the boundary could be increased by before hitting a datapoint.

# Maximum Margin

denotes +1

denotes -1

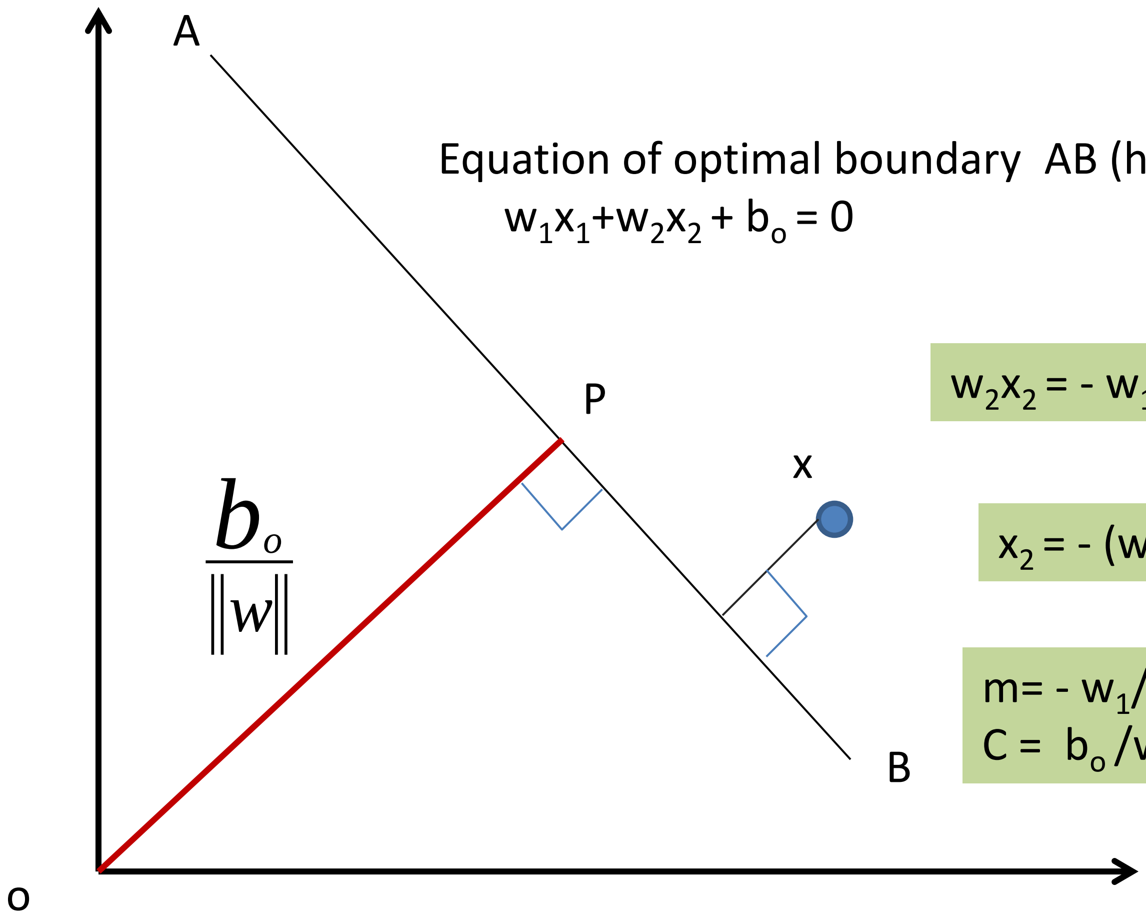
Support Vectors are those datapoints that the margin pushes up against




The maximum margin linear classifier is the linear classifier with the maximum margin.

This is the simplest kind of SVM (Called an LSVM)

# Distance of a point on the decision boundary from origin




Equation of optimal boundary AB (hyperplane /line) :

$$w_1x_1 + w_2x_2 + b_o = 0$$

$$w_2x_2 = - w_1x_1 - b_o$$

$$x_2 = - (w_1/w_2)\, x_1 - (b_o/w_2)$$

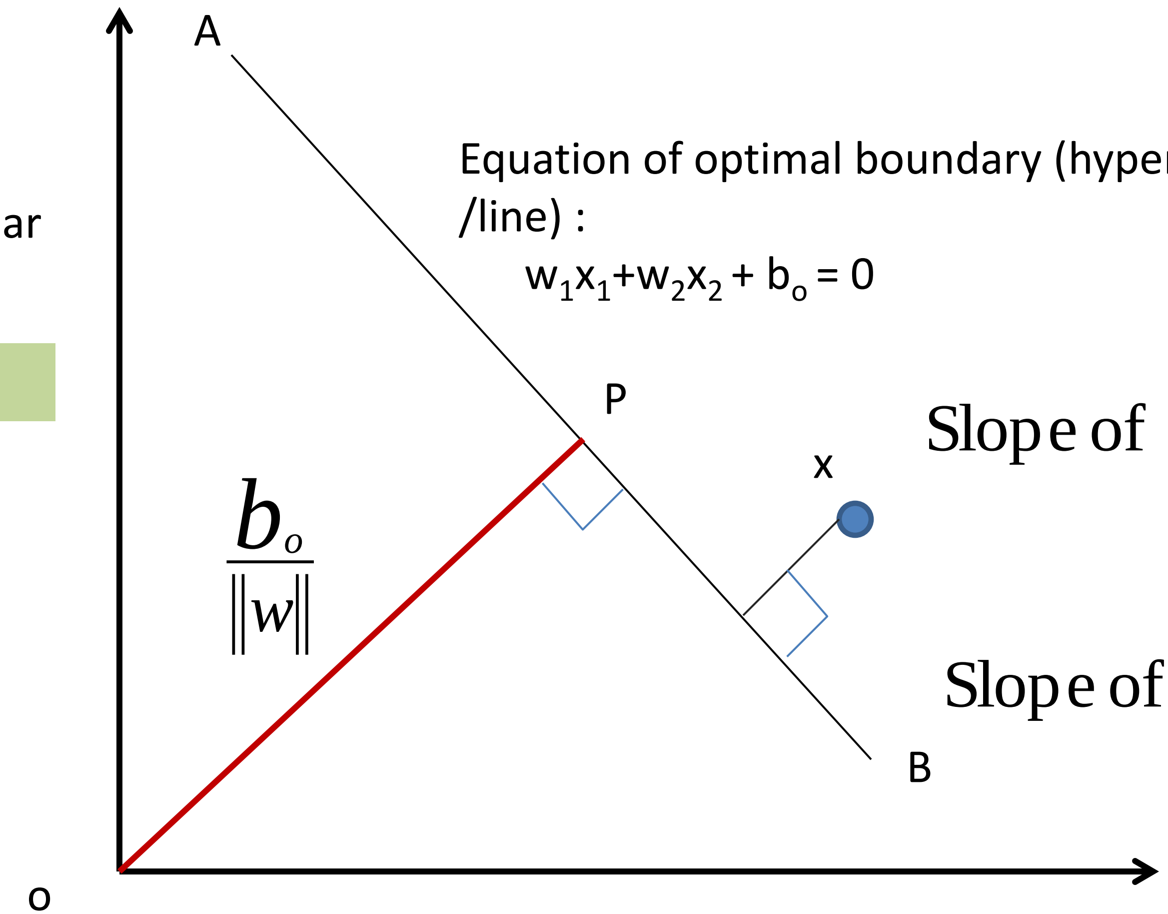
$m = - w_1/w_2$ and

$C = b_o/w_2$

# Distance of a point on the decision boundary from origin

Perpendicular lines

$m_1 m_2 = -1$



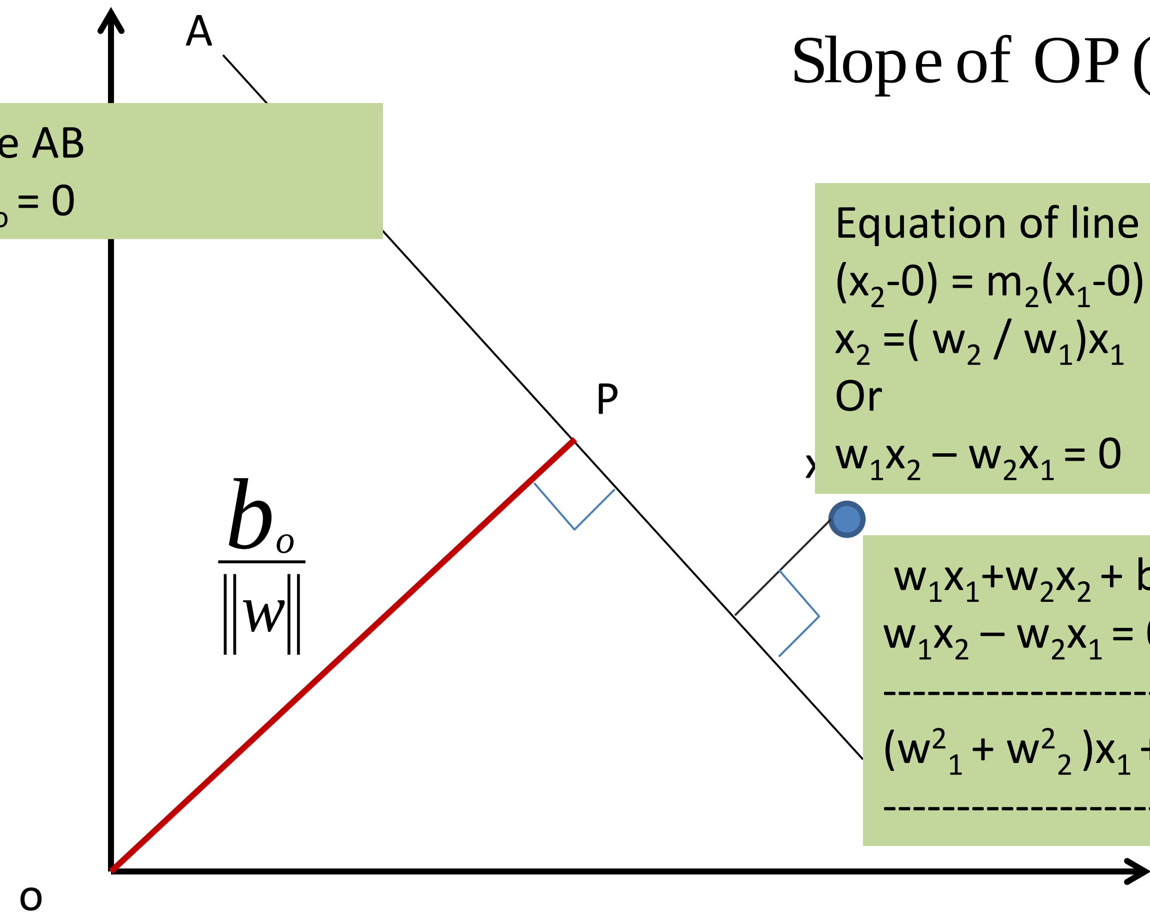

Equation of optimal boundary (hyperplane /line) :

$w_1x_1 + w_2x_2 + b_o = 0$

$$\text{Slope of AB (m1)} = -\frac{w_1}{w_2}$$

$$\text{Slope of OP (m2)} = \frac{w_2}{w_1}$$

# Distance of a point on the decision boundary from origin




$$\text{Slope of OP (m2)} = \frac{w_2}{w_1}$$

Equation of line AB
$w_1x_1+w_2x_2 + b_o = 0$

Equation of line OP
$(x_2-0) = m_2(x_1-0)$
$x_2 =( w_2 / w_1)x_1$
Or
$w_1x_2 – w_2x_1 = 0$

$w_1x_1+w_2x_2 + b_o = 0$ (multiply by $w_1$)
$w_1x_2 – w_2x_1 = 0$ (multiply by $w_2$)
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$(w^2{}_1 + w^2{}_2 )x_1 + b_o w_1 = 0$
----------------------------------------

# Distance of a point on the decision boundary from origin

Equation of line OP

$(x_2\text{-}0) = m_2(x_1\text{-}0)$

$x_2 = (w_2 / w_1)x_1$

Or

$w_1x_2 - w_2x_1 = 0$

$w_1x_1 + w_2x_2 + b_o = 0$ (multiply by $w_1$)

$w_1x_2 - w_2x_1 = 0$ (multiply by $w_2$)

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$(w^2_1 + w^2_2)x_1 + b_o w_1 = 0$

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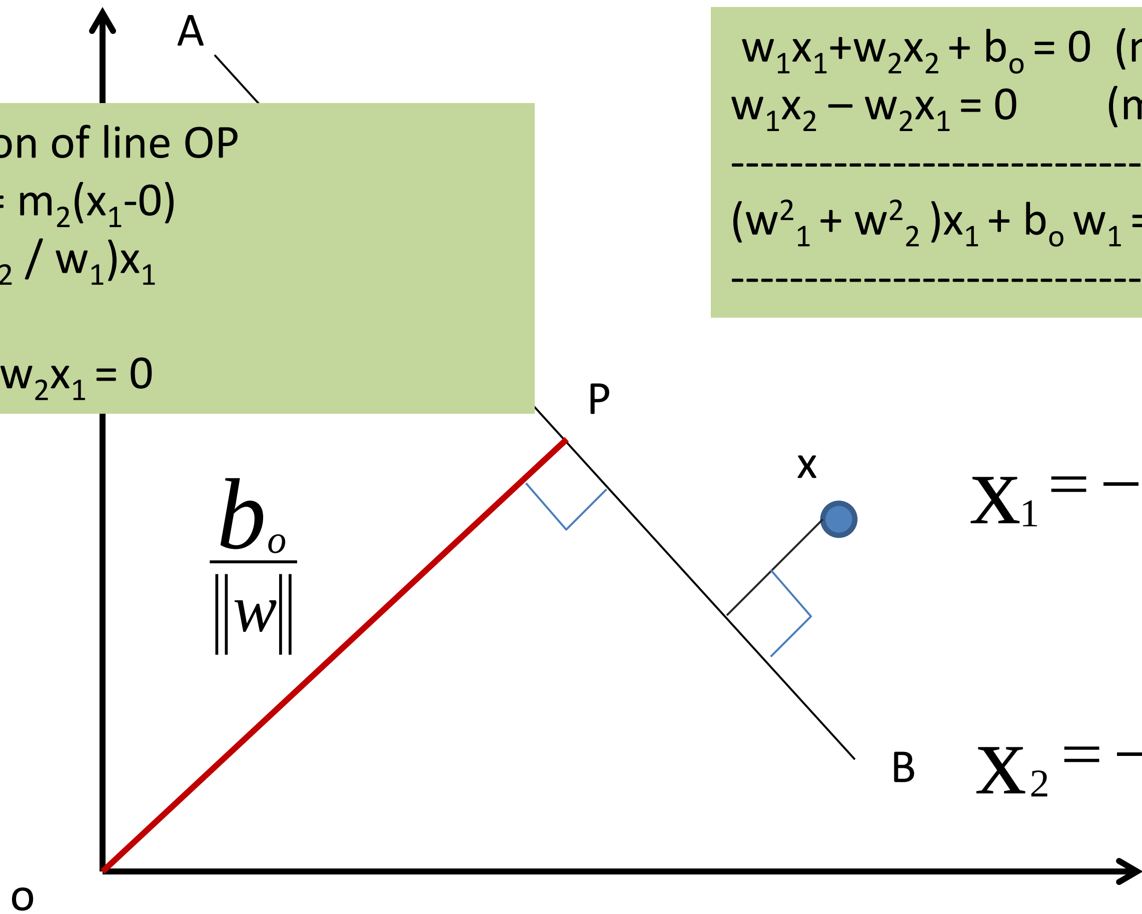


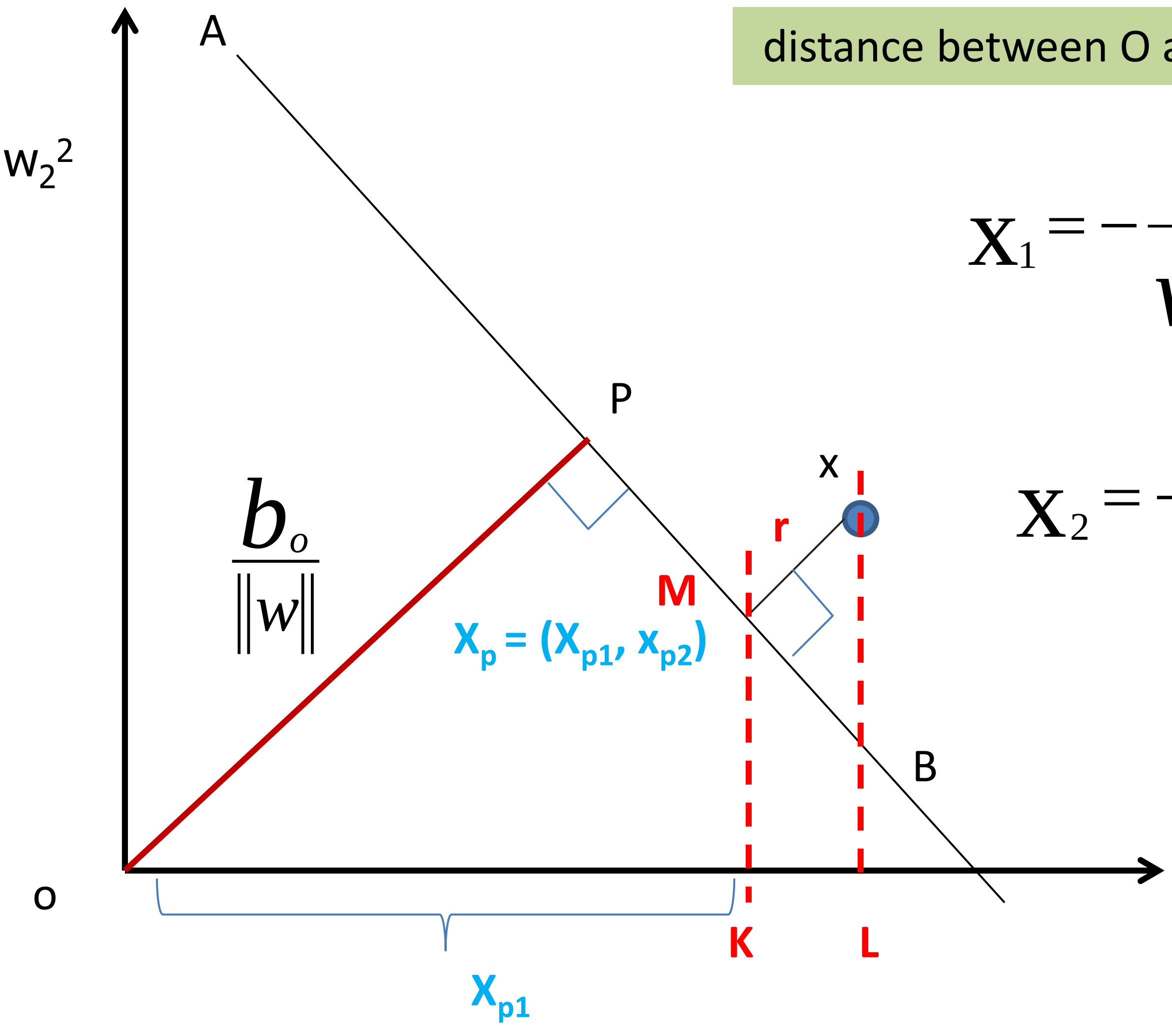

$$X_1 = -\frac{b_o w_1}{w_1^2 + w_2^2}$$

$$X_2 = -\frac{b_o w_2}{w_1^2 + w_2^2}$$

# Distance of a point on the decision boundary from origin

distance between O and P = d (say)

$\|w\|^2 = w_1^2 + w_2^2$




$$X_1 = -\frac{b_o w_1}{w_1^2 + w_2^2}$$

$$X_2 = -\frac{b_o w_2}{w_1^2 + w_2^2}$$

$$d = \frac{b_o}{\sqrt{w_1^2 + w_2^2}}$$

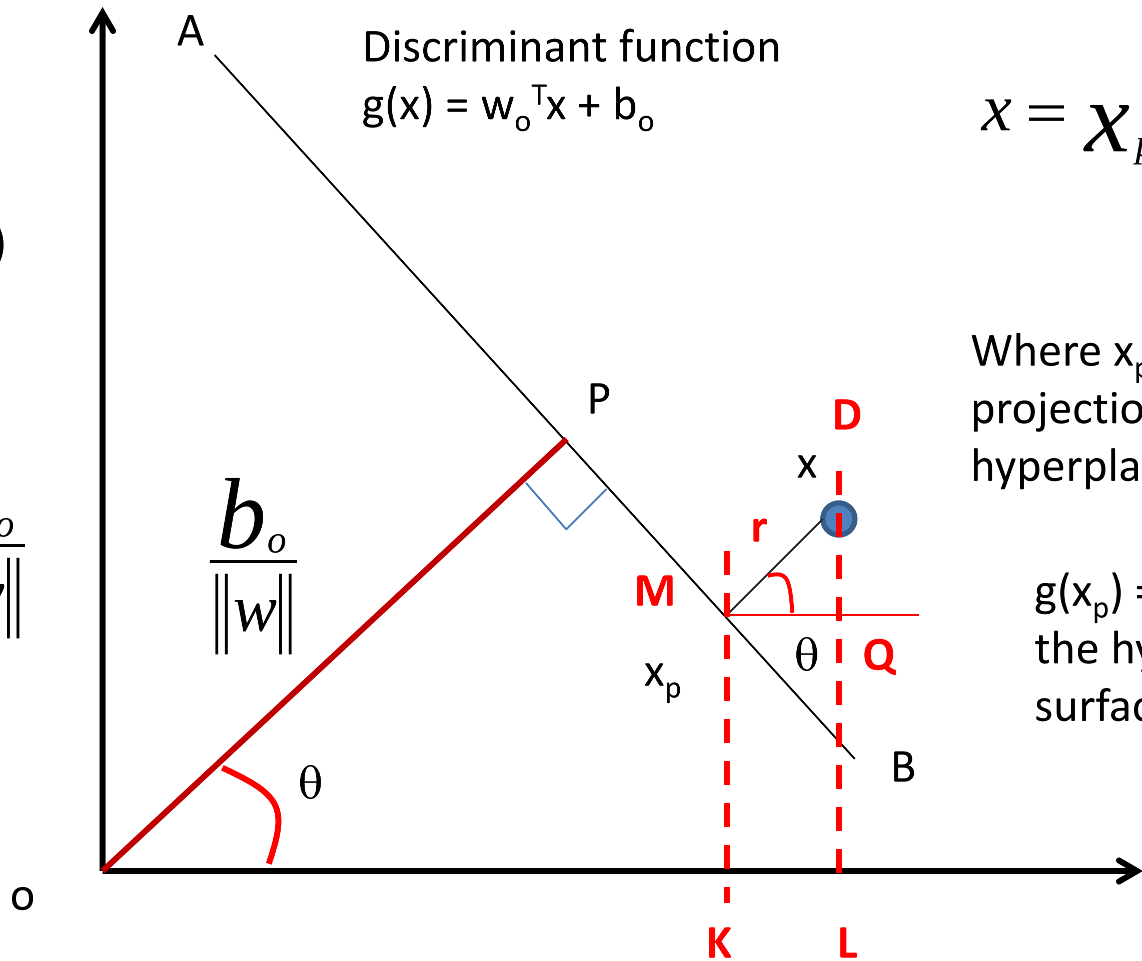
# Distance of a point in a class from optimal hyperplane

From DMQ

MQ = $r\cos(\theta)$
DQ = $r\sin(\theta)$

Also, from KOP
$\cos(\theta) \; \alpha \; \dfrac{b_o}{\|w\|}$

Discriminant function
$g(x) = w_o^T x + b_o$



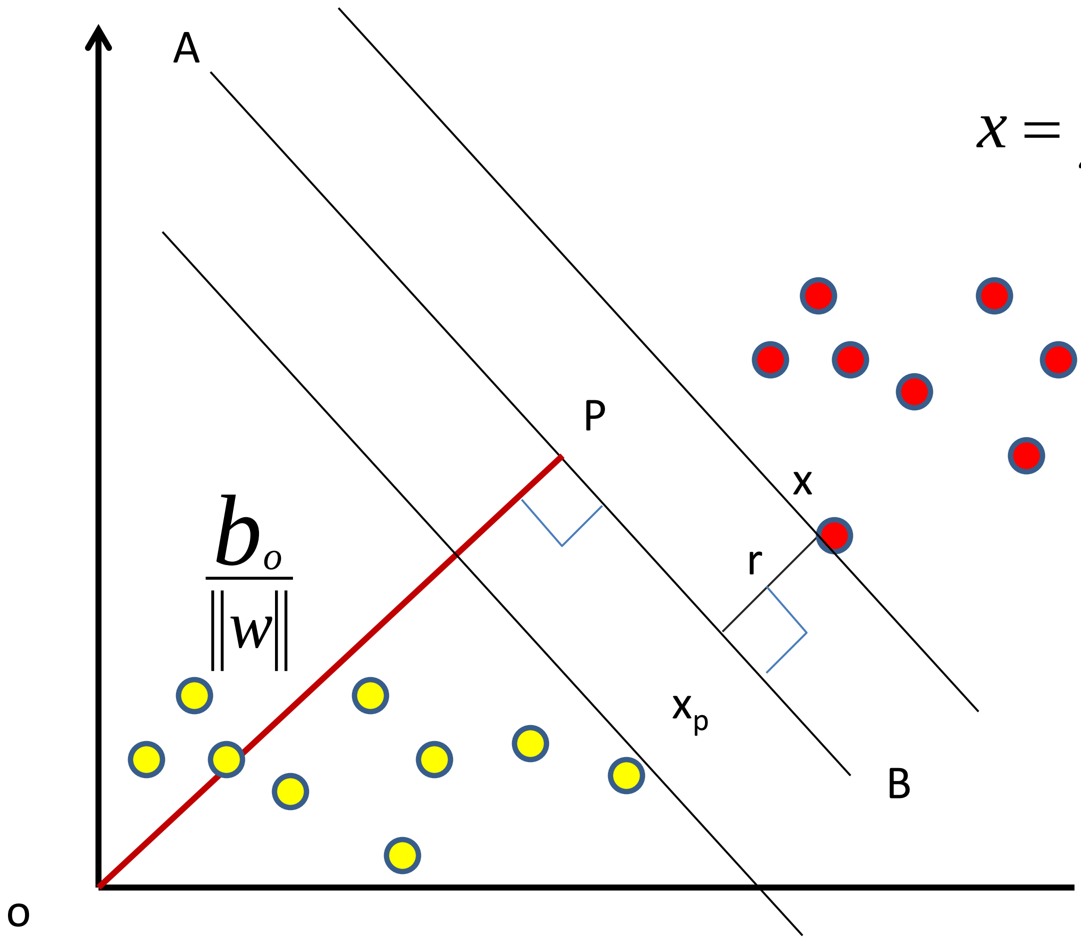

$$x = x_p + r\frac{w_o}{\|w\|}$$

Where $x_p$ is the normal projection of x on the hyperplane

$g(x_p) = w^T x + b_o = 0$ on the hyperplane/Decision surface/line

# Margin

$g(x) = w_o^T x + b_o$



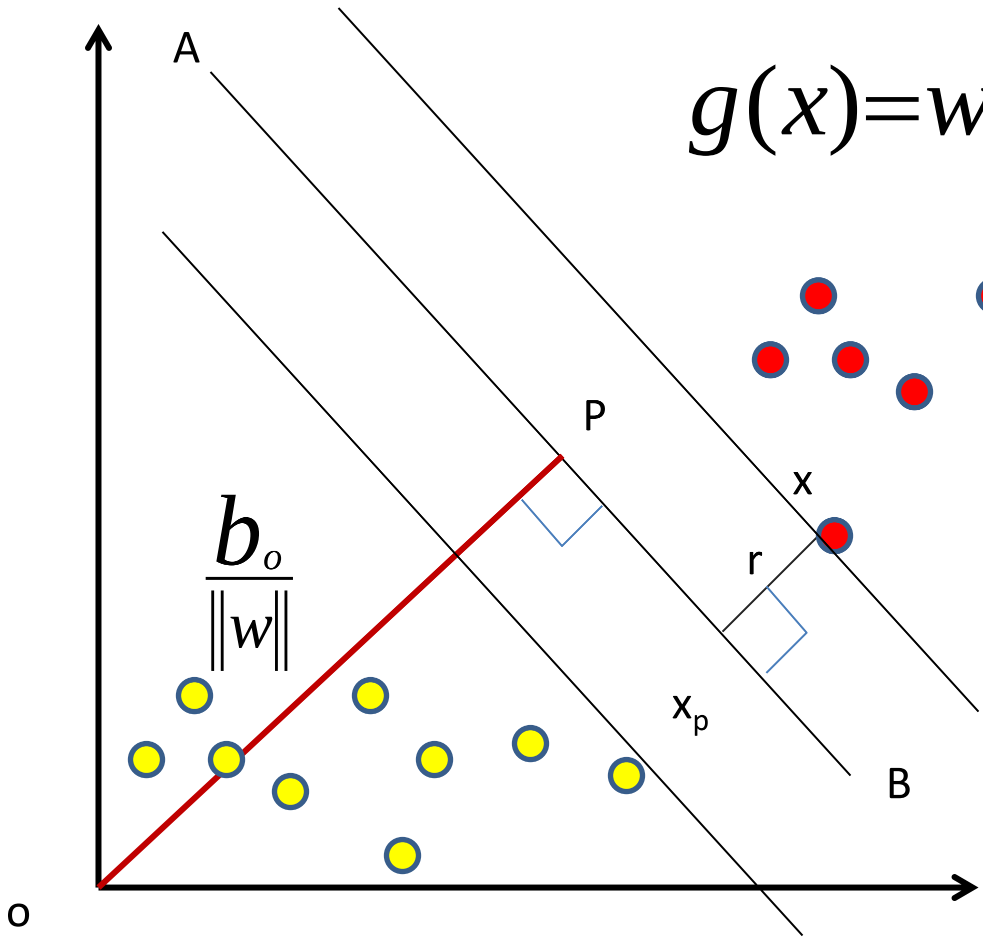

$$x = x_p + r\frac{w_o}{\|w\|}$$

r: positive if x is on the positive side of the hyperplane

r: negative if x is on the negative side of the hyperplane

# Margin

$g(x) = w_o^T x + b_o$



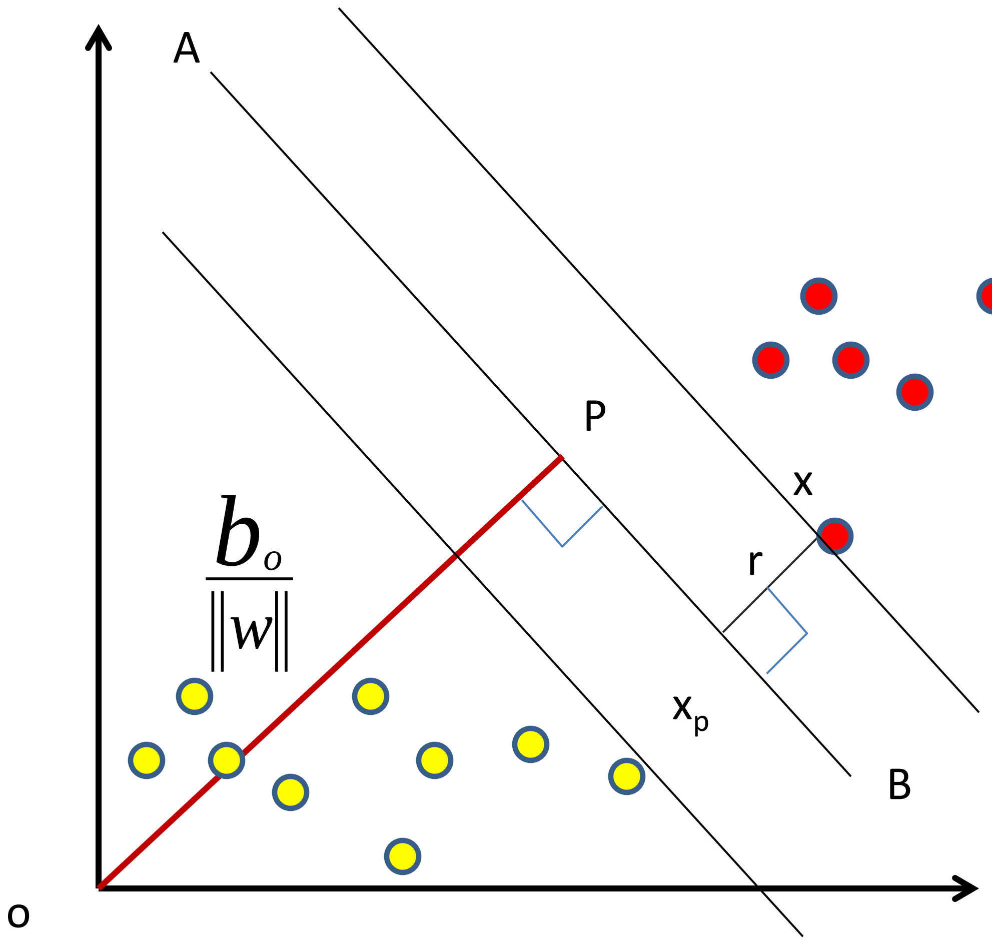

$$g(x) = w_o^T x_p + r\, w_o^T \frac{w_o}{\|w\|} + b_o$$

Since xp is on the hyperplane,
$g(x_p) = w_o^T x_p + b_o = 0$

Therefore

$$g(x) = r\, w_o^T \frac{w_o}{\|w\|}$$

# Margin




$$g(x) = r\, w_o^T \frac{w_o}{\|w\|}$$

$$g(x) = r\|w_o\|$$

$$r = \frac{g(x)}{\|w_o\|}$$

# Total margin to optimize

Considering $g(x^s) = +1$ for the support vector $x^s$ for which the class is +1

Similarly,

$g(x^s) = -1$ for the support vector $x^s$ for which the class is -1

Therefore, for both support vectors

$$r = \frac{+1}{\|w_o\|} \qquad r = \frac{-1}{\|w_o\|}$$

# Total margin to optimize

Total margin to optimize is

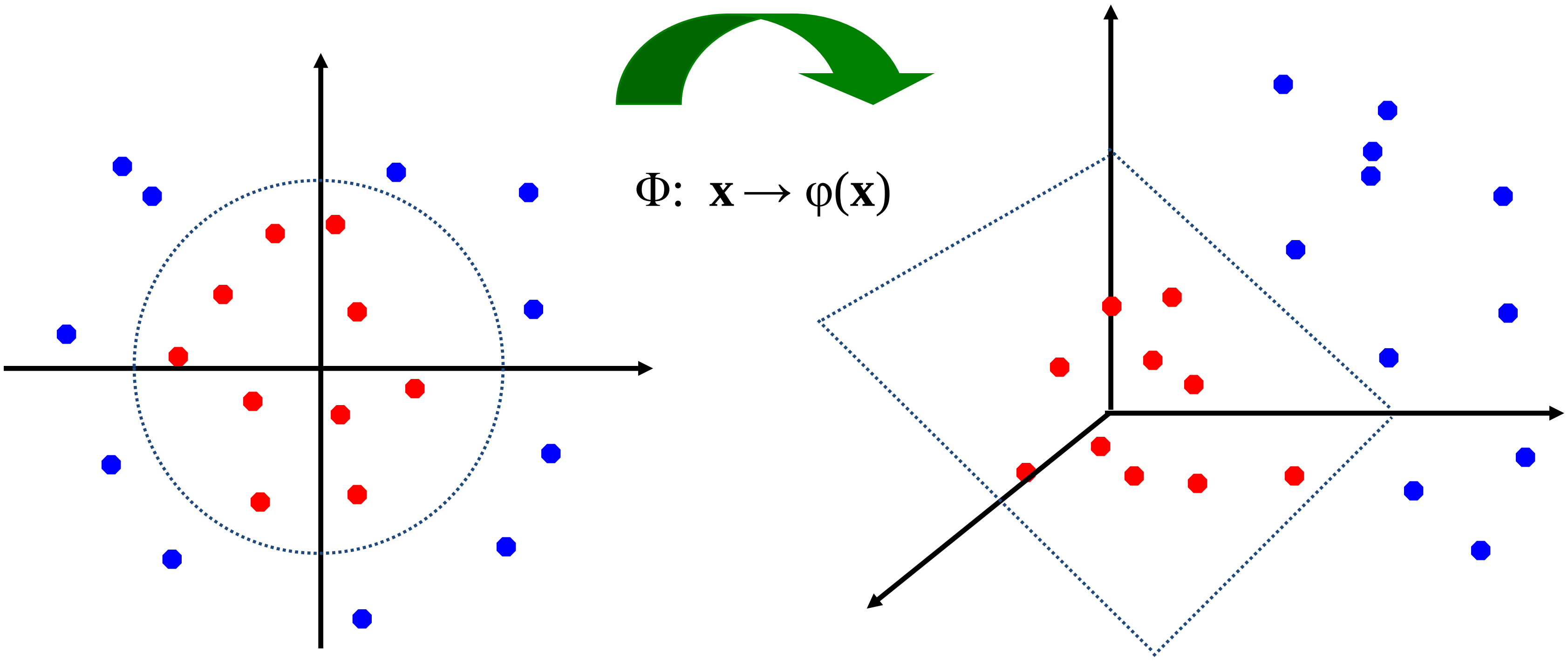
$$\rho = \frac{2}{\|w_o\|}$$

Maximize $\rho$

Or equivalently Minimize the Euclidean norm of weight vector w

# Non-linear SVMs: Feature spaces

- General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable:



# Examples of Kernel Functions

- Linear: $K(\mathbf{x_i},\mathbf{x_j}) = \mathbf{x_i}^T\mathbf{x_j}$

- Polynomial of power $p$: $K(\mathbf{x_i},\mathbf{x_j}) = (1 + \mathbf{x_i}^T\mathbf{x_j})^p$

- Gaussian (radial-basis function network):

$$K(\mathbf{x_i}, \mathbf{x_j}) = \exp\left(-\frac{\left\|\mathbf{x_i} - \mathbf{x_j}\right\|^2}{2\sigma^2}\right)$$

- Sigmoid: $K(\mathbf{x_i},\mathbf{x_j}) = \tanh(\beta_0\mathbf{x_i}^T\mathbf{x_j} + \beta_1)$

# SVM Applications

- **SVM has been used successfully in many real-world problems**

  **- text (and hypertext) categorization**

  **- image classification**

  **- bioinformatics (Protein classification, Cancer classification)**

  **- hand-written character recognition**

# Some Issues

- **Choice of kernel**
  - Gaussian or polynomial kernel is default
  - if ineffective, more elaborate kernels are needed
  - domain experts can give assistance in formulating appropriate similarity measures

- **Choice of kernel parameters**
  - e.g. σ in Gaussian kernel
  - σ is the distance between closest points with different classifications
  - In the absence of reliable criteria, applications rely on the use of a validation set or cross-validation to set such parameters.