(Q 1) $f: [a,b] \to \mathbb{R}$ bounded, $P = \{x_0 \cdots x_n\}$

$I_i = [x_{i-1}, x_i]$ try $m_i(f) = \inf_{t \in I_i} f(t)$, $M_i(f) = \sup_{t \in I_i} f(t)$

(i) $1 \le i \le n$, $M_i(f) - m_i(f) = \sup\{|f(t) - f(s)| : t \in I_i\}$

$f(t) \le M_i(f)$

$f(s) \le$

Ily $m_i(f) \ge f(s)$

$f(t)$

$\Rightarrow M_i(f) - m_i(f) \ge |f(t) - f(s)|$

$\therefore M_i(f) - m_i(f) > 0$ for some ε

then $M_i(f) - m_i(f) - \varepsilon > 0$

$\delta < (M_i f - \varepsilon/2) - (m_i f + \varepsilon/2)$

(ii) $|f|(x) = |f(x)|$, $M_i(|f|) - m_i(|f|) \le M_i(f) - m_i(f)$

$U(|f|, P) - L(|f|, P) \le U(f, P) - L(f, P)$

(iii) $M_i(f^2) = (M_i(f))^2$

$M_i(f^2) - m_i(f^2) = M_i(|f|)^2 - m_i(|f|)^2$

$= (M_i(|f|) + m_i(|f|))(M_i(|f|) - m_i(|f|))$

$\le 2M(M_i(|f|) - m_i(|f|))$

$fg = \frac{1}{2}[(f+g)^2 - f^2 - g^2]$

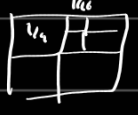
$L(-f, P) = -U(f, P)$ check.

bounded

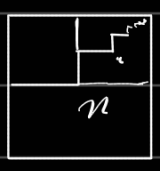
$S \subseteq \mathbb{R}$ $\inf(-S) = -\sup(S)$

refinement.

Q) iv) $f:[0,1] \to$



(1/4)



$(1/4 - x/4) + x + 1/4 = 1$

$+ 3x/4 = 1/2$

$x = \frac{4}{3} \times 1/2 = 2/3$

continuous, riemann integrable, bdd

GD set