

Chain complexes

Definition and basic properties

Throughout, we work over a commutative ring R .

Definition 1 (Chain complex). $d^2 = 0$.

Definition 2 (Cycles). $Z_n = \ker(d_n)$

Definition 3 (Boundaries). $B_n = \operatorname{im}(d_{n+1})$

Note: $d^2 = 0$ implies that $B_n \subset Z_n$

[Pause for effect]

Definition 4 (Homology). $H_n = Z_n/B_n$

Example 1. $C_n = \mathbb{Z}/8$, $d_n = [\text{multiplication by } 4.]$

0.1 The category of chain complexes

Objects are chain complexes, morphisms are level-wise maps that commute with the differential.

[diagramhere]