Topic 03:

Two-View Geometry

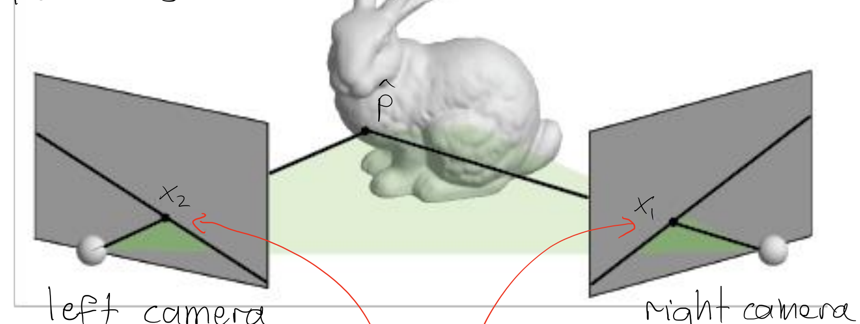
- epipolar lines & epipolar planes
- the fundamental matrix (F-matrix)
- geometric interpretation of the F-matrix
- the essential matrix
- fundamental matrix estimation:
  the normalized 8-point algorithm
- algorithms & applications of stereo reconstruction
- image homographies
- stereo image rectification

Piazza : Please enroll
(or monitor it)

Office hours this week:
12:30 - 2pm tomorrow

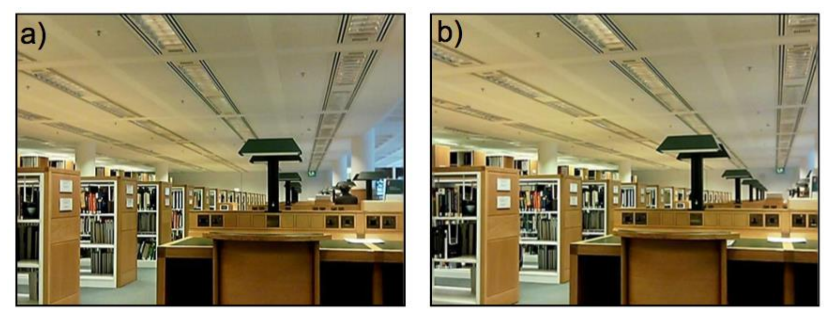
Stereo triangulation

if we know the camera centers & corresponding pixels (in homogeneous coordinates) we can compute the homogeneous coordinates of $\hat{P}$



The stereo correspondence problem: given x_1, find x_2

a stereo image pair



Left Right (Prince, 2011)

two-view geometry

Goal: Explore basic geometry of multiple images to support the inference of 3D scene structure from two or more images of the scene.

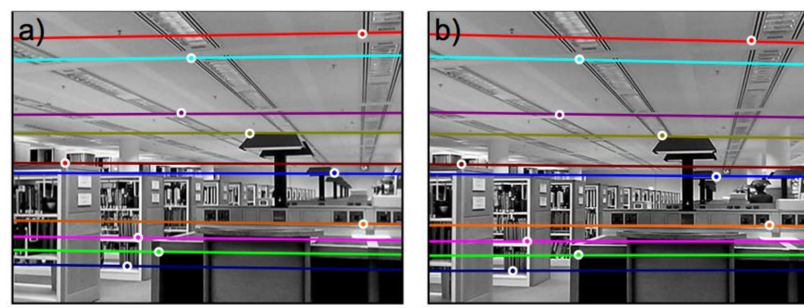
Consider two perspective images of a scene as taken from a stereo pair of cameras (or equivalently, assume the scene is rigid and imaged with a single camera from two different locations).

- Given a scene point $\vec{X}^p$ which is imaged in the "left" camera at $\vec{p}^L$, where could the image of the same point be in the right camera?
- The relationship between such *corresponding image points* turns out to be both simple and useful; i.e., the corresponding point in the "right" camera, $\vec{p}^R$, is constrained to lie on a line.

a stereo image pair

Left Right (Prince, 2011)

the epipolar constraint

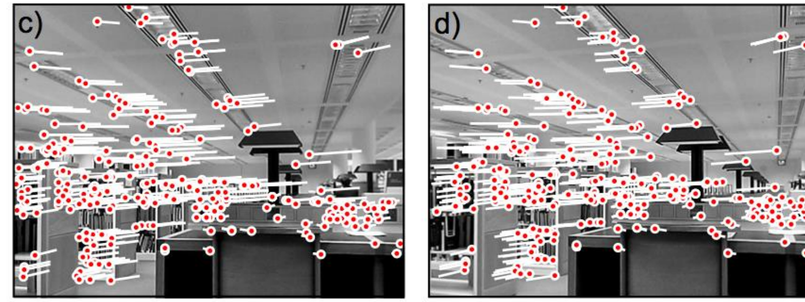


(Prince, 2011)

The Epipolar Constraint

For every pair of perspective photos, there exists a line-to-line mapping between them such that scene points projecting to one line will also project to the other

the epipolar constraint: significance



(Prince, 2011)

The Epipolar Constraint

⇒ correspondence-finding between two (rigid) views is a 1D problem

⇒ it allows us to infer relative camera motion

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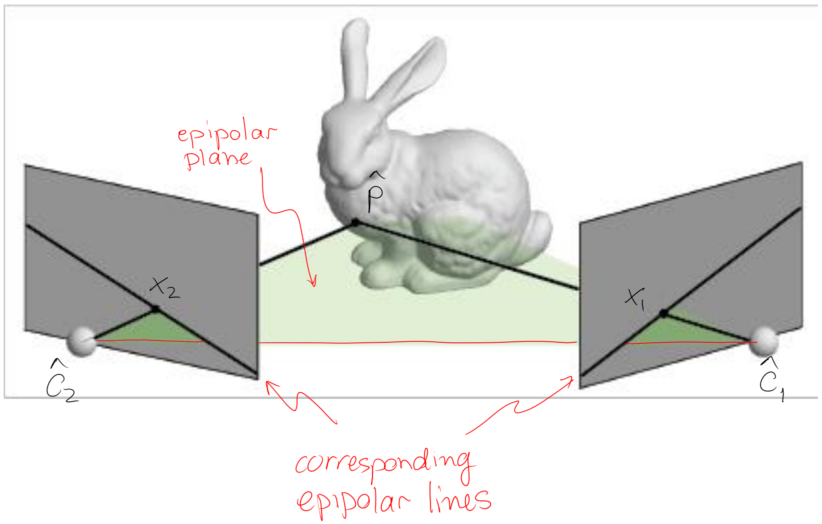
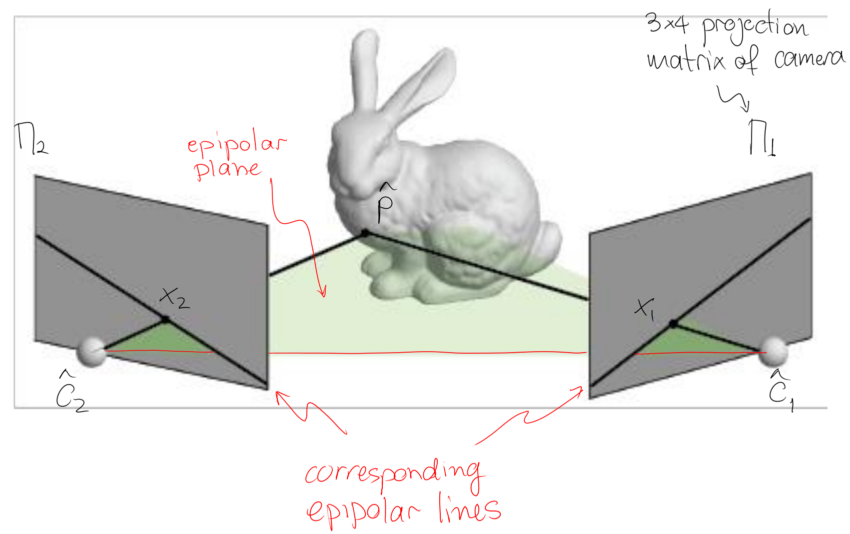
epipolar lines & epipolar planes



epipolar lines & epipolar planes



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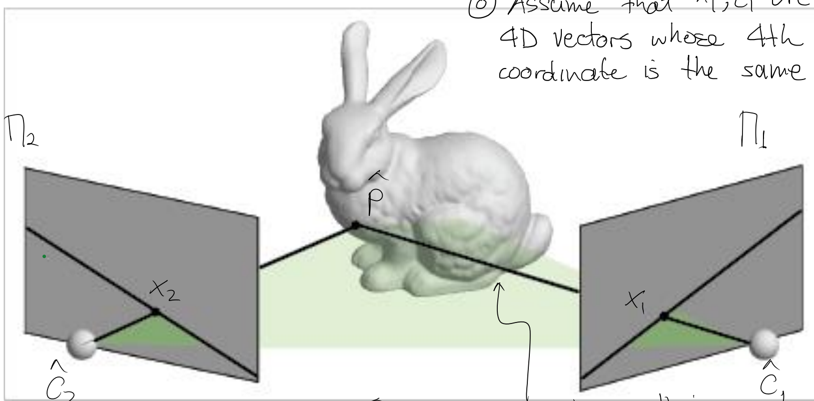
geometry of the epipolar constraint

(0) Assume that $\hat{x}_1, \hat{c}_1$ are 4D vectors whose 4th coordinate is the same



$x_2 = \Pi_2 \hat{p}$

(1) Typical point on this ray

$\hat{p} = \lambda \hat{c}_1 + (1-\lambda)\hat{x}_1$

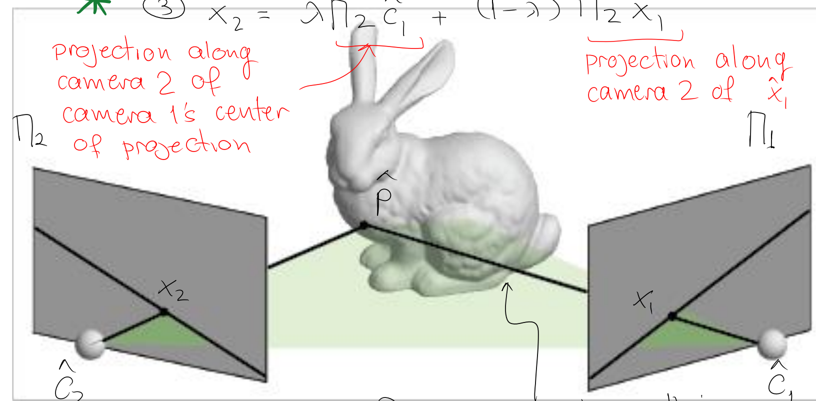
homogeneous 3D coords of image point x_1

geometry of the epipolar constraint

(3) $x_2 = \underbrace{\lambda \Pi_2 \hat{c}_1}_{} + (1-\lambda)\underbrace{\Pi_2 \hat{x}_1}_{}$

projection along camera 2 of camera 1's center of projection

projection along camera 2 of $\hat{x}_1$



(2) Its projection in image 2:

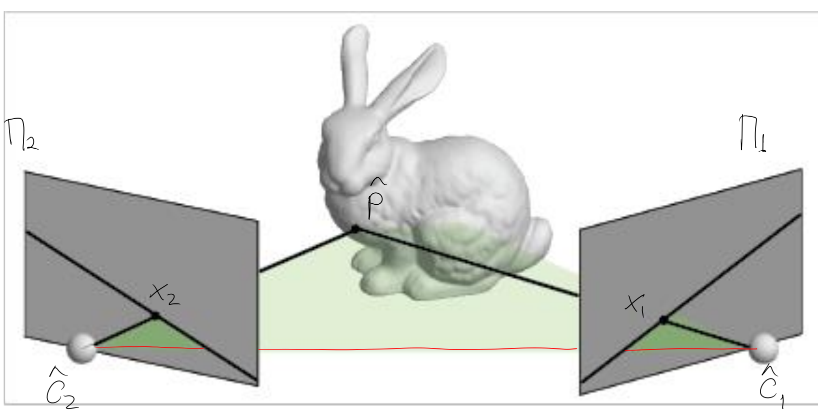
$\Pi_2 (\lambda \hat{c}_1 + (1-\lambda)\hat{x}_1)$

(1) Typical point on this ray

$\hat{p} = \lambda \hat{c}_1 + (1-\lambda)\hat{x}_1$

homogeneous 3D coords of image point x_1

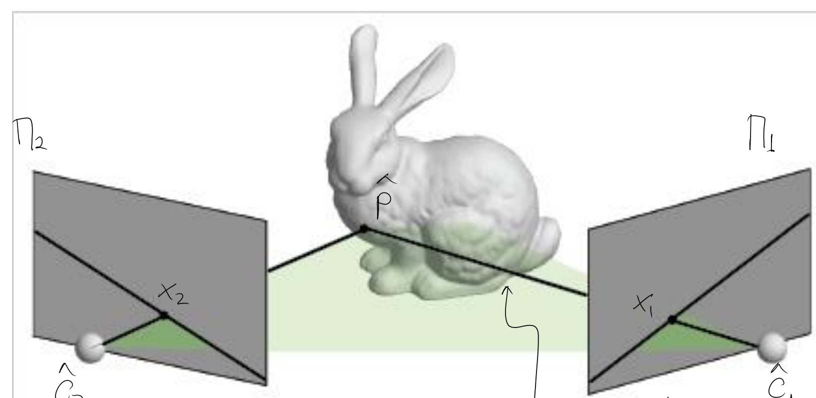
geometry of the epipolar constraint



$x_2 = \Pi_2 \hat{p}$

$x_1 = \Pi_1 \hat{p}$

geometry of the epipolar constraint



(2) Its projection in image 2:

$\Pi_2 (\lambda \hat{c}_1 + (1-\lambda)\hat{x}_1)$

(1) Typical point on this ray

$\hat{p} = \lambda \hat{c}_1 + (1-\lambda)\hat{x}_1$

homogeneous 3D coords of image point x_1

geometry of the epipolar constraint

③ $x_2 = \lambda \underbrace{\Pi_2 \hat{c}_1}_{} + (1-\lambda) \underbrace{\Pi_2 \hat{x}_1}_{}$

projection along camera 2 of camera 1's center of projection

projection along camera 2 of $\hat{x}_1$

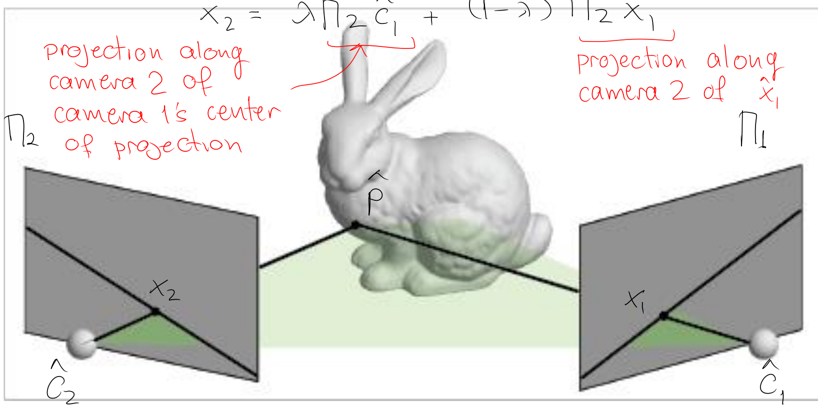
This is always a line, called the epipolar line

derivation of the epipolar constraint

$x_2 = \lambda \underbrace{\Pi_2 \hat{c}_1}_{} + (1-\lambda) \underbrace{\Pi_2 \hat{x}_1}_{}$

projection along camera 2 of camera 1's center of projection

projection along camera 2 of $\hat{x}_1$



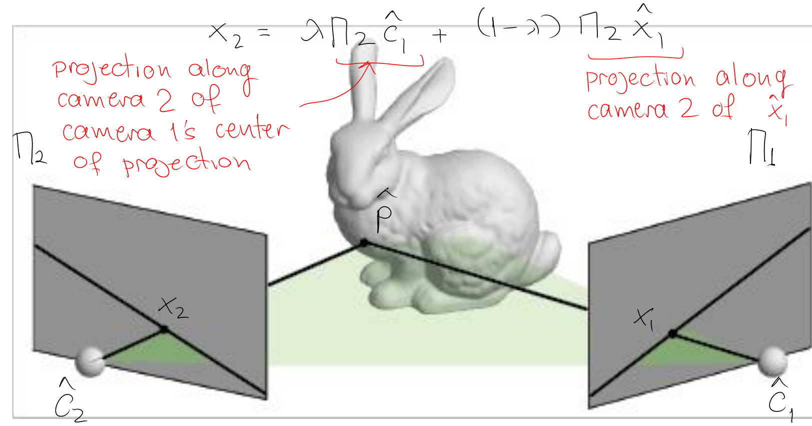
Key observation: we can compute this line knowing just Π_1, Π_2 and (image point) x_1

derivation of the epipolar constraint

$x_2 = \lambda \underbrace{\Pi_2 \hat{c}_1}_{} + (1-\lambda) \underbrace{\Pi_2 \hat{x}_1}_{}$

projection along camera 2 of camera 1's center of projection

projection along camera 2 of $\hat{x}_1$



1. Π_1 maps world points to image points

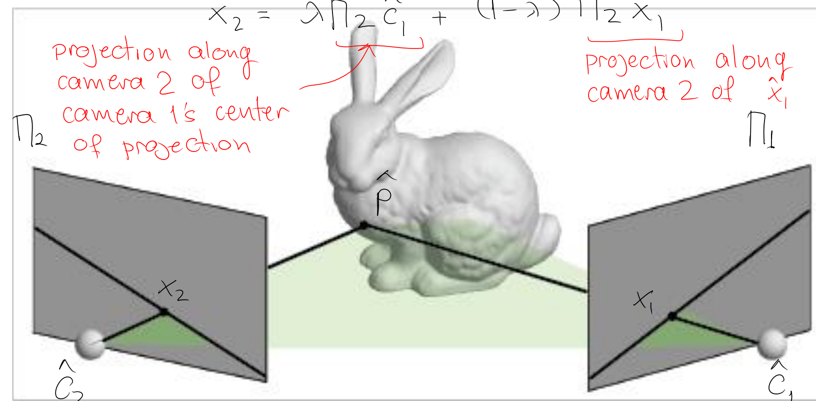
   Question: How do we go from image point x_1 to world point $\hat{x}_1$?

derivation of the epipolar constraint

$x_2 = \lambda \underbrace{\Pi_2 \hat{c}_1}_{} + (1-\lambda) \underbrace{\Pi_2 \hat{x}_1}_{}$

projection along camera 2 of camera 1's center of projection

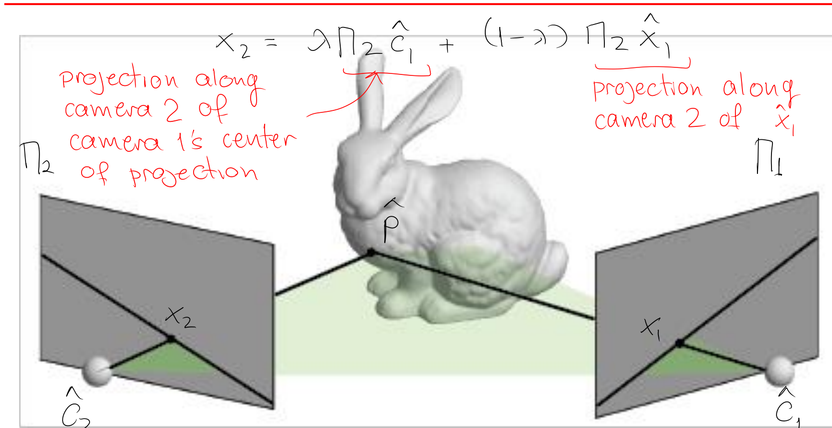
projection along camera 2 of $\hat{x}_1$



1. Question: How do we go from image point x_1 to world point $\hat{x}_1$?

   Question: Can we invert Π_1?

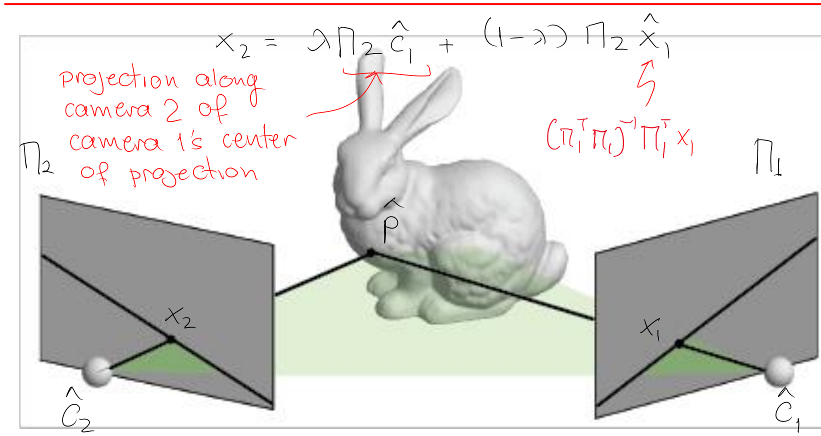
derivation of the epipolar constraint



1. Question: How do we go from image point x_1 to world point $\hat{x}_1$?

Ans: $x_1 = \Pi_1 \hat{x}_1$ so define $\hat{x}_1 = (\Pi_1^T \Pi_1)^{-1} \Pi_1^T x_1$ *

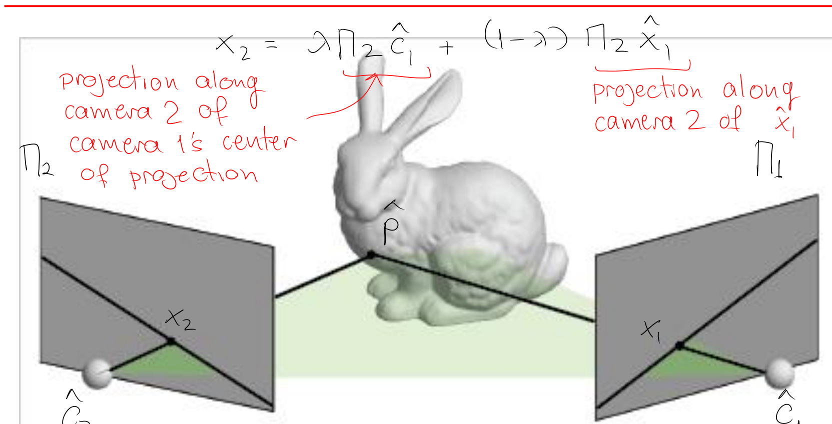
derivation of the epipolar constraint



2. Question: What is $\hat{c}_1$?

The projection of $\hat{c}_1$ along camera 1 is degenerate

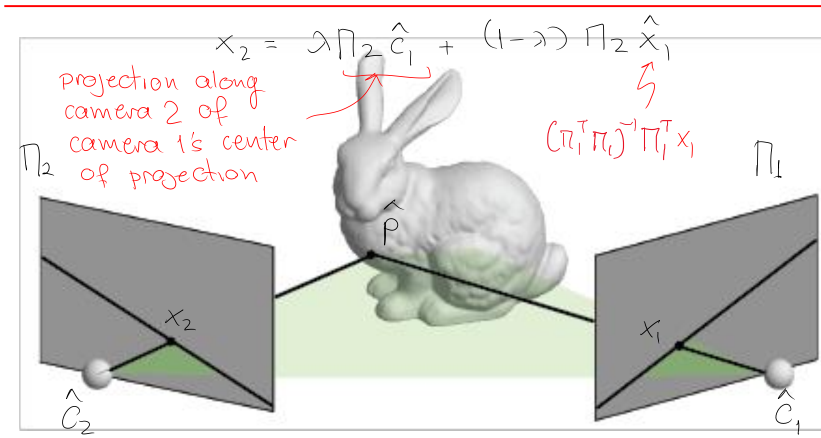
derivation of the epipolar constraint



1. Question: How do we go from image point x_1 to world point $\hat{x}_1$?

Ans: No, but we can compute Π_1's pseudoinverse

derivation of the epipolar constraint



2. Question: What is $\hat{c}_1$?

Can we use the pseudoinverse here?

Ans: No, $\hat{c}_1$ is not on the image plane

derivation of the epipolar constraint

- a point on the ray containing $\hat{x}_1$ and $\hat{c}_1$:
  $\lambda \hat{c}_1 + (1-\lambda)\hat{x}_1$
- its projection is $\Pi_1(\lambda\hat{c}_1 + (1-\lambda)\hat{x}_1) \simeq x_1$

  (*) $\lambda(\Pi_1 \hat{c}_1) + (1-\lambda)(\Pi_1 \hat{x}_1) \simeq x_1$
- but we also know that by construction

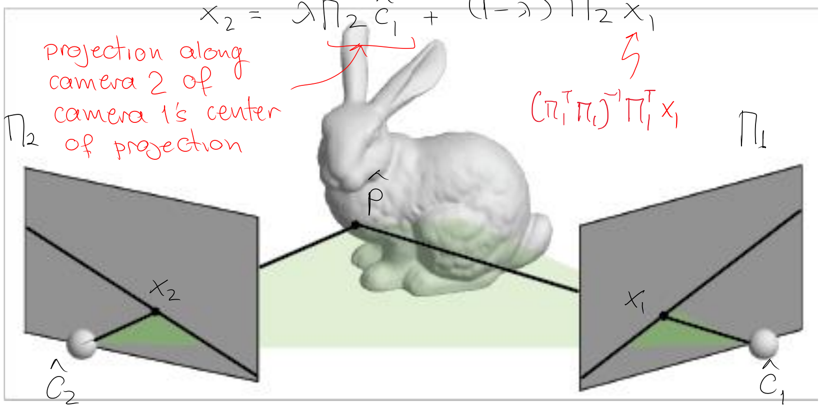
  $\Pi_1 \hat{x}_1 \simeq x_1$
- since (*) holds for all points $x_1, \hat{x}_1$
- it follows that $\Pi_1 \hat{c}_1 = 0$

  $\hat{c}_1$ is in Π_1's null space

derivation of the epipolar constraint

$x_2 = \lambda \Pi_2 \hat{c}_1 + (1-\lambda)\Pi_2 \hat{x}_1$

$(\Pi_1^T\Pi_1)^{-1}\Pi_1^T x_1$



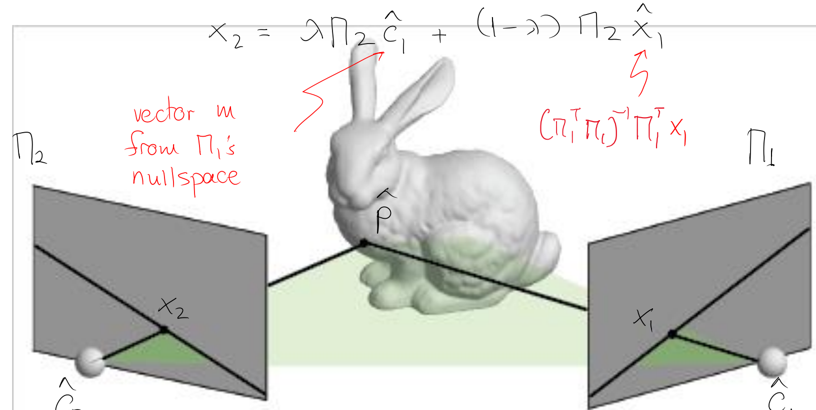
2. Question: What is $\hat{c}_1$?

   $\Pi_1 \hat{c}_1 = 0 \iff \hat{c}_1$ is in the nullspace of Π_1

derivation of the epipolar constraint

$x_2 = \lambda \Pi_2 \hat{c}_1 + (1-\lambda)\Pi_2 \hat{x}_1$

$(\Pi_1^T\Pi_1)^{-1}\Pi_1^T x_1$



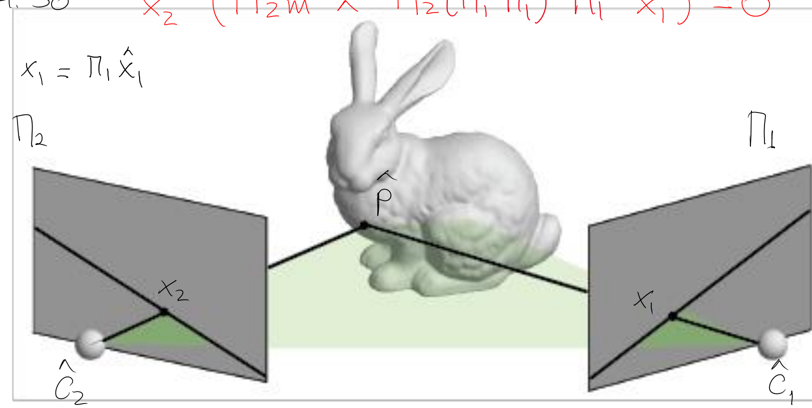
3. Putting it all together, the epipolar line passes through points

   $\Pi_2 m$ and $\Pi_2(\Pi_1^T\Pi_1)^{-1}\Pi_1^T x_1$

derivation of the epipolar constraint

4. So $x_2^T\left(\Pi_2 m \wedge \Pi_2(\Pi_1^T\Pi_1)^{-1}\Pi_1^T x_1\right) = 0$

$x_1 = \Pi_1 \hat{x}_1$



3. Putting it all together, the epipolar line passes through points

   $\Pi_2 m$ and $\Pi_2(\Pi_1^T\Pi_1)^{-1}\Pi_1^T x_1$

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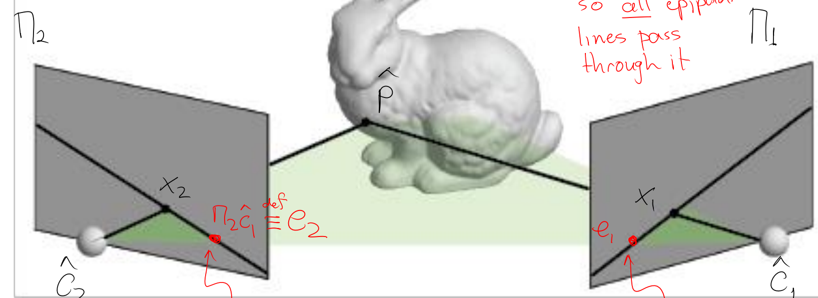
geometric interpretation of the F-matrix: epipoles

$x_2^T \underbrace{F}_{\substack{3\times 3\\ \text{matrix}}} x_1 = 0$

$x_2 = \lambda \underbrace{\Pi_2 \hat{c}_1}_{} + (1-\lambda)\, \Pi_2 \hat{x}_1$

does not depend on x_1 so all epipolar lines pass through it



called the epipole in image 2 *

* epipole in image 1

the fundamental matrix

4. So $x_2^T \left(\Pi_{2m} \wedge \Pi_2(\Pi_1^T\Pi_1)^{-1}\Pi_1^T x_1\right) = 0$

$$\Longleftrightarrow x_2^T \underbrace{[\Pi_{2m}]_\times}_{\text{cross-product matrix}} \Pi_2 (\Pi_1^T \Pi_1)^{-1} \Pi_1^T x_1 = 0$$

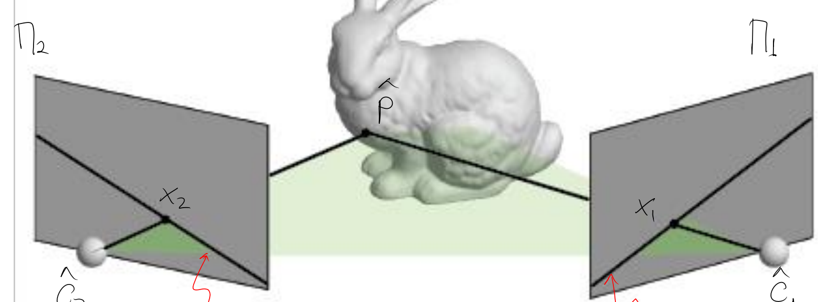
$$a \wedge b = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} b$$

$$\Longleftrightarrow \boxed{x_2^T F x_1 = 0}$$

F is a 3x3 matrix called the Fundamental Matrix

geometric interpretation of the F-matrix

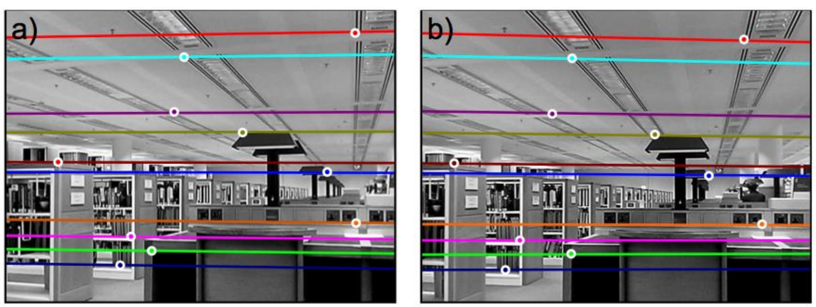
$x_2^T \underbrace{F}_{\substack{3\times 3\\ \text{matrix}}} x_1 = 0$



line whose homogeneous coordinates are Fx_1

line whose homogeneous coordinates are $x_2^T F$

the epipoles

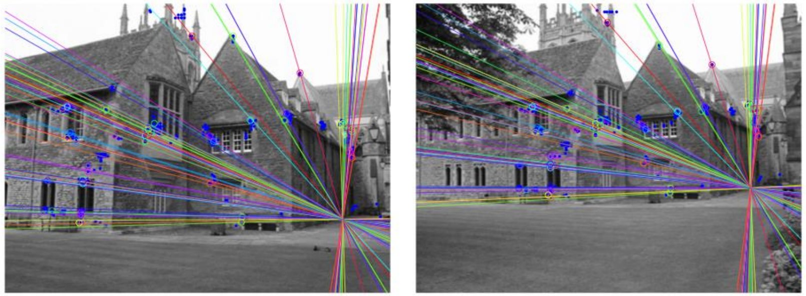


(Prince, 2011)

Question: Where are the epipoles?

What does this tell you about the camera's motion between views?

the epipoles



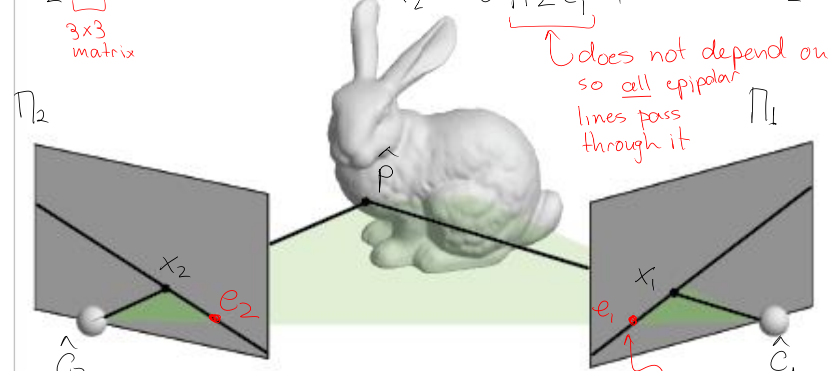
Question: Where are the epipoles?

What does this tell you about the camera's motion between views?

geometric interpretation of the F-matrix: epipoles

$x_2^T \underbrace{F}_{\substack{3\times3\\ \text{matrix}}} x_1 = 0$ $\qquad x_2 = \lambda \underbrace{\Pi_2 \hat{c}_1} + (1-\lambda)\Pi_2 \hat{x}_1$

does not depend on x_1 so all epipolar lines pass through it



epipole in image 1

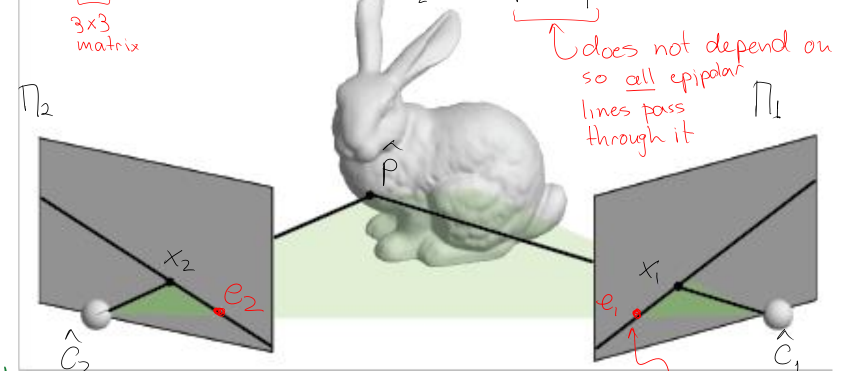
So $e_2^T F x_1 = 0$ for all x_1

It follows $e_2^T F = 0 \Rightarrow$ F has rank 2 and e_2 is its (left) null vector

geometric interpretation of the F-matrix: epipoles

$x_2^T \underbrace{F}_{\substack{3\times3\\ \text{matrix}}} x_1 = 0$ $\qquad x_2 = \lambda \underbrace{\Pi_2 \hat{c}_1} + (1-\lambda)\Pi_2 \hat{x}_1$

does not depend on x_1 so all epipolar lines pass through it



epipole in image 1

So $x_2^T F e_1 = 0$ for all x_2

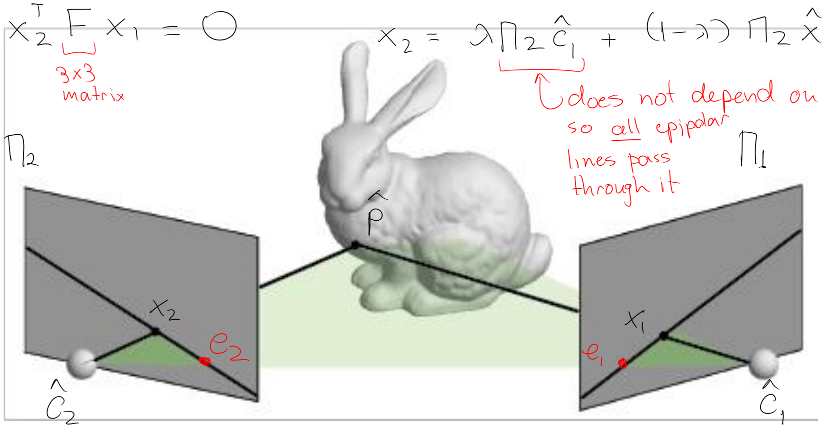
It follows $Fe_1 = 0 \Rightarrow$ F has rank 2 and e_1 is its (right) null vector

basic properties of the fundamental matrix *

$x_2^T \underbrace{F}_{\substack{3\times3\\ \text{matrix}}} x_1 = 0$

$x_2 = \lambda \underbrace{\Pi_2 \hat{c}_1} + (1-\lambda)\Pi_2 \hat{x}_1$

does not depend on x_1 so all epipolar lines pass through it



① rank(F) = 2

② null vectors are the epipoles

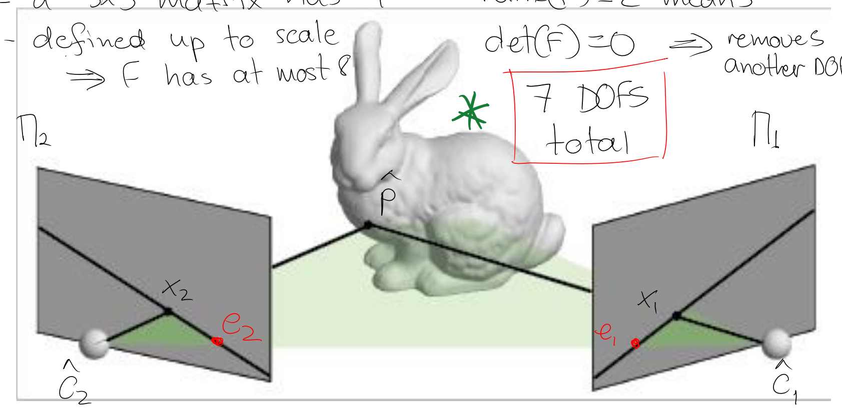
③ defined up to scale

how many degrees of freedom does F have?

- a 3x3 matrix has 9
- defined up to scale
  $\Rightarrow$ F has at most 8

- rank(F) = 2 means det(F) = 0 $\Rightarrow$ removes another DOF

$\boxed{\text{7 DOFs total}}$ *



① rank(F) = 2

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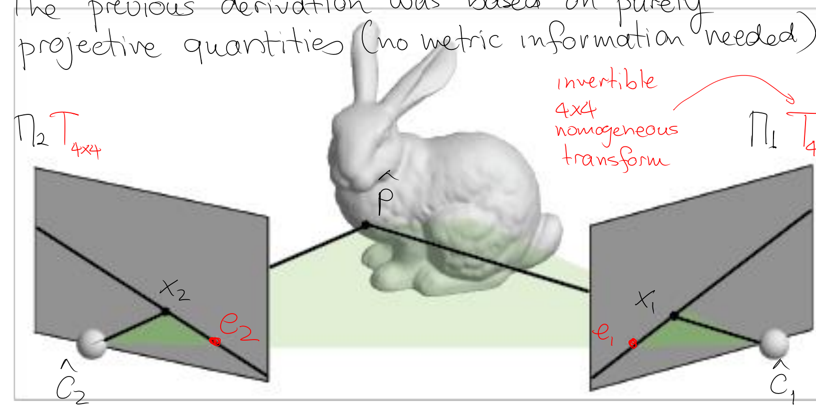
projective invariance of the F-matrix

The previous derivation was based on purely projective quantities (no metric information needed)

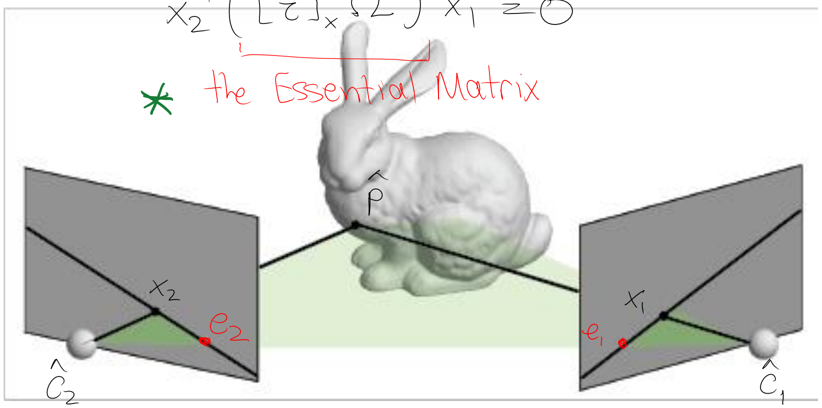
invertible 4x4 homogeneous transform



* If we multiply both Π_1 and Π_2 by T, nothing changes (it cancels out in the formula for F)

the Essential matrix

$x_2^T \underbrace{\left([\tau]_\times \Omega \right)}_{\text{the Essential Matrix}} x_1 = 0$

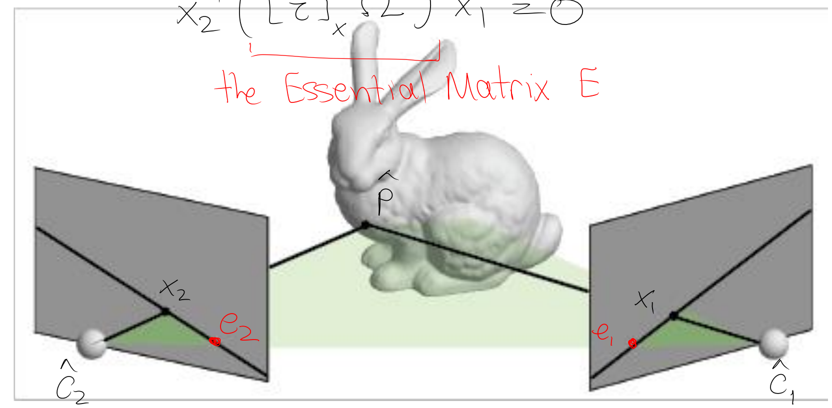


Epipolar line passes through τ and $[\Omega\ \tau]\begin{bmatrix} x_1 \\ 1 \end{bmatrix} = \Omega x_1 + \tau$

Line coordinates are $\tau \wedge (\Omega x_1 + \tau) = \tau \wedge \Omega x_1$

the Relative Orientation Problem

$x_2^T \underbrace{\left([\tau]_\times \Omega \right)}_{\text{the Essential Matrix } E} x_1 = 0$



Once we know E, we can compute Ω and τ

This is known as the relative orientation problem

details in (p.431 in Prince)

the epipolar constraint with metric camera info

Eg. assume square pixels of unit size for both images and focal length 1

(5) Let us write Π_2 as $\Pi_2 \stackrel{def}{=} [\underbrace{\Omega}_{3\times3 \text{ matrix}}\ \underbrace{\tau}_{3\times1 \text{ vector}}]$

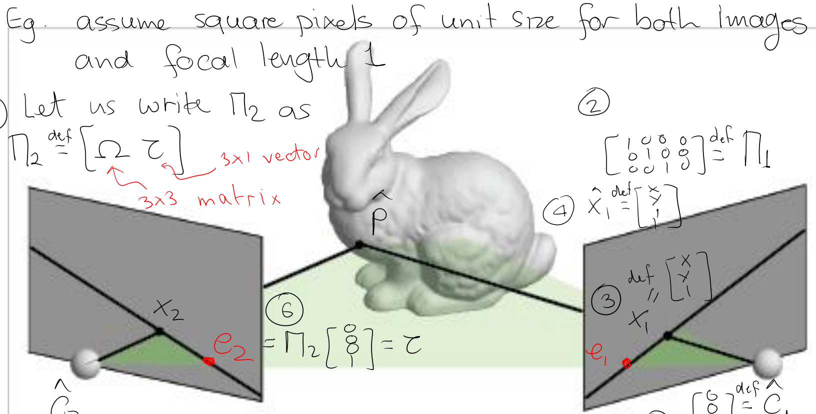
(2) $\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \end{bmatrix} \stackrel{def}{=} \Pi_1$

(4) $\hat{x}_1 \stackrel{def}{=} \begin{bmatrix} x \\ y \\ 1 \\ 1 \end{bmatrix}$

(3) $x_1 \stackrel{def}{=} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

(6) $e_2 = \Pi_2 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \tau$

(1) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \stackrel{def}{=} \hat{C}_1$

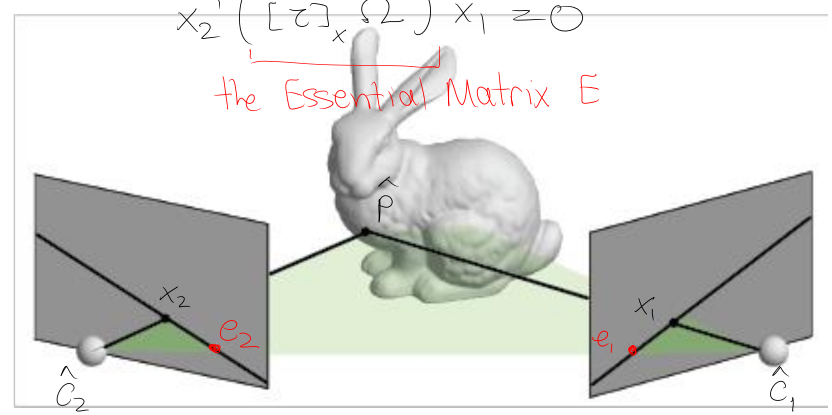


(7) Epipolar line passes through τ and $[\Omega\ \tau]\begin{bmatrix} x_1 \\ 1 \end{bmatrix} = \Omega x_1 + \tau$

* Line coordinates are $\tau \wedge (\Omega x_1 + \tau) = \tau \wedge \Omega x_1$

the Essential matrix

$x_2^T \underbrace{\left([\tau]_\times \Omega \right)}_{\text{the Essential Matrix } E} x_1 = 0$

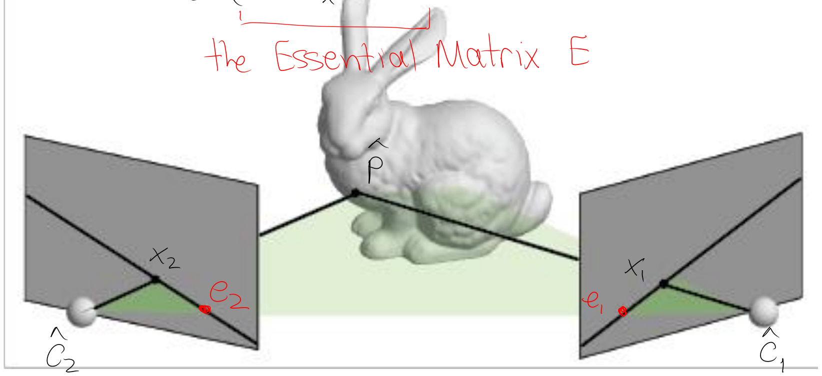


Unlike F, E has 5 degrees of freedom

* (3 translation + 3 rotation − 1 rank)

the Relative Orientation Problem

$$x_2^T \underbrace{\left([\tau]_\times \Omega \right)}_{\text{the Essential Matrix } E} x_1 = 0$$



$\Rightarrow$ just from knowing E, we can establish a 3D coordinate system in which the 3D position and orientation of both cameras is known!!

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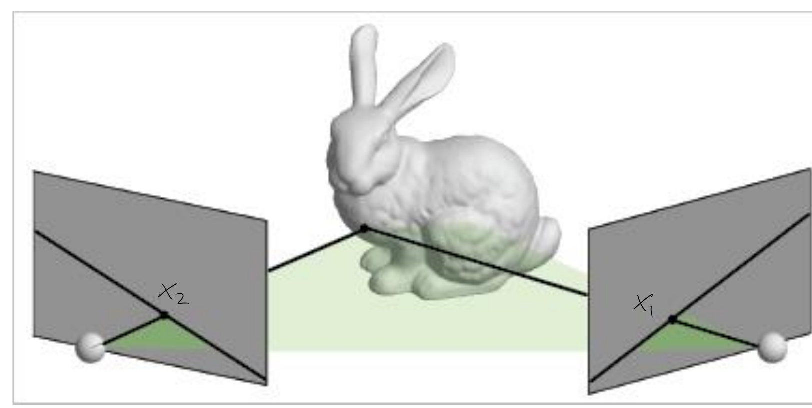
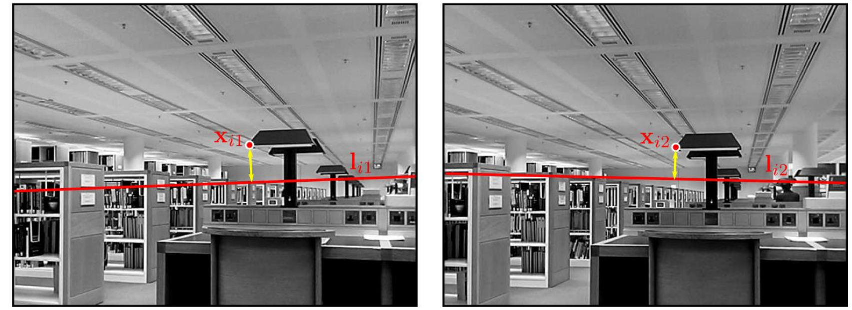
estimating the fundamental matrix



Since F (and E) are relations strictly between image points, we can estimate them from image correspondences

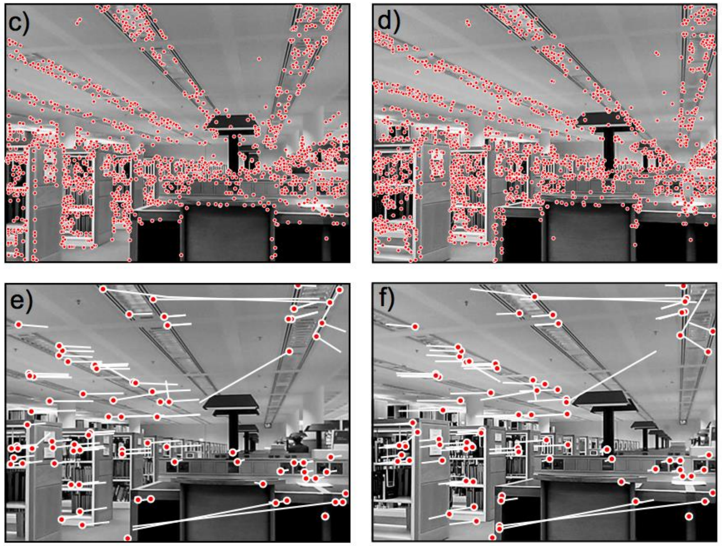
estimating the fundamental matrix



(Prince, 2011)

Since F (and E) are relations strictly between image points, we can estimate them from image correspondences

input: feature detection & approx. matches



F-matrix estimation: gold-standard algorithm

Gold Standard Approach: Suppose the noise in the point positions $\vec{x}_k^\mu$, for $\mu = L, R$ is additive, independent and normally distributed with mean zero and covariance Σ_k^μ:

$$\vec{x}_k^\mu = \vec{m}_k^\mu + \vec{n}_k^\mu, \tag{15}$$

where $\vec{m}_k^\mu$ is the true position and $\vec{n}_k^\mu$ is the noise. Note: there is no noise in the third component of $\vec{x}_k^\mu$ in homogeneous coordinates.

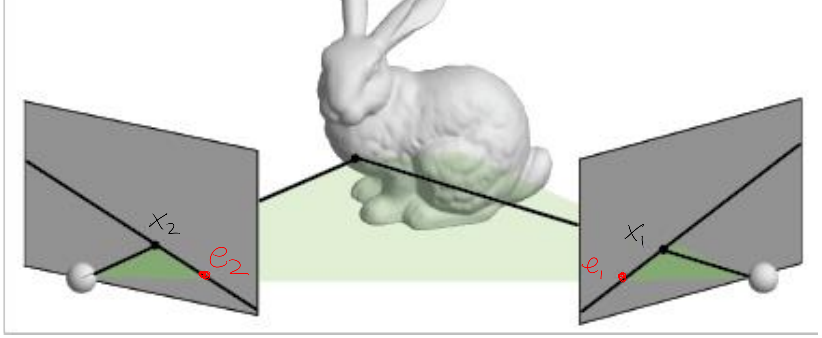
Then the (maximum likelihood) problem is to find $F \in \Re^{3\times3}$ and $\vec{m}_k^\mu$, for $k = 1, \ldots, K$ and $\mu = L, R$, such that the following objective function is minimized:

$$* \quad \mathcal{O} \equiv \sum_{\mu\in\{L,R\}} \sum_{k=1}^{K} (\vec{x}_k^\mu - \vec{m}_k^\mu)^T (\Sigma_k^\mu)^\dagger (\vec{x}_k^\mu - \vec{m}_k^\mu) \tag{16}$$

where $(\Sigma_k^\mu)^\dagger$ denotes the pseudo-inverse.

minimum # of point correspondences

What is the minimum number of correspondences to estimate F and E?



F has 7 DOFs $\Rightarrow$ minimum 7 *

E has 5 DOFs $\Rightarrow$ minimum 5 *

* Each correspondence gives 1 constraint equation $x_2^T F x_1 = 0$

gold standard: minimizing “reprojection error”

Gold Standard Approach: Suppose the noise in the point positions $\vec{x}_k^\mu$, for $\mu = L, R$ is additive, independent and normally distributed with mean zero and covariance Σ_k^μ:

$$\vec{x}_k^\mu = \vec{m}_k^\mu + \vec{n}_k^\mu, \tag{15}$$

where $\vec{m}_k^\mu$ is the true position and $\vec{n}_k^\mu$ is the noise. Note: there is no noise in the third component of $\vec{x}_k^\mu$ in homogeneous coordinates.

F-matrix estimation: gold-standard algorithm

subject to

$$(\vec{m}_k^L)^T F \vec{m}_k^R = 0, \quad k = 1, \ldots, K, \tag{17}$$

$$\mathrm{rank(F)} = 2 \tag{18}$$

Thus, $\mathcal{O}$ is a quadratic objective function for the $\vec{m}_k^\mu$'s, with nonlinear constraints (17) and (18).

Then the (maximum likelihood) problem is to find $F \in \Re^{3\times3}$ and $\vec{m}_k^\mu$, for $k = 1, \ldots, K$ and $\mu = L, R$, such that the following objective function is minimized:

$$\mathcal{O} \equiv \sum_{\mu\in\{L,R\}} \sum_{k=1}^{K} (\vec{x}_k^\mu - \vec{m}_k^\mu)^T (\Sigma_k^\mu)^\dagger (\vec{x}_k^\mu - \vec{m}_k^\mu) \tag{16}$$

where $(\Sigma_k^\mu)^\dagger$ denotes the pseudo-inverse.

⇒ this is a non-linear problem

easier alternative: minimizing algebraic error

Going back to the epipolar constraint

$$X_2^T \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} X_1 = 0$$

- Ignoring noise, this constraint is <u>linear</u> in the elements of F

the 8-point algorithm

We have a matrix A determined from the point correspondences

$$* \quad A \begin{bmatrix} F_{11} \\ F_{12} \\ \vdots \\ F_{33} \end{bmatrix} = 0$$

- Ignoring noise, this constraint is <u>linear</u> in the elements of F

the 8-point algorithm

We have a matrix A determined from the point correspondences

$$A \begin{bmatrix} F_{11} \\ F_{12} \\ \vdots \\ F_{33} \end{bmatrix} = 0$$

... but scaling is poor because typically pixels have $(\underset{[0,1000]}{x}, \underset{[0,1000]}{y}, 1)$ always 1

the normalized 8-point algorithm ✳

Hartley (PAMI, 1997) introduced the following algorithm. Given corresponding points $\{(\vec{x}_k^L, \vec{x}_k^R)\}_{k=1}^K$ with $K \geq 8$,

1. Recenter and rescale the image points using M^μ, $\mu = L, R$, such that

$$M^\mu = \begin{pmatrix} s^\mu & 0 & b_1^\mu \\ 0 & s^\mu & b_2^\mu \\ 0 & 0 & 1 \end{pmatrix}, \qquad (24)$$

with

$$\frac{1}{K}\sum_{k=1}^{K} M^\mu \vec{x}_k^\mu = (0,0,1)^T, \qquad (25)$$

$$\frac{1}{K}\sum_{k=1}^{K} [M^\mu \vec{x}_k^\mu - (0,0,1)^T]_*^2 = (\sigma_1^2, \sigma_2^2, 0)^T, \qquad (26)$$

where $\sigma_1^2 + \sigma_2^2 = 2$. Here $[...]_*^2$ denotes the square of each element. Define $\vec{r}_k^\mu = M^\mu \vec{x}_k^\mu$ for $k = 1, \ldots, K$ and $\mu = L, R$.

the normalized 8-point algorithm

2. Minimize the objective function $\mathcal{O}(\hat{F})$

$$\mathcal{O}(\hat{F}) \equiv \sum_{k=1}^{K} \left[(\vec{r}_k^L)^T \hat{F} \vec{r}_k^R \right]^2. \qquad (27)$$

Note this is a linear least squares problem for the elements of $\hat{F}$.

the normalized 8-point algorithm

2. Minimize the objective function $\mathcal{O}(\hat{F})$

$$\mathcal{O}(\hat{F}) \equiv \sum_{k=1}^{K} \left[(\vec{r}_k^L)^T \hat{F} \vec{r}_k^R \right]^2. \qquad (27)$$

Note this is a linear least squares problem for the elements of $\hat{F}$.

3. Project $\hat{F}$ to the nearest rank 2 matrix (with the error measured in the Frobenius norm):
   (a) Form the SVD of $\hat{F} = U\Sigma V^T$. In general $\Sigma = \mathrm{diag}[\sigma_1^2, \sigma_2^2, \sigma_3^2]$ with $\sigma_i^2 \geq \sigma_{i+1}^2$ for $i = 1, 2$.
   (b) Reset $\sigma_3 = 0$.
   (c) Assign $\hat{F}$ to be $U\Sigma V^T$.
4. Undo the normalization of the image points,

$$F = (M^L)^T \hat{F} M^R \qquad (28)$$

F-estimation in the presence of outlies

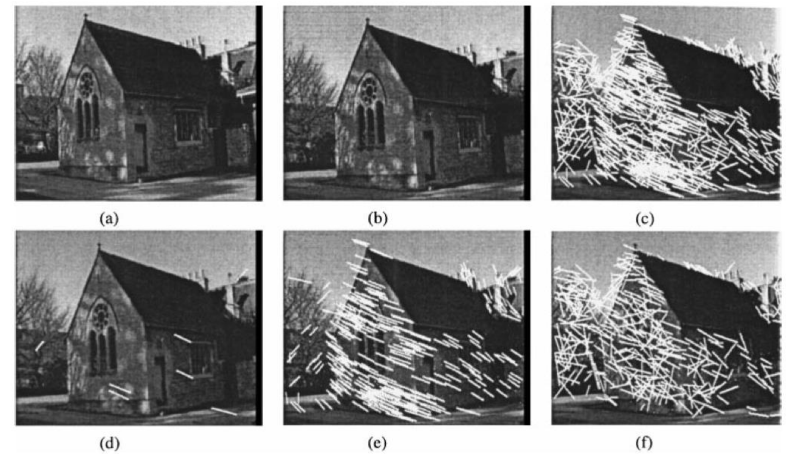
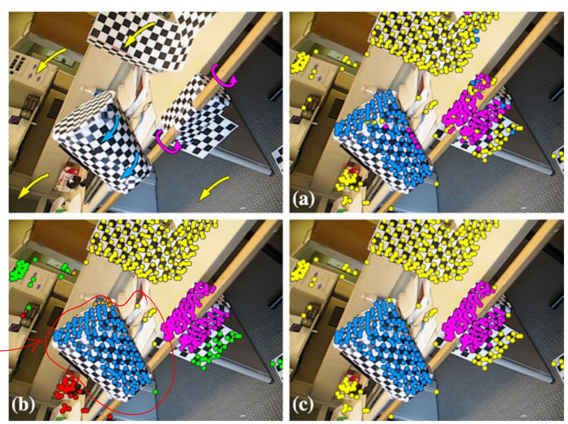


FIG. 4. (a), (b) First two images of chapel sequence, (c) matches, (d) basis supplied by MLESAC, (e) the inliers, and (f) outliers.

From Torr & Zissermann, MLESAC, CVIU v.78, n.1, 2000
(we will discuss this method later in the course)

F-matrix estimation & 3D motion analysis

image points consistent with rigid-body 3D motion



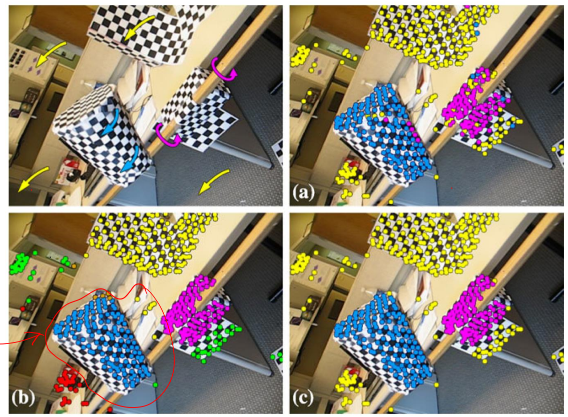
(Delong et al, IJCV12)

Static scene + moving camera $\Longleftrightarrow$ static camera + one rigidly-moving object

one F-matrix applies to any pair of images

F-matrix estimation & motion segmentation

image points consistent with rigid-body 3D motion



(Delong et al, IJCV12)

K F-matrices for every image pair (one per object)

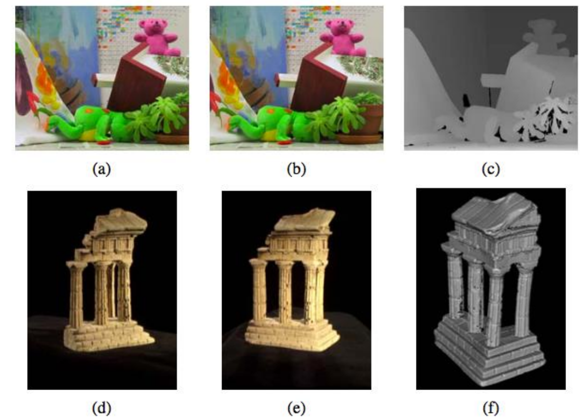
static camera + K rigidly-moving objects

Topic 03:

Two-View Geometry

- epipolar lines & epipolar planes
- the fundamental matrix (F-matrix)
- geometric interpretation of the F-matrix
- the essential matrix
- fundamental matrix estimation: the normalized 8-point algorithm
- algorithms & applications of stereo reconstruction
- image homographies
- stereo image rectification

Algorithms for stereo-based 3D reconstruction



Ch.11 of Szeliski's book discusses many techniques (not required reading)

Topic 03:

Two-View Geometry

- epipolar lines & epipolar planes
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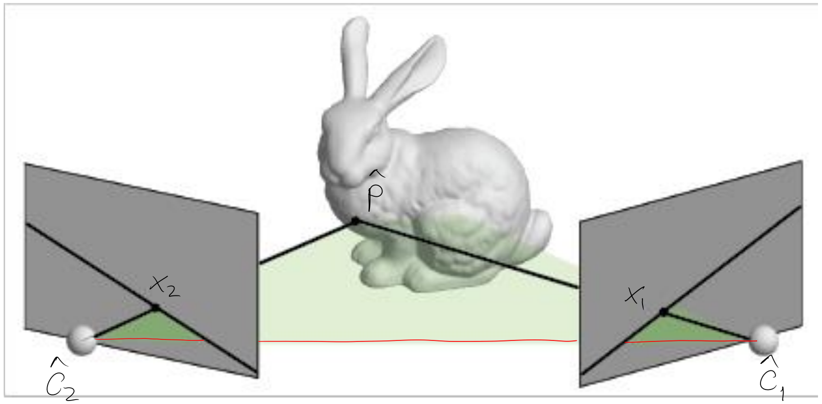
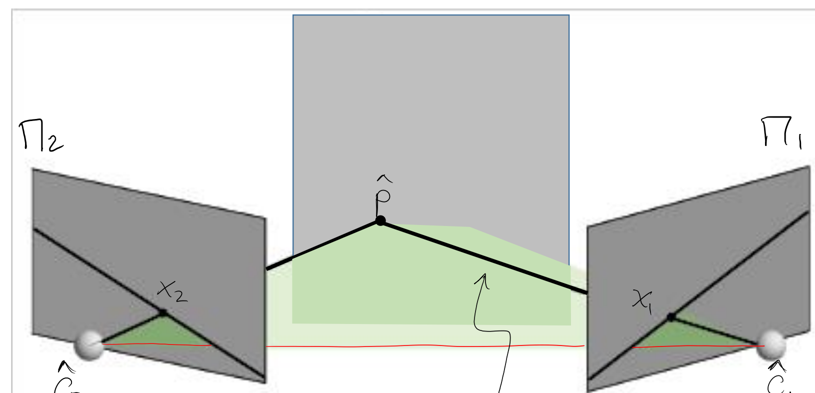
- fundamental matrix estimation: the normalized 8-point algorithm
- algorithms & applications of stereo reconstruction
- **image homographies**
- stereo image rectification

Special case: geometry of two views of a plane



Special case: geometry of two views of a plane

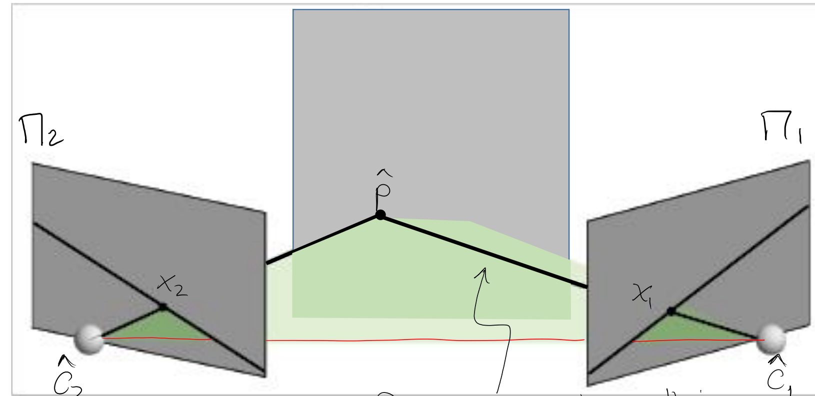


① Typical point on this ray

$$\hat{P} = \lambda \hat{C}_1 + \hat{x}_1$$

homogeneous 3D coords of image point x_1

Special case: geometry of two views of a plane



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$$\hat{P} = \lambda \hat{C}_1 + \hat{x}_1$$

homogeneous 3D coords of image point x_1

② More generally:

$$\hat{P} = \lambda \hat{C}_1 + \Pi_1^{*} x_1$$

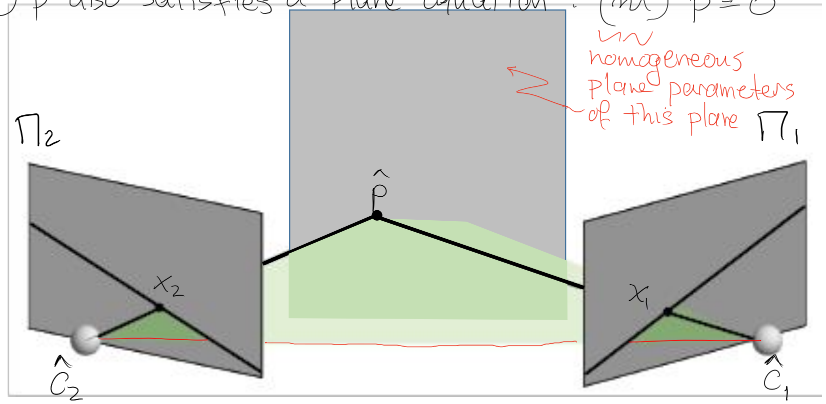
vector in nullspace of Π_1

pseudo-inverse of Π_1

Special case: geometry of two views of a plane

③ $\hat{p}$ also satisfies a plane equation : $(\hat{m})^T \hat{p} = 0$

homogeneous plane parameters of this plane



② More generally:

$\hat{p} = \lambda \hat{c}_1 + \Pi_1^* x_1$

vector in nullspace of Π_1

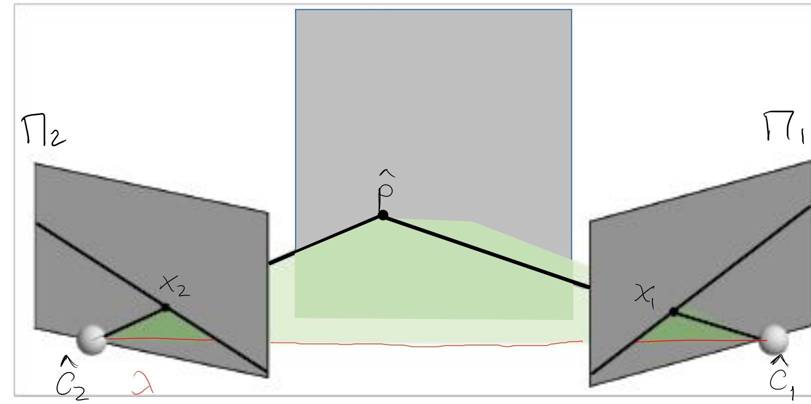
pseudo-inverse of Π_1

④ Combining relations:

$(\hat{m})^T (\lambda \hat{c}_1 + \Pi_1^* x_1) = 0$

$\Rightarrow \lambda = -(\hat{m})^T \Pi_1^* x_1 / (\hat{m})^T \hat{c}_1$

Special case: geometry of two views of a plane

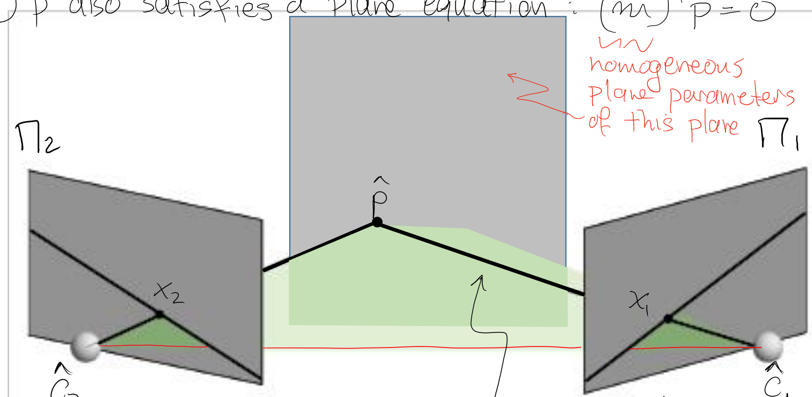


$$\hat{p} = \underbrace{-\frac{1}{(\hat{m})^T \hat{c}_1} (\hat{m})^T \Pi_1^* x_1}_{\lambda} \hat{c}_1 + \Pi_1^* x_1 = \underbrace{\left[I - \frac{1}{(\hat{m})^T \hat{c}_1} \cdot \hat{c}_1 (\hat{m})^T \right] \Pi_1^*}_{3\times 4 \text{ matrix}} x_1$$

Special case: geometry of two views of a plane

③ $\hat{p}$ also satisfies a plane equation : $(\hat{m})^T \hat{p} = 0$

homogeneous plane parameters of this plane



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$\hat{P} = \lambda \hat{c}_1 + \hat{x}_1$

homogeneous 3D coords of image point x_1

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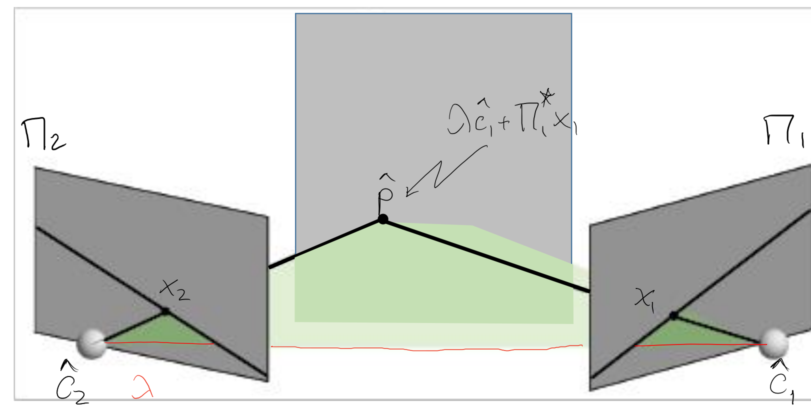
Special case: geometry of two views of a plane



$$\hat{p} = \underbrace{-\frac{1}{(\hat{m})^T \hat{c}_1} (\hat{m})^T \Pi_1^* x_1}_{\lambda} \hat{c}_1 + \Pi_1^* x_1$$

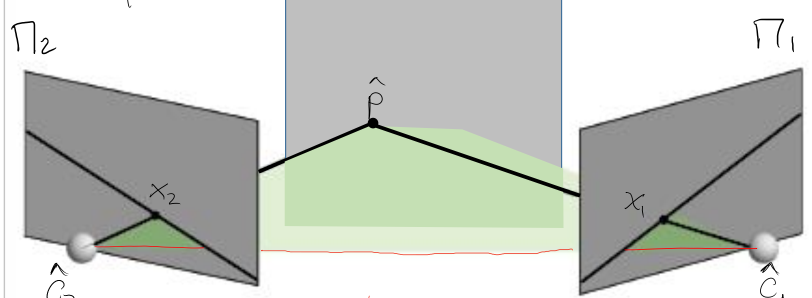
④ Combining relations:

$(\hat{m})^T (\lambda \hat{c}_1 + \Pi_1^* x_1) = 0$

$\Rightarrow \lambda = -(\hat{m})^T \Pi_1^* x_1 / (\hat{m})^T \hat{c}_1$

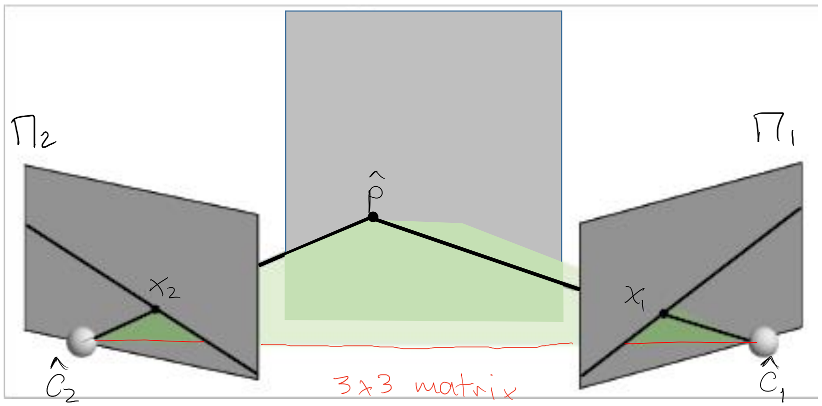
The image-to-3D-plane transformation

This 3x4 matrix maps homogeneous image coordinates to the homogeneous 3D coordinates of a <u>unique</u> point on the 3D plane



$$\underbrace{\left[I - \frac{1}{(\hat{m})^T \hat{C}_1} \hat{C}_1 (\hat{m})^T \right] \Pi_1^*}_{3\times 4 \text{ matrix}}$$

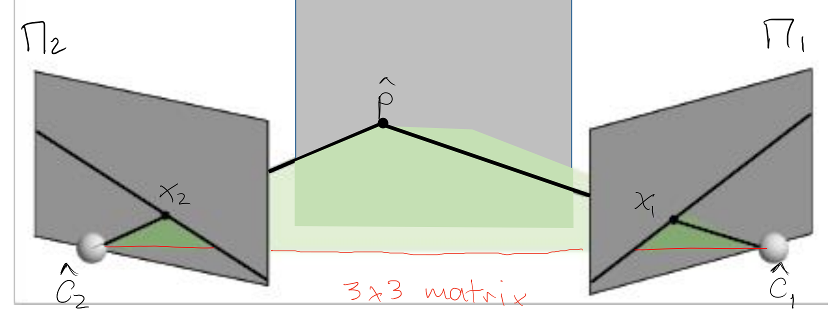
The image of a plane point in a second camera



$$x_2 = \underbrace{\Pi_2 \left[I - \frac{1}{(\hat{m})^T \hat{C}_1} \hat{C}_1 (\hat{m})^T \right] \Pi_1^*}_{3\times 3 \text{ matrix}} x_1$$

The homography between two views of a plane

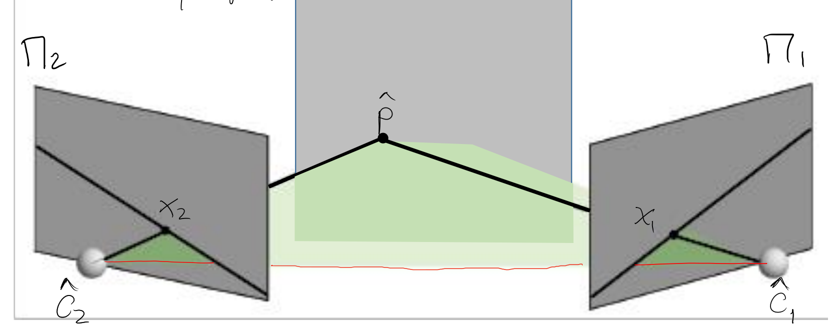
The matrix H defines an invertible linear mapping from the projection of plane points in image 1 to their projection in image 2.



$$H = \underbrace{\Pi_2 \left[I - \frac{1}{(\hat{m})^T \hat{C}_1} \hat{C}_1 (\hat{m})^T \right] \Pi_1^*}_{3\times 3 \text{ matrix}}$$

The homography between two views of a plane

The matrix H defines an invertible linear mapping from the projection of plane points in image 1 to their projection in image 2.



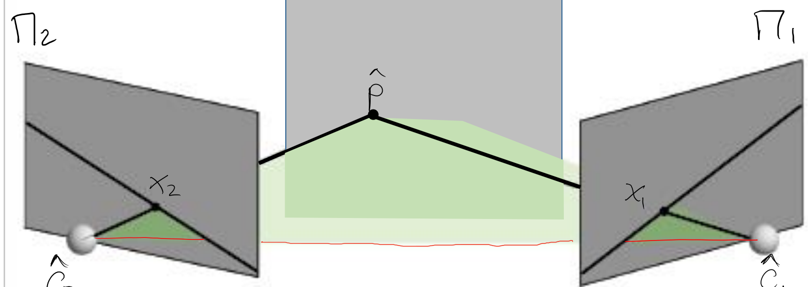
homogeneous coords of $\hat{p}$'s projection in image 2

$$x_2 = \underbrace{H}_{\substack{3\times 3 \text{ homography} \\ \text{matrix}}} x_1$$

homogeneous 2D coords of $\hat{p}$'s projection in image 1

Computing the homography matrix

Corollary (prove it!): we can compute H by solving a linear system if we know the projection of at least 4 points on the plane in the two views.



homogeneous coords of $\hat{p}$'s projection in image 2

$$x_2 = H x_1$$

3x3 homography matrix

homogeneous 2D coords of $\hat{p}$'s projection in image 1

Example: creating an image mosaic

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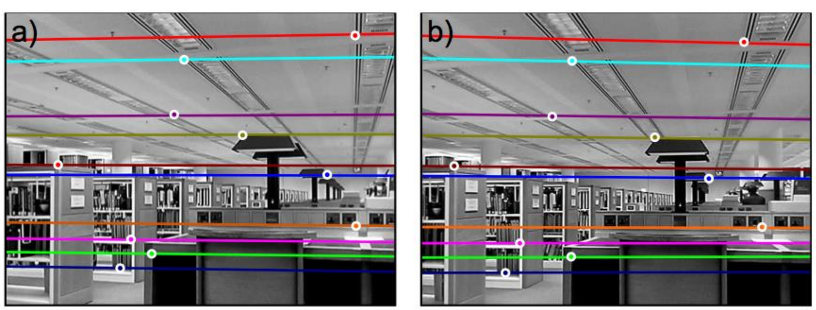
a stereo image pair

Left

Right

(Prince, 2011)

the epipolar constraint



(Prince, 2011)

The Epipolar Constraint

For every pair of perspective photos, there exists a line-to-line mapping between them such that scene points projecting to one line will also project to the other

stereo image rectification

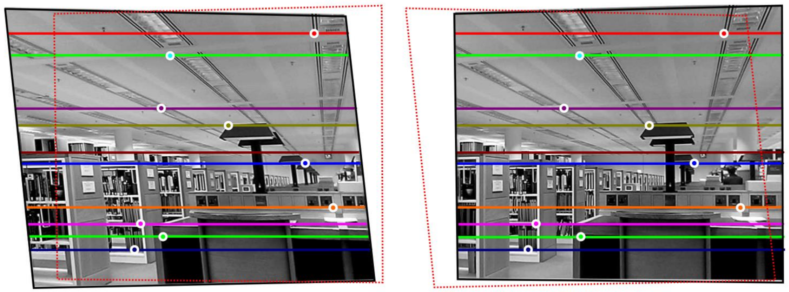
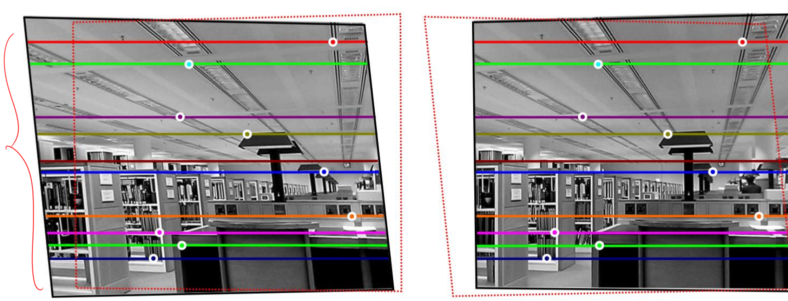


Figure 16.9 Planar rectification. The images from figures 16.6 and 16.7 have been rectified by applying homographies. After rectification, each point induces an epipolar line in the other image that is horizontal and on the same scanline (compare to figure 16.7a). This means that the match is guaranteed to be on the same scanline. In this figure, the red dotted line is the superimposed outline of the other image.

Once F (or E) are known, we can warp the images so that corresponding epipolar lines line up

stereo image rectification

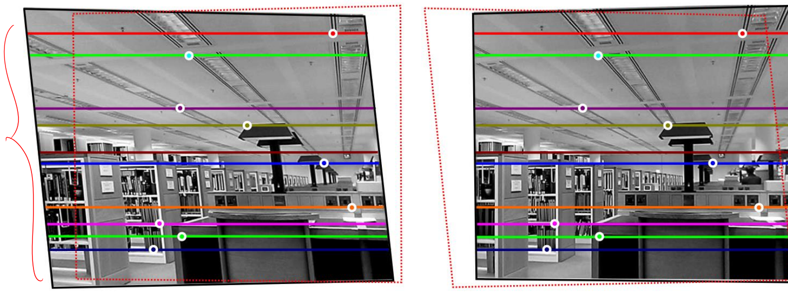


the warp sends both epipoles to infinity

Idea: find homographies H_1, H_2 such that

$$H_2^T F H_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

stereo image rectification



the warp sends both epipoles to infinity

see 16.5.1 in Prince for details.