

Apply local and global common subexpression elimination to quicksort code flowgraph.

Apply local and global common sub expression elimination to quicksort code flowgraph

STEP 1:

apply local common sub expression elimination to all basic blocks

use AEB algorithm

STEP 2:

apply global common sub expression elimination

- find EVAL(i), KILL(i)
- Use them to find in(i) and out(i) [iterative algorithm]
- Use AEin(i) for global common sub expression elimination

- 1. i = m 1
- 2. j = n
- 3. t1 = 4 * n
- 4. v = a[t1]

- 1. AEB={<1, m, -, 1, nil>}
- 2. AEB={<1, m, -, 1, nil>}
- 3. AEB={<1, m, -, 1, nil>, <3, 4, *, n, t1>}
- 4. AEB={<1, m, -, 1, nil>, <3, 4, *, n, t1>}

No change in block

1.
$$i = i + 1$$

2.
$$t2 = 4 * i$$

3.
$$t3 = a[t2]$$

4. if
$$t3 < v$$
 goto B2

3.
$$AEB=\{<2, 4, *, i, t2>\}$$

No change in block

1.
$$j = j - 1$$

- 2. t4 = 4 * j
- 3. t5 = a[t4]
- 4. if t5 > v goto B3

- 1. AEB=∅
- 2. $AEB=\{<2, 4, *, j, t4>\}$
- 3. $AEB=\{<2, 4, *, j, t4>\}$
- 4. AEB={<2, 4, *, j, t4>, <4, t5, >, v, nil>}

No change in block

1. if
$$i \ge j$$
 goto B6



1.
$$t6 = 4 * i$$

2.
$$x = a[t6]$$

3.
$$t7 = 4 * i \rightarrow t7 = t6$$

4.
$$t8 = 4 * i$$

5.
$$t9 = a[t8]$$

6.
$$a[t7] = t9$$

7.
$$t10 = 4 * j \rightarrow t10 = t8$$

8.
$$a[t10] = x$$

3.
$$AEB=\{<1, 4, *, i, t6>\}$$

7.
$$AEB=\{<1, 4, *, i, t6>, <4, 4, *, j, t8>\}$$



1.
$$t6 = 4 * i$$

2.
$$x = a[t6]$$

3.
$$t7 = 4 * i$$

4.
$$t8 = 4 * j$$

5.
$$t9 = a[t8]$$

6.
$$a[t7] = t9$$

7.
$$t10 = 4 * j$$

8.
$$a[t10] = x$$

1.
$$t6 = 4 * i$$

2.
$$x = a[t6]$$

3.
$$t7 = t6$$

4.
$$t8 = 4 * j$$

5.
$$t9 = a[t8]$$

6.
$$a[t7] = t9$$

7.
$$t10 = t8$$

8.
$$a[t10] = x$$

9. goto B2



1.
$$t6 = 4 * i$$

2.
$$x = a[t6]$$

3.
$$t7 = 4 * i$$

4.
$$t8 = 4 * j$$

5.
$$t9 = a[t8]$$

6.
$$a[t7] = t9$$

$$7. \quad t10 = 4 * j$$

8.
$$a[t10] = x$$

1.
$$t6 = 4 * i$$

2.
$$x = a[t6]$$

3.
$$t8 = 4 * j$$

4.
$$t9 = a[t8]$$

5.
$$a[t6] = t9$$

6.
$$a[t8] = x$$

7. goto B2

As, these calculation were not requested explicitly by the programmer, they are **eliminated**.*

NOTE:

- Compilers need to be judicious about the number of temporaries created to hold values.
- An excessive number of temporary values creates register pressure possibly resulting in spilling registers to memory, which may take longer than simply recomputing an arithmetic result when it is needed.



2.
$$x = a[t11]$$

3.
$$t12 = 4 * i \rightarrow t12 = t11$$

5.
$$t14 = a[t13]$$

6.
$$a[t12] = t14$$

7.
$$t15 = 4 * n \rightarrow t15 = t13$$

8.
$$a[t15] = x$$

3.
$$AEB=\{<1, 4, *, i, t11>\}$$



2.
$$x = a[t11]$$

3.
$$t12 = 4 * i$$

5.
$$t14 = a[t13]$$

6.
$$a[t12] = t14$$

7.
$$t15 = 4 * n$$

8.
$$a[t15] = x$$

2.
$$x = a[t11]$$

3.
$$t12 = t11$$

5.
$$t14 = a[t13]$$

6.
$$a[t12] = t14$$

7.
$$t15 = t13$$

8.
$$a[t15] = x$$



2.
$$x = a[t11]$$

$$3. +12 = 4 * i$$

5.
$$t14 = a[t13]$$

6.
$$a[t12] = t14$$

$$7. \quad t15 = 4 * n$$

8.
$$a[t15] = x$$

2.
$$x = a[t11]$$

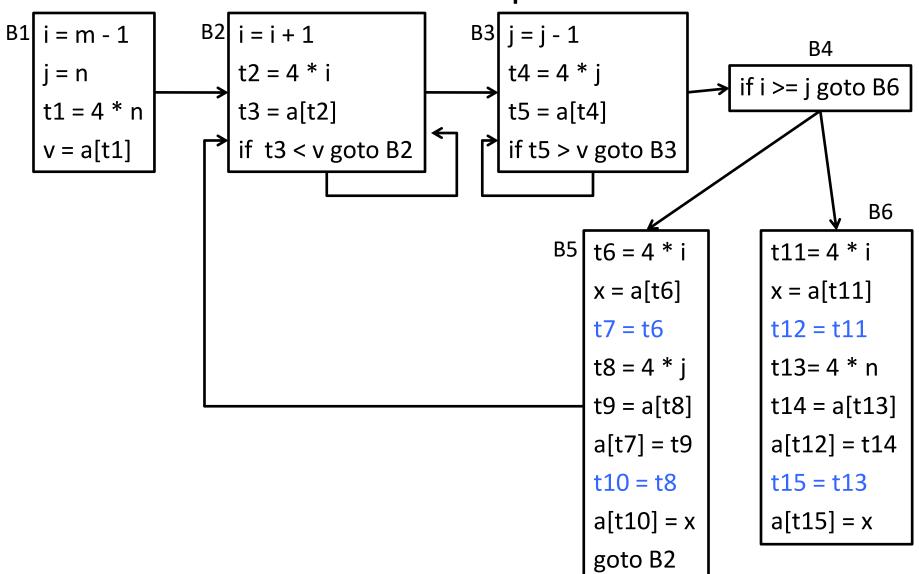
4.
$$t14 = a[t13]$$

5.
$$a[t11] = t14$$

6.
$$a[t13] = x$$



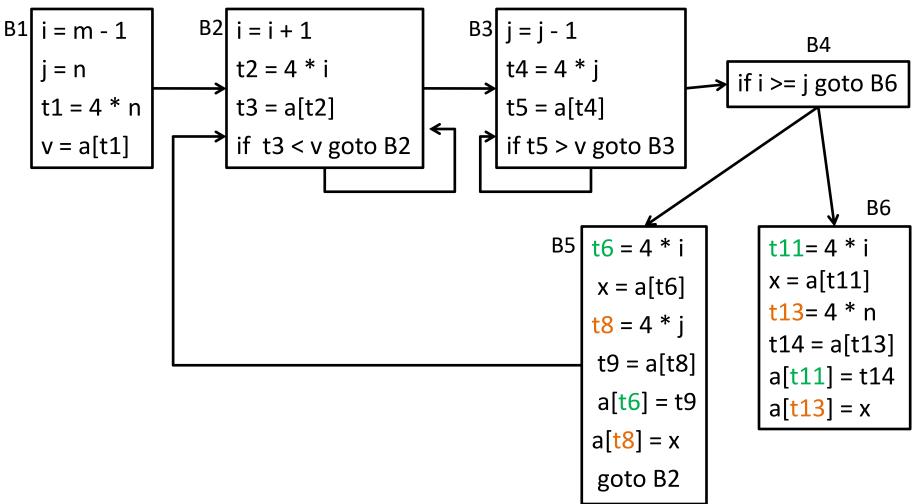
After local common subexpression elimination



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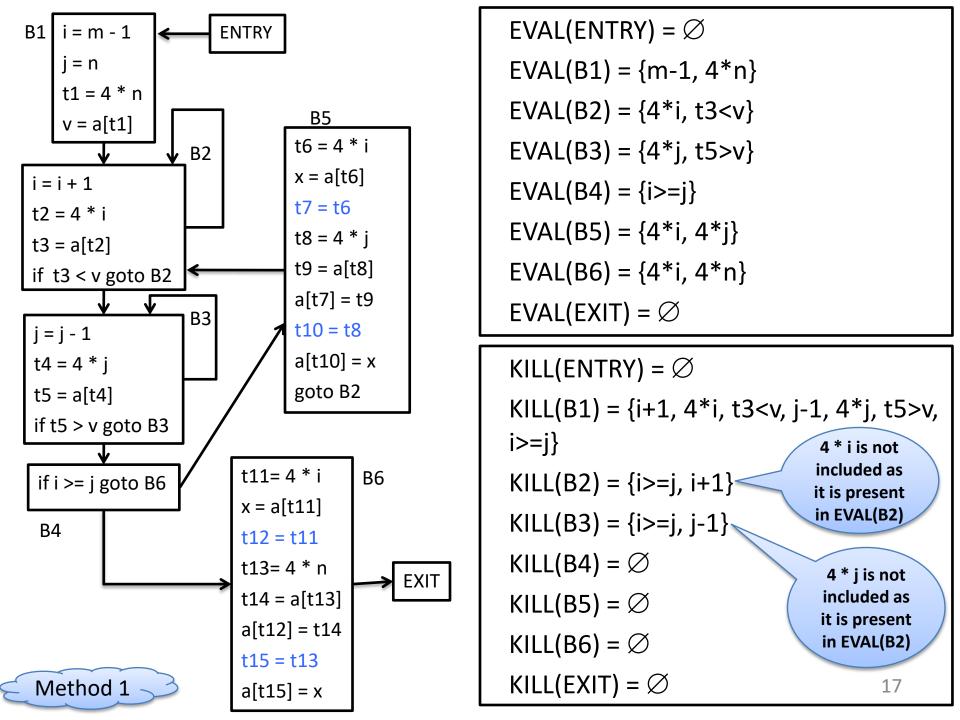


After local common subexpression elimination



Global Common subexpression elimination

- Find EVAL(i), KILL(i)
- Use them to find in(i) and out(i) [iterative algorithm]
- Use AEin(i) for global common sub expression elimination



EVAL(ENTRY) =
$$\emptyset$$

EVAL(B1) = {m-1, 4*n}
EVAL(B2) = {4*i, t3v}
EVAL(B4) = {i>=j}
EVAL(B5) = {4*i, 4*j}
EVAL(B6) = {4*i, 4*n}
EVAL(EXIT) = \emptyset

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KILL(ENTRY) = \emptyset

KILL(B1) = {i+1, 4*i, t3<v, j-1, 4*j, t5>v, i>=j}

KILL(B2) = {i>=j, i+1}

KILL(B3) = {i>=j, j-1}

KILL(B4) = \emptyset

KILL(B5) = \emptyset

KILL(B6) = \emptyset

KILL(B6) = \emptyset

KILL(EXIT) = \emptyset

U = U EVAL(i) = {m-1, 4*n, 4*i, t3<v, 4*j, t5>v, i>=j}
```

	Initial	Iteration 1	
	OUT(i) = U - KILL(i)	$IN(i) = \cap out(j) j \in Pred(i)$	$OUT(i) = EVAL(i) \cup (IN(i) - KILL(i))$
ENTRY	Ø	Ø	Ø
B1	{m-1, 4*n}	Ø	{m-1, 4*n}
B2	{m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v }</v,>	{m-1, 4*n}	{4*i, t3 <v, 4*n}<="" m-1,="" th=""></v,>
В3	{m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>	{m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v }</v,>	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< th=""></v}<>
B4	U	{m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
B5	U	{m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v, i>=j}</v,>	{4*i, 4*j, m-1, 4*n, t3 <v, t5="">v, i>=j}</v,>
В6	U	{m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v, i>=j}</v,>	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>
EXIT	U	{m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v, i>=j}</v,>	not required

EVAL(ENTRY) = \emptyset	
EVAL(B1) = {m-1, 4*r	ո}
EVAL(B2) = {4*i, t3<\	/ }
EVAL(B3) = {4*j, t5>\	/ }
$EVAL(B4) = \{i >= j\}$	
EVAL(B5) = {4*i, 4*j}	
EVAL(B6) = {4*i, 4*n	}
EVAL(EXIT) = \emptyset	
Iteration 1	
old OUT(i)	IN

4*i, t3<v}

t3<v, 4*j, t5>v}

t3 < v, t5 > v, i > = j

{4*i, 4*n, m-1,

not required

{4*i, 4*j, m-1, 4*n,

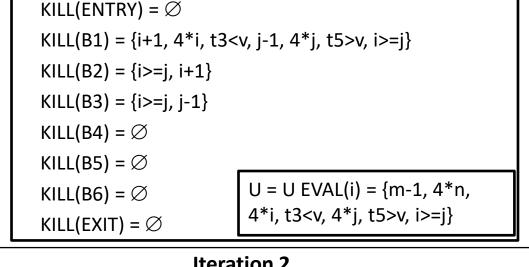
t3<v, 4*j, t5>v, i>=j}

B4

B5

B6

EXIT



|{i>=j, m-1, 4*n, 4*i, t3<v, 4*j, t5>v} |{4*i, 4*j, i>=j, m-1, 4*n, t3<v,

{i>=j, m-1, 4*n, 4*i, t3<v, 4*j, t5>v} |{4*i, 4*n, i>=j, m-1, t3<v, 4*j,

{4*i, 4*n, m-1, t3<v, 4*j, t5>v, i>=j}|not required

t5>v}

t5>v}

{i>=j, 4*j, t5>v, m-1, 4*n, 4*i, t3<v}

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EVAL(D4) = {1>=J}		$KILL(B4) = \emptyset$			
$EVAL(B5) = {4*i, 4*j}$		$KILL(B5) = \emptyset$			
	$EVAL(B6) = {4*i, 4*n}$	}	KILL(B6) = ∅	U = U EVAL(i) = {m-1, 4*n,	
	EVAL(EXIT) = ∅		$KILL(EXIT) = \varnothing$	4*i, t3 <v, 4*j,="" t5="">v, i>=j}</v,>	
	Iteration 1		Itera	ation 2	
	old_OUT(i)	IN(i)	= ∩ out(j) j∈ Pred(i)	$OUT(i) = EVAL(i) \cup (IN(i) - KILL(i)$	
ENTRY \varnothing		Ø		Ø	
B1 {m-1, 4*n}		Ø		{m-1, 4*n}	
B2 {4*i, t3 <v, 4*n<="" 4*n}="" m-1,="" th="" {m-1,=""><th>}</th><th colspan="2">{4*i, t3<v, 4*n}<="" m-1,="" th=""></v,></th></v,>		}	{4*i, t3 <v, 4*n}<="" m-1,="" th=""></v,>		
	Casha - a ash	C a viva	a auto 3	fadia – adia a	

	EVAL(B6) = $\{4^*i, 4^*n\}$ EVAL(EXIT) = \emptyset		$KILL(B6) = \emptyset$ $KILL(EXIT) = \emptyset$	4*i, t3 <v, 4*j,="" t5="">v, i>=j}</v,>	
	Iteration 1		Iteration 2		
old_OUT(i)		IN(i)	= ∩ out(j) j∈ Pred(i)	OUT(i) = EVAL(i) U (IN(i) - KIL	_L(i
ENTRY	Ø	Ø		Ø	
B1	{m-1, 4*n}	Ø		{m-1, 4*n}	
B2	{4*i, t3 <v, 4*n}<="" m-1,="" th=""><th>{m-1, 4*n</th><th>}</th><th>{4*i, t3<v, 4*n}<="" m-1,="" th=""><th></th></v,></th></v,>	{m-1, 4*n	}	{4*i, t3 <v, 4*n}<="" m-1,="" th=""><th></th></v,>	
В3	{4*j, t5>v, m-1, 4*n, {4*i, t3 <v< th=""><th>m-1, 4*n}</th><th>4*j, t5>v, 4*i, t3<v, 4*r<="" m-1,="" th=""><th>1}</th></v,></th></v<>		m-1, 4*n}	4*j, t5>v, 4*i, t3 <v, 4*r<="" m-1,="" th=""><th>1}</th></v,>	1}

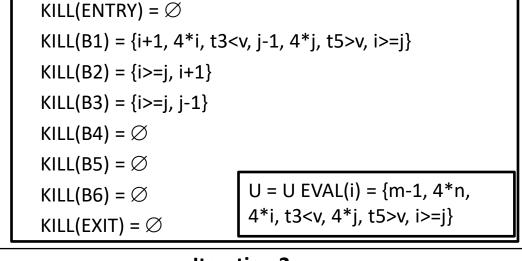
 $\{i > = j, m-1, 4*n, 4*i, | \{4*j, t5 > v, m-1, 4*n, 4*i, t3 < v\}$

	Iteration 1	
L	EVAL(EXIT) = \emptyset	
	$EVAL(B6) = {4*i, 4*n}$	
	$EVAL(B5) = {4*i, 4*j}$	
	$EVAL(B4) = \{i >= j\}$	
	$EVAL(B3) = {4*j, t5>v}$	
	$EVAL(B2) = {4*i, t3< v}$	
	$EVAL(B1) = \{m-1, 4*n\}$	
	EVAL(ENTRY) = \emptyset	

not required

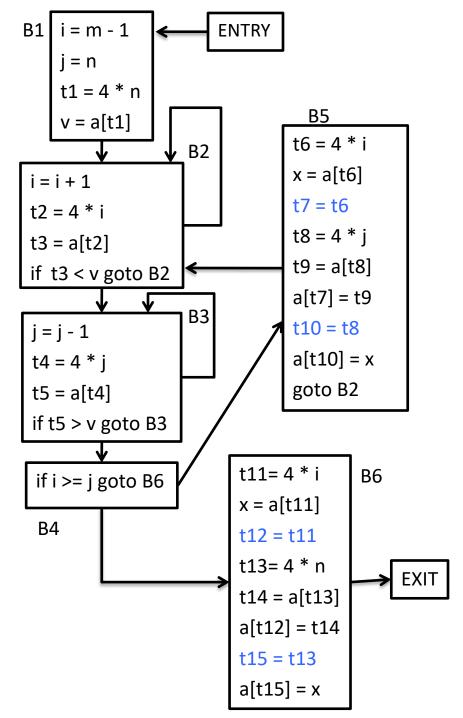
EXIT

t3<v, 4*j, t5>v, i>=j}

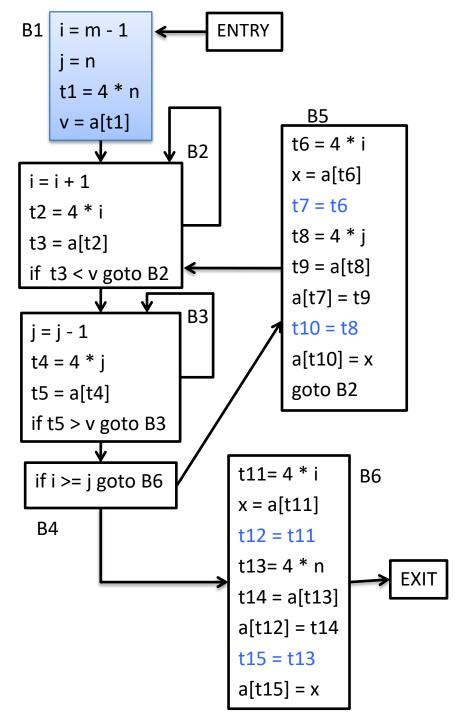


	EVAL(B6) = $\{4^{+}1, 4^{+}1\}$ EVAL(EXIT) = \emptyset	$KILL(Bb) = \emptyset$ $KILL(EXIT) = \emptyset$	4*i, t3 <v, 4*j,="" t5="">v, i>=j}</v,>
	Iteration 1	Iterat	tion 2
	old_OUT(i)	$IN(i) = \cap out(j) j \in Pred(i)$	$OUT(i) = EVAL(i) \cup (IN(i) - KILL(i))$
ENTRY	Ø	Ø	Ø
B1	{m-1, 4*n}	Ø	{m-1, 4*n}
B2	{4*i, t3 <v, 4*n}<="" m-1,="" th=""><th>{m-1, 4*n}</th><th>{4*i, t3<v, 4*n}<="" m-1,="" th=""></v,></th></v,>	{m-1, 4*n}	{4*i, t3 <v, 4*n}<="" m-1,="" th=""></v,>
В3	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< th=""><th>{4*i, t3<v, 4*n}<="" m-1,="" th=""><th>{4*j, t5>v, 4*i, t3<v, 4*n}<="" m-1,="" th=""></v,></th></v,></th></v}<>	{4*i, t3 <v, 4*n}<="" m-1,="" th=""><th>{4*j, t5>v, 4*i, t3<v, 4*n}<="" m-1,="" th=""></v,></th></v,>	{4*j, t5>v, 4*i, t3 <v, 4*n}<="" m-1,="" th=""></v,>
B4	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< th=""><th>{i>=j, 4*j, t5>v, m-1, 4*n, 4*i, t3<v}< th=""></v}<></th></v}<>	{i>=j, 4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< th=""></v}<>
B5	{4*i, 4*j, m-1, 4*n, t3 <v, t5="">v, i>=j}</v,>	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>	{4*i, 4*j, i>=j, m-1, 4*n, t3 <v, t5="">v}</v,>
В6	{4*i, 4*n, m-1,	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>	{4*i, 4*n, i>=j, m-1, t3 <v, 4*j,<="" th=""></v,>

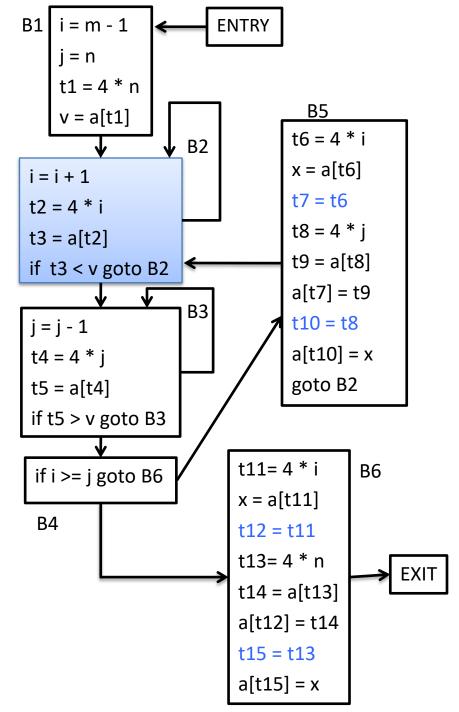
Basic Block	AEin(i)
ENTRY	\varnothing
B1	Ø
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>



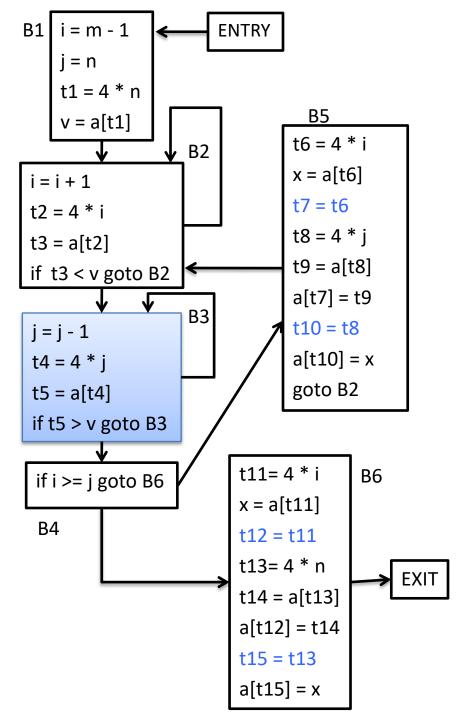
Basic Block	AEin(i)
ENTRY	\varnothing
B1	Ø
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>



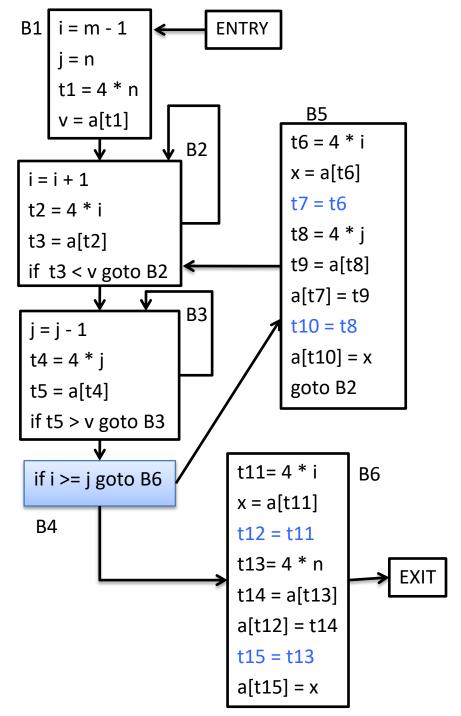
Basic Block	AEin(i)
ENTRY	\varnothing
B1	\varnothing
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>



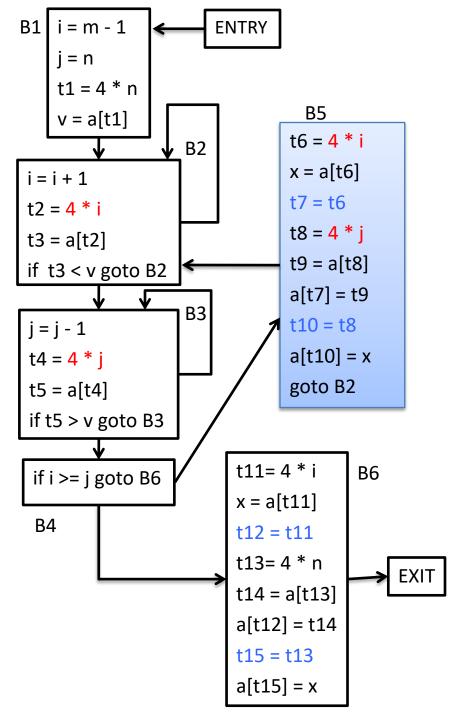
Basic Block	AEin(i)
ENTRY	\varnothing
B1	Ø
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>



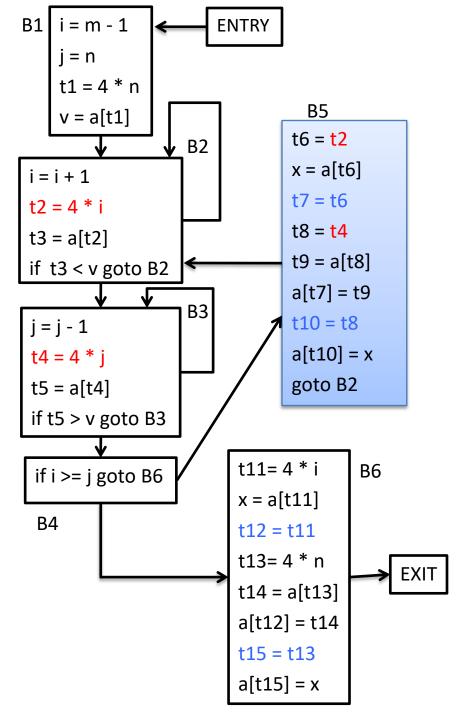
Basic Block	AEin(i)
ENTRY	\varnothing
B1	\varnothing
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>



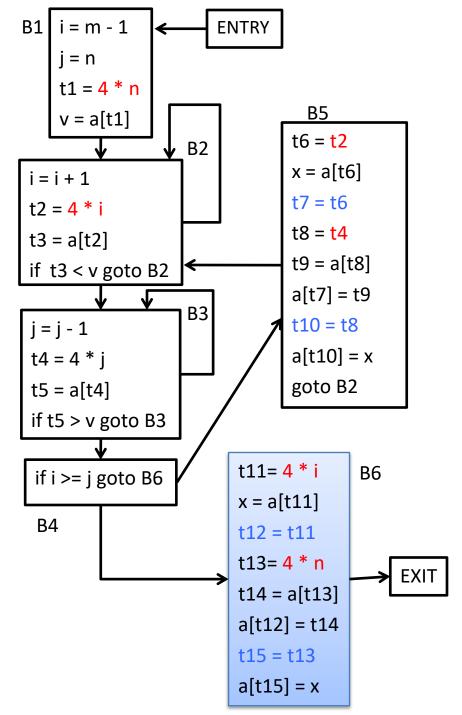
Basic Block	AEin(i)
ENTRY	\varnothing
B1	\varnothing
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>



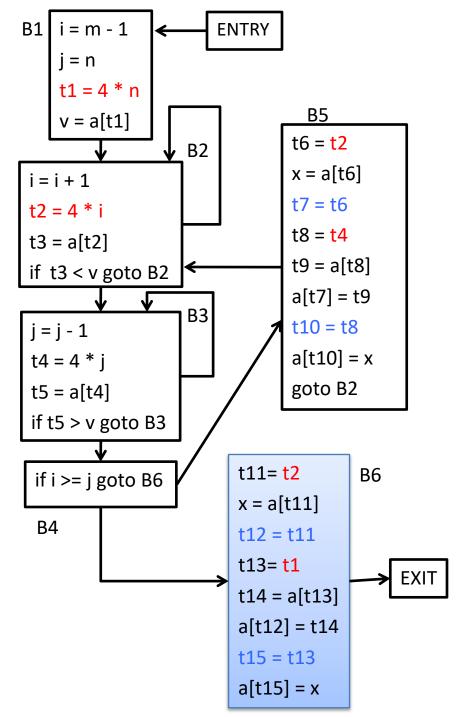
Basic Block	AEin(i)
ENTRY	\varnothing
B1	\varnothing
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>



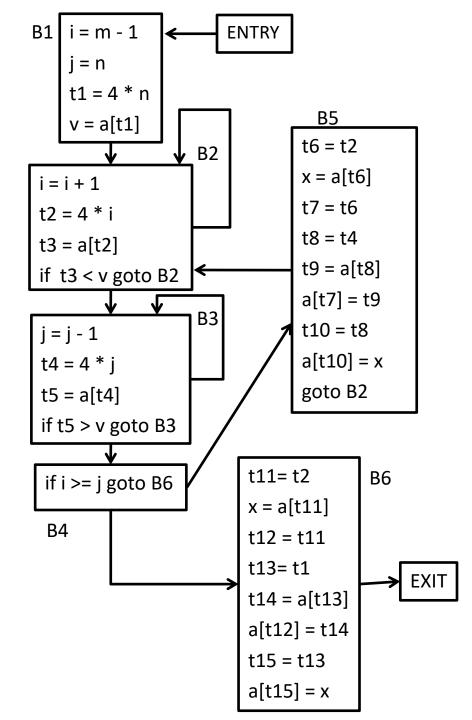
Basic Block	AEin(i)
ENTRY	\varnothing
B1	\varnothing
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>

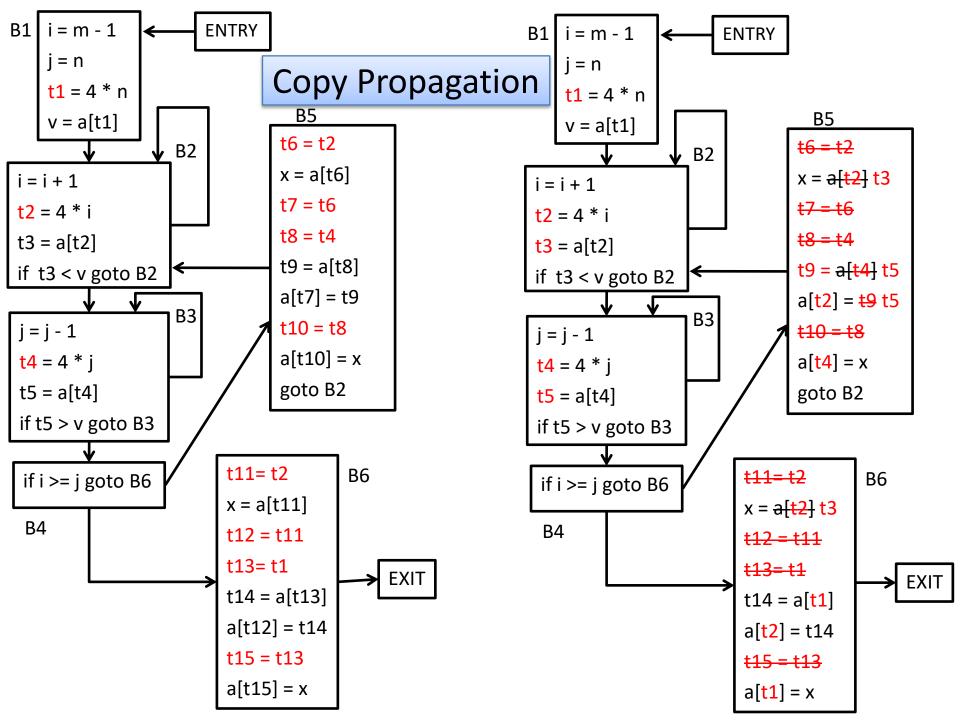


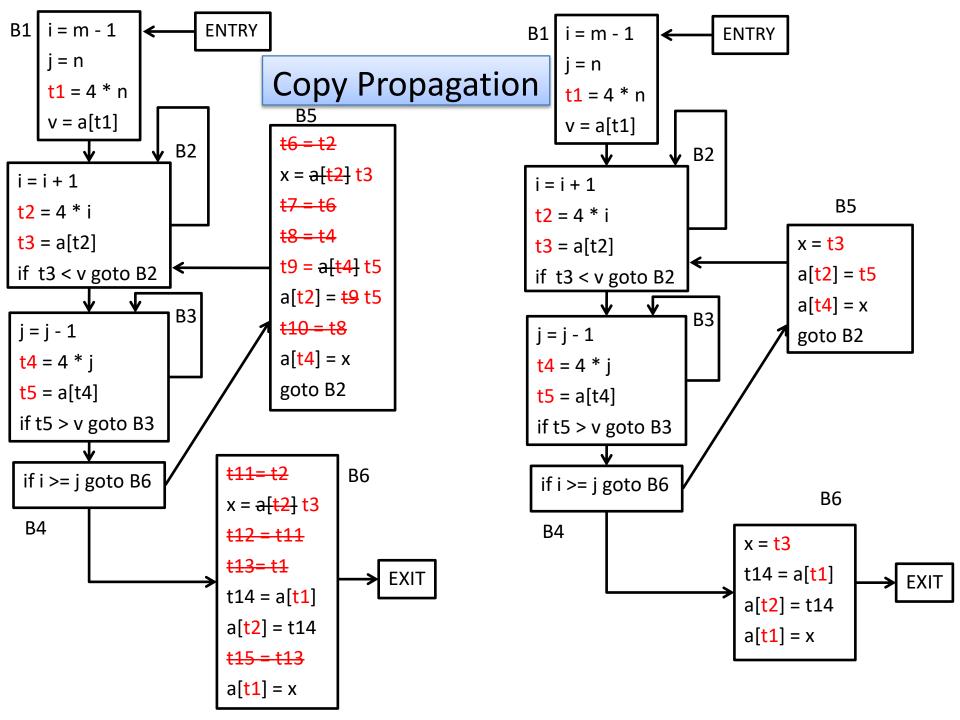
Basic Block	AEin(i)
ENTRY	\varnothing
B1	\varnothing
B2	{m-1, 4*n}
В3	{4*i, t3 <v, 4*n}<="" m-1,="" td=""></v,>
B4	{4*j, t5>v, m-1, 4*n, 4*i, t3 <v}< td=""></v}<>
B5	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
В6	{i>=j, m-1, 4*n, 4*i, t3 <v, 4*j,="" t5="">v}</v,>
EXIT	{4*i, 4*n, m-1, t3 <v, 4*j,="" t5="">v, i>=j}</v,>

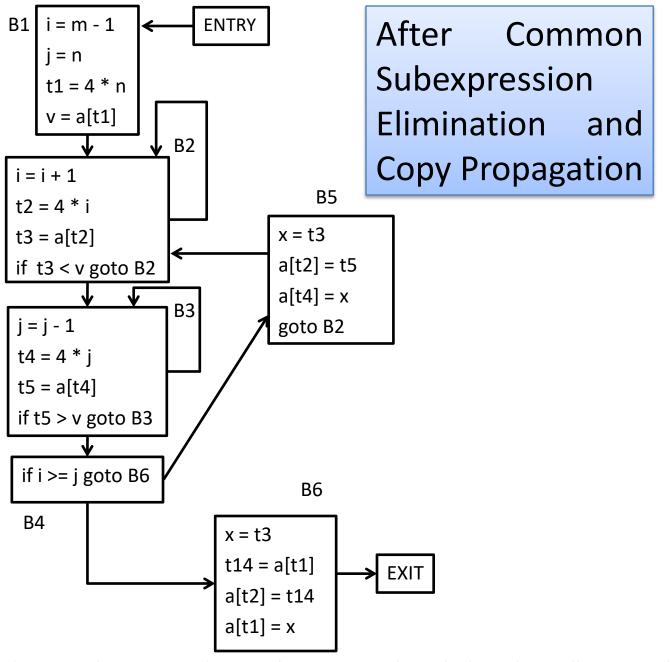


After global common subexpression elimination









Same graph as in the book Compiler: Principles, techniques and tools by Aho, Ullman and Sethi, 2nd Ed., Pearson Education page no. 589