Chain complexes

Definition and basic properties

Throughout, we work over a commutative ring R.

Definition 1 (Chain complex)**.** $d^2 = 0$.

Definition 2 (Cycles)**.** $Z_n = ker(d_n)$

Definition 3 (Boundaries)**.** $B_n = im(d_{n+1})$

Note: $d^2 = 0$ implies that $B_n \subset Z_n$

Pause for effect

Definition 4 (Homology)**.** $H_n = Z_n/B_n$

Example 1. $C_n = \mathbb{Z}/8$, $d_n =$*(multiplication by 4).*

0.1 The category of chain complexes

Objects are chain complexes, morphisms are level-wise maps that commute with the differential.

diagram here

0.2 $Ch(R)$ is an abelian category