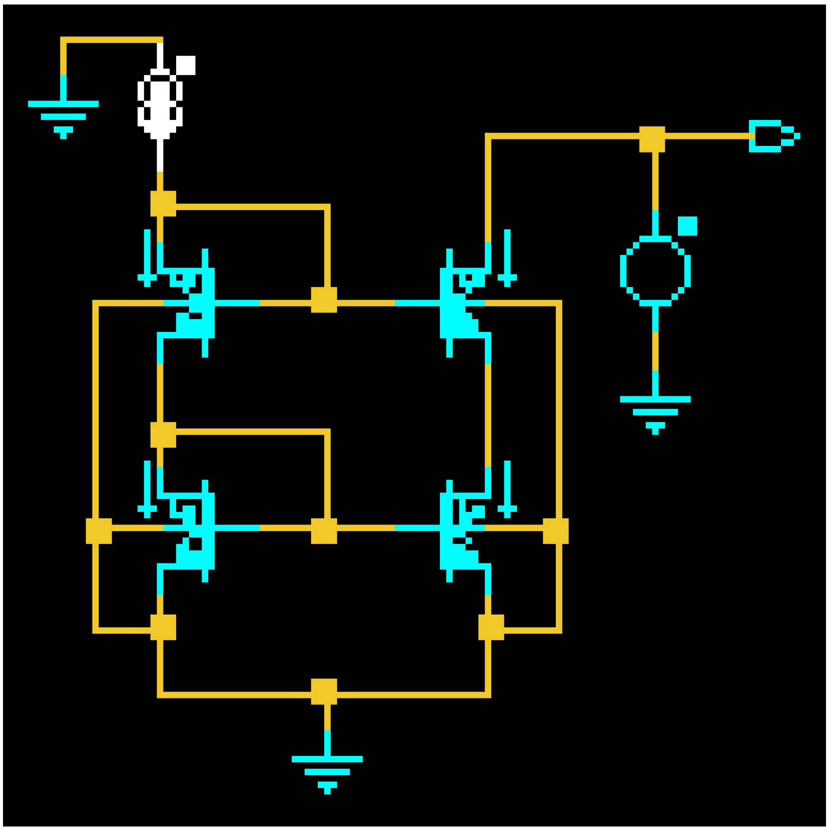
CIRCUIT DIAGRAM



Cascode Current Mirror

Theory:

One way to add emitter resistance without using actual resistors or generating a large voltage drop is to stack one current mirror on top of another, as shown to the left. This arrangement is known as a *cascode current mirror*. The term "cascode" dates back to the days of vacuum tubes, before any sort of semiconductor devices were invented. In a vacuum tube, the equivalent of the emitter is called the "cathode," and the configuration of Q2 and Q4 to the left was described as "cascade to cathode," which got shortened to "cascode." The name remains unchanged regardless of the specific components connected in this way.

In this circuit, Q1 and Q2 form a current mirror in which both collectors are held to a voltage of V_{BE}, so they show a close match in operating conditions. At the same time, they serve as emitter resistances for a second current mirror (Q3 and Q4).

Generally, all four transistors are matched, but that isn't strictly necessary. The transistors of each mirror pair should be matched to each other, of course, but Q1 and Q2 don't have to match Q3 and Q4. Each mirror operates independently of the other, although they necessarily interact with each other.

Base Currents

The lower current mirror, Q1 and Q2, operates normally, with $I_{C1} = I_{C2}$, $I_{E1} = I_{E2}$, and $I_{B1} = I_{B2}$. This is not true of the current mirror composed of Q3 and Q4. In the upper current mirror, I_{E3} must include the base currents for Q1 and Q2. Thus, $I_{E4} = I_{C2}$, but $I_{E3} = I_{C1} + I_{B1} + I_{B2}$. For the same reason, $I_{B3} > I_{B4}$.

When comparing I_O to I_{REF}, it is easiest to relate both currents to I_{E1}, which is also I_{E2}. Assuming that all four transistors are matched, and keeping in mind that $I_C/I_E = \beta/(\beta + 1)$, we get:

$$I_O = I_{C4} = \frac{\beta}{\beta + 1} I_{E4} = \frac{\beta^2}{(\beta + 1)^2} I_{E2} = \frac{\beta^2}{(\beta + 1)^2} I_{E1}$$

$$I_{REF} = I_{C3} + I_{B3} + I_{B4} = I_{E3} + I_{B4} = I_{C1} + 2I_{B1} + I_{B4} = \frac{\beta}{\beta + 1} I_{E1} + \frac{2}{\beta + 1} I_{E1} + \frac{\beta}{(\beta + 1)^2} I_{E1}$$

With everything expressed in terms of I_{E1}, we can calculate the ratio I_O/I_{REF}. This expression simplifies to:

$$\frac{I_O}{I_{REF}} = \frac{\beta^2}{\beta^2 + 4\beta + 2} = \frac{1}{1 + 4/\beta + 2/\beta^2}$$

Output Resistance

Because Q2 is not connected as a diode, but is fully operational as a transistor, its output resistance r_O serves as the emitter resistance for Q4. This means that R_{OUT} for Q4 can be quite large. The mathematical expression for R_{OUT} is:

$$R_{OUT} = r_O(2 + g_m r_O) \approx g_m r_O^2$$

Keep in mind that one r_O is for Q2 while the other is for Q4. If the Early voltages of these two transistors aren't equal, the r_O values must be handled separately. Either way, the effective output resistance is much higher than for the simple current mirror.

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