Q1)

(v) $mat(A) = [a_{ij}]_{m\times n}$

$$x = \sum_{j=1}^{n} x_j e_j \in \mathbb{R}^n$$

$$Ax = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} a_j x_j \right) u_i \in \mathbb{R}^m$$

$$\| Ax \|^2 = \sum_{j=1}^{m} \left(\sum_{j=1}^{n} a_{ij} x_2 \right)^2 \leq \sum_{i=1}^{m} \left(\sum_{j=1}^{n} a_{ij}^2 \right) \|x\|^2$$

$$\| Ax \| \leq \left(\sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2} \right) \|x\|$$

$$\|A\| \leq$$

Q4) Banach contraction theorem

Q3) (Mean value theorems)

$$\gamma : [0,1] \longrightarrow \mathbb{R}^n \xrightarrow{\langle \, , u \rangle} \mathbb{R}$$

$$\gamma(t) = (\gamma_1(t), \ldots \; \gamma_n(t))$$

$$g(t) = \langle \gamma(t), u \rangle = \sum_{i=1}^{n} \gamma_i(t) u_i$$

$$|g(1) - g(0)| = |g'(t)|$$

$$|g'(t)| = |\langle \gamma'(t), u \rangle| \leq \| \gamma'(t) \| \|u\| \leq C\|u\|$$

$$|(\gamma(1) - \gamma(0)) u| = |\gamma'(t) u|$$

$$\Rightarrow \| \gamma(1) - \gamma(0) \| \leq C$$

$$(D_u \phi)(x) = (\nabla \phi(x)) . u.$$