Resolutions

Replace a chain complex with an equivalent one that is better-behaved.

0.1 Motivation

There are pathologies in the category of chain complexes. For example, $Hom_{\mathbb{Z}}(C, \mathbb{Z})$ could have nontrivial homology even if C has trivial homology! Same problem, e.g., for $- \otimes \mathbb{Z}/p$.

This means that a category of chain complexes modulo weak equivalence wouldn't have well-defined hom objects or tensor products.

This kind of problem occurs for topological spaces too: The Warsaw circle, W, has trivial homotopy groups, but the mapping space from W to the circle has nontrivial homotopy!

[*picture*]

0.2 Solution for spaces

In spaces, the CW complexes form a well-behaved subcategory, and every space is weakly equivalent to a CW complex. So one could "restrict" to CW complexes. We still do want to consider all spaces, so develop a system for coherently replacing any space with a CW complex . . .

The analogous solution in chain complexes comes in the form of resolutions! This can be done in two different (dual) ways . . .

0.3 Projective resolutions

0.4 Injective resolutions