$\mathbb{R}^n \xrightarrow{f} \mathbb{R}^m$ $\qquad r \le \min(m,n)$

$(x_1, \dots x_n) \longrightarrow (x_1 \dots x_r, 0, 0 \dots 0)$

Rank of f at x = $rk(f'(x))$ $\qquad$ Find Jacobian

$$\left[\begin{array}{cccc|c} 1 & 0 & \cdots & & - \\ 0 & 1 & \cdots & & - \\ & \underbrace{\quad\quad}_{r} & & 1 & 00 \\ 0 & & & & \\ 0 & & \cdots & & 0 \end{array}\right]$$

$$\phi(x,y) = \begin{cases} \dfrac{x^n y}{x^2+y^2} & ; (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$$

$x \qquad x^1 y$

$D_1 \phi(x,y)$; D

$$\frac{d\phi}{dx} \quad \frac{y(x^2+y^2) - xy(2x)}{(x^2+y^2)^2} = \frac{y^3 - x^2 y}{(x^2+y^2)^2}$$

$$\frac{d\phi}{dy} \quad \frac{x(x^2+y^2) - xy(2y)}{(x^2+y^2)^2} = \frac{x^3 - y^2 x}{(x^2+y^2)^2}$$

$$\frac{Df}{dx} = \frac{(m x^{m-1} y (x^2+y^2) - (2x\, x^m y)}{(x^2+y^2)^2} = \frac{m x^{x-1} y^3 + (m-2) x^{x+1} y}{(x^2+y^2)^2}$$

$$\frac{Df}{dy} = \frac{x^m (x^2+y^2) - (2y) x^m y}{(x^2+y^2)^2} = \frac{(x^2-y^2) x^m}{(x^2+y^2)^2}$$

$$\frac{xy}{x^2+y^2} \le 1/2 \qquad |f(x,y) \le 1/2 |x|$$

$$\frac{f(h,k)}{\sqrt{h^2+k^2}} = \frac{h^2 k}{(h^2+k^2)^{3/2}}$$

If your function is partial derivative, and they are continuous then total derivative exist and it's continuous

iii) $f: E \longrightarrow \mathbb{R}^m$, $\quad E \subseteq \mathbb{R}^n$

$$f(x) = \sum_{i=1}^{m} f_i(x) u_i$$

If $D_j f_i(x)$ are bounded on E, for $1 \le i \le m$, $1 \le j \le n$

Show f is continuous, & continuously differentiable

Ans) Since $D_j f_i(x)$ are bounded.

$$\begin{aligned} f(x+h) - f(x) &= f(x_1+h_1, x_2+h_2) - f(x_1, x_2) \\ &= f(x_1+h_1, x_2+h_2) - f(x_1, x_2+h_2) + f(x_1, x_2+h_2) - f(x_1, x_2) \\ &= h_1 D_1 f(x_1+\theta_1 h_1, x_2+h_2) + h_2 D_2 f(x_1, x_2+\theta_2 h_2) \\ &\le |h_1| (\qquad) + |h_2| (\qquad) \end{aligned}$$

Since partial der are bounded,

$$f : \overset{U, \mathbb{R}^n}{E} \longrightarrow \mathbb{R}^n$$

$f'(a)$ exist

$$f'(a) e_j = \sum D_j f_i(a) u_i$$

$$|D_j f_i(x) - D_j f_i(y)| = |\langle f'(x) e_j, u_i \rangle - \langle f'(y) e_j, u_i \rangle|$$
$$= |\langle (f'(x) - f'(y)) e_j, u_i \rangle|$$
$$\leq \| f'(x) - f'(y) \| \to 0 \text{ as } y \to x$$

$$f = (f_1, f_2)$$

$$(D_1 f_1(x) + D_2 f_1(x)) \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$f_1(x+h) - f_1(x) - [h_1 D_1 f_1(x) + h_2 D_2 f_1(x)]$$

$$f_1(x + V_2) - f_1(x + V_1) + f_1(x + V_1) + f_1(x + V_0)$$
$$- (h_1 D_1 f_1(x) + h_2 D_2 f_1(x))$$

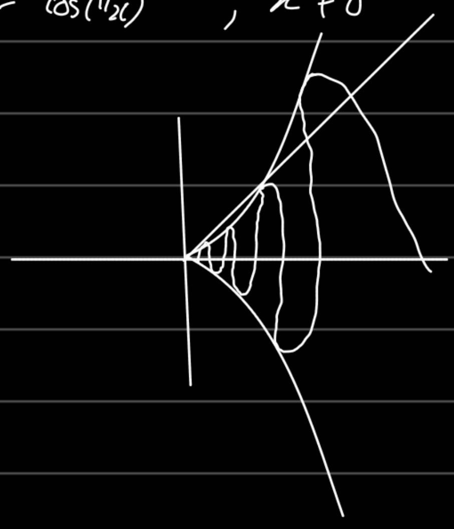
$V_0 = 0$
$V_1 = h_1 v_1$
$V_2 = h$

Q2.

$$f(x) = \begin{cases} x + x^2 \sin(1/x) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

x^{-1}

$$f'(x) = 1 + 2x \sin(1/x) - \cos(1/x) \quad ; x \neq 0$$



this function is ~~not~~ injective.

$$f(x) = x^3$$
$$f'(x) = 3x^2$$

$$f : \mathbb{R}^2 \to \mathbb{R}^2$$
$$|f'(z)|^2 = \det(J_f(z))$$

$$\det \begin{bmatrix} & \\ & \end{bmatrix} = e^{2x_1}$$

$$f(x_1, x_2) = f(x_1 + 2n\pi x_2)$$

$$f(x_1, x_2) = \left(\frac{x_1}{x_1^2 + x_2^2}, \frac{x_2}{x_1^2 + x_2^2} \right)$$

$$f\left(\frac{x_1}{x}, \frac{x_1}{x}, \left(\frac{x^1}{} \right. \right.$$

Q3) i) $f(x_1, x_2) = (e^{x_1}\cos x_2,\ e^{x_1}\sin x_2)$ $f(z) = e^z$

ii) $f(x_1, x_2) = (x_1^2 - x_2^2,\ 2x_1x_2)$ $f(z) = z^2$

iii) $f(x_1, x_2) = \left(\frac{x_1}{x_1^2+x_2^2},\ \frac{x_2}{x_1^2+x_2^2}\right)$ $f(z) = 1/z$ $(x_1, x_2) \neq (0,0)$

↳ both inversion & convulution same.

$f(x,y) = (x_1y_1^2 + x_2,\ x_1x_2^2 + y_1^2y_2^2)$

$\mathbb{R}^n \times \mathbb{R}^m$
$\cup$
$f: E \longrightarrow \mathbb{R}^n$
(a,b)

Supp $f(a,b) = 0$, mat $(f'(a,b))$

A_x A_y

$x_1y_1^2 + x_2,\ x_1x_2^2 + y_1^2y_2^2$

$$f'(x,y) = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc|cc} dx_1 & dx_2 & dy_1 & dy_2 \end{array}$$

$$f'(x,y) = \left[\begin{array}{cc|cc} y_1^2 & 1 & 2x_1y_1 & 0 \\ x_2^2 & 2x_1x_2 & 2y_1y_2^2 & 2y_2y_1^2 \end{array}\right] \begin{array}{c} a \\ b \end{array}$$

$a = (-1, 1)$ $b = (1, 1)$
$\ \ x_1\ x_2$ $\ \ y_1\ y_2$

$$= \left[\begin{array}{cc|cc} 1 & 1 & -2 & 0 \\ 1 & -2 & 2 & 2 \end{array}\right]$$

$x_1y_1^2 + x_2 = 0$ $y_1 = \pm\sqrt{\frac{x_2}{(-x_1)}}$

$x_1x_2^2 - \frac{x_2}{x_1}y_1^2 =$

$y_1^2 = x_1x_2 \times x_1$
$= x_1^2x_2$

$y_2 = |x_1|\sqrt{x_2}$

$g'(a) = -A_y^{-1}A_x$

$$= \begin{bmatrix} -2 & 0 \\ 2 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}$$

$=$

$f_1 = 3x + y - z + u^2 = 0$
$f_2 = x - y + 2z + u = 0$
$f_3 = 2x + 2y - 3z + 2u = 0$

$$J_f(x,y,z,u) = \begin{bmatrix} 3 & 1 & -1 & 2u \\ 1 & -1 & 2 & 1 \\ 2 & 2 & -3 & 2 \end{bmatrix}$$

$u^2 - 3u = 0$ $u = 0\ u = 3.$

$A(x,y,u) =$

$f : \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}$

$$f(x, y_1, y_2) = x^2 y_1 + e^{x+y_2}$$