

Chain homotopy

An important equivalence relation on chain maps

The notion of chain homotopy can be explained by an "interval" chain complex, I , and map

$$M \otimes I \rightarrow N$$

Suppose R is the ground ring. The chain complex I is nontrivial in two degrees:

$$\begin{aligned} I_0 &= R \oplus R \\ I_1 &= R \end{aligned}$$

Suppose the two generators for I_0 are a and b , and suppose the generator for I_1 is c . Then the differential is given by

$$d(c) = b - a.$$

It's a fun exercise to check that the usual definition of chain homotopy equivalence is the same as the one determined by a map out of $M \otimes I$. (The part that shifts degree by 1 is the part coming from I_1).

In fact, this I is the chain complex you would get from the cellular chains on the standard interval I , so from that point of view it's not so surprising that it plays such a crucial role.