Homological dimension

Dimension is determined by length of a minimal resolution

Definition 1.

- *Projective dimension*
- *Injective dimension*
- *Flat dimension*

Theorem 1 (Homological dimension)**.** *Let R be a commutative ring, the following are equivalent:*

(a) sup_B { *inj.dim(B), B an R-module* }

(b) sup_A {*proj.dim(A), A an R-module* }

(c) sup_I {*proj.dim(R/I), I an ideal* }

(d) *sup* { *d such that $Ext^d_R(A, B) \neq 0$ for some A, B* }

Example 1.

- *Every field has homological dimension 0.*
- *$R = \mathbb{Z}$ has homological dimension 1.*
- *$R = \mathbb{Z}/m$ with $p^2|m$ has homological dimension ∞ (consider $Ext_{\mathbb{Z}/p^2}(\mathbb{Z}/p, \mathbb{Z}/p)$).*