Special Issue Letters

Comments on "Editorial: Fuzzy Models—What Are They and Why?"

William H. Woodall and Robert E. Davis

This comment is in reference to this issue's editorial.[1] In the editorial, an example is presented to demonstrate differences between fuzzy membership and probability.

In Example 1 of the Editorial, a thirsty person must choose between two bottles of liquid—Bottle A and Bottle B. Bottle A has membership 0.91 in the set of potable liquids, while Bottle B has a probability of 0.91 of being potable. The argument is that the logical choice is Bottle A because its contents are at least "reasonably similar" to a perfectly potable liquid and Bottle B has a 9% chance of being unsavory or even deadly. Under this scenario, the fuzzy modeler has had the opportunity to sample the liquid in Bottle A, while the person evaluating Bottle B knows only that its contents were randomly selected from some population of liquids, 91% of which are potable.

Probability can be used, however, in a way that is much more closely analogous to this use of membership. In this alternative use of probability, the contents of Bottle B are examined in the same manner as those of Bottle A. A person then assigns a value to the probability that a person, perhaps randomly selected from some population, would find the contents potable. This value could be determined subjectively, but also checked later by random sampling if desired. If the value of this probability were also 0.91, then there is no clear preference between the two bottles. Observation of the contents of the bottles would change neither the membership nor the probability. In addition, changing the numerical value of the membership and the probability, e.g., from 0.91 to 0.50, has no effect on the decision. Thus, this use of probability makes the argument for the usefulness of memberships much less convincing.

We have found that many of those advocating the use of fuzzy logic have justified their methods by offering very limited views of probability. In our opinion, probability can be used to represent the information claimed to be provided only by memberships. The references below, which illustrate these points in detail, are available upon request.

References

[1] D. Barrett and W. H. Woodall, "A probabilistic alternative to the fuzzy logic controller," *Tech. Rep.*, Appl. Stat. Program, Univ. Alabama, 1993.

[2] D. Redden and W. H. Woodall, "Properties of fuzzy linear regression," *Tech. Rep.*, Appl. Stat. Program, Univ. Alabama, submitted to *Fuzzy Sets Syst.*, 1993.

[3] J. Sullivan and W. H. Woodall, "A comparison of fuzzy forecasting and Markov modeling," *Tech. Rep.*, Appl. Stat. Program, Univ. Alabama, submitted to *Fuzzy Sets Syst.*, 1993.

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[1] J. C. Bezdek, *IEEE Trans. Fuzzy Syst.*, vol. 1, no. 1, pp. 1–6, Feb. 1993.

The Thirsty Traveler Visits Gamont: A Rejoinder to "Comments on Fuzzy Sets — What are They and Why?"

James C. Bezdek

Here is the example [1]:

Example 1

One of the first questions asked about this scheme, and the one that is still asked most often, concerns the relationship of fuzziness to probability. Are fuzzy sets just a clever disguise for statistical models? Well, in a word, NO. Perhaps an example will help.

Let the set of all liquids be the universe of objects, and let fuzzy subset L = { all potable (= "suitable for drinking") liquids }. Suppose you had been in the desert for a week without drink and you came upon two bottles A and B, marked as in the left half of Fig. 1 (memb = "membership", and prob="probability").

Confronted with this pair of bottles, and given that you must drink from the one that you chose - which would you choose to drink from first ? Most readers familiar with the basic ideas of fuzzy sets, when presented with this experiment, immediately see that while A could contain, say, swamp water, it would not (discounting the possibility of a Machiavellian fuzzy modeler) contain liquids such as hydrochloric acid. That is, membership of 0.91 means that the contents of A are "fairly similar" to perfectly potable liquids (pure water). On the other hand, the probability that B is potable = 0.91 means that over a long run of experiments, the contents of B are expected to be potable in about 91% of the trials; and the other 9%? In these cases the contents will be unsavory (indeed, possibly deadly)- about 1 chance in 10. Thus, most subjects will opt for a chance to drink swamp water, and will choose bottle A.

Another facet of this example concerns the idea of observation. Continuing then, suppose that we examine the contents of A and B, and discover them to be as shown in the right half of Fig. 1 - that is, A contains beer, while B contains hydrochloric acid. After observation then, the membership value for A will be unchanged, while the probability value for B clearly drops from 0.91 to 0.0.

Finally, what would be the effect of changing the numerical information in this example? Suppose that the membership and probability values were both 0.5 - would this influence your choice? Almost certainly it would. In this case many observers would switch to bottle B, since it offers a 50% chance of being drinkable, whereas a membership value this low would presumably indicate a liquid unsuitable for drinking (this depends, of course, entirely on the MF of the fuzzy set L).

In summary, Example 1 shows that these two types of models possess philosophically different kinds of information; fuzzy memberships, which represent similarities of objects to imprecisely defined properties; and probabilities, which convey information about relative frequencies. Moreover, interpretations about and decisions based on

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