

Universal coefficient theorem

Theorem 1. *Let P be a flat complex with each $d(P_n)$ flat. Then for all R -modules M we have a short exact sequence*

$$0 \rightarrow H_n(P) \otimes M \rightarrow H_n(P \otimes M) \rightarrow \text{Tor}_1(H_{n-1}(P), M) \rightarrow 0$$

Proof (proof goes here) ■

Theorem 2. *Let P be a flat complex with each $d(P_n)$ flat and let Q be any chain complex. Then we have a short exact sequence*

$$0 \rightarrow (H_*(P) \otimes H_*(Q))_n \rightarrow H_n(P \otimes Q) \rightarrow \bigoplus_{p+q=n-1} \text{Tor}_1(H_p(P), H_q(Q)) \rightarrow 0$$

Proof Same idea as above. ■