Topic 08:

Radiometry

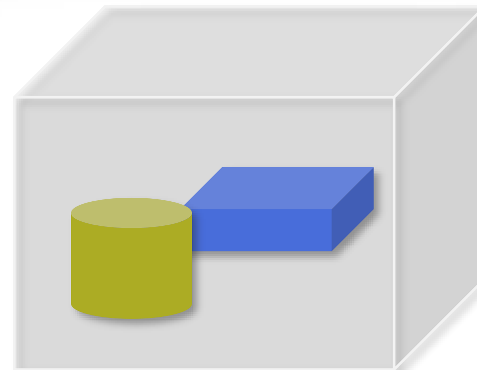
- The big picture
- Measuring light coming from a light source
  - Measurements for a “2D world”
  - Generalization to 3D
- Measuring light falling onto a patch: Irradiance
- Measuring light leaving a patch: Radiance
- The Bidirectional Reflectance Distribution Function
- Phong Reflectance Model

image formation

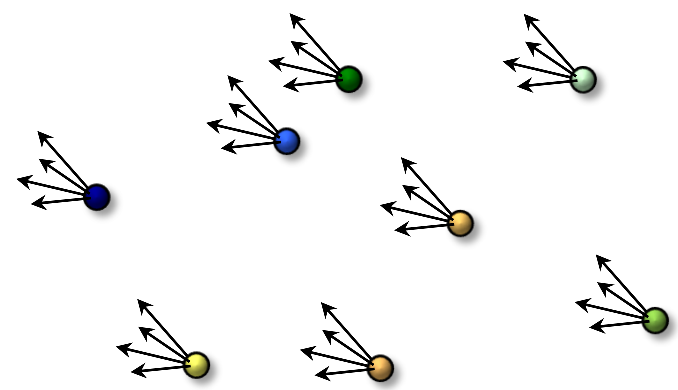
and a camera

how do we express mathematically
the captured photo?

given a (static) 3D scene

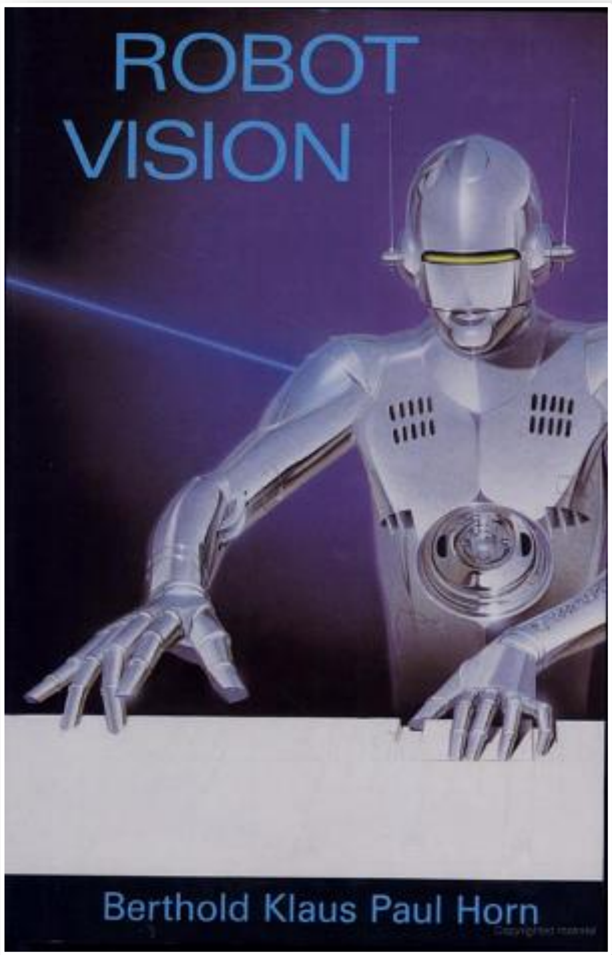


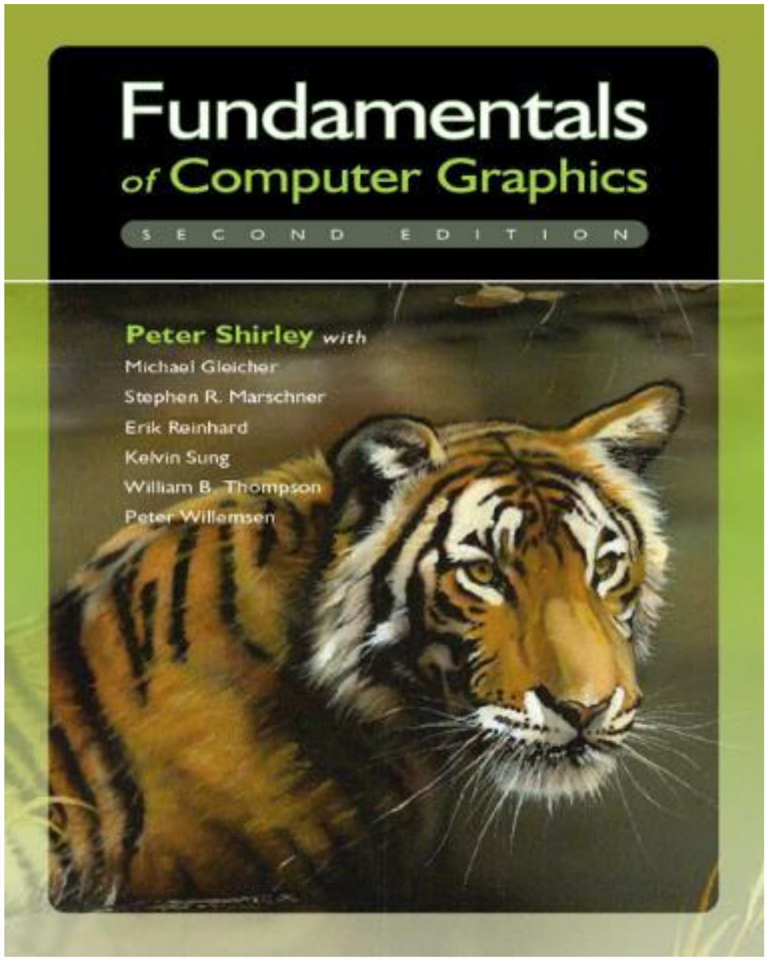
a set of light sources



what is a photo?

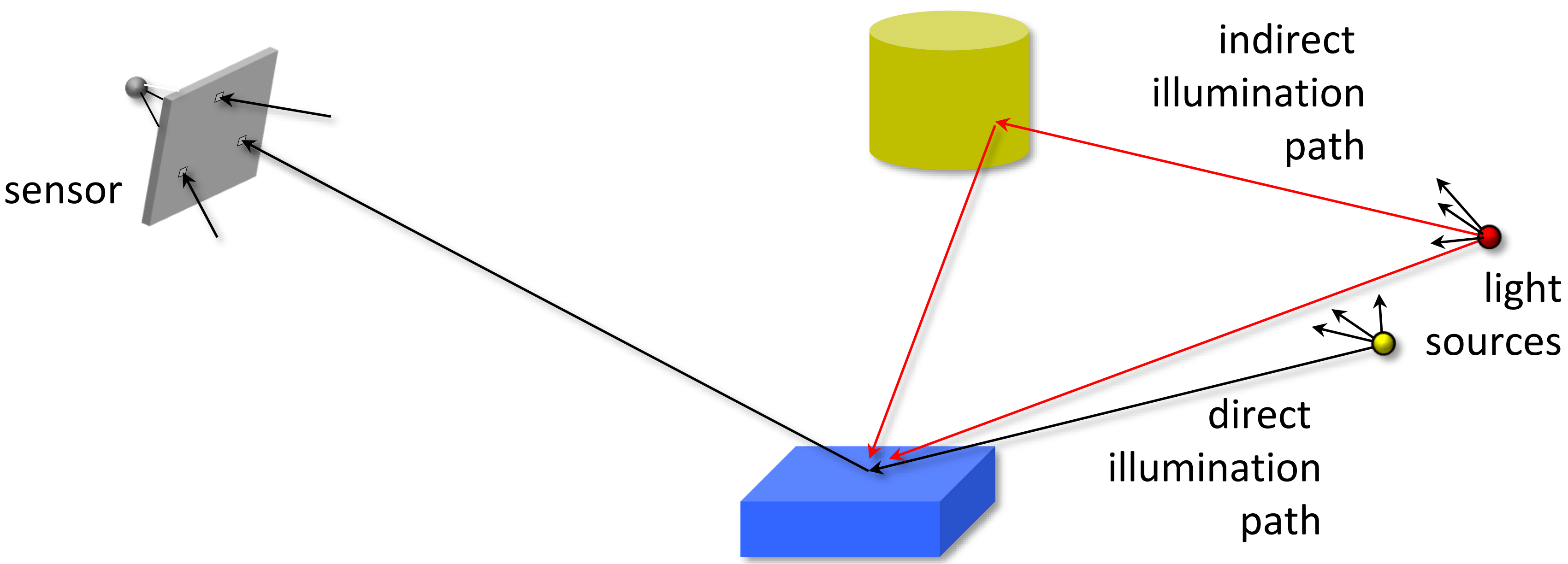
answer 1: the result of light transport from light sources to pixels





what is a photo?

answer 1: the result of light transport from light sources to pixels

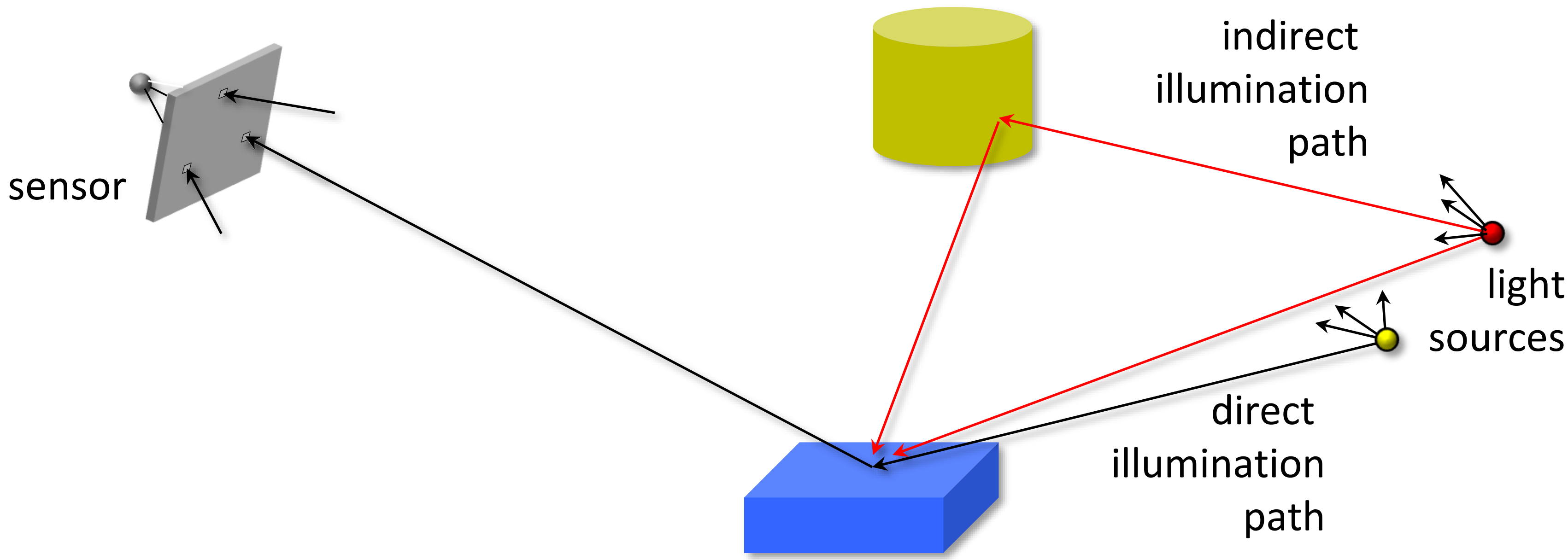


what is a photo?

answer 2: a 2D array of photon counts recorded by camera sensor

in this course...

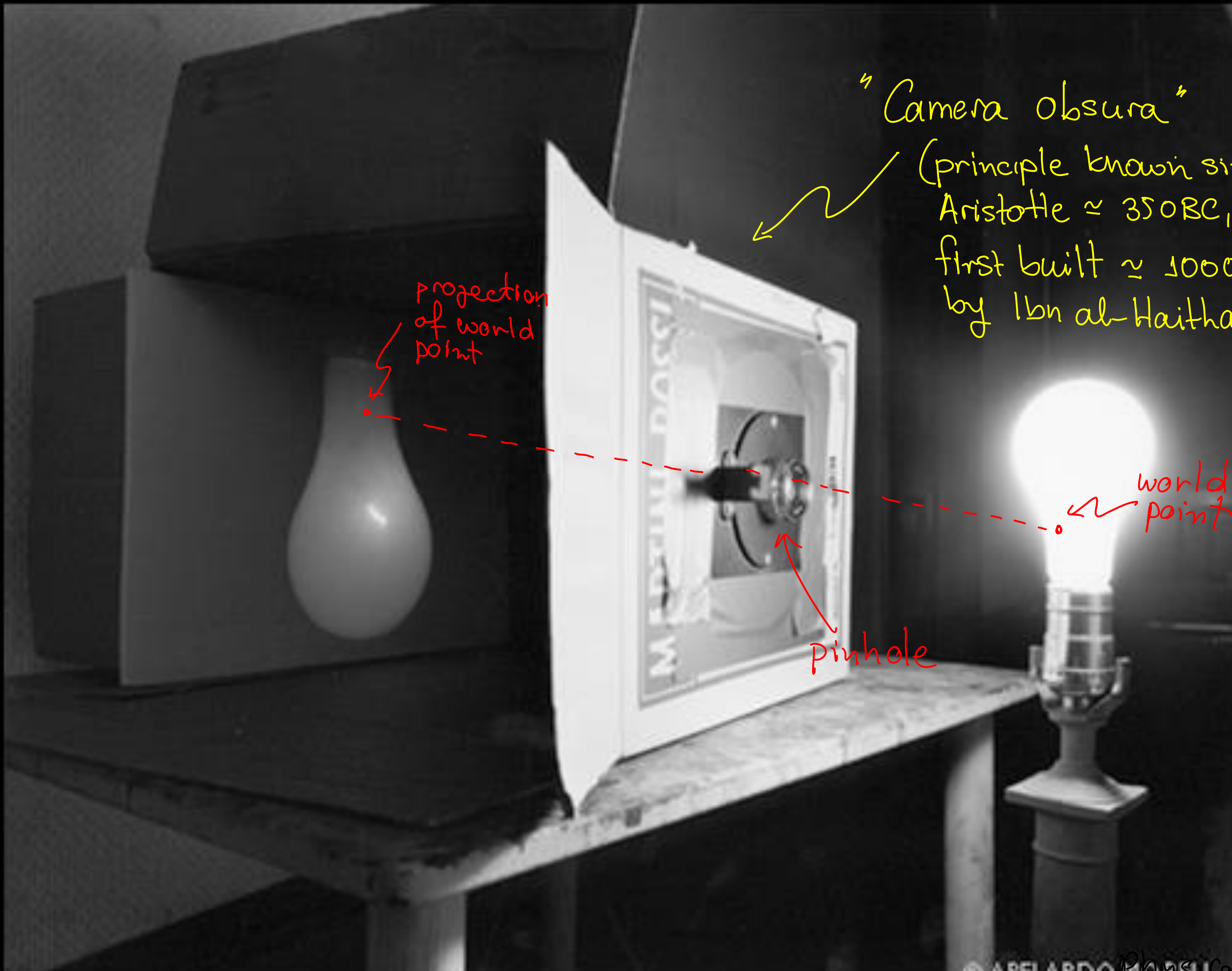
ignore quantum nature of light, ignore indirect illumination (mostly)



Two steps in modeling image formation:

projection: quantifying the 2D image location of a scene point

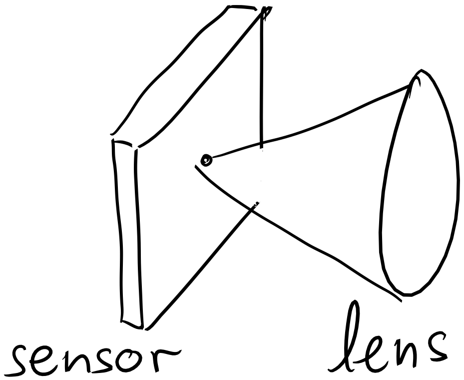
radiometry: quantifying the appearance of a scene point

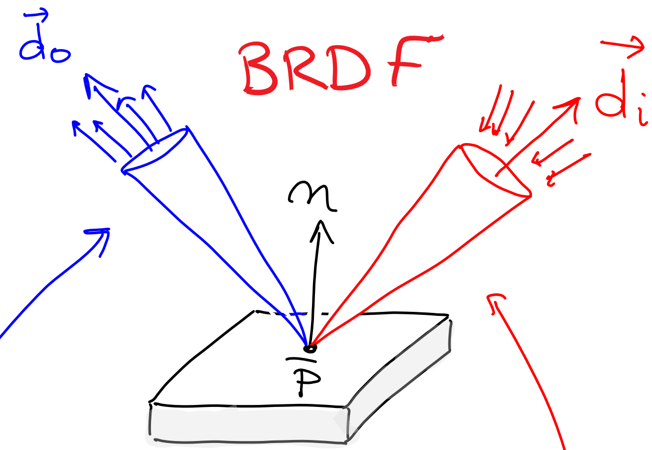


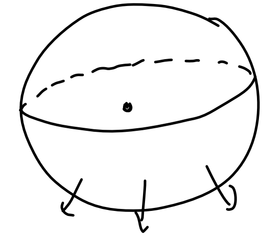
Light Bulb, 1991

www.lensculture.com/morell.html

The Basic "Light Transport" Path







① light sources emit light along a continuum of directions

radiant intensity

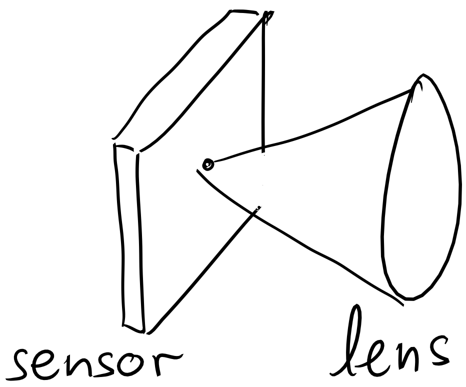
② light converges onto a point from a continuous "cone" of directions

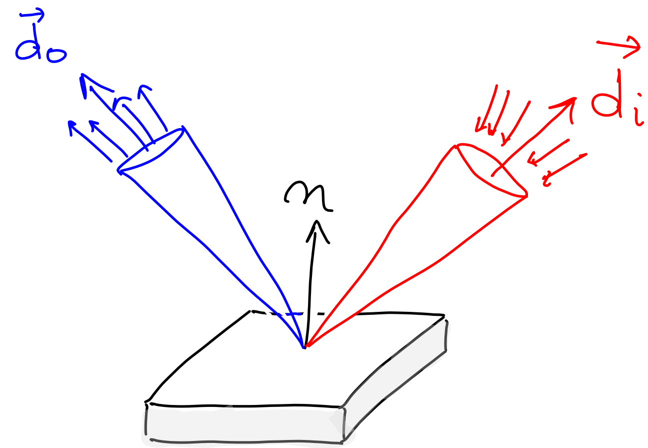
irradiance

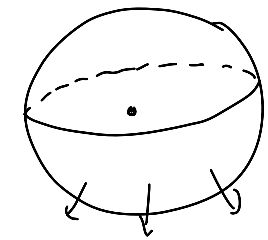
③ light is reflected from a point to a continuous "cone" of directions

radiance

Radiometry: Getting the Physics Right







Radiometry: measurement of electromagnetic radiation

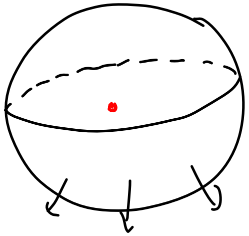
Physics:
- Joules
- Watts

Geometry:

differential
- patches
- directions
- solid angles
- foreshortened area

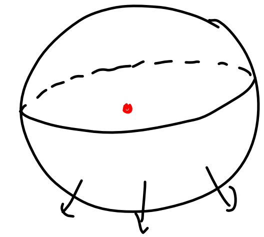
Radiometry:
- radiant energy
- radiant flux
- irradiance
- radiance
- radiant exitance
- BRDF

The Basic "Light Transport" Path



light sources emit light along a continuoum of directions

Radiant Energy & Radiant Flux

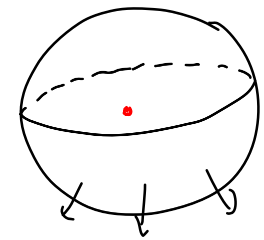


light source emits radiant flux Φ (in Watts)

- Light is energy (i.e. photons)
  - ⇒ measured in Joules (J) (4×10^{-19} J per photon)
  - ⇒ energy emitted by a light source is called radiant energy
- We are interested in "steady-state" conditions (energy per unit time interval)
  - ⇒ energy per unit time (a.k.a power)
  - ⇒ measured in Watts (= J/sec)
  - ⇒ called radiant flux Φ

Measuring Light Emitted from a (Point) Source

light emitted
by the light source falls
onto an object surface.



light source
emits radiant
flux Φ (in Watts)

Q: How do we measure
emission along specific
directions?

Q: How do we measure
light received at a
point on a surface?

Q: What units should we use?

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Flux Through an Arc (2D world, Uniform Source)

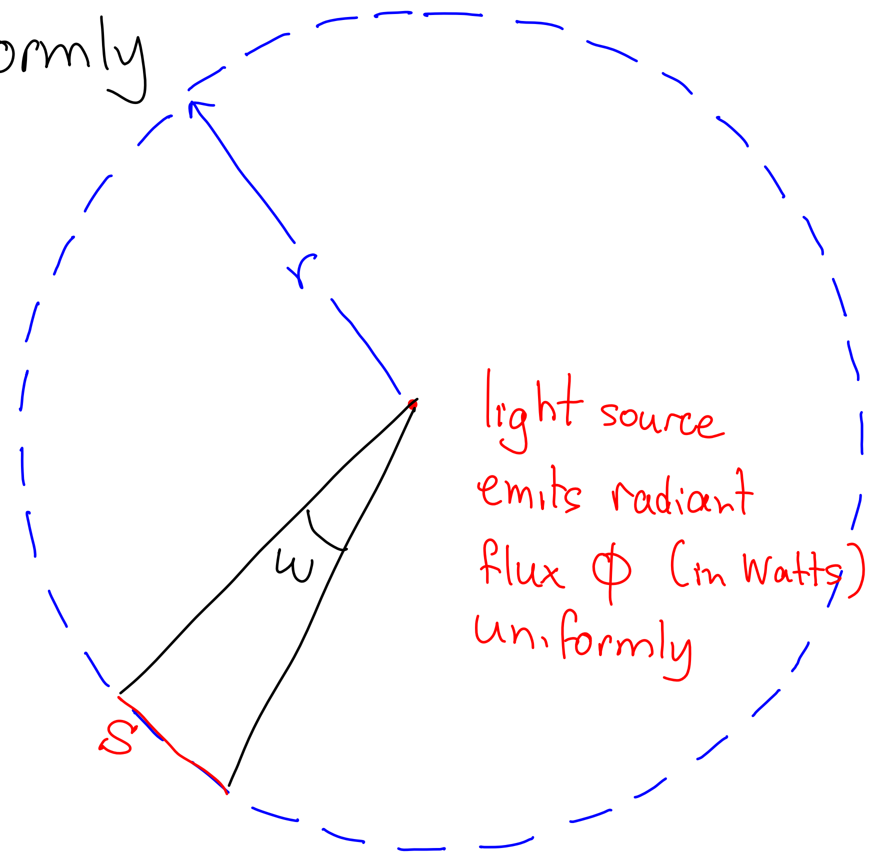
- Suppose source emits uniformly in all directions
- How much flux flows through arc S'?

$$\Phi \, \frac{\text{length of } S'}{\text{perimeter}} =$$

$$\Phi \, \frac{\omega \cancel{r}}{2\pi \cancel{r}} = \omega \, \frac{\Phi}{2\pi}$$

measured in radians (rad)

measured in Watts per radian (W/rad)



light source emits radiant flux Φ (in Watts) uniformly

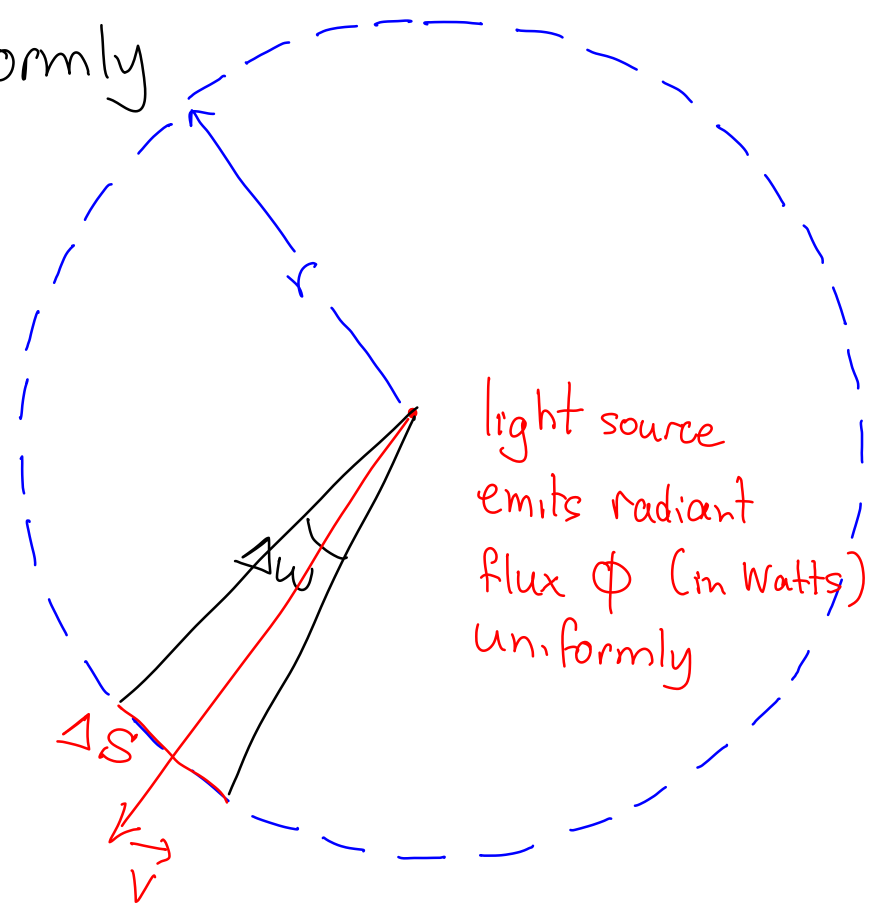
Flux Along a Direction (2D world, Uniform Source)

- Suppose source emits uniformly in all directions
- How much flux flows along direction v?
  - Consider a small arc ΔS centered at $\vec{v}$

$$\Delta\Phi = \Phi \frac{\text{length } \Delta S}{\text{perimeter}} = \Delta\omega \frac{\Phi}{2\pi}$$

- Define flux along $\vec{v}$ to be the limit of $\Delta\Phi$ as $\Delta\omega \to 0$:

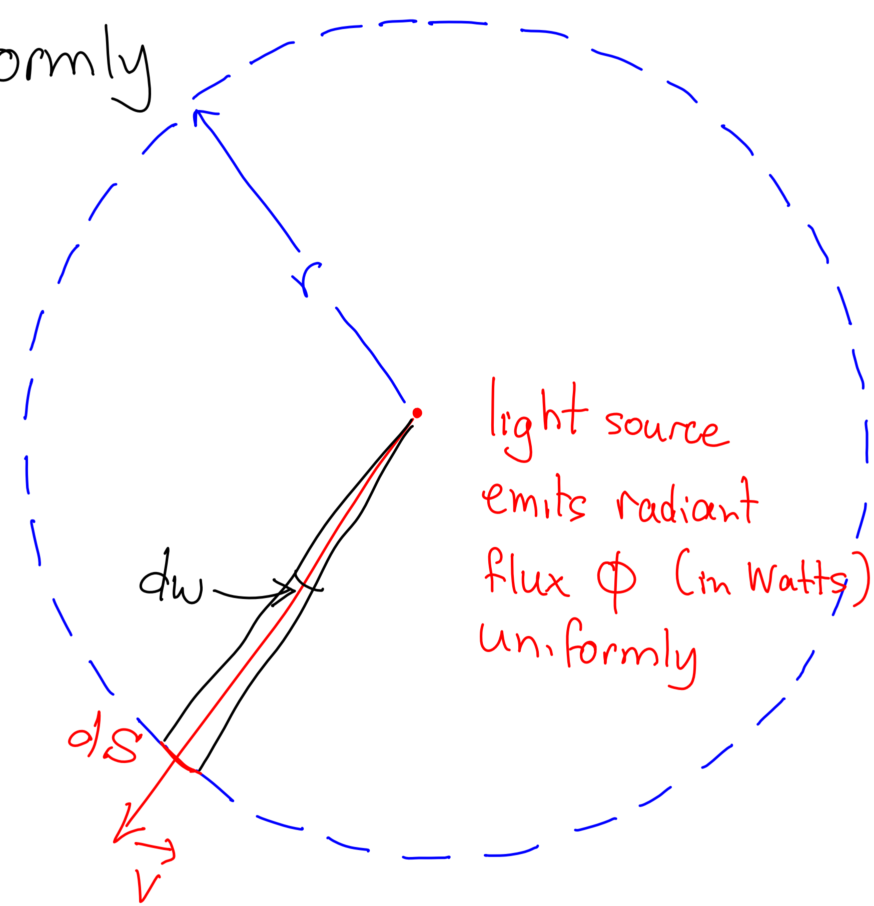
$$d\Phi = d\omega \frac{\Phi}{2\pi} \stackrel{\text{def}}{=} \lim_{\Delta\omega \to 0} \Delta\omega \frac{\Phi}{2\pi}$$



Flux Along a Direction (2D world, Uniform Source)

- Suppose source emits uniformly in all directions

- How much flux flows along direction $\vec{v}$?



* Ans: A differential flux $d\Phi$:

$$\underbrace{d\Phi}_{\text{differential flux}} = \underbrace{d\omega}_{\text{differential angle}} \frac{\Phi}{2\pi}$$

differential flux

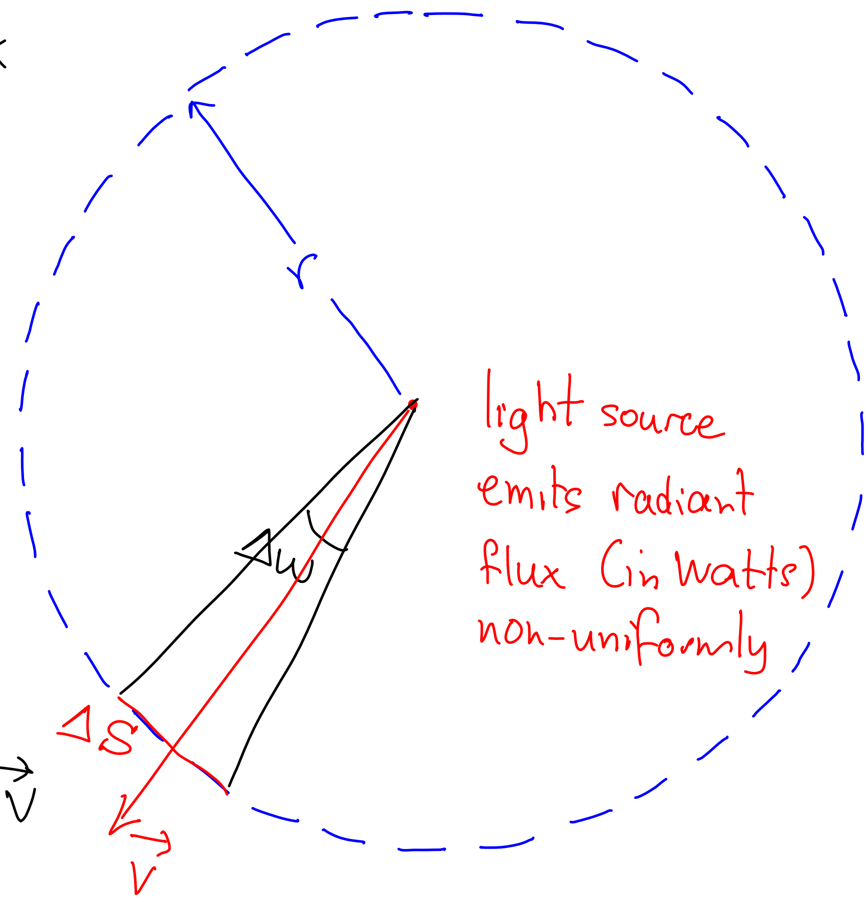
differential angle

these are both infinitesimal quantities!

$\frac{\Phi}{2\pi}$ measured in Watts per radian (W/rad)

Flux Along a Direction (2D, Non-Uniform Src)

- Suppose source emits flux non-uniformly.
- How do we quantify the source's emission "strength" in a given direction?
  - Consider a small arc ΔS centered at $\vec{v}$
  - To describe the light source's emission along $\vec{v}$ we need the fraction



$$\frac{\text{watts}}{\text{radian}} \left\{ \frac{\Delta\Phi}{\Delta\omega} \right.$$

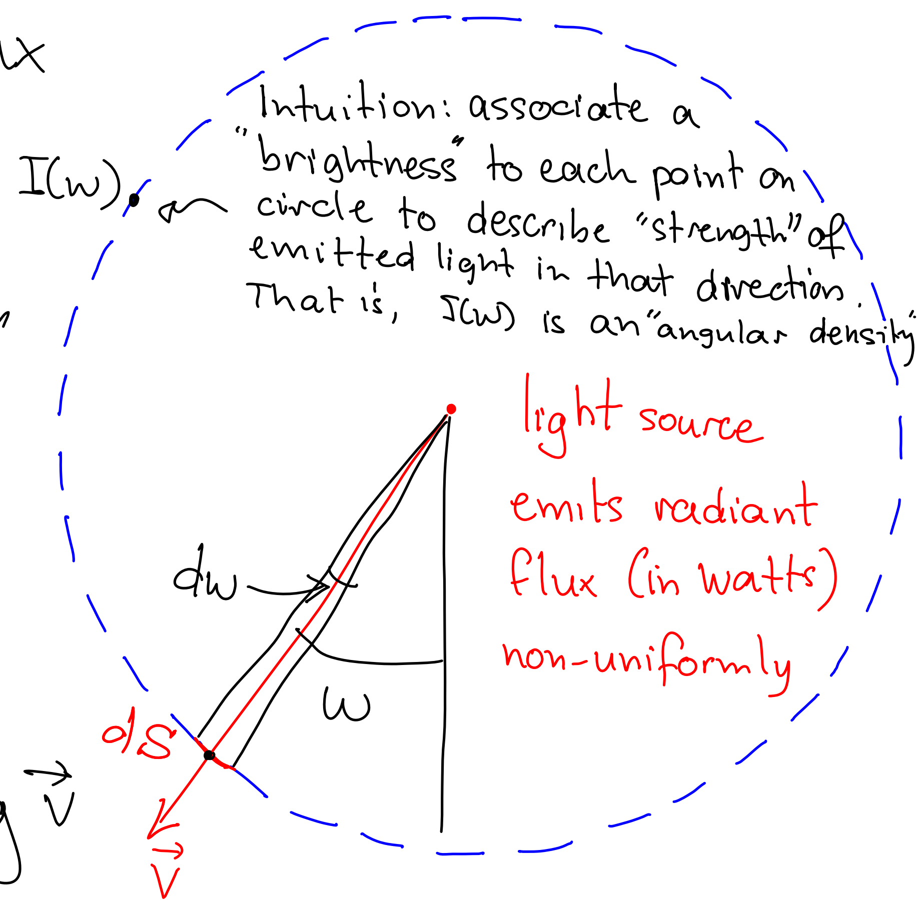
$\Delta\Phi$ ← flux "sent" through angle $\Delta\omega$

$\Delta\omega$ ← angle $\Delta\omega$

as $\Delta\omega \to 0$

Flux Along a Direction: Radiant Intensity

- Suppose source emits flux non-uniformly.
- How do we quantify the source's emission "strength" in a given direction?
  - Consider a small arc ΔS centered at $\vec{v}$
  - To describe the light source's emission along $\vec{v}$ we need the limit



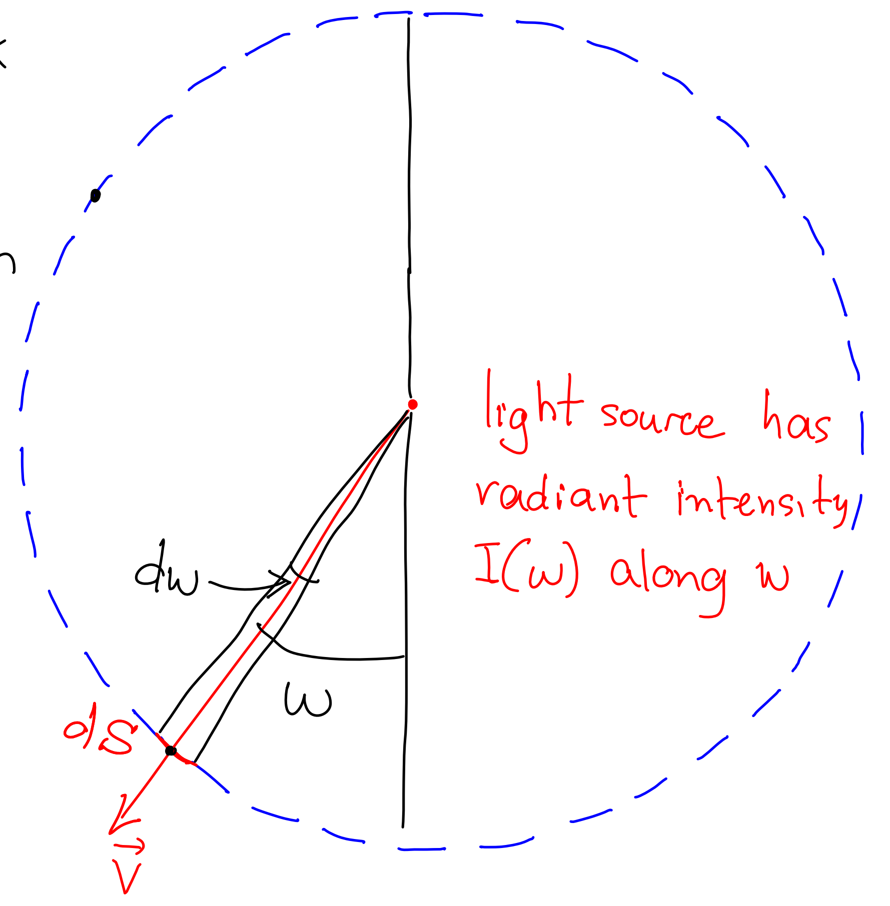
$$\lim_{\Delta \omega \to 0} \frac{\Delta \Phi}{\Delta \omega} = \boxed{\frac{d\Phi}{d\omega} = I(\omega)}$$

Radiant intensity of source in direction ω $\left(\frac{\text{watts}}{\text{radian}}\right)$

Flux Along a Direction: Radiant Intensity

- Suppose source emits flux non-uniformly.
- Its emission strength given by radiant intensity:

$$I(\omega) = \frac{d\Phi}{d\omega} \quad \left(\frac{\text{watts}}{\text{radian}}\right)$$



- Differential flux given by

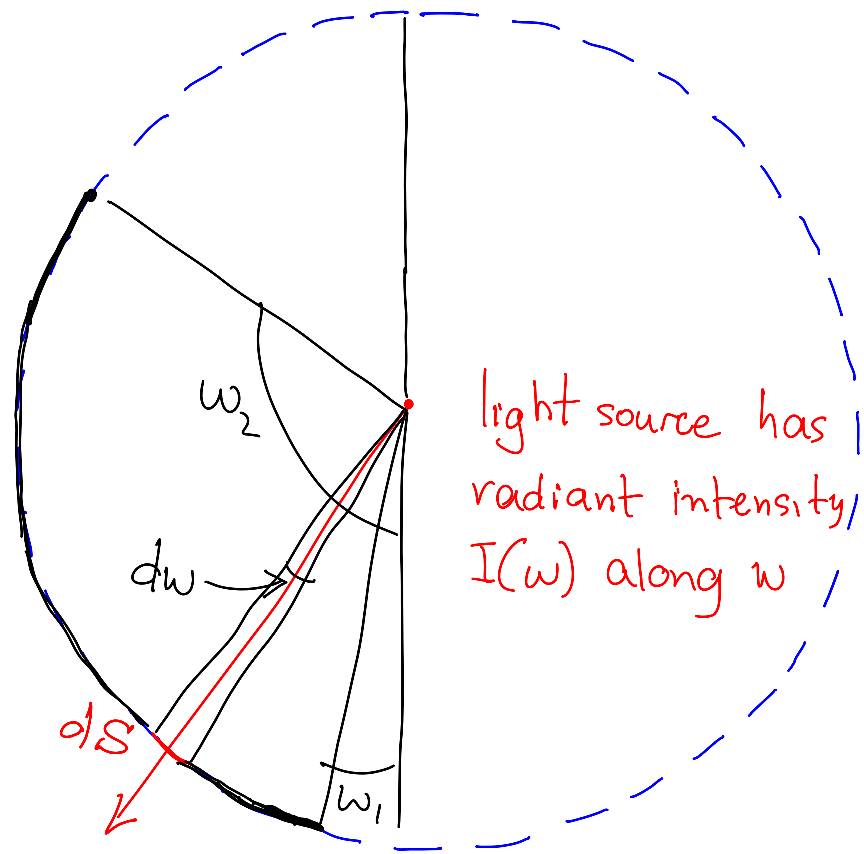
$$d\Phi = d\omega \cdot I(\omega)$$

- Compare to a uniformly-emitting source of flux Φ

$$d\Phi = d\omega \cdot \frac{\Phi}{2\pi}$$

Flux Through a General Arc (2D, Non-Uniform)

- What is the total flux through the arc $[\omega_1, \omega_2]$?



- Differential flux given by

$$d\Phi = d\omega \cdot I(\omega)$$

$\Rightarrow$ Compute the integral over $[\omega_1, \omega_2]$

$$\boxed{\Phi_{\omega_1\omega_2} = \int_{\omega_1}^{\omega_2} d\Phi = \int_{\omega_1}^{\omega_2} I(\omega)\, d\omega}$$

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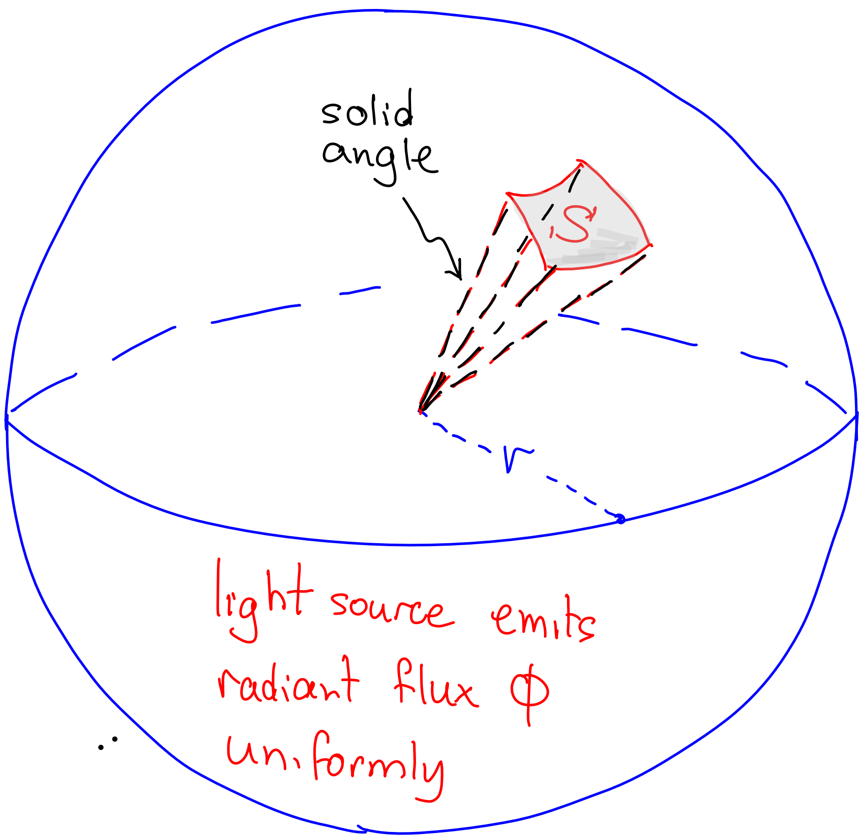
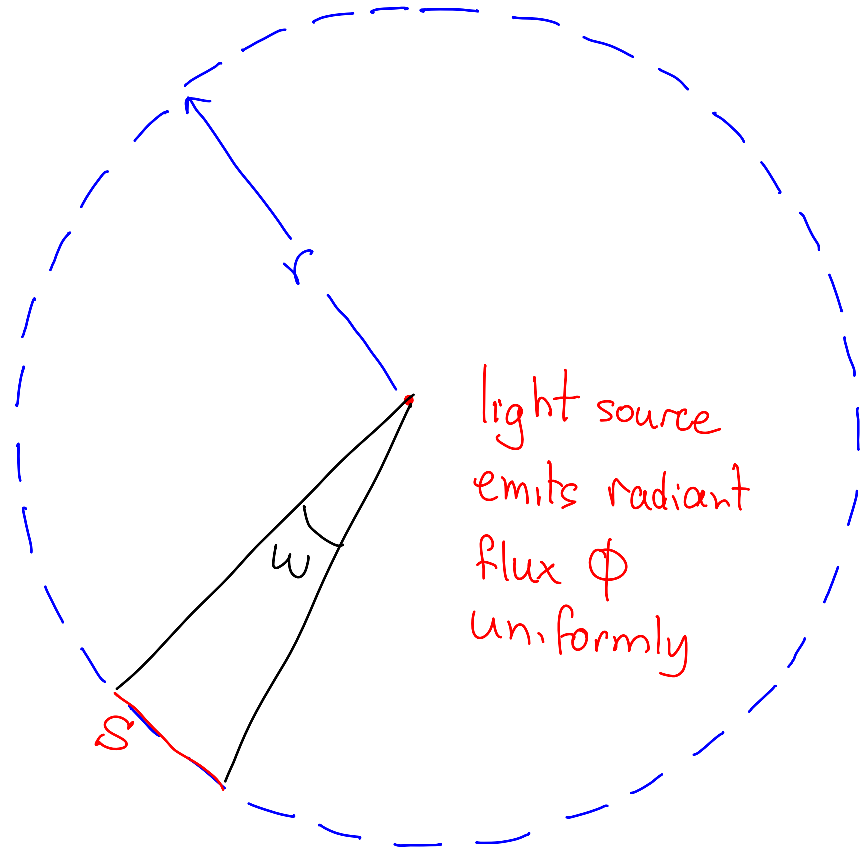
Flux Through an Arc (for 3D, Uniform Source)





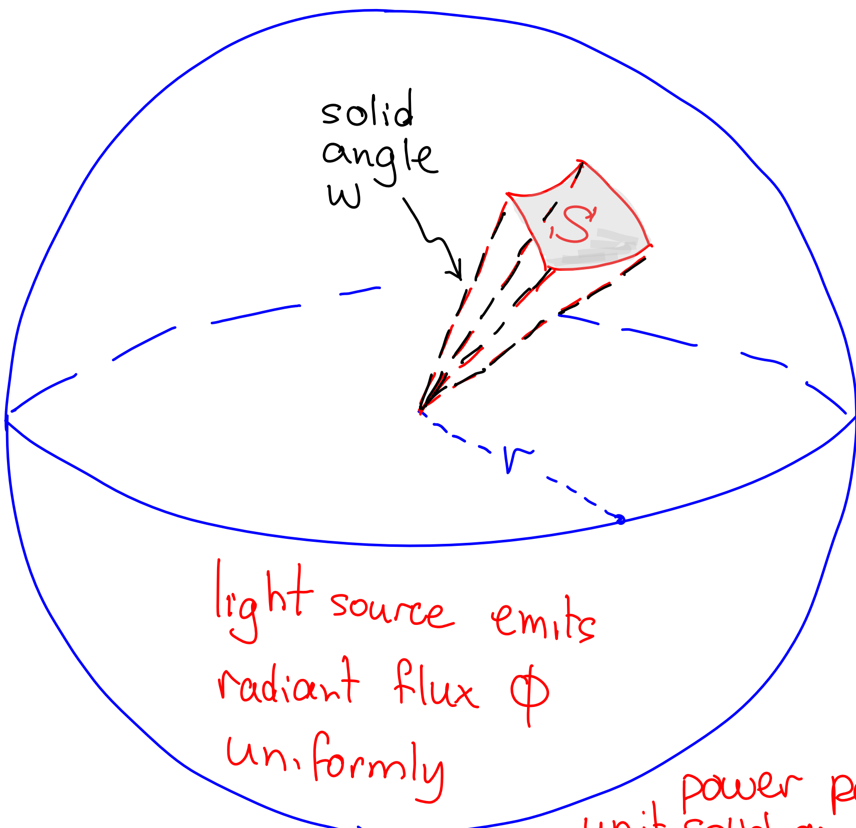
flux through S:

$$\Phi \frac{\text{area}(S)}{4\pi r^2} = \frac{\text{area}(S)}{r^2} \cdot \frac{\Phi}{4\pi}$$

flux through S:

$$\Phi \frac{\text{length}(S)}{\text{perimeter}} = \omega \cdot \frac{\Phi}{2\pi} \; (W/\text{rad})$$

Arcs/Angles in 2D ⇔ Areas/Solid Angles in 3D



- Definition:

  Solid angle ω of a patch S' on a sphere of radius r:

  $$\boxed{\omega = \frac{\text{area}(S')}{r^2}}$$ *

- Solid angles are measured in steradians (sr)

- The solid angle of a full sphere is 4π

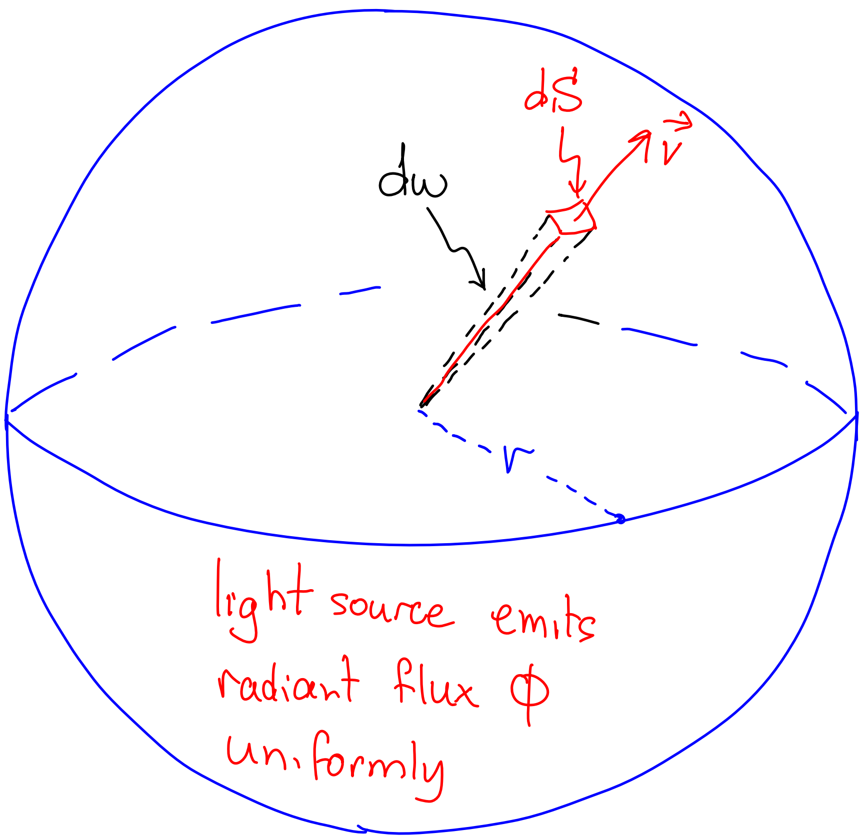
power per unit solid angle (W/sr)

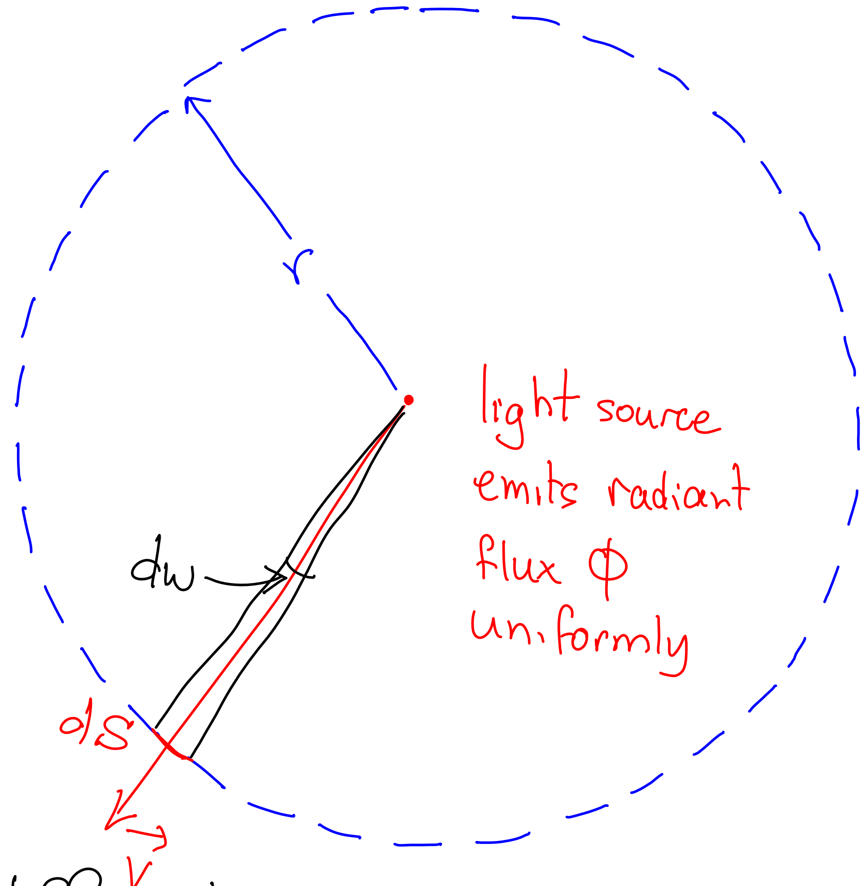
flux through S':

$$\Phi \frac{\text{area}(S')}{4\pi r^2} = \omega \frac{\Phi}{4\pi}$$

solid angle (sr)

Flux Along a Direction (3D, Uniform Source)





differential flux along direction $\vec{v}$:

$$d\Phi = \underline{d\omega}\,\frac{\Phi}{4\pi}$$

differential solid angle

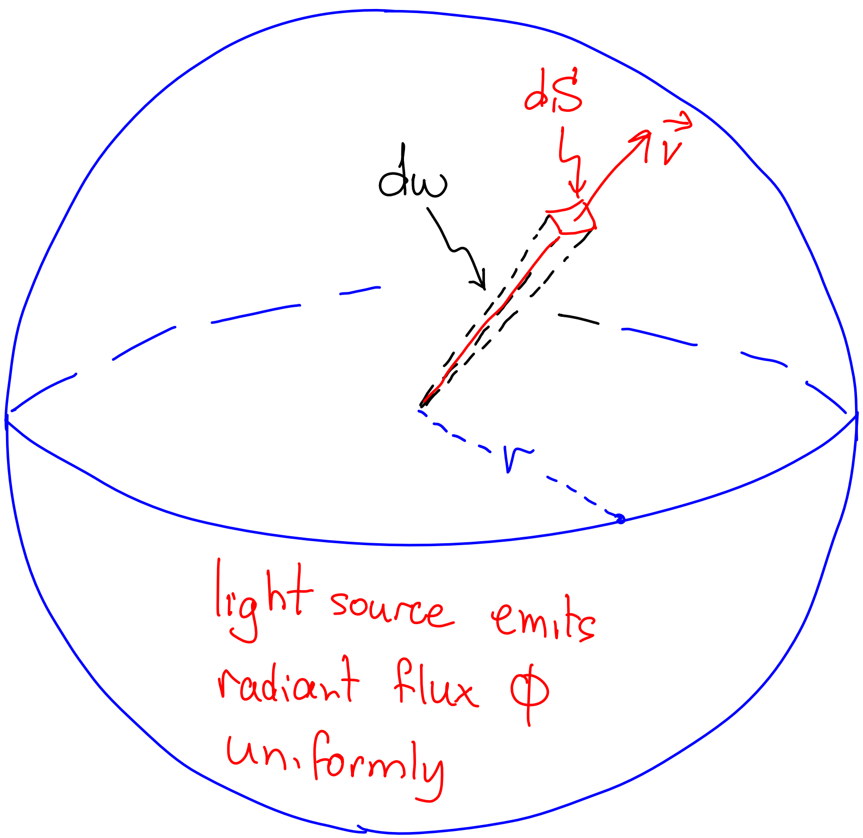
power per unit solid angle

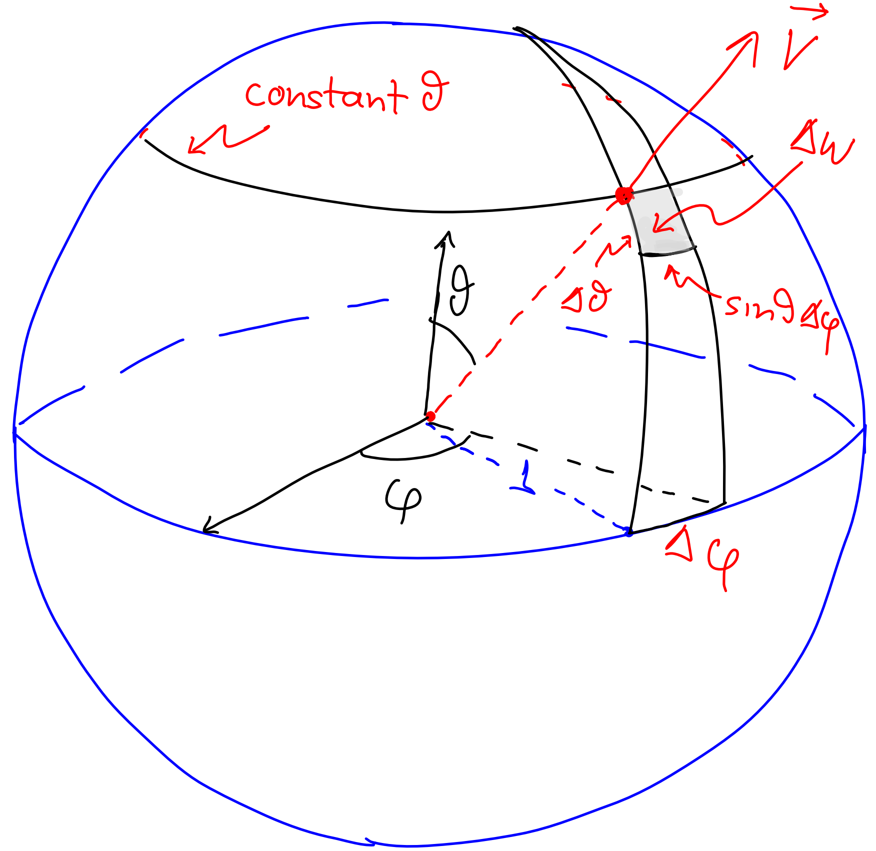
differential flux along direction $\vec{v}$:

$$d\Phi = \underline{d\omega}\,\frac{\Phi}{2\pi}$$

differential angle

Differential Solid Angles ⇔ Spherical Coords





differential flux along direction $\vec{v}$:

$$d\Phi = d\omega \, \frac{\Phi}{4\pi}$$

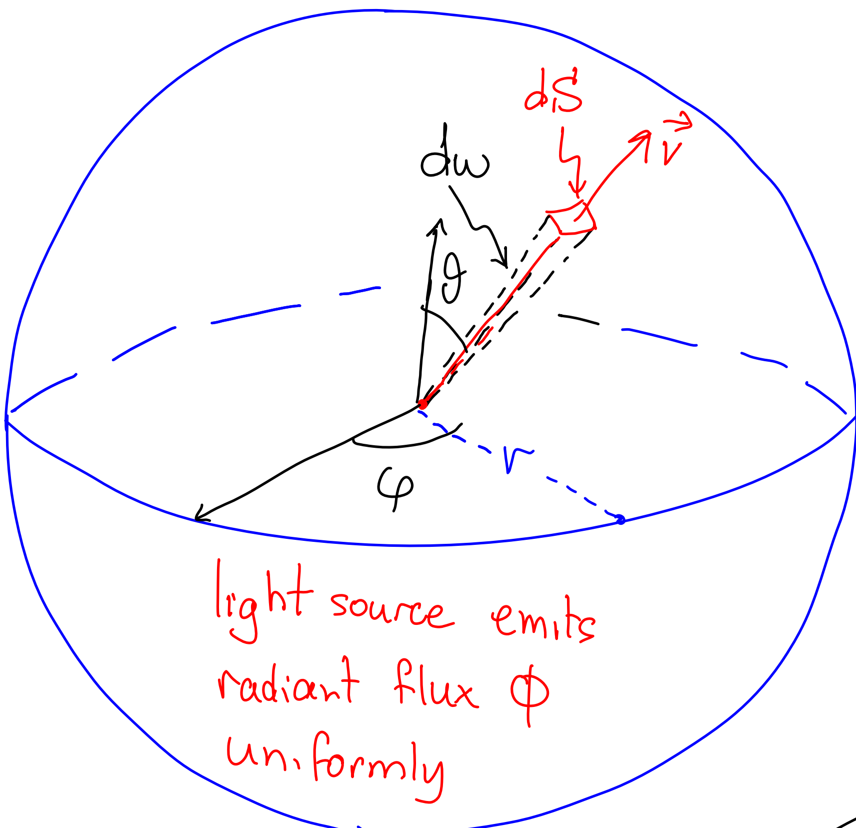
← power per unit solid angle

if $v = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$

$$\text{area}(\Delta\omega) \simeq \Delta\vartheta \cdot \Delta\varphi \sin\vartheta$$

when $\Delta\omega \to 0$, $\boxed{d\omega = d\vartheta \, d\varphi \sin\vartheta}$

Flux Along a Direction (3D, Uniform Source)



Differential flux along direction (ϑ, φ) for a source emitting uniformly in all directions: ✱

$$\boxed{d\Phi = d\vartheta\, d\varphi \sin\vartheta \frac{\Phi}{4\pi}}$$

combining boxed expressions

differential flux along direction $\vec{v}$:

$$\boxed{d\Phi = d\omega \frac{\Phi}{4\pi}}$$

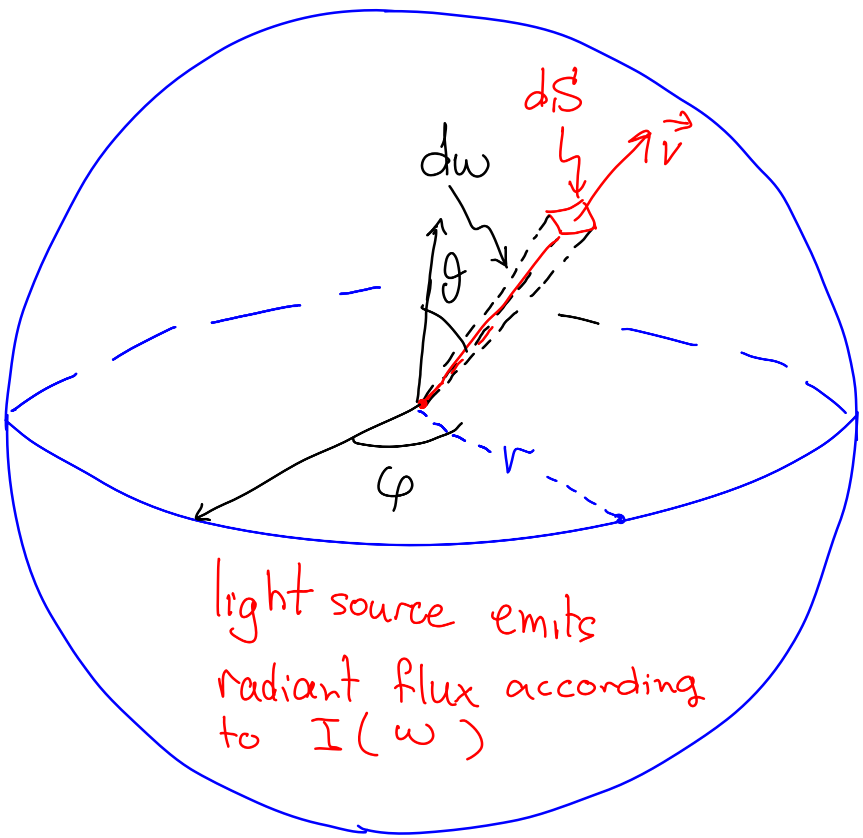
power per unit solid angle

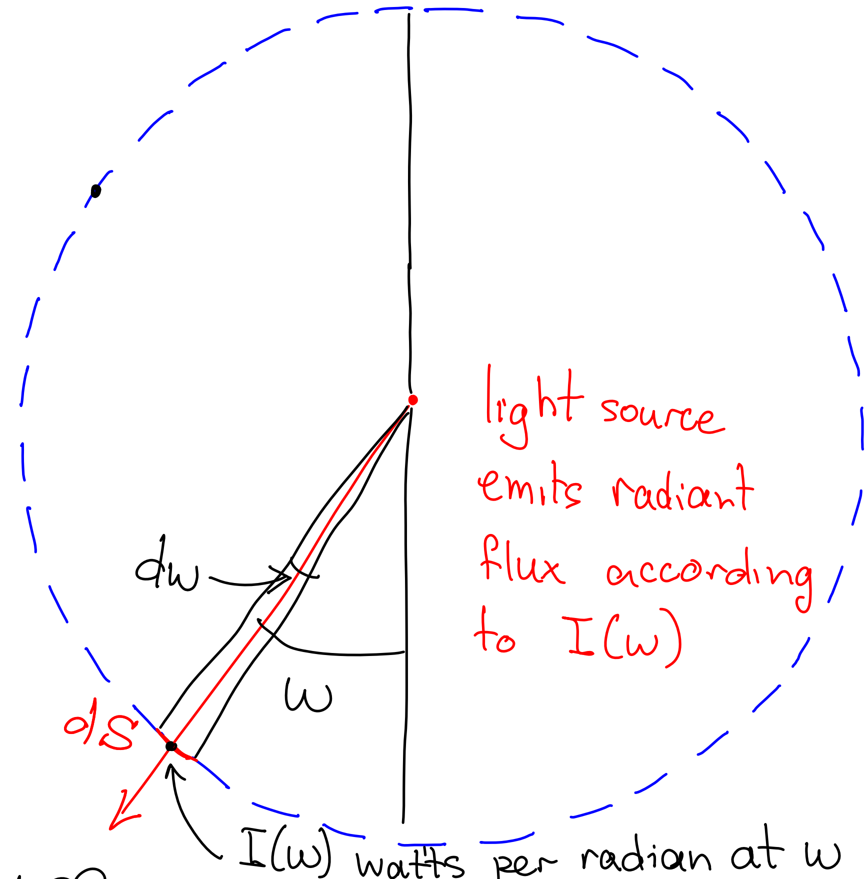
if $v = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$

$\text{area}(\Delta\omega) \simeq \Delta\vartheta \cdot \Delta\varphi \sin\vartheta$

when $\Delta\omega \to 0$, $\boxed{d\omega = d\vartheta\, d\varphi \sin\vartheta}$

Radiant Intensity (3D, Non-Uniform Src)





differential flux along direction $\vec{v}$:

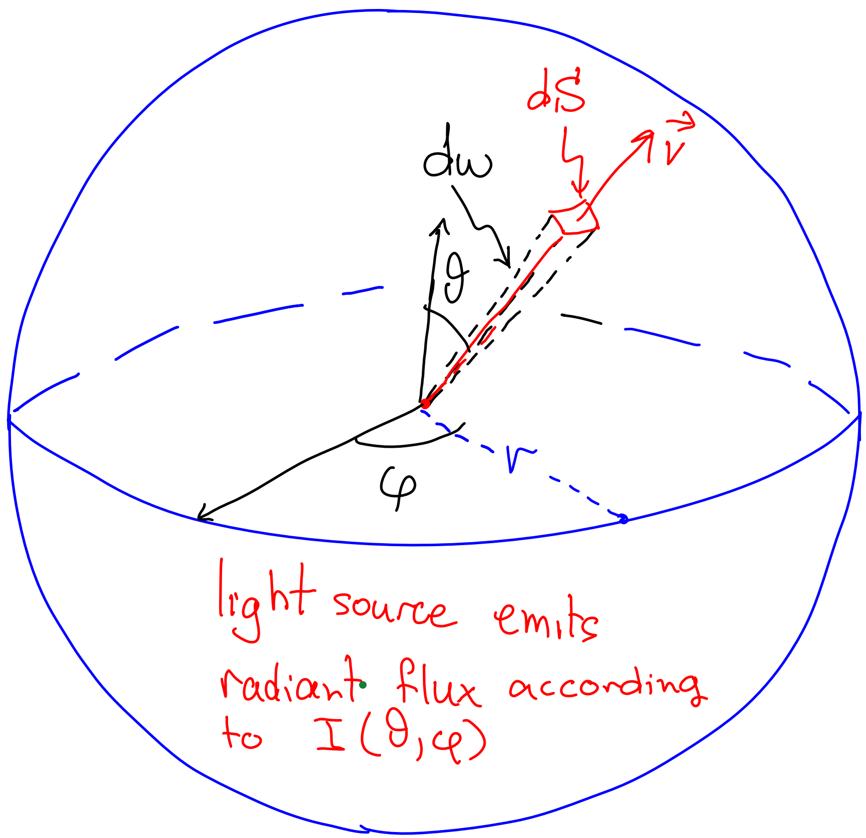
$$d\Phi = d\omega\, I(\omega)$$

Radiant intensity (Watts/steradian)

differential flux along direction $\vec{v}$:

$$d\Phi = d\omega \cdot I(\omega)$$

Radiant Intensity (3D, Non-Uniform Src)



- $I(\omega)$ is called radiant intensity (flux along specific direction)
- Measured using W/sr
- Can also be written as a function of ϑ, φ: $I(\vartheta,\varphi)$

differential flux along direction $\vec{v}$:

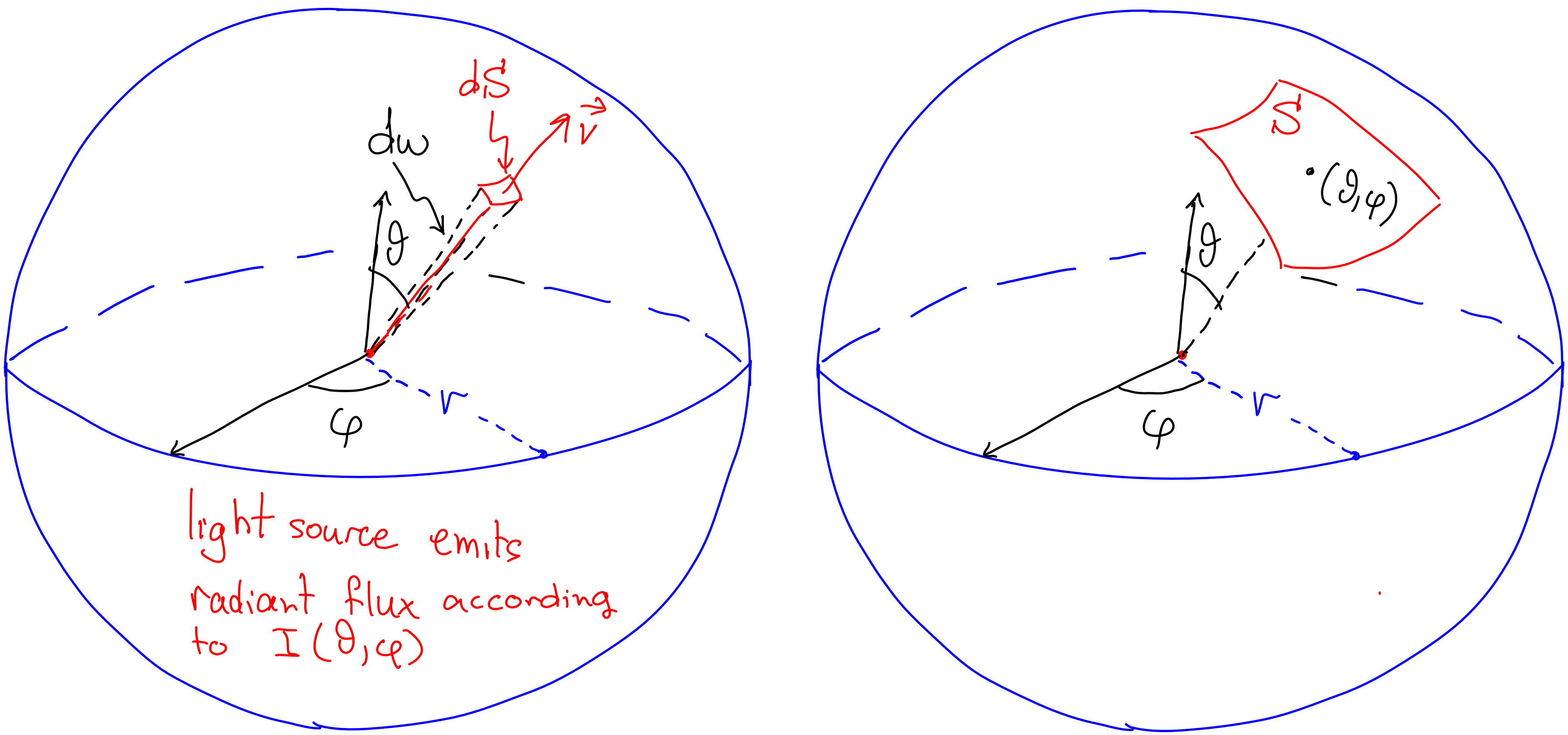
$$d\Phi = d\omega\, I(\omega)$$

- differential flux along (ϑ,φ)

*

$$\boxed{d\Phi = d\vartheta\, d\varphi \sin\vartheta\, I(\vartheta,\varphi)}$$

Flux Through a General Patch (3D, Non-Uniform)



differential flux along direction $\vec{v}$:

$$d\Phi = d\vartheta\, d\varphi \sin\vartheta\, I(\vartheta,\varphi)$$

total flux through region S:

$$\Phi_S = \iint_{(\vartheta,\varphi)\in S} \sin\vartheta\, I(\vartheta,\varphi)\, d\vartheta\, d\varphi$$

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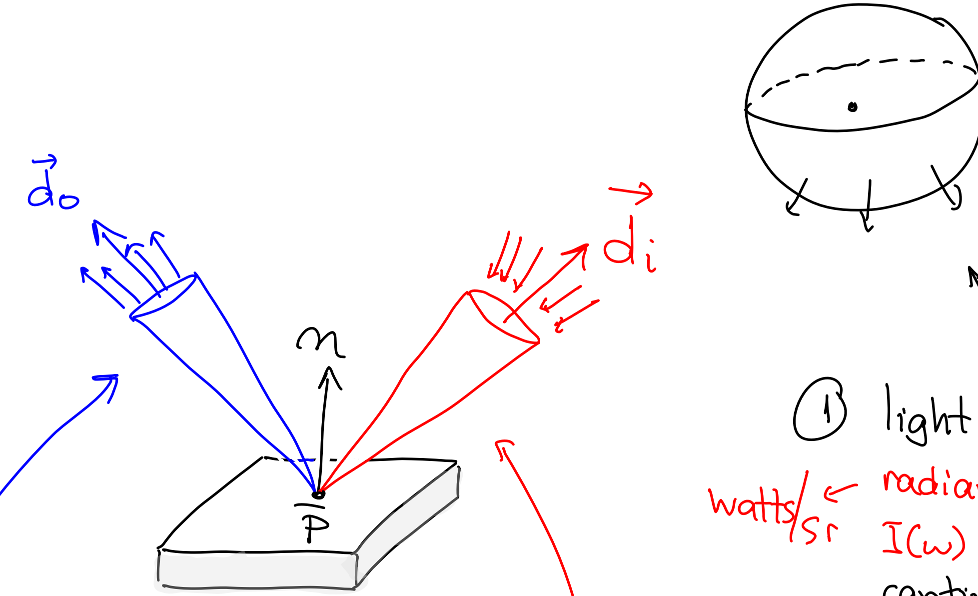
The Basic "Light Transport" Path



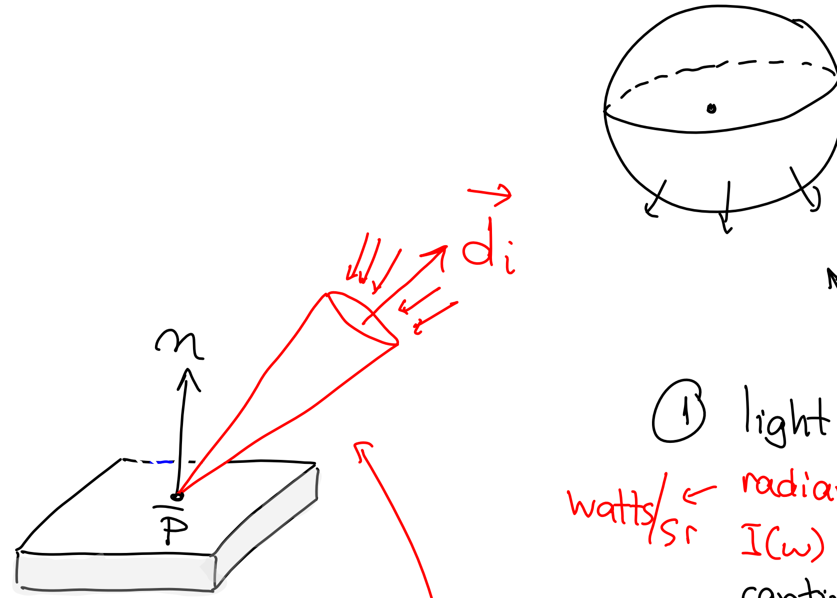
① light source has radiant intensity $I(\omega)$ along a continuum of directions

watts/sr ← radiant intensity

② light converges onto a point from a continuous "cone" of directions

③ light is reflected from a point to a continuous "cone" of directions

The Basic "Light Transport" Path



① light source has radiant intensity $I(\omega)$ along a continuum of directions

watts/sr

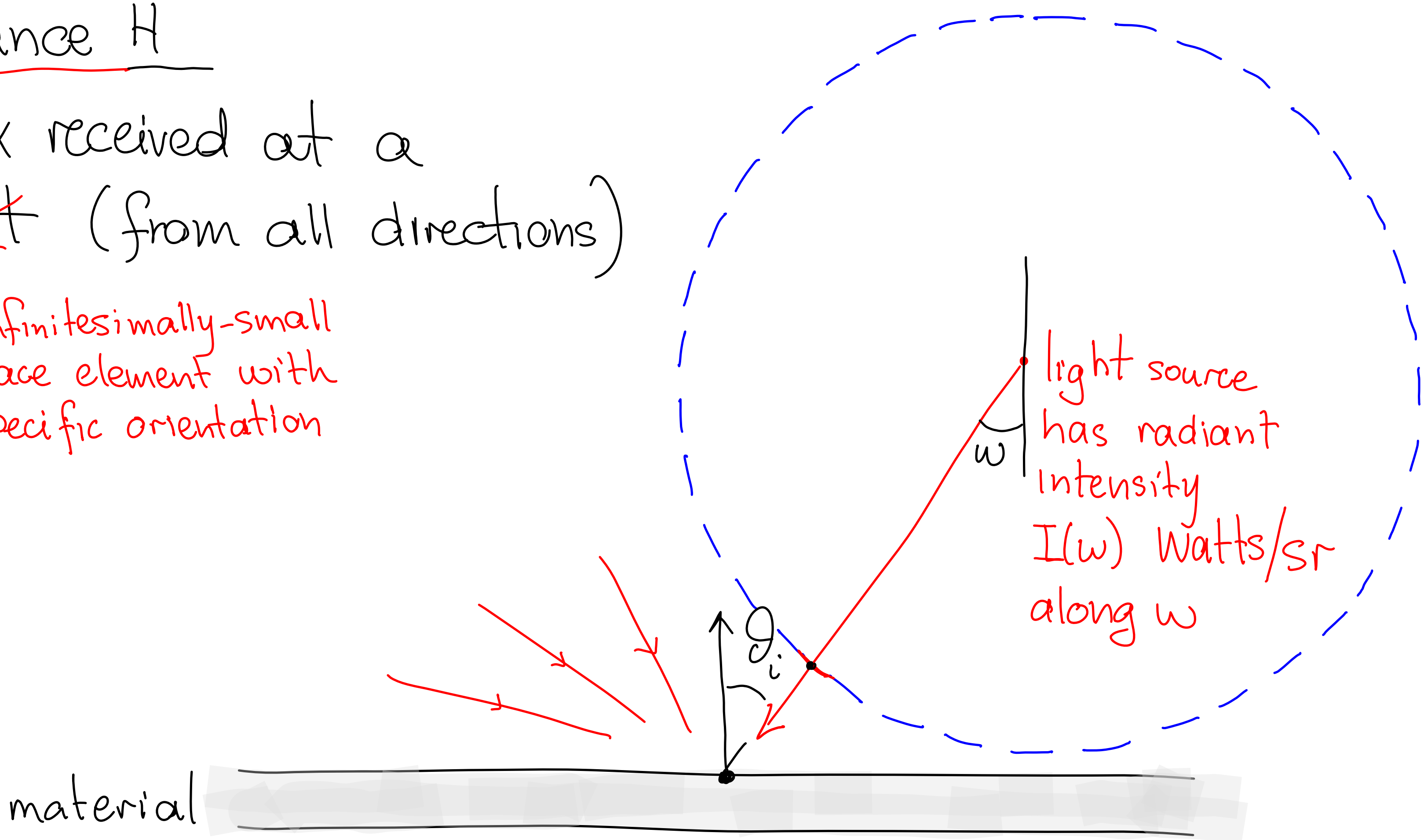
② light converges onto a point from a continuous "cone" of directions

Measuring Incident Light: Irradiance

Irradiance H

Flux received at a ~~point~~ (from all directions)

" an infinitesimally-small surface element with a specific orientation



Definition of Irradiance (for "small patches")

Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all direction)}}{\text{area of patch}}$$

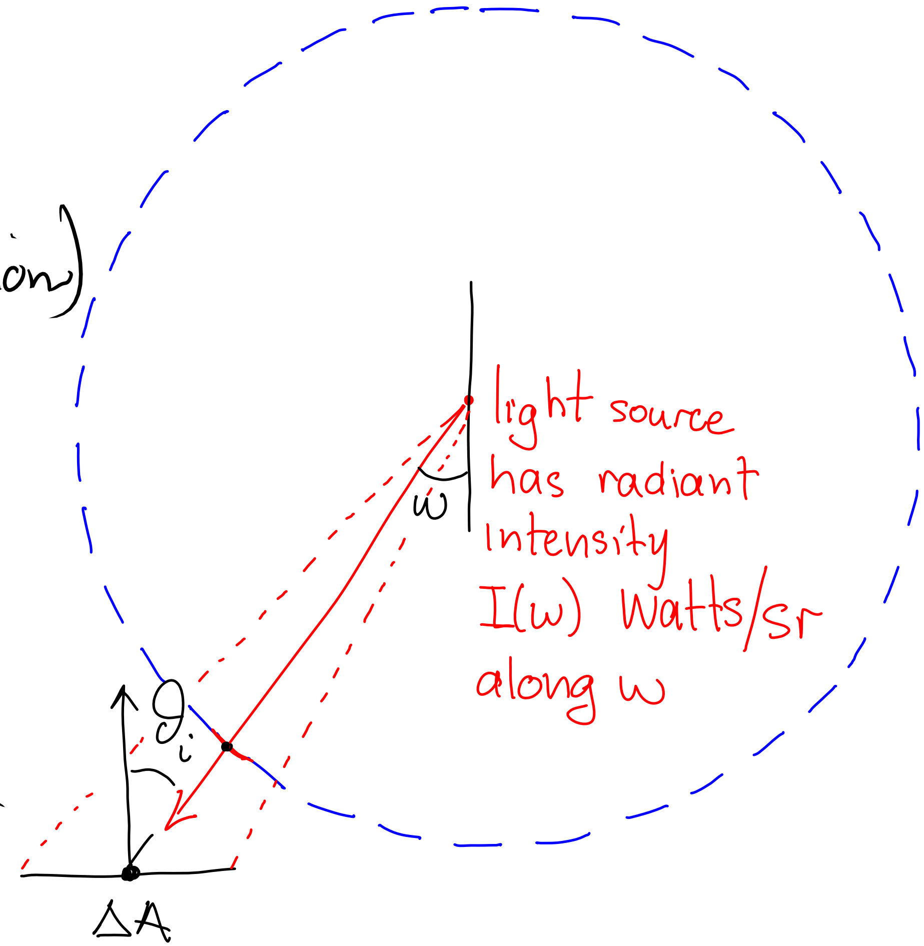
(measured in Watts/m²)

For small patches

$$H = \frac{\Delta\Phi}{\Delta A}$$

$\Delta\Phi$ ← total flux received from all directions

ΔA ← area of patch



Definition of Irradiance (for differential areas)

Irradiance H

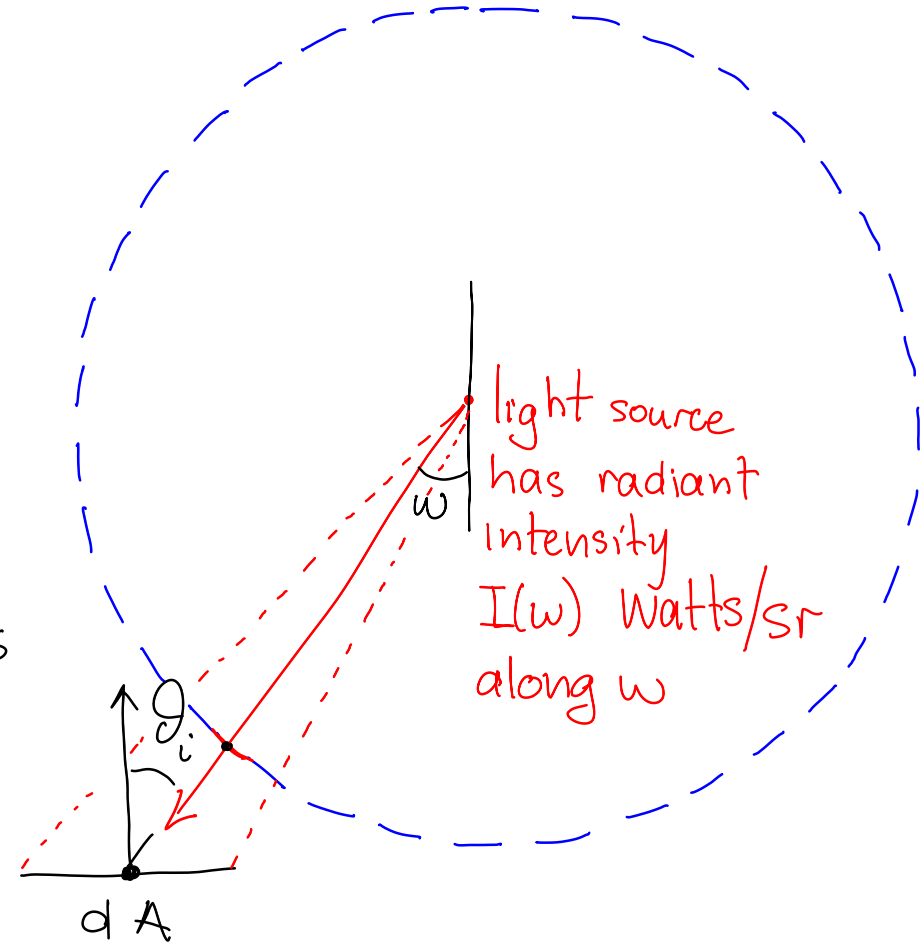
Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)

For infinitesimal patches

$$H = \lim_{\Delta A \to 0} \frac{\Delta \Phi}{\Delta A} = \frac{d\Phi}{dA}$$



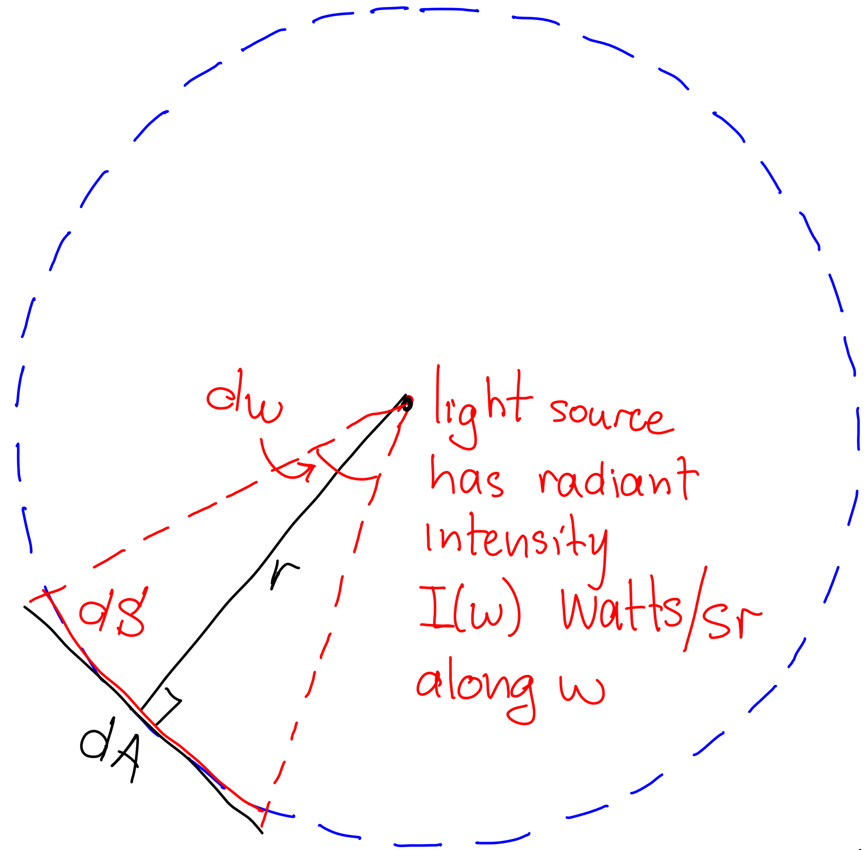
Computing Irradiance: Normal Incidence

Irradiance H

Flux received per unit area:

- $H = \dfrac{\text{flux received (all directions)}}{\text{area of patch}}$

(measured in Watts/m^2)



Example 1: * Calculate the irradiance at a planar patch dA that faces the source and is distance r away from it

- let $d\Phi$ be the flux through dA
- $d\Phi$ = flux through $dS = dw \cdot I(w) = \dfrac{dS}{r^2} \cdot I(w)$
- for infinitesimal patches $dA \simeq dS \Rightarrow d\Phi = \dfrac{dA}{r^2} I(w)$

$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(w)}{r^2}}$$

Computing Irradiance: Normal Incidence

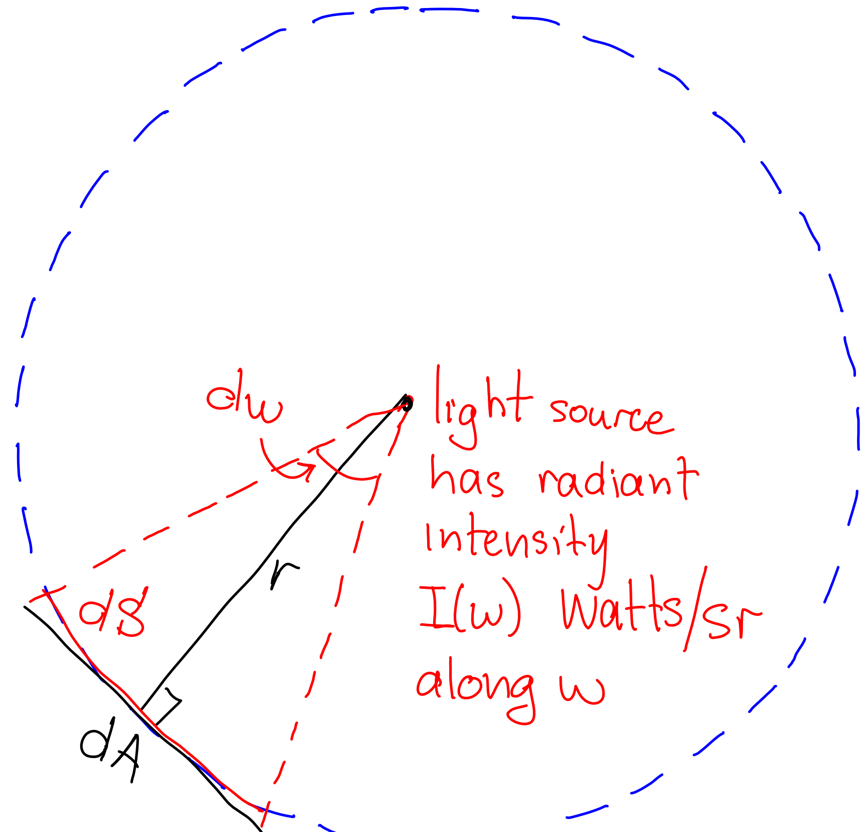
- Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in Watts/m^2)

Example 1: Calculate the irradiance at a planar patch dA that faces the source and is distance r away from it



$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}}$$

⇒ Irradiance decreases quadratically with distance ("squared-distance fall-off")

⇒ the farther the patch is, the less light it gets

Computing Irradiance: Tilted Patches

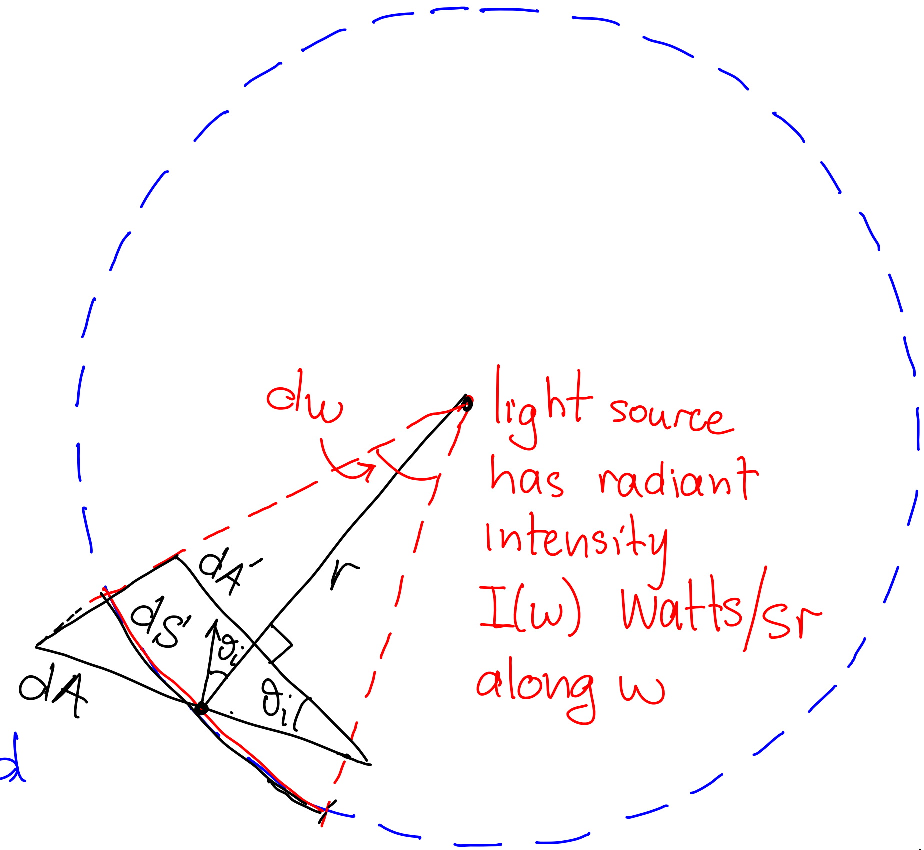
- Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in Watts/m^2)

Example 2: Calculate the irradiance at a planar patch dA at angle θ with source and distance r away from it



- define dA' to be the patch at distance r that faces the light source

$$\boxed{H = \frac{d\Phi}{dA'} = \frac{I(\omega)}{r^2}}$$

- for infinitesimal patches, $dA' \simeq dA \cos\theta \Rightarrow$

$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}\cos\theta_i}$$

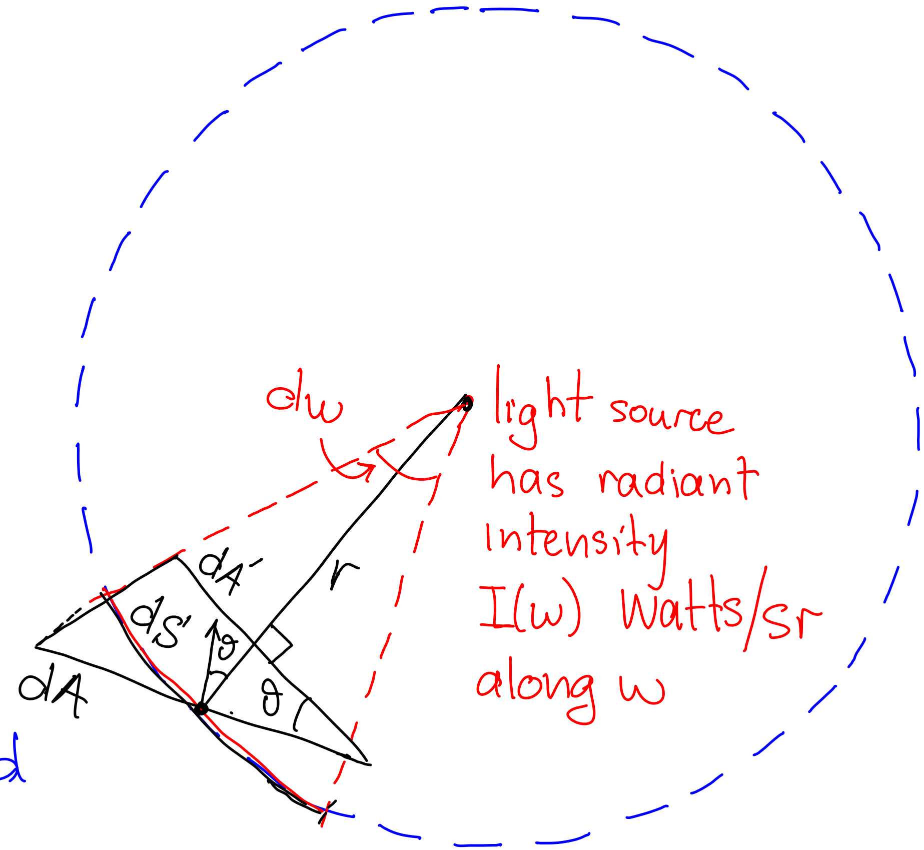
Computing Irradiance: Foreshortening

- Irradiance H

Flux received per unit area:

$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)



Example 2: * Calculate the irradiance at a planar patch dA at angle ϑ with source and distance r away from it

The foreshortening effect:

patches tilted relative to the source receive less light per unit area

$$\boxed{H = \frac{d\Phi}{dA} = \frac{I(\omega)}{r^2}\cos\vartheta}$$

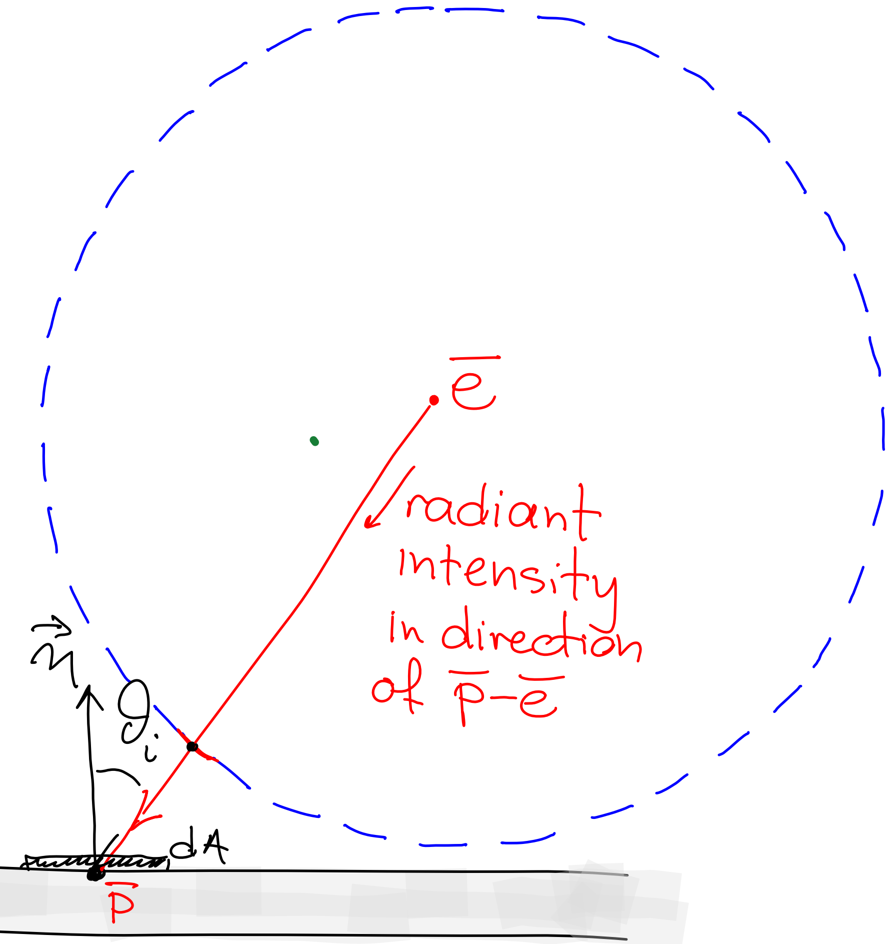
Example: Irradiance due to Point Light Source

Irradiance:

Flux received per unit area:

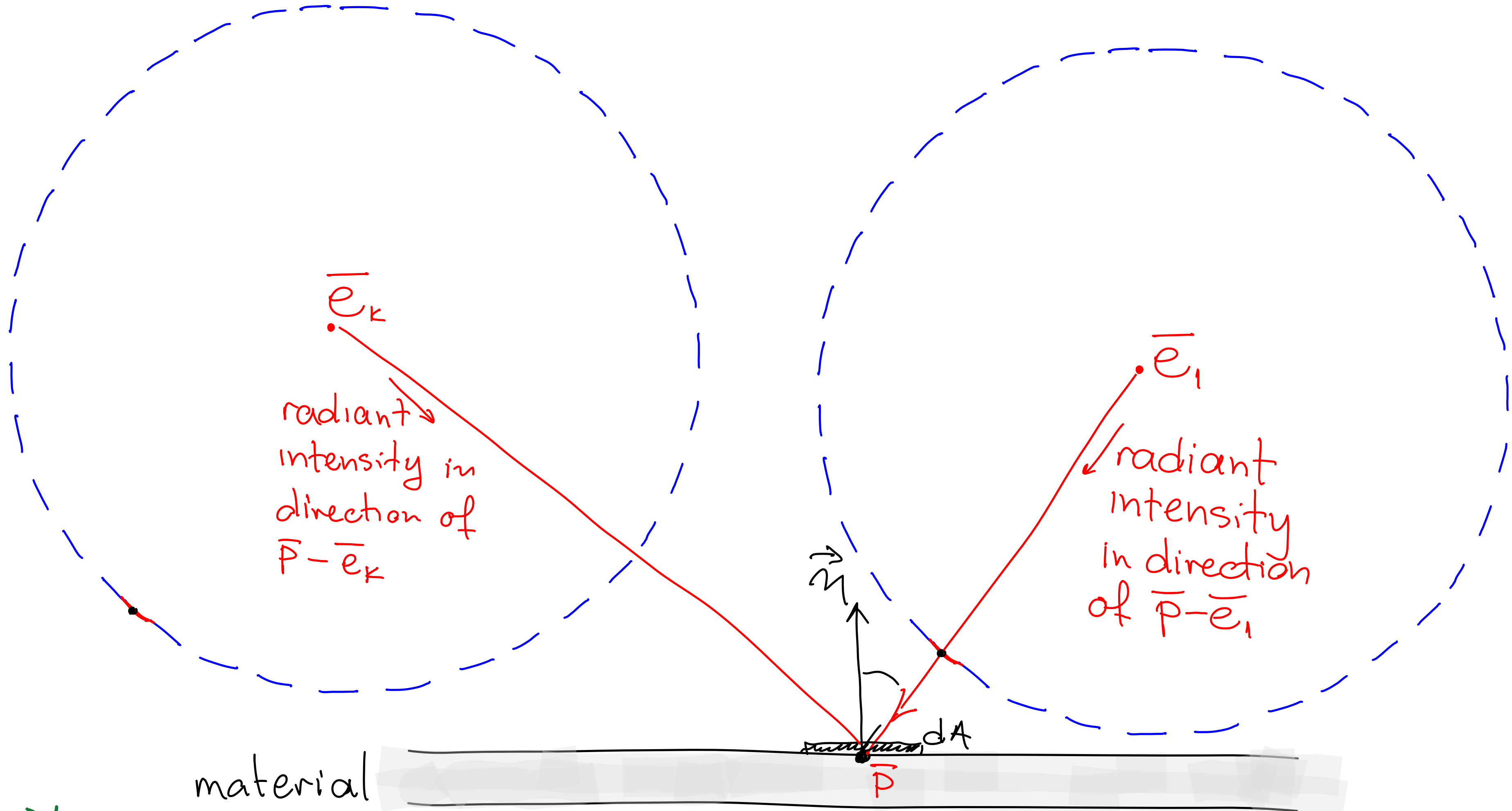
$$H = \frac{\text{flux received (all directions)}}{\text{area of patch}}$$

(measured in watts/m^2)



$*$ Irradiance at $\bar{p}$ (one light): $H(\bar{p}) = \dfrac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2}\, \vec{n} \cdot \dfrac{(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|}$

Example: Irradiance due to Multiple Sources



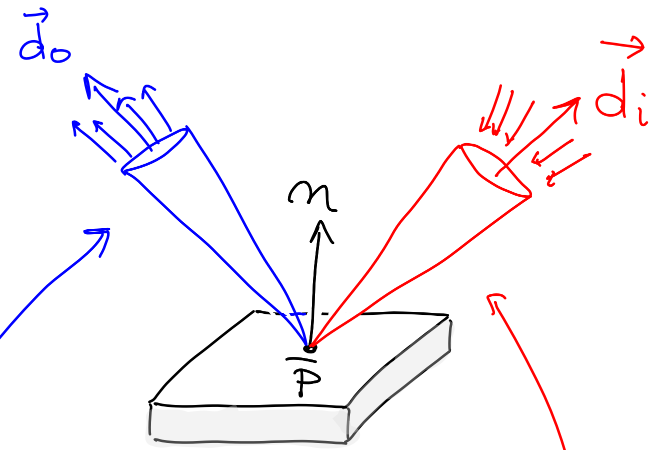
Irradiance at $\bar{p}$ (K lights): $H(\bar{p}) = \sum_{k=1}^{K} \frac{I(\bar{p}-\bar{e}_k)}{\|\bar{p}-\bar{e}_k\|^2} \, \vec{n} \cdot \frac{(\bar{p}-\bar{e}_k)}{\|\bar{p}-\bar{e}_k\|}$

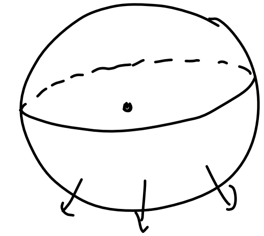
Topic 08:

Radiometry

- The big picture
- Measuring light coming from a light source
    - Measurements for a “2D world”
    - Generalization to 3D
- Measuring light falling onto a patch: Irradiance
- Measuring light leaving a patch: Radiance
- The Bidirectional Reflectance Distribution Function
- Phong Reflectance Model

The Basic "Light Transport" Path





① light source has radiant intensity $I(\omega)$ along a continuum of directions

watts/sr

② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

watts/m^2

③ light is reflected from a point to a continuous "cone" of directions

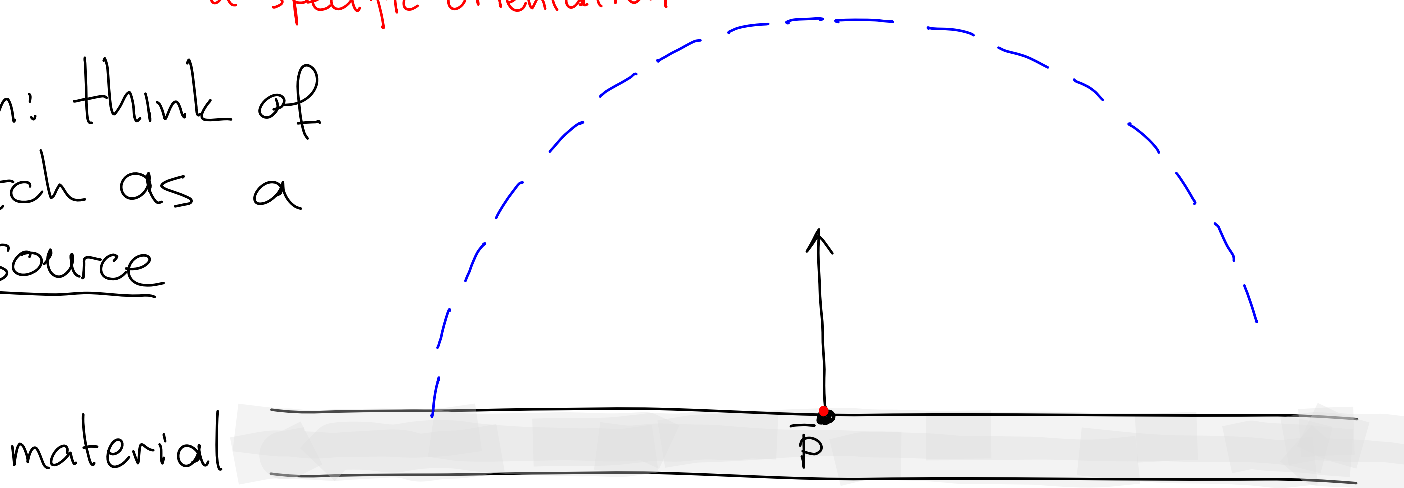
Measuring Outgoing Light: Radiance

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by a ~~surface point~~

an infinitesimally-small surface element with a specific orientation

Intuition: think of the patch as a light source



Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

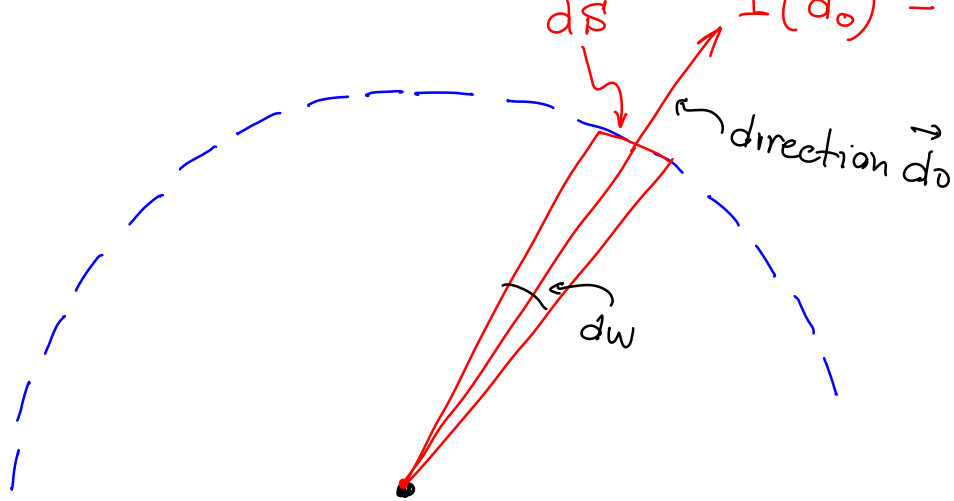
Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source

flux through dS'

$$I(\vec{d}_o) = \frac{d\Phi}{d\omega}$$

solid angle of dS'



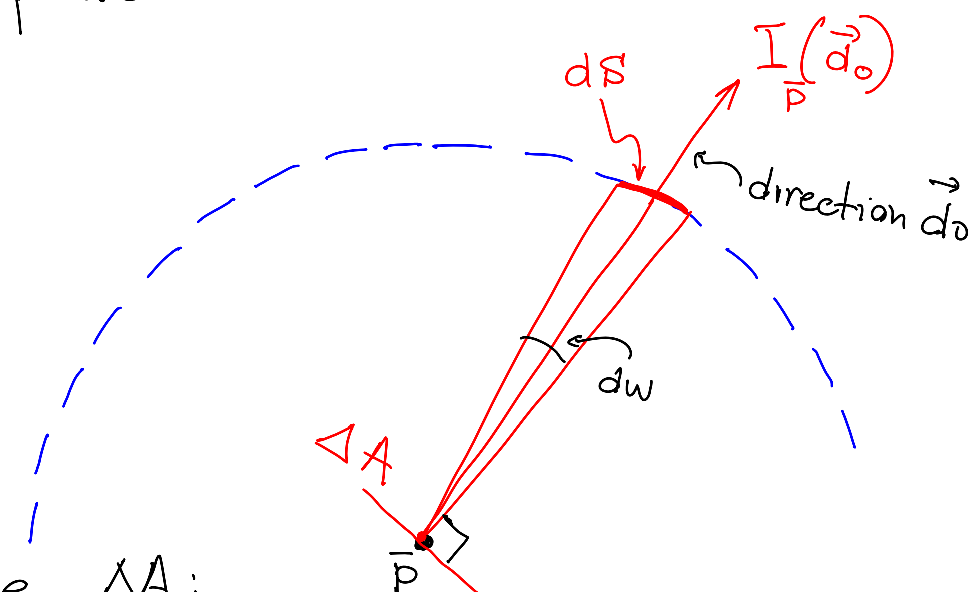
When the light source is a point its emission is quantified using its radiant intensity $I(\vec{d}_o)$

Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source



For a patch source ΔA:

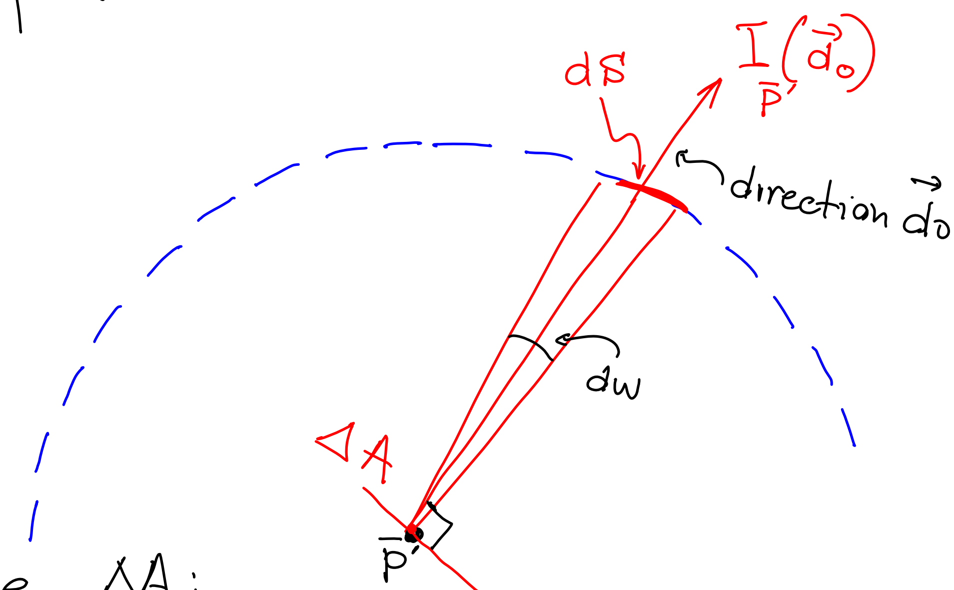
- Measure total radiant intensity ΔI through dS due to ΔA

Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source



For a patch source ΔA:

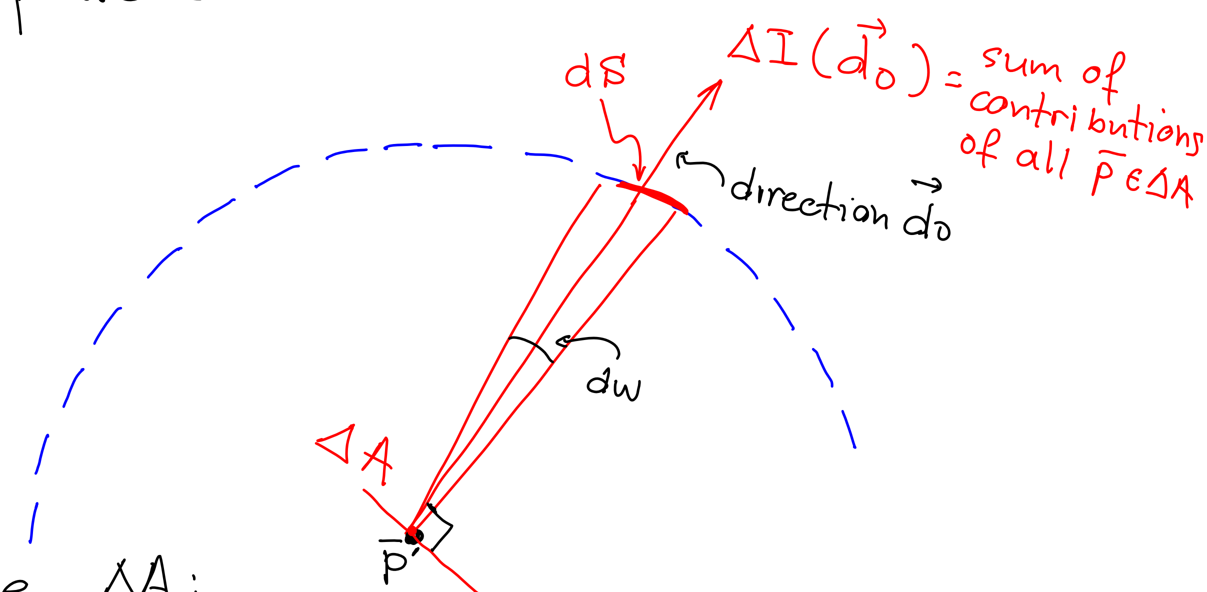
- Measure total radiant intensity ΔI through dS due to ΔA

Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

Intuition: think of the patch as a light source



For a patch source ΔA:

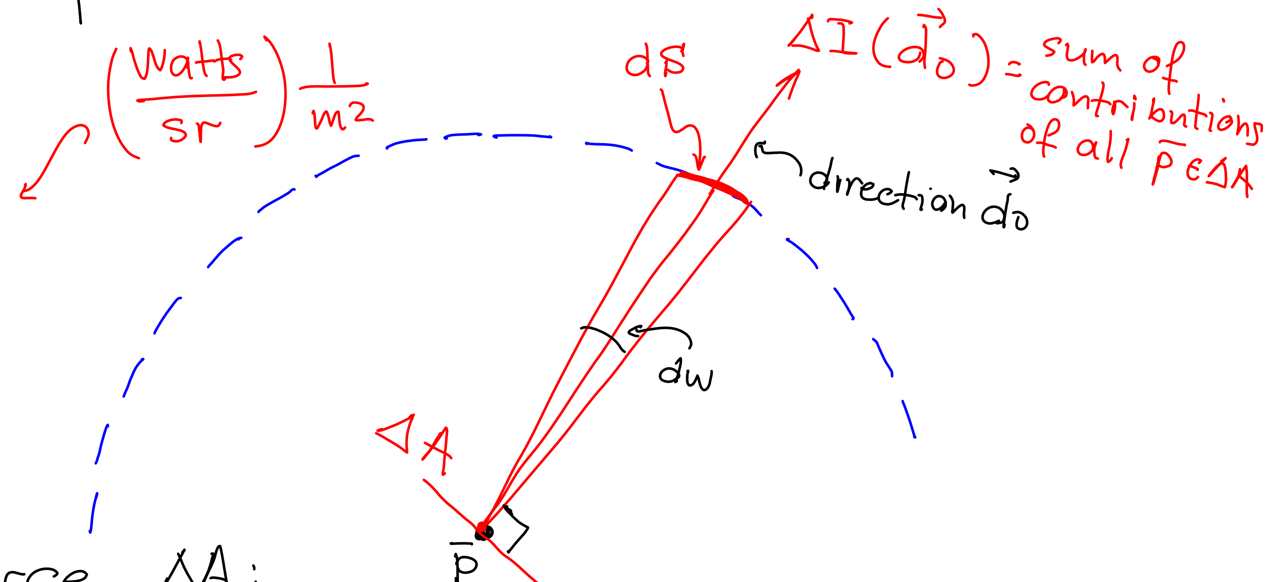
- Measure total radiant intensity ΔI through dS due to ΔA
- Divide by the area of the patch

Defining Radiance: Basic Intuition

Surface Radiance $L(\bar{p}, \vec{d_o})$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d_o}) = \lim_{\Delta A \to 0} \frac{\Delta I(\vec{d_o})}{\Delta A}$$



For a patch source ΔA:

- Measure total radiant intensity ΔI through dS due to ΔA
- Divide by the area of the patch (and take limit)

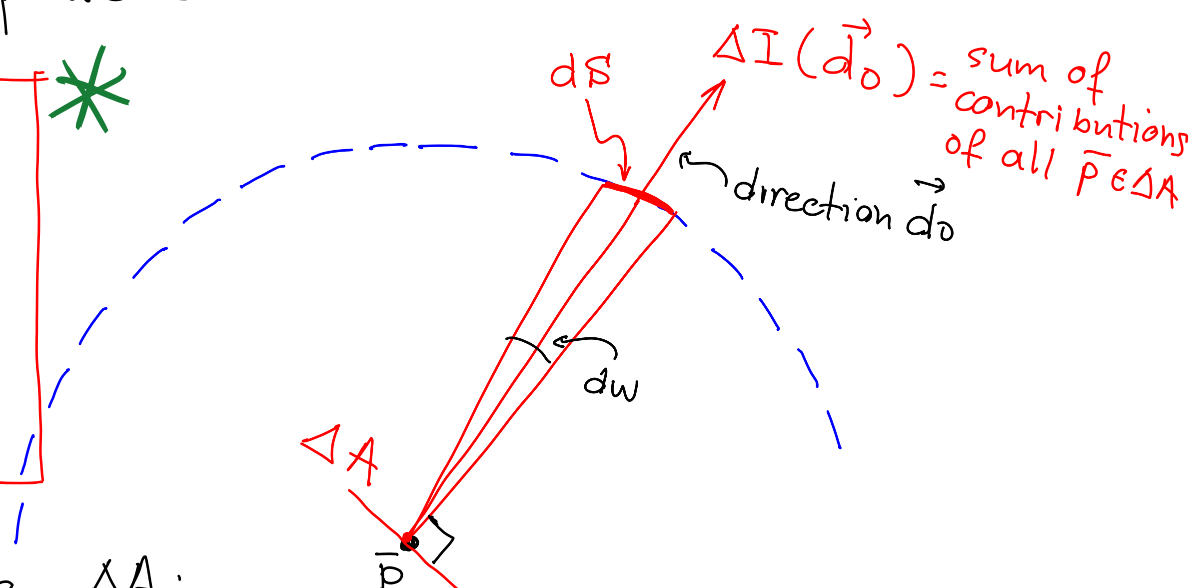
Definition of Radiance

Surface Radiance $L(\bar{p}, \vec{d}_o)$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$\boxed{\begin{aligned} L(\bar{p}, \vec{d}_o) &= \frac{dI}{dA} \\ &= \frac{d}{dA}\left(\frac{d\Phi}{d\omega}\right) = \frac{d^2\Phi}{dA\,d\omega} \end{aligned}}$$

(in Watts/sr·m²)



For a patch source ΔA:

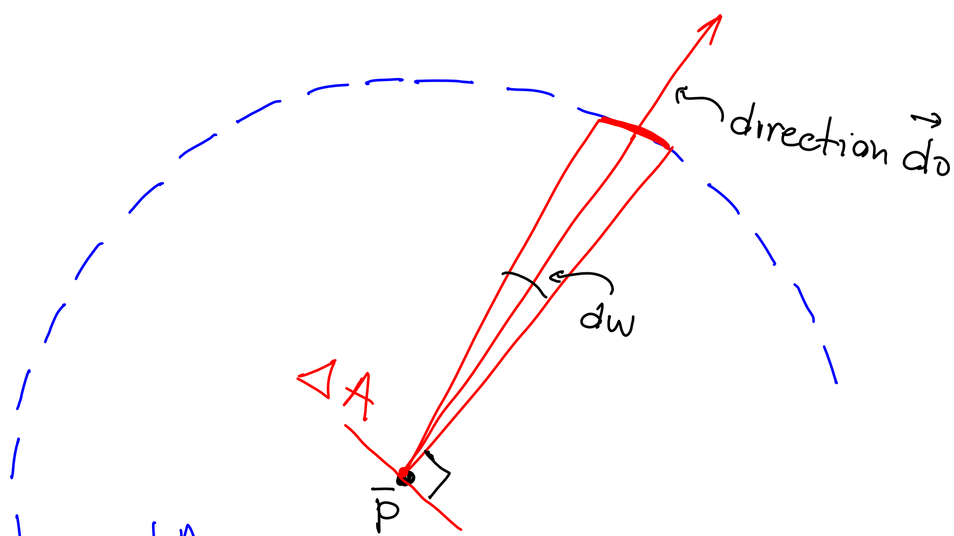
- Measure total radiant intensity ΔI through dS due to ΔA
- Divide by the area of the patch

Definition of Radiance Assumes Perpendicular Patch

Surface Radiance $L(\bar{p}, \vec{d_o})$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d_o}) = \frac{dI}{dA} = \frac{d}{dA}\left(\frac{d\Phi}{dw}\right) = \frac{d^2\Phi}{dA\,dw}$$



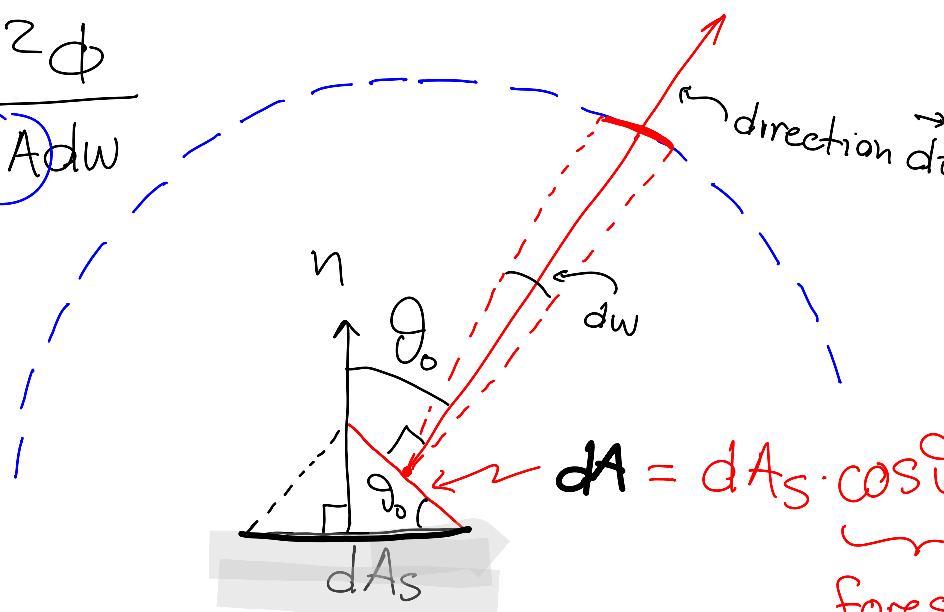
Attention: Division by dA assumes that patch is perpendicular to emission direction $\vec{d_o}$

Radiance for a Tilted Patch

Surface Radiance $L(\bar{p}, \vec{d_o})$

Flux emitted in a particular direction by an infinitesimally small surface patch

$$L(\bar{p}, \vec{d_o}) = \frac{dI}{dA} = \frac{d^2\Phi}{dA\, d\omega}$$



$$dA = \underbrace{dA_s \cdot \cos\vartheta_o}_{\text{foreshortening term}}$$

dA: patch orthogonal to $\vec{d_o}$

dA_s: tilted surface patch

Radiance for a Tilted Patch

Surface Radiance for a tilted patch ✱

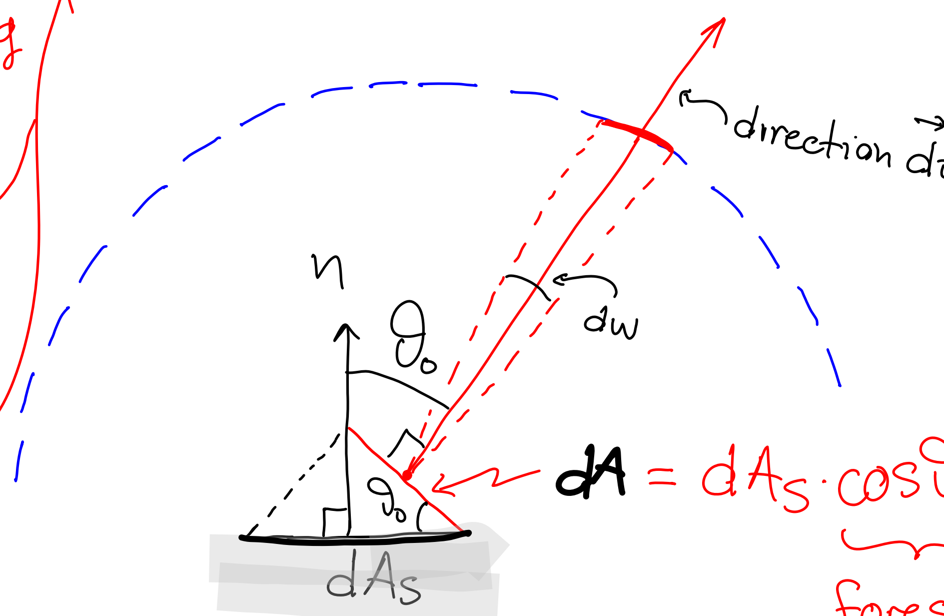
$$\frac{d^2\phi}{dA_s\,d\omega}(\bar{p},\vec{d}_o) = \cos\vartheta_o \cdot L(\bar{p},\vec{d}_o)$$
$$= (\vec{m}\cdot\vec{d}_o)\,L(\bar{p},\vec{d}_o)$$

combining (✱), (✱✱)

So we have:

$$(*)\quad L(\bar{p},\vec{d}_o) = \frac{d^2\phi}{dA\,d\omega}$$

$$(**)\quad dA_s = \frac{1}{\cos\vartheta_o}\cdot dA$$



$$dA = dA_s\cdot\underbrace{\cos\vartheta_o}_{\text{foreshortening term}}$$

dA: patch orthogonal to $\vec{d}_o$

dA_s: tilted surface patch

Measuring All Outgoing Light: Radiant Exitance

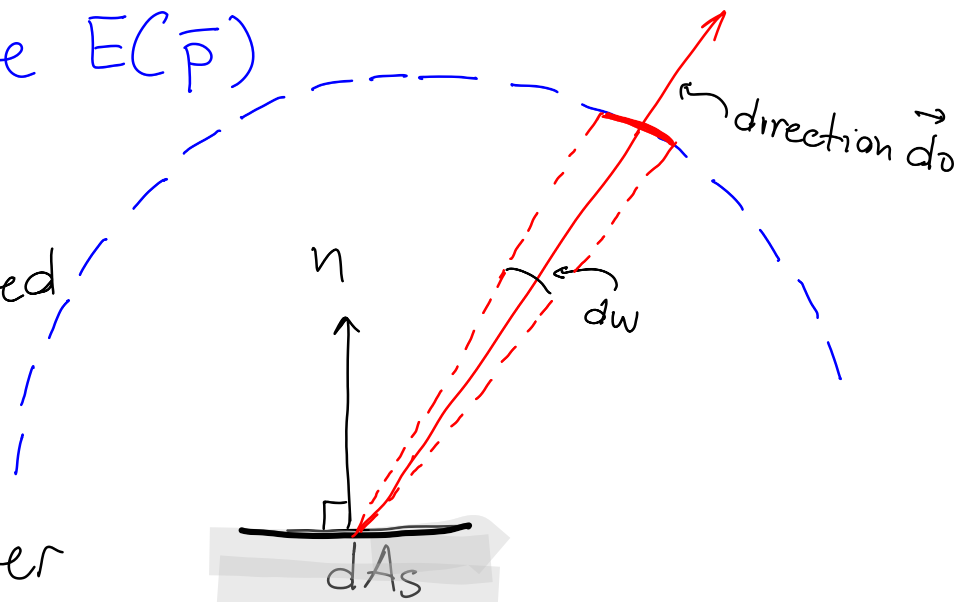
$\ast$ Surface Radiance for a tilted patch

$$\frac{d^2\Phi}{dA_s\, d\omega}(\bar{p}, \vec{d}_o) = (\vec{n} \cdot \vec{d}_o)\, L(\bar{p}, \vec{d}_o)$$

Radiant exitance $E(\bar{p})$
(aka Radiocity)

Total flux emitted from dA_s in <u>all</u> directions

$\Rightarrow$ an integral over directions:



$\ast$ $$E(\bar{p}) = \int_{\vec{d}_o} (\vec{n} \cdot \vec{d}_o)\, L(\bar{p}, \vec{d}_o)\, d(\vec{d}_o)$$

Topic 08:

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- Measuring light leaving a patch: Radiance
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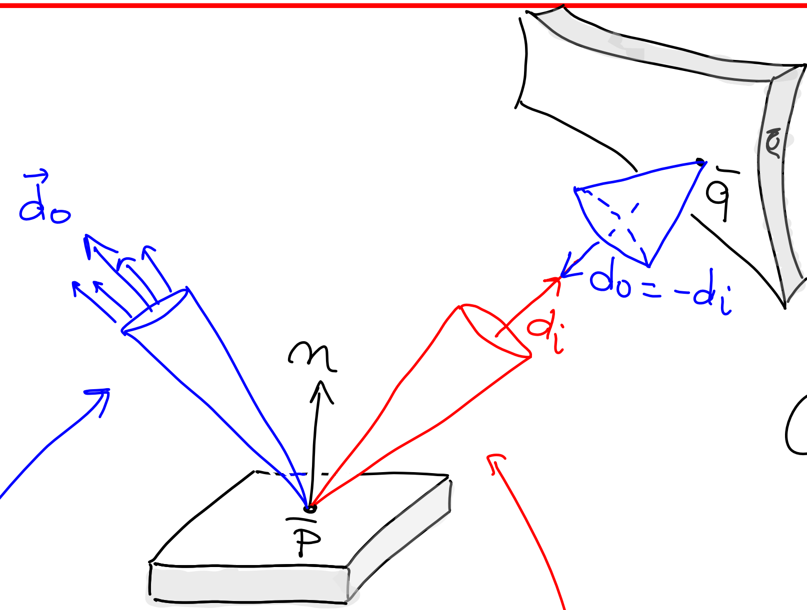
- Phong Reflectance Model

Completing the Light Transport Path: Reflectance



① radiance emitted at $\bar{q}$ is received as incident flux at $\bar{p}$

Watts/(sr·m²)

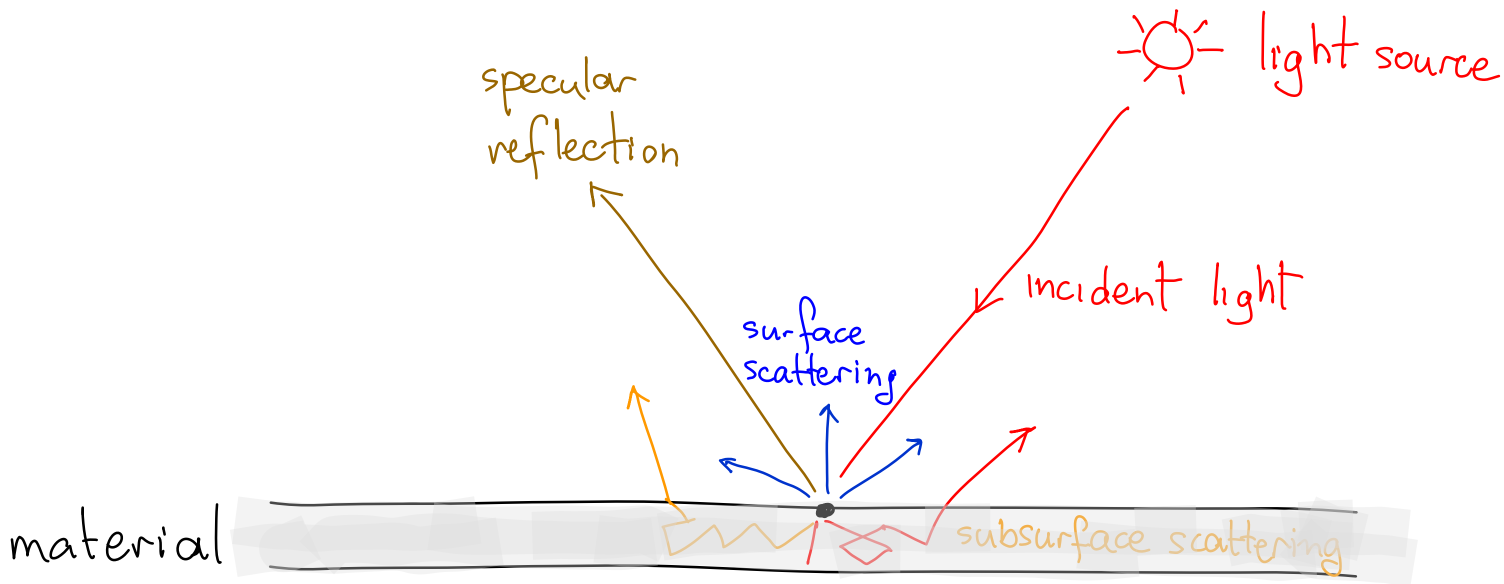
③ radiance $L(\bar{p}, \vec{d}_o)$ emitted from infinitesimal patch at $\bar{p}$ in direction $\vec{d}_o$

Watts/m²

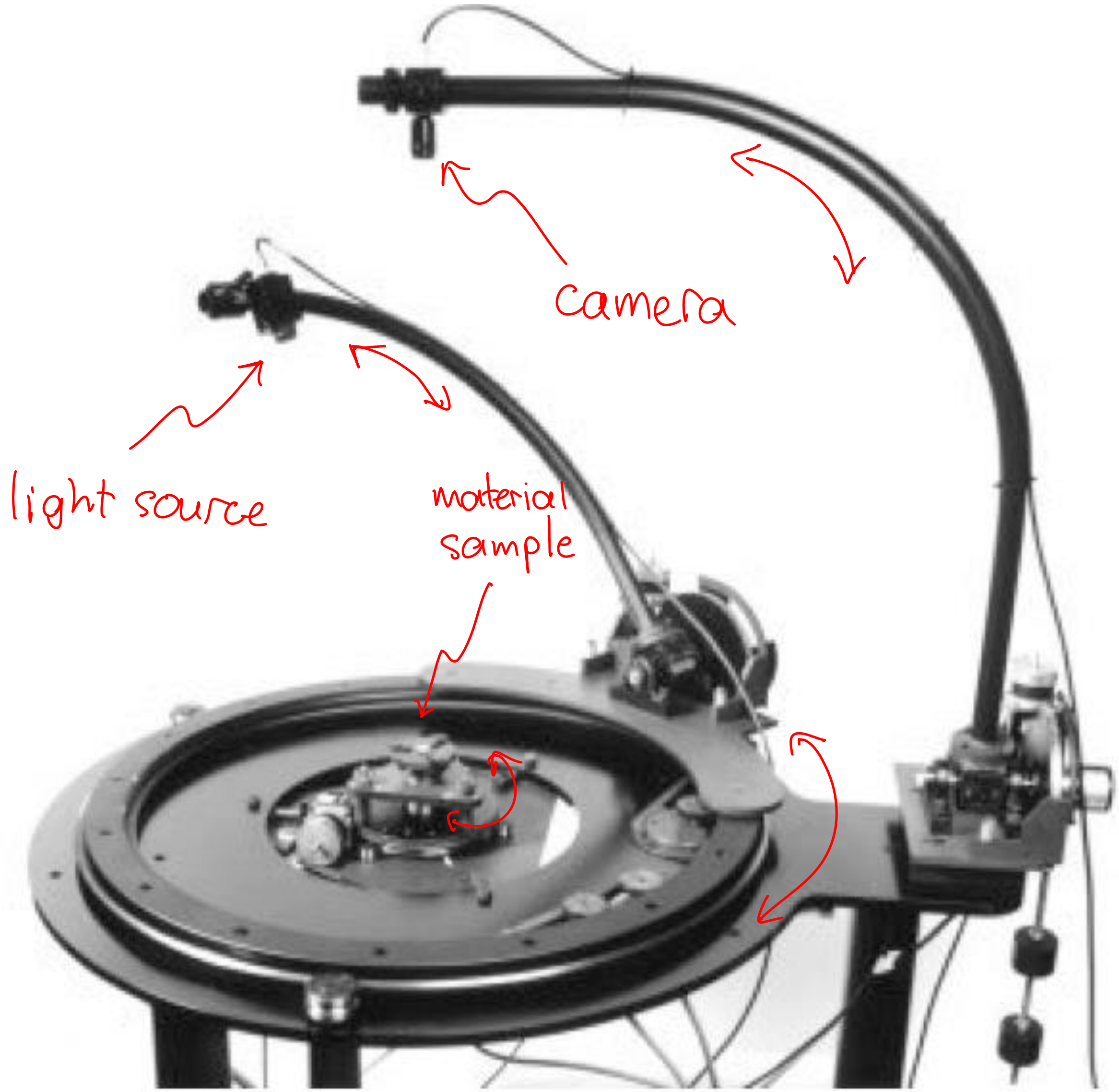
② irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

Q: How is incident light "transformed" to outgoing light?

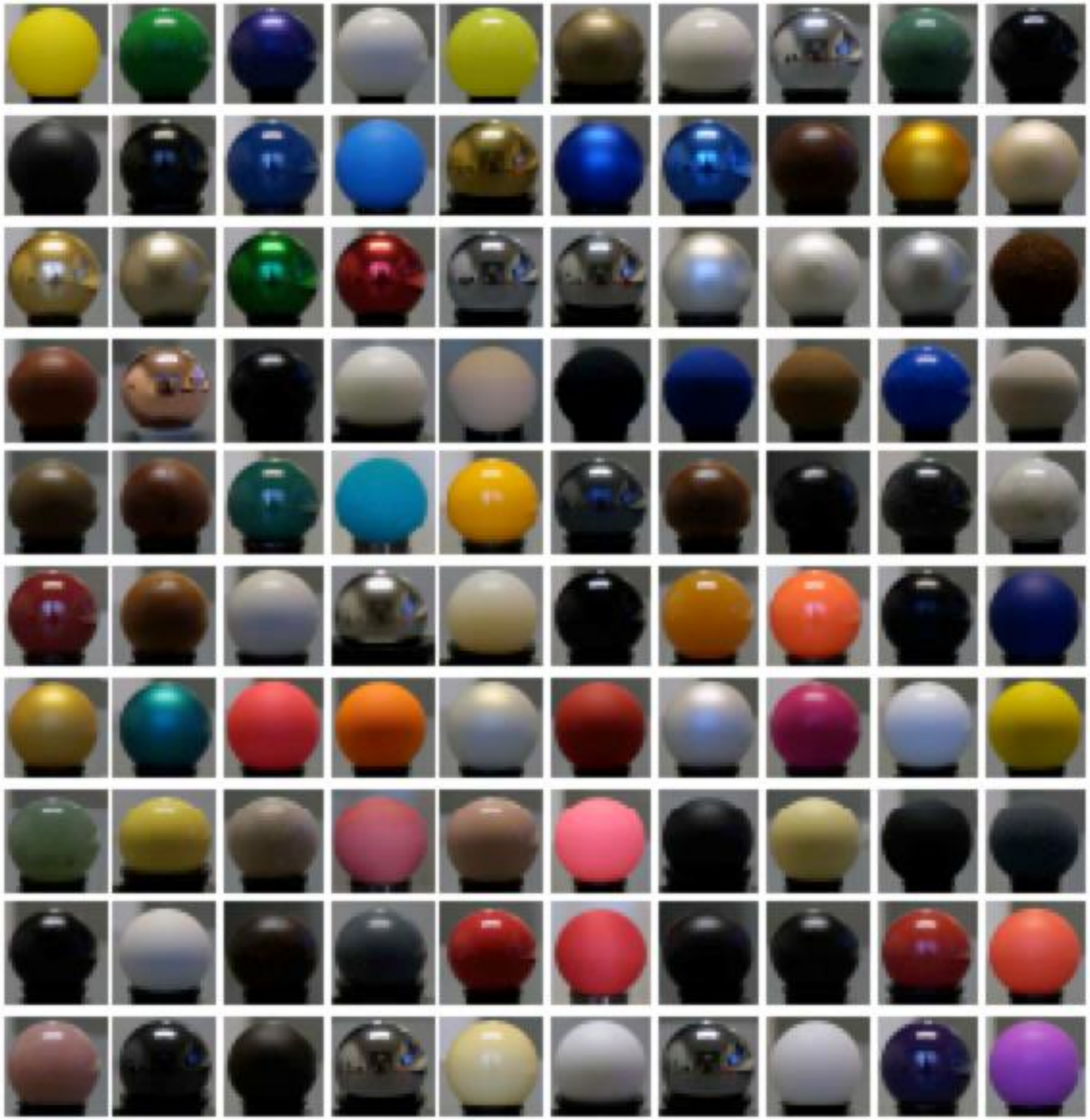
Common Modes of Local Light Transport



Empirical Measurement of Reflectance



Visualizing Surface Reflectance Properties

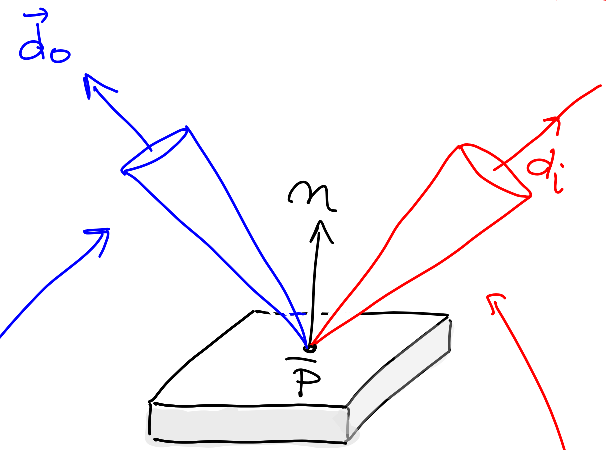


General Definition: The BRDF of a Point

$$\text{BRDF}: \quad \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

in direction $\vec{d}_o$

due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



Watts/(sr·m²)

(3) radiance $L(\bar{p}, \vec{d}_o)$ emitted from infinitesimal patch at $\bar{p}$ in direction $\vec{d}_o$

Watts/m²

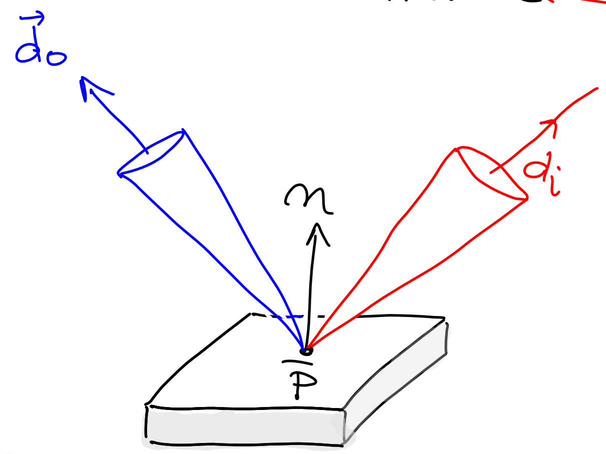
(2) irradiance $H(\bar{p})$ at an infinitesimal patch due to light received along one or more (or a continuum of) directions

General Definition: The BRDF of a Point

BRDF: $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

in direction $\vec{d}_o$

due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



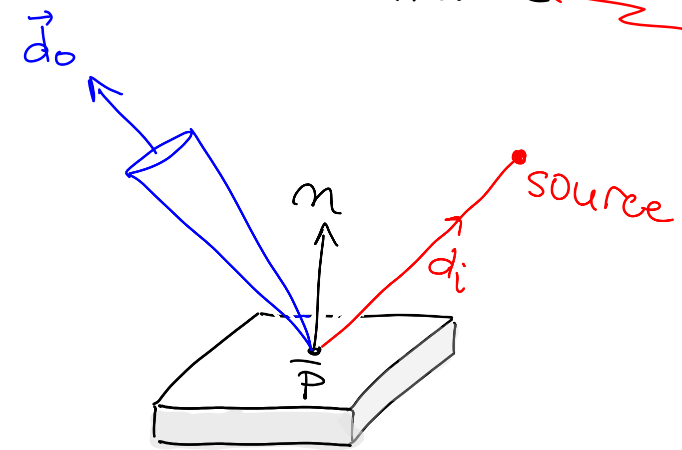
Intuition: The BRDF tells us how bright $\bar{p}$ will appear if viewed along $\vec{d}_o$ when it receives light from a small cone of directions along $\vec{d}_i$

General Definition: The BRDF of a Point

$$\text{BRDF}: \ \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around $\vec{d}_i$



Simpler: Suppose we only have a point light source

Intuition: The BRDF tells us how bright $\bar{p}$ will appear if viewed along $\vec{d}_o$ and the source is along $\vec{d}_i$

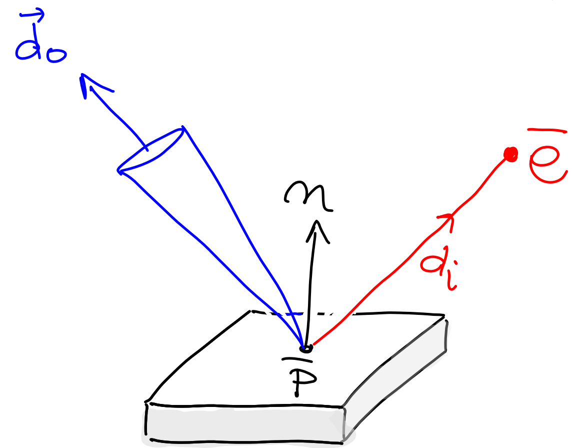
Radiance Due to a Point Light Source

* BRDF : $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

in direction $\vec{d}_o$

due to flux arriving from an infinitesimal solid angle around

has units of $\dfrac{1}{sr}$



Example #1: * Source is at $\bar{e}$

Radiant intensity is $I(\bar{p}-\bar{e})$

BRDF at $\bar{p}$ is $\rho_{\bar{p}}$

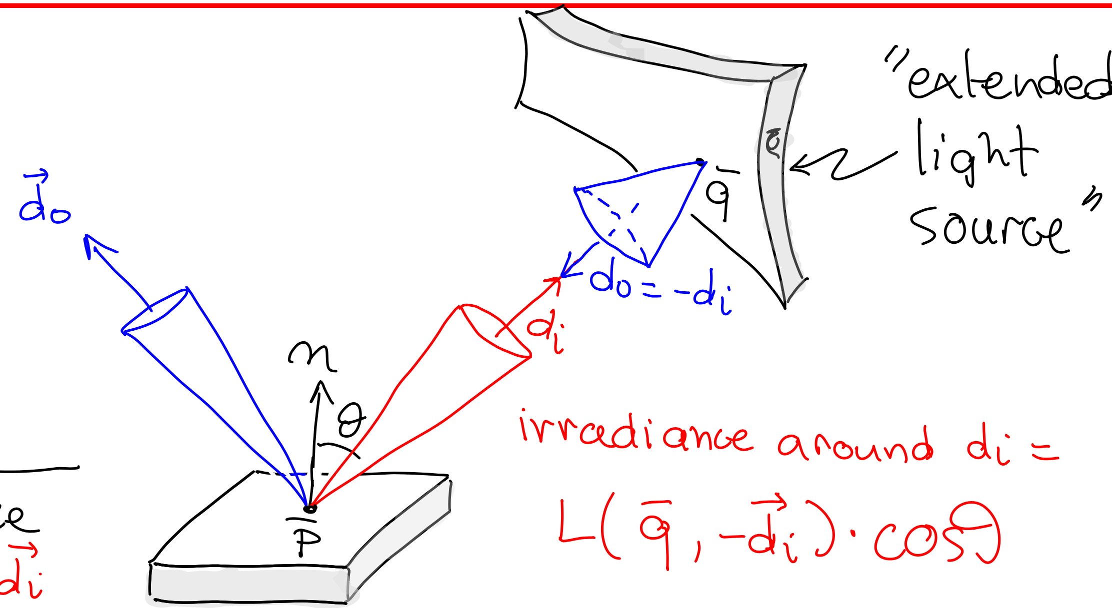
Q: What is the radiance along $\vec{d}_o$?

Reminder: $H(\bar{p}) = \dfrac{I(\omega)}{r^2}\cos\theta$

Ans: $L(\bar{p}, \vec{d}_o) = \rho(\vec{d}_i, \vec{d}_o)\, H(\bar{p}) = \rho(\vec{d}_i, \vec{d}_o)\, \dfrac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2}\cos\theta$

Radiance Due to an Extended Source

$$\rho_{\bar{p}}(\vec{d}_i,\vec{d}_o) = \frac{\text{radiance along } \vec{d}_o}{\text{irradiance around } \vec{d}_i}$$



irradiance around $\vec{d}_i$ = $L(\bar{q},-\vec{d}_i)\cdot\cos\theta$

Example #2: * Extended source with radiance $L(\bar{q},\vec{d}_i)$

BRDF at $\bar{p}$ is $\rho_{\bar{p}}$

Q: What is the radiance along $\vec{d}_o$?

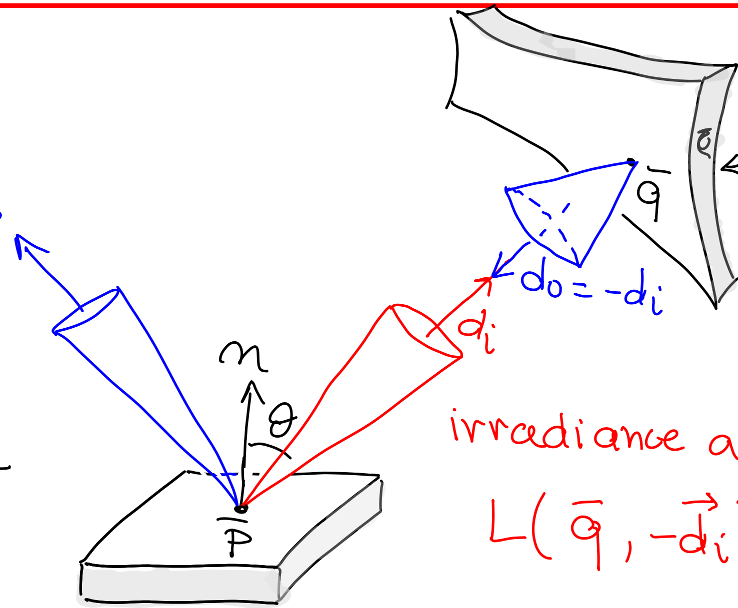
Ans: $$L(\bar{p},\vec{d}_o) = \int_{\vec{d}_i} \rho_{\bar{p}}(\vec{d}_i,\vec{d}_o)\cdot\underbrace{(\text{irradiance around } \vec{d}_i)}_{= L(\bar{q},-\vec{d}_i)(\vec{n}\cdot\vec{d}_i)}\, d(\vec{d}_i)$$

Radiance Due to an Extended Source

$$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

along $\vec{d}_o$ (radiance)

around $\vec{d}_i$ (irradiance)



"extended light source"

irradiance around d_i =

$L(\bar{q}, -\vec{d}_i) \cdot \cos\theta$

Example #2: Extended source with radiance $L(\bar{q}, \vec{d}_i)$

BRDF at $\bar{p}$ is $\rho_{\bar{p}}$

Q: What is the radiance along $\vec{d}_o$?

Using spherical coords (θ, φ) for $\vec{d}_i$:

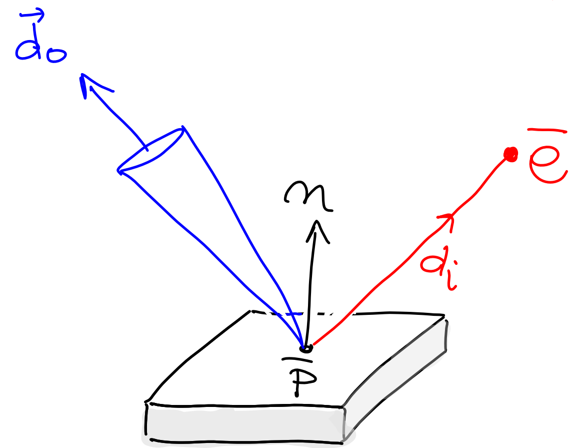
Ans: $$L(\bar{p}, \vec{d}_o) = \int_\theta \int_\varphi \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) \cdot \underbrace{(\text{irradiance around } \vec{d}_i)}_{= L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i)} \sin\theta \, d\theta \, d\varphi$$

The BRDF of a Diffuse Point

BRDF : $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around



Example #3: * What is the BRDF of a diffuse surface point?

For diffuse points:

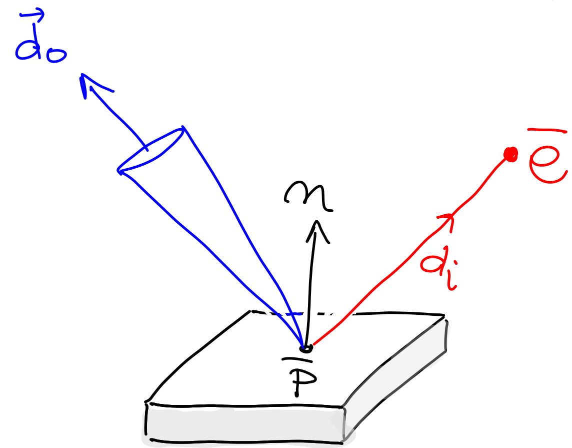
- brightness independent of $\vec{d}_o$
- brightness depends only on total incident flux (i.e. irradiance) not illumination dir

The BRDF of a Diffuse Point

$$\text{BRDF}: \ \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around



Example #3: What is the BRDF of a diffuse surface point?

For diffuse points:

radiance = constant fraction of irradiance

$$\Rightarrow \quad \rho(\vec{d}_i, \vec{d}_o) = \text{constant}$$

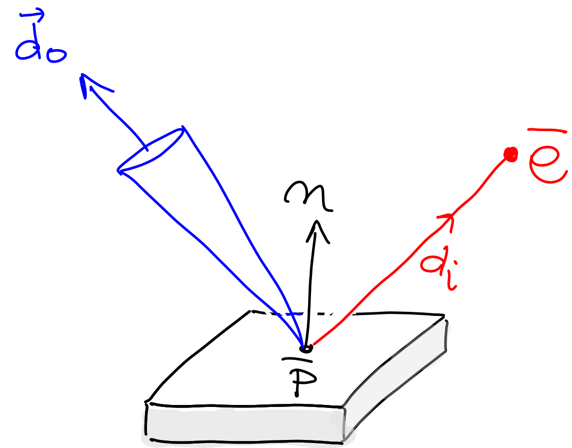
↑ what is it equal to?

The BRDF of a Diffuse Point

BRDF : $\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \dfrac{\text{radiance}}{\text{irradiance}}$

radiance ← in direction $\vec{d}_o$

irradiance ← due to flux arriving from an infinitesimal solid angle around $\bar{e}$



Example # 3: What is the BRDF of a diffuse surface point?

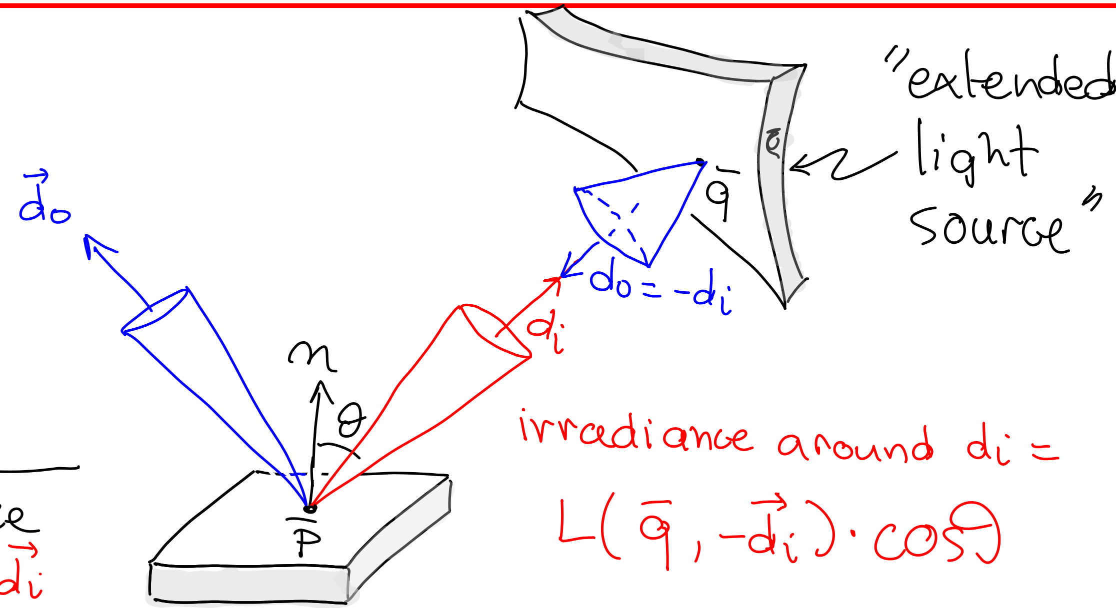
For diffuse points:

total light coming in (irradiance) = total light going out (radiant exitance)

conservation of energy

can show that $\rho = \frac{1}{\pi}$

Radiance of a Diffuse Point Due to Extended Src



along $\vec{d}_o$

$$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

around $\vec{d}_i$

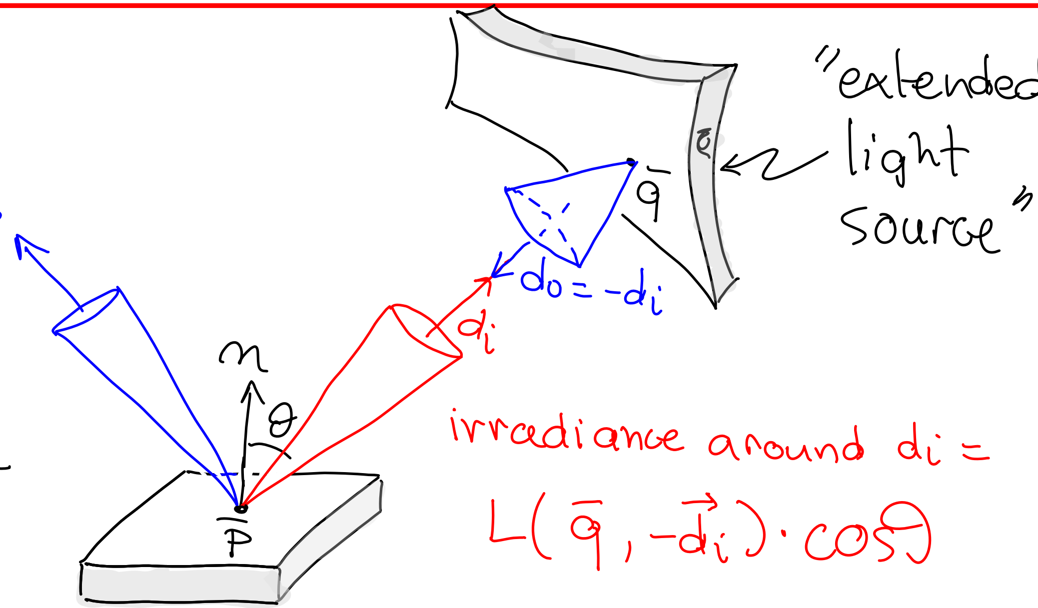
irradiance around d_i = $L(\bar{q}, -\vec{d}_i) \cdot \cos\theta$

Example #4: Extended source with radiance $L(\bar{q}, \vec{d}_i)$
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p}, \vec{d}_o) = \int_{\vec{d}_i} \rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) \cdot \underbrace{(\text{irradiance around } \vec{d}_i)}_{= L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i)} \, d(\vec{d}_i)$$

Radiance of a Diffuse Point Due to Extended Src



along $\vec{d}_o$

$$\rho_{\bar{p}}(\vec{d}_i, \vec{d}_o) = \frac{\text{radiance}}{\text{irradiance}}$$

around $\vec{d}_i$

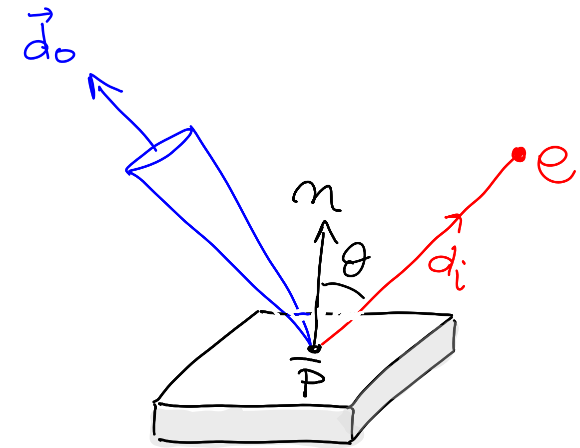
Example #4: Extended source with radiance $L(\bar{q}, \vec{d}_i)$

$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p}, \vec{d}_o) = \frac{1}{\pi} \int_{\vec{d}_i} L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i)\, d(\vec{d}_i)$$

Radiance of Diffuse Point due to Point Light Src



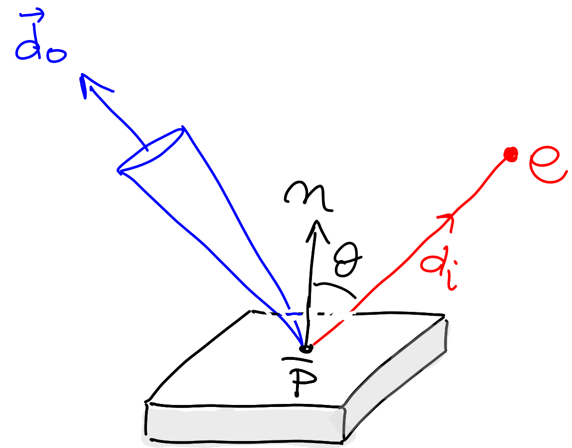
Example # 5 : * Point light source at distance r
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $L(\bar{p}, \vec{d}_o) = \frac{1}{\pi} \cancel{\int_{\vec{d}_i}} L(\bar{q}, -\vec{d}_i)(\vec{n} \cdot \vec{d}_i) \cancel{d(\vec{d}_i)}$ only one direction

$L(\bar{q}, -\vec{d}_i) = \frac{I(\bar{p}-\bar{e})}{\|\bar{p}-\bar{e}\|^2}$

Radiance of Diffuse Point due to Point Light Src



Example # 5 : Point light source at distance r
$\bar{p}$ is a diffuse point

Q: What is the radiance along $\vec{d}_o$?

Ans: $$L(\bar{p},\vec{d}_o) = \frac{1}{\pi} \cdot \frac{I(\bar{p}-\bar{e})}{\cancel{\|\bar{p}-\bar{e}\|^2}} \cdot (\vec{n}\cdot\vec{d}_i) = \frac{1}{\pi} I(\bar{p}-\bar{e})(\vec{n}\cdot\vec{d}_i)$$

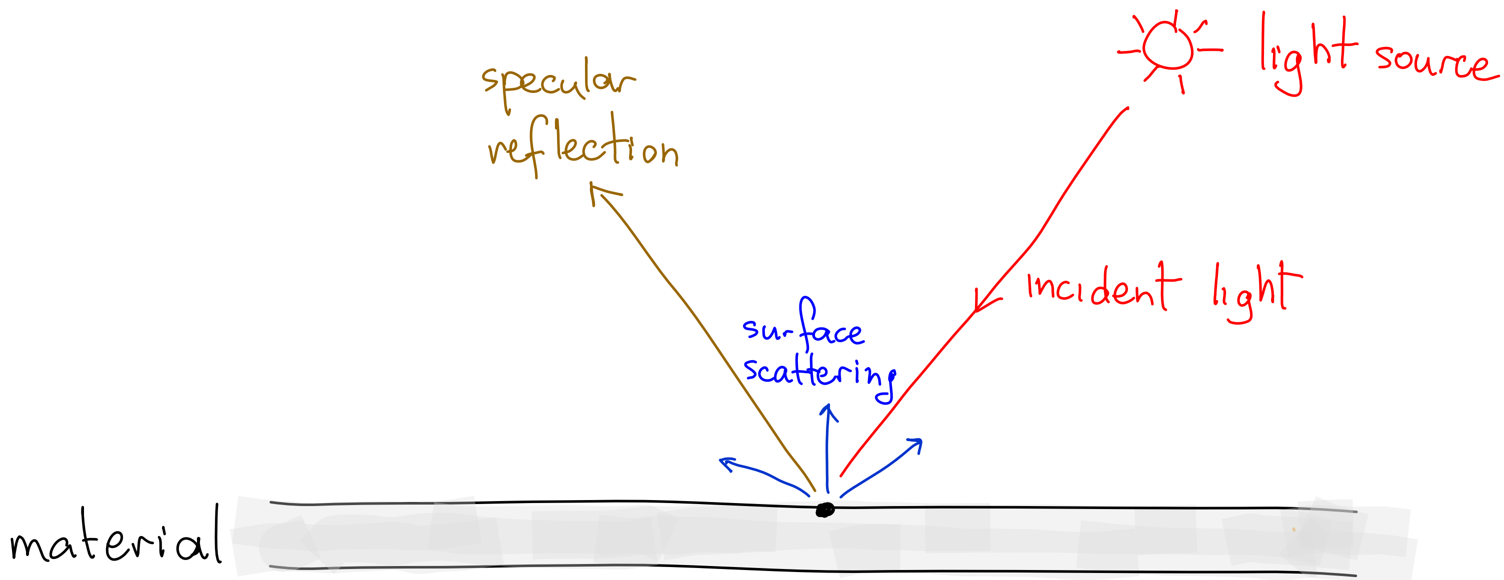
can be ignored if light very far away

Topic 08:

Radiometry

- The big picture
- Measuring light coming from a light source
  - Measurements for a “2D world”
  - Generalization to 3D
- Measuring light falling onto a patch: Irradiance
- Measuring light leaving a patch: Radiance
- The Bidirectional Reflectance Distribution Function
- Phong Reflectance Model

A Simple Parametric Model: Phong Reflectance

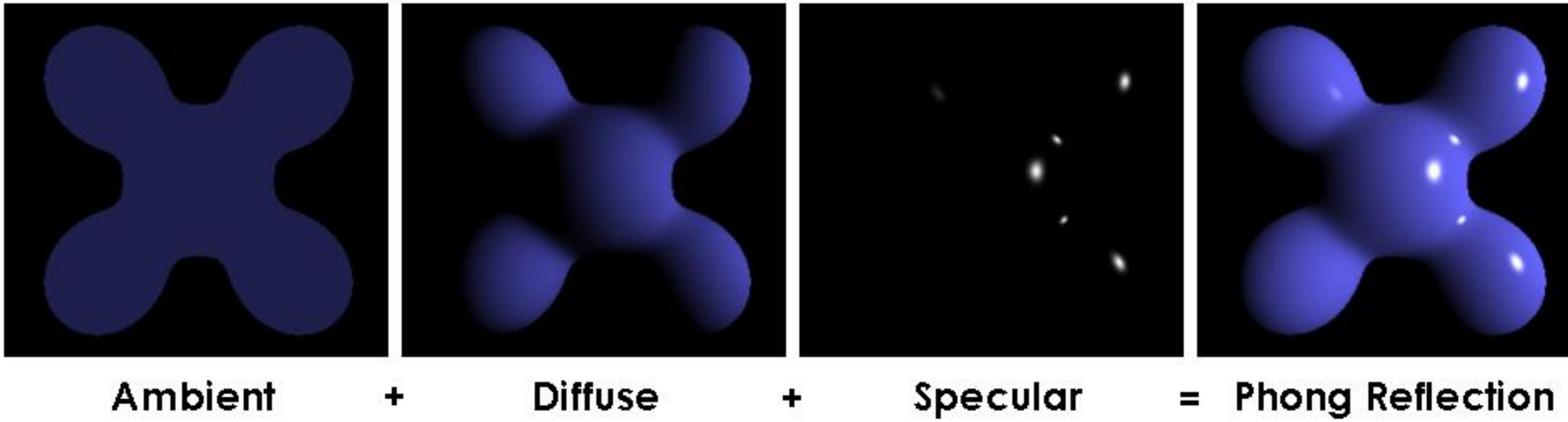


Phong model: A simple, computationally-efficient model that has 3 components:

- Diffuse
- Ambient
- Specular

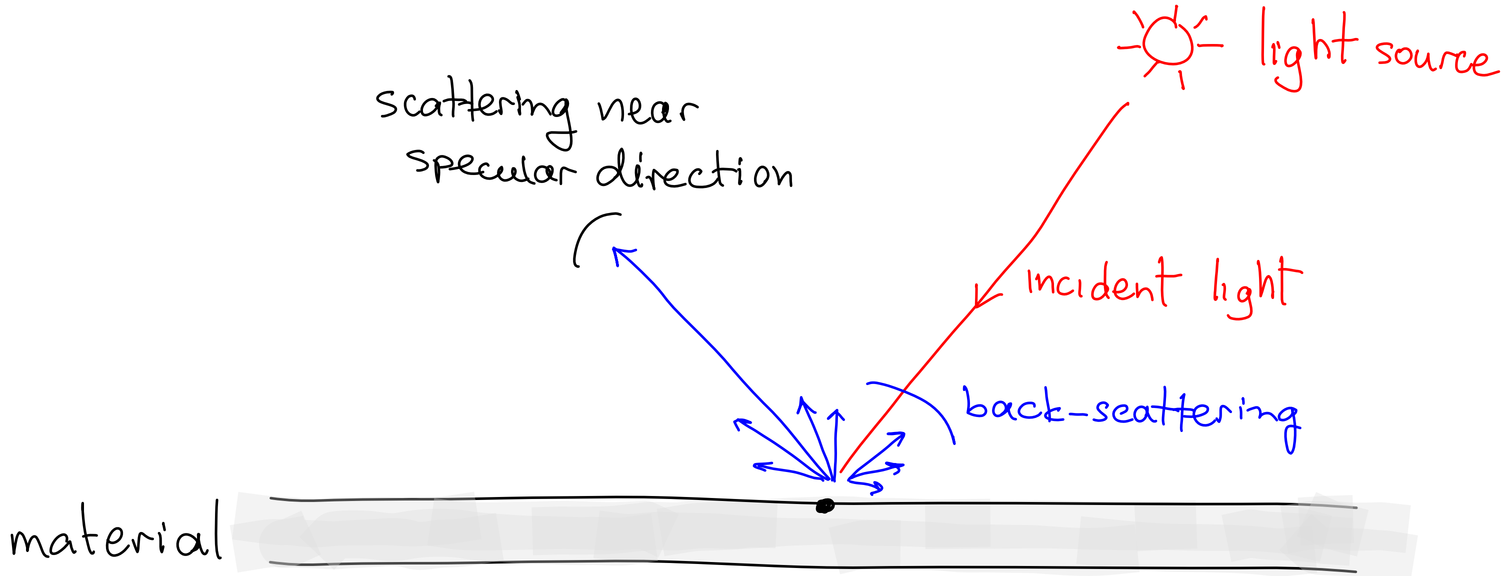
Phong Reflectance: The General Equation *

Brad Smith, Wikipedia



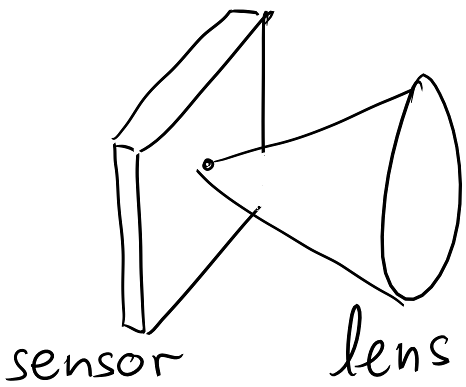
$$\underbrace{L(\vec{b},\vec{n},\vec{s})}_{\substack{\text{intensity at}\\ \text{projection of}\\ \text{point } \bar{p}}} = \underbrace{r_a I_a}_{\text{ambient}} + \underbrace{r_d I_d \max(0, \vec{n}\cdot\vec{s})}_{\text{diffuse}} + \underbrace{r_s I_s \max(0, \vec{r}\cdot\vec{b})^{\alpha}}_{\text{specular}}$$

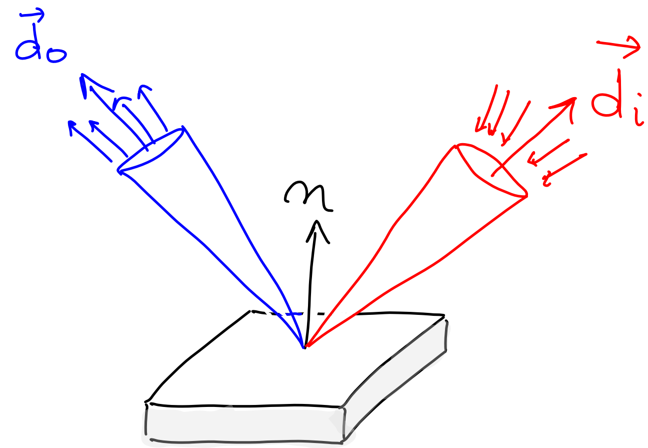
Generalizing the Phong Model

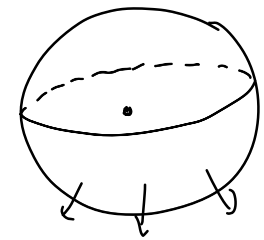


- All reflected light can be thought of as a form of scattering
- For most real materials, the Phong-based distinction into specular + diffuse reflection is a crude approximation

Radiometry: Getting the Physics Right







Radiometry: measurement of electromagnetic radiation

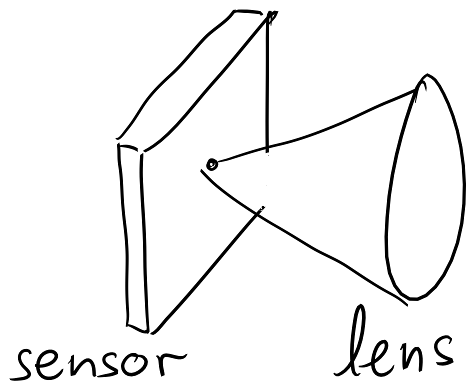
Physics:
- Joules
- Watts

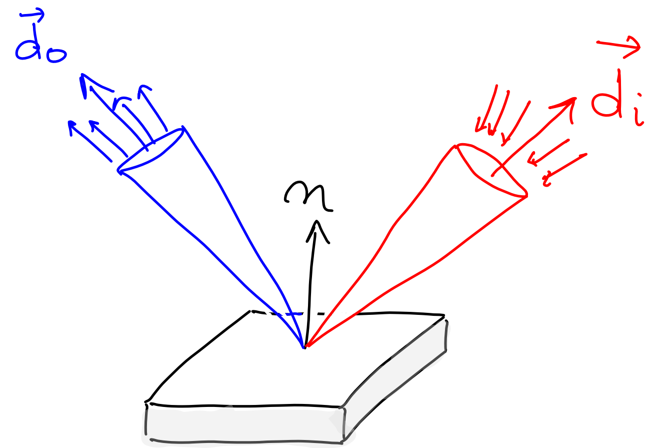
Geometry:

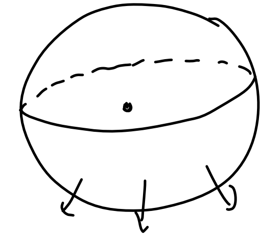
differential
- patches
- directions
- solid angles
- foreshortened area

Radiometry:
- radiant energy
- radiant flux
- irradiance
- radiance
- radiant exitance
- BRDF

Radiometry & Color (see lecture notes)







Radiometry : measurement of electromagnetic radiation

Physics:
- Joules
- Watts

+ wavelength λ

Geometry:

differential
- patches
- directions
- solid angles
- foreshortened area

all quantities are functions of wavelength too!

Radiometry:
- radiant energy
- radiant flux
- irradiance
- radiance
- radiant exitance
- BRDF