Continuous Space MDPs

Last Time

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- What are the differences between online and offline solutions?
- Are there solution techniques that are *independent* of the state space size?

Guiding Questions

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- What tools do we have to solve MDPs with continuous S and A?

Current Tool-Belt

Continuous S and A

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The old rules still work!

Today: Four Tools

1. Linear Dynamics, Quadratic Reward

2. Value Function Approximation

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while not converged

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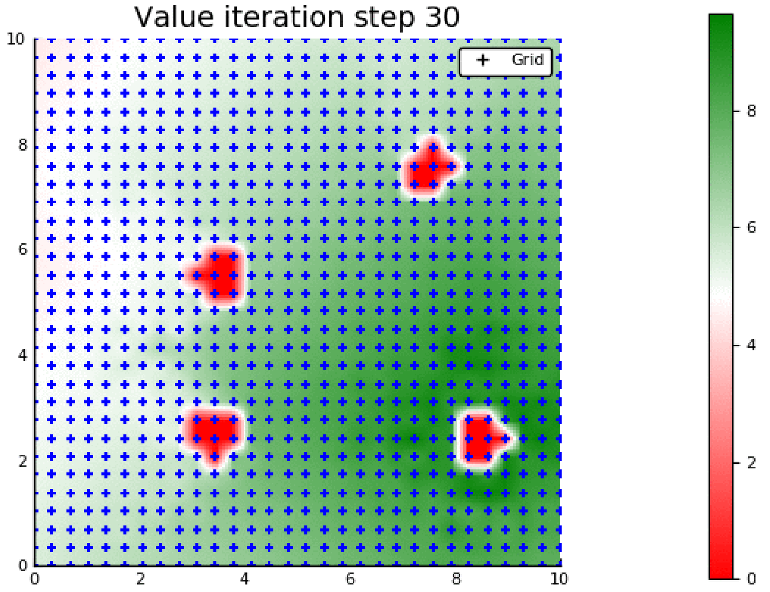

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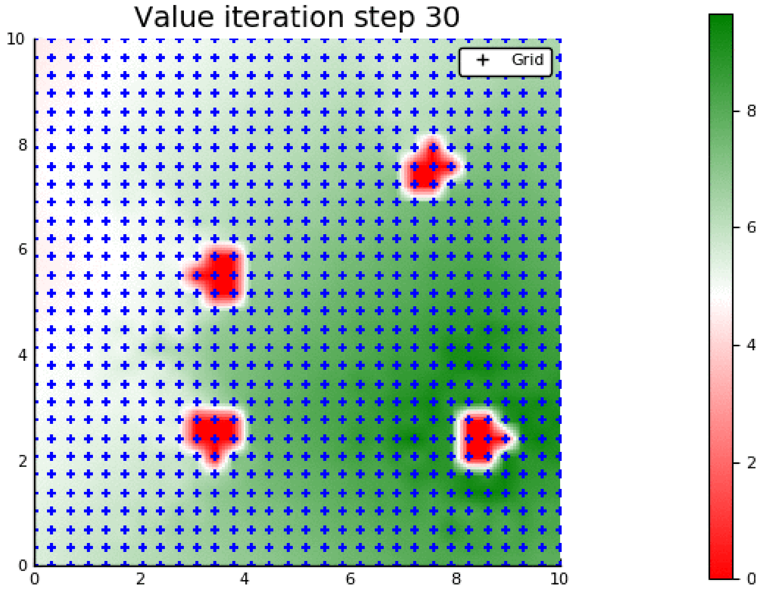

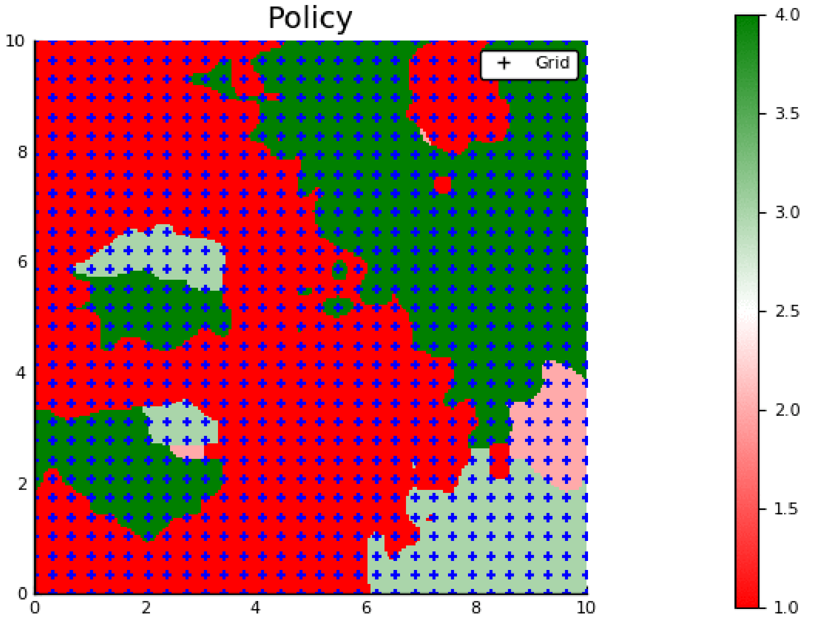

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Function Approximation

Weighting of 2^d points

Weighting of only $d + 1$ points!

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- Local: (e.g. simplex interpolation)

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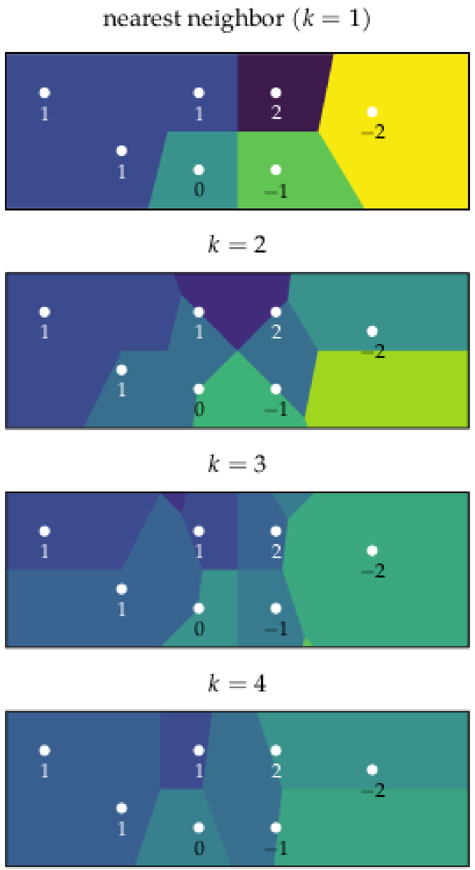

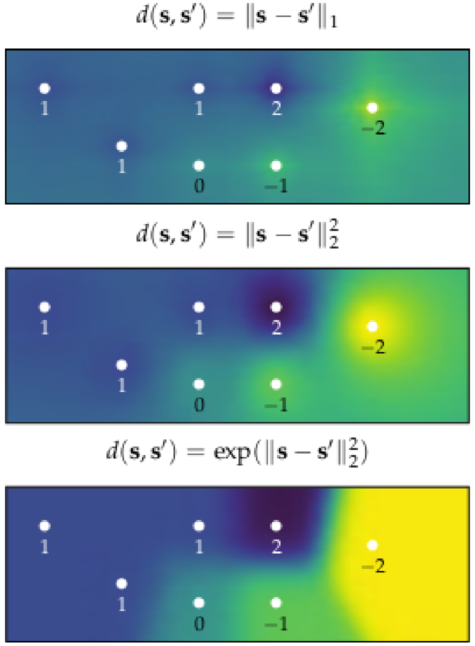

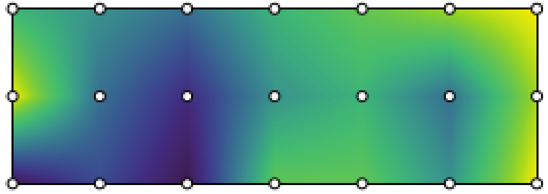

Figure 8.9. Two-dimensional linear interpolation over a 3×7 grid.

Weighting of 2^d points

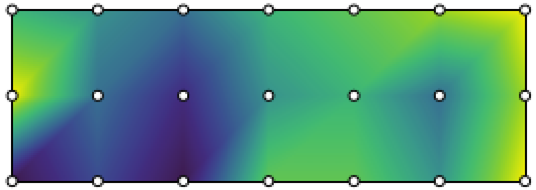

Figure 8.10. Two-dimensional simplex interpolation over a 3×7 grid.

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Function Approximation: Mountain Car

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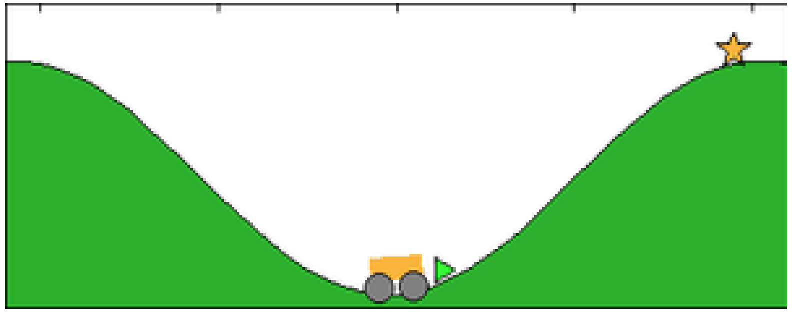

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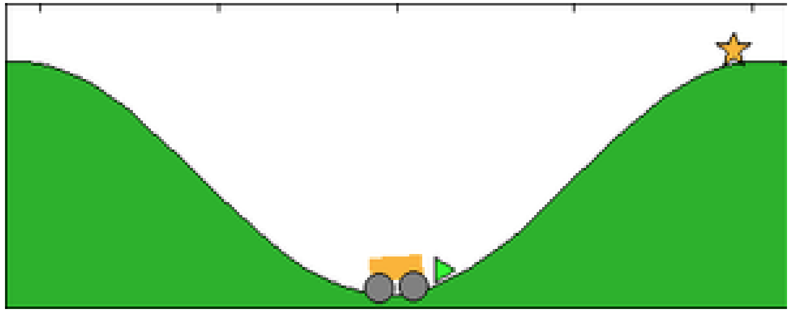

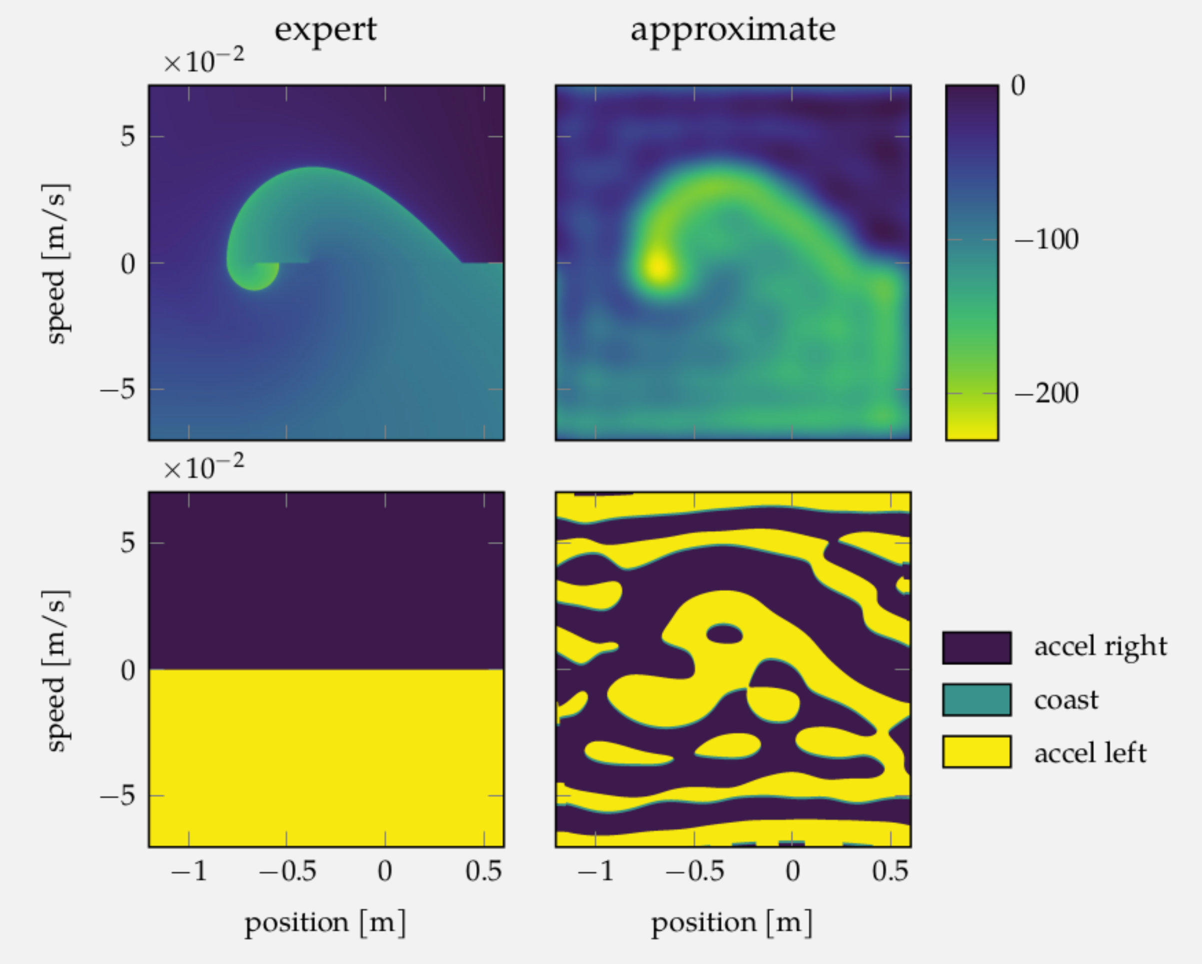

(Fourier, 17 params)

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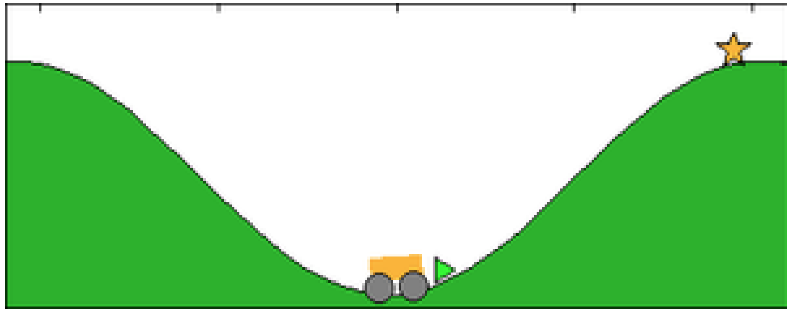


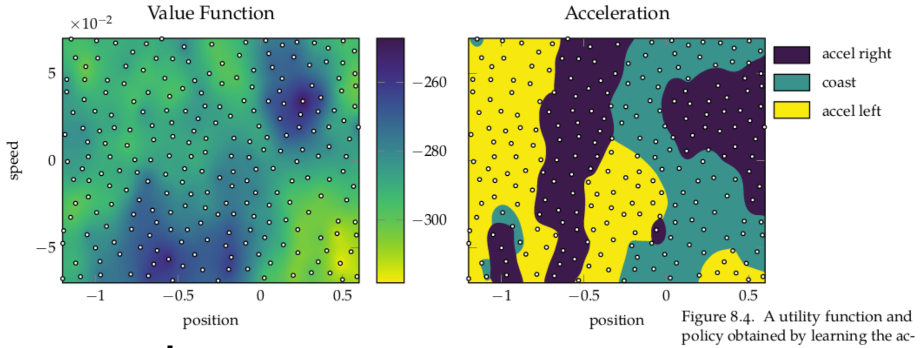
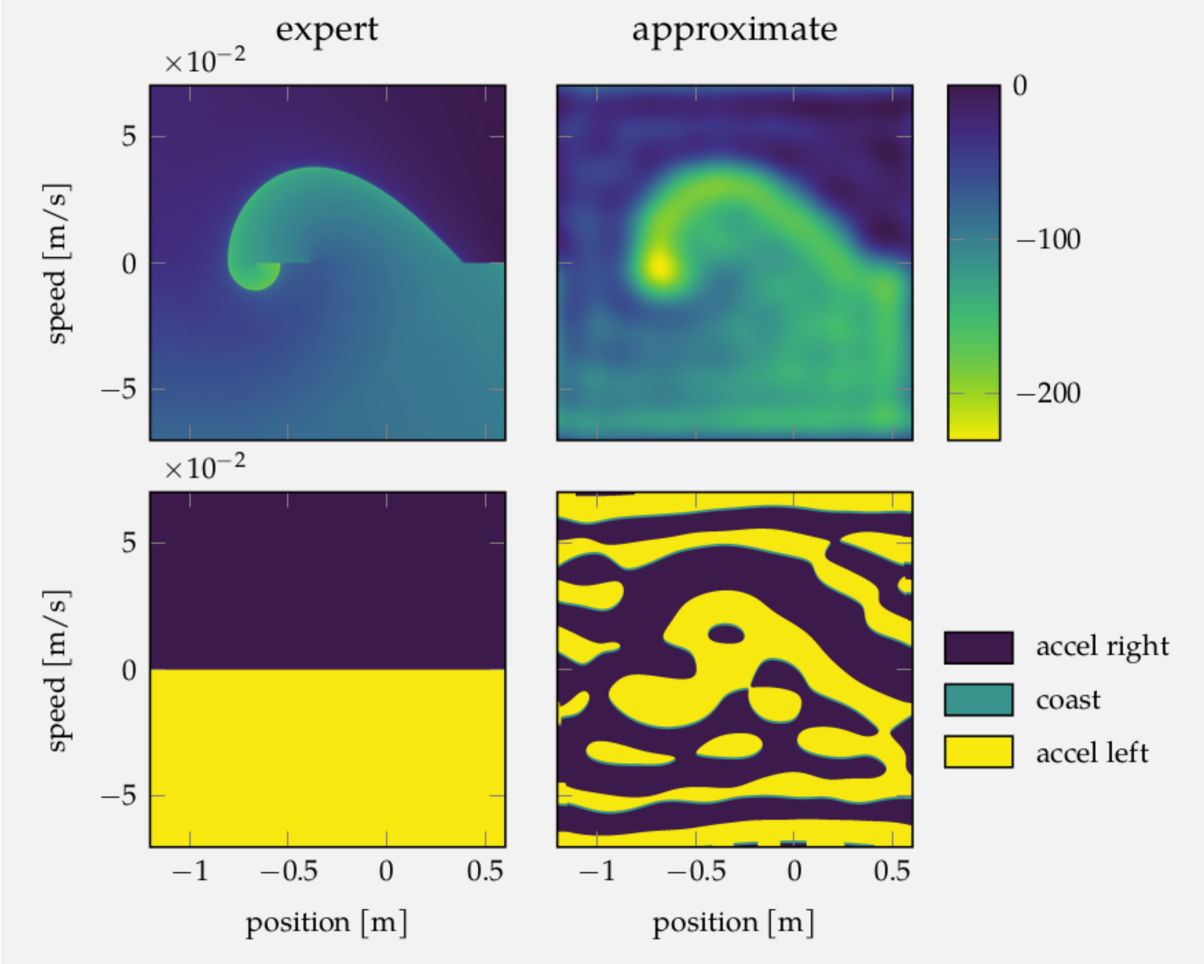


Figure 8.4. A utility function and policy obtained by learning the action values for a finite set of states (white) in the mountain car problem using the distance function $\|\mathbf{s} - \mathbf{s}'\|_2 + 0.1$.

(Kernel, > 100 params)



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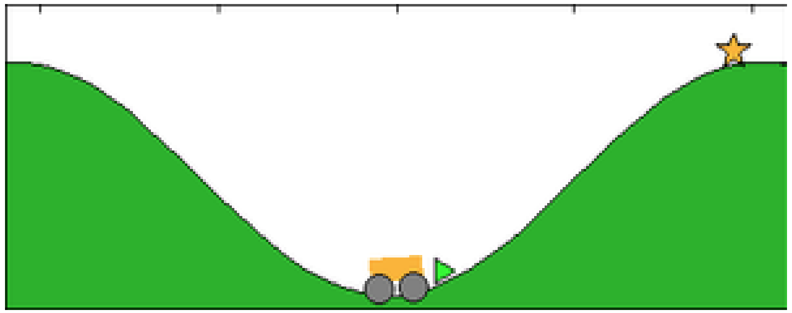

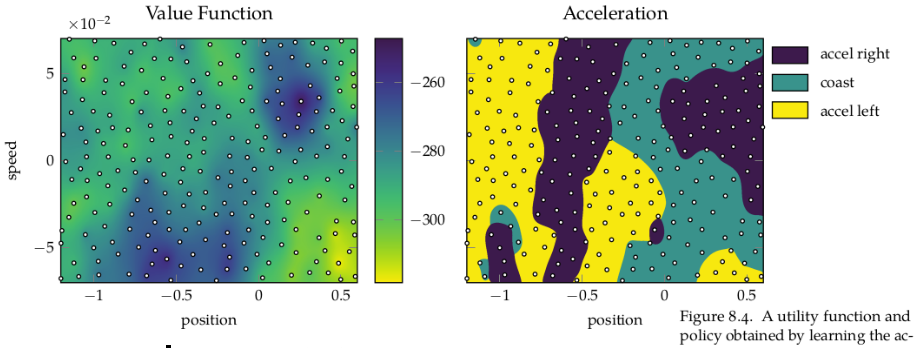

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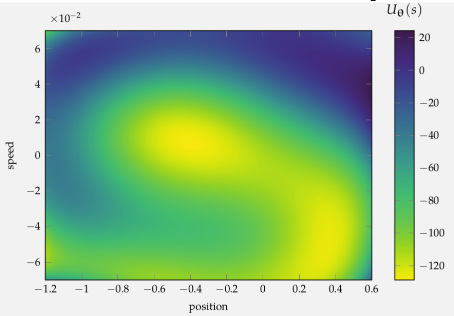
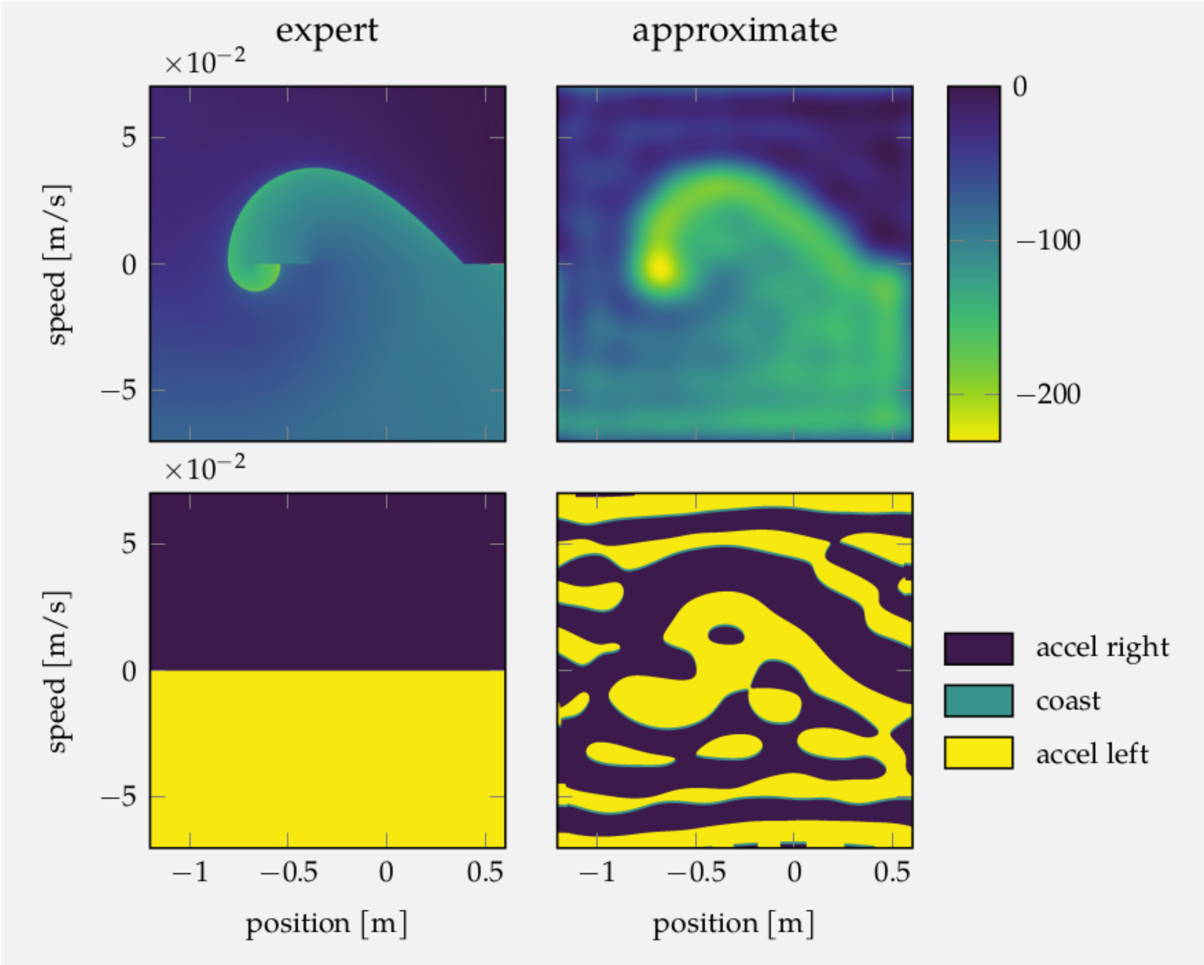


(Polynomial, 28 params)



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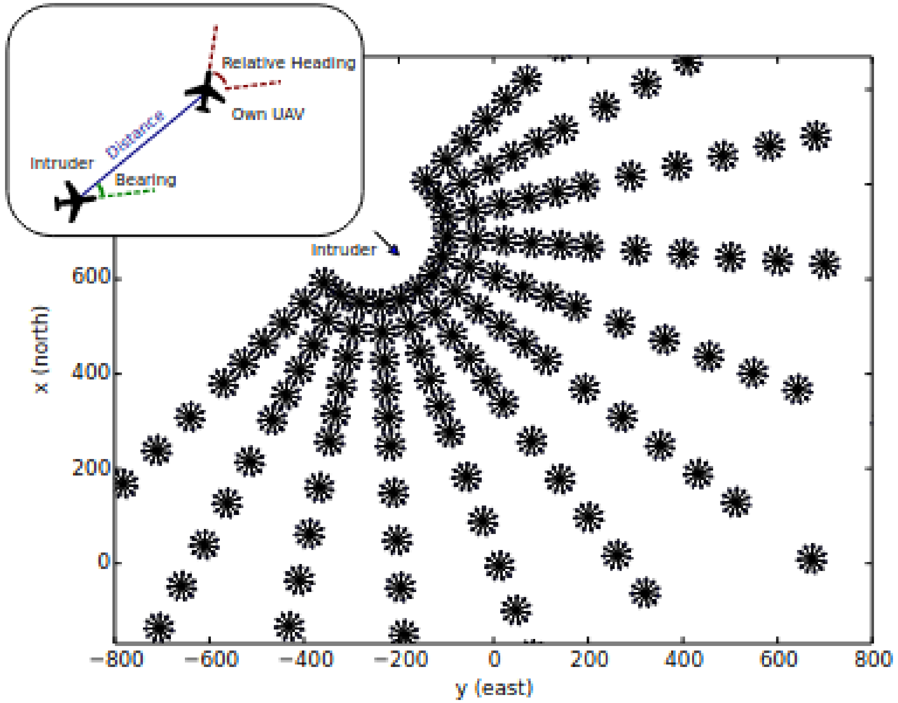

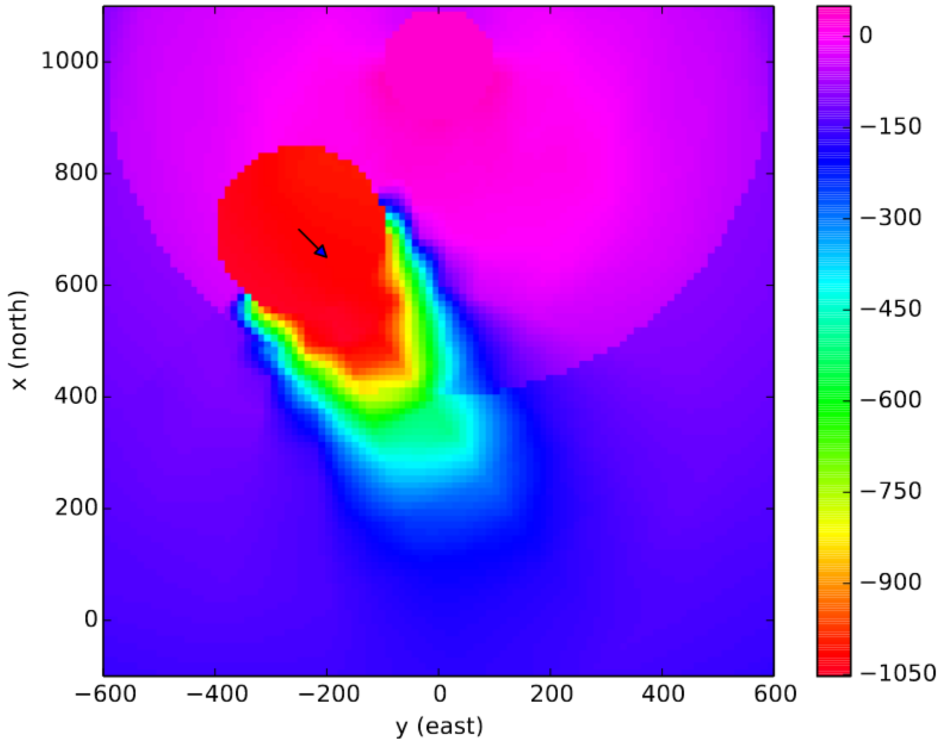

Break

What will a Monte Carlo Tree Search tree look like if run on a problem with continuous spaces?

3. Sparse Tree Search/Progressive Widening

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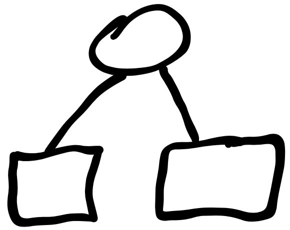

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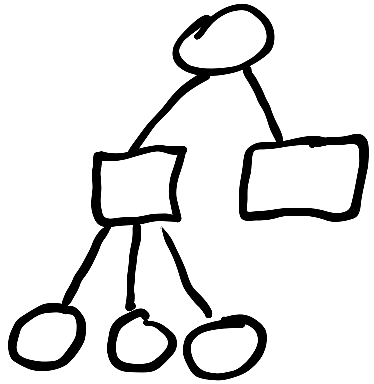

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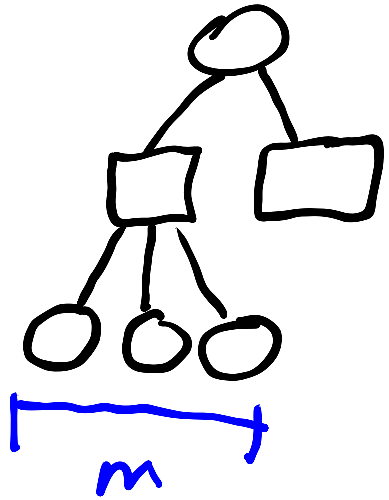

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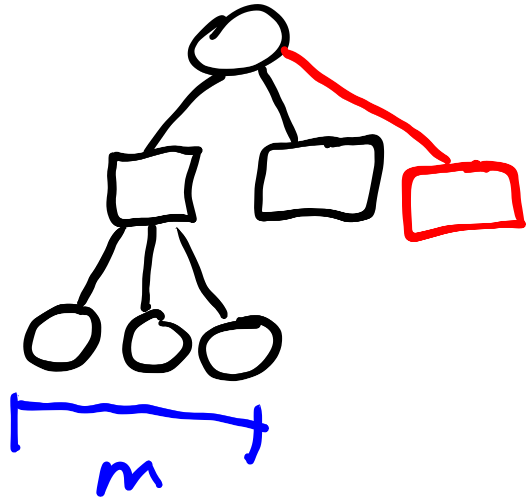

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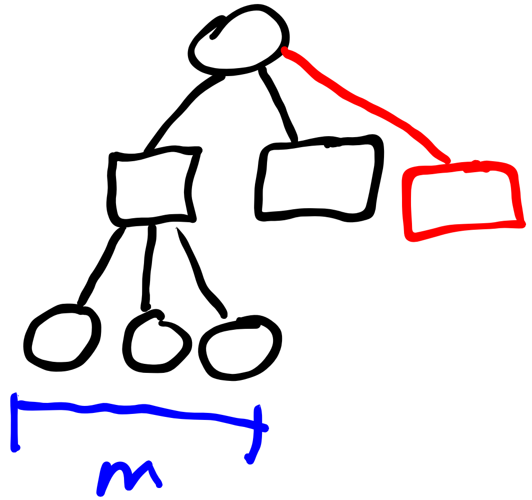

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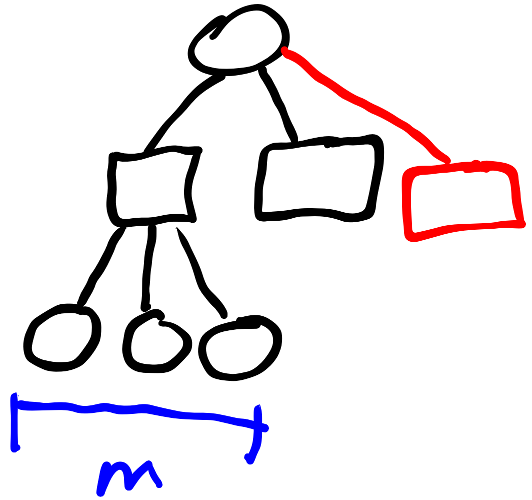

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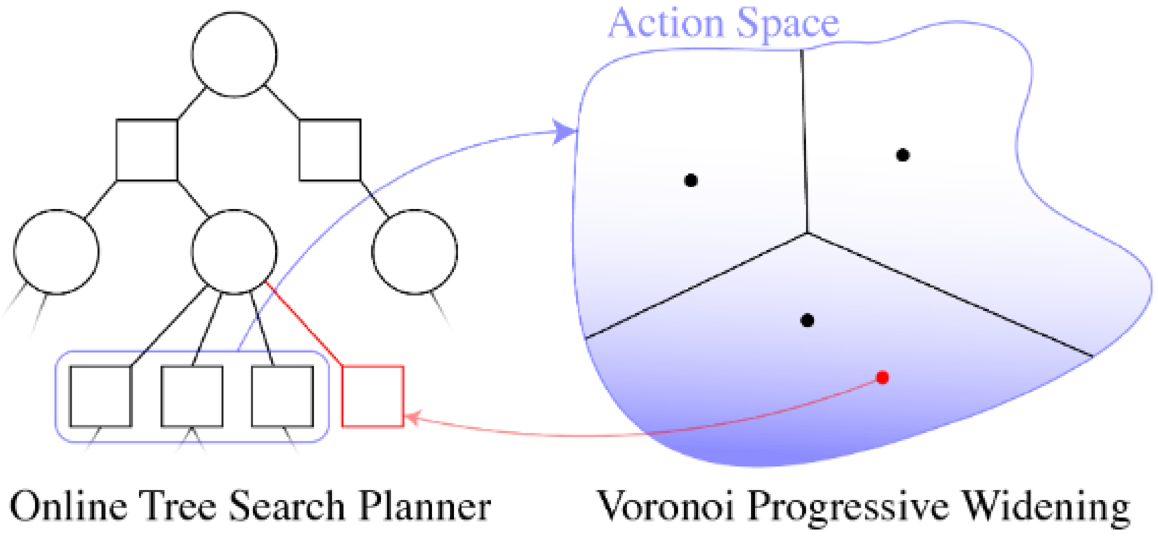

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(Use off-the-shelf optimization software, e.g. Ipopt)

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$$\begin{aligned} \underset{a_{1:d}, s_{1:d}}{\text{maximize}} \quad & \sum_{t=1}^{d} \gamma^t R(s_t, a_t) \\ \text{subject to} \quad & s_{t+1} = \mathrm{E}[T(s_t, a_t)] \quad \forall t \end{aligned}$$

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Hindsight Optimization

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Guiding Questions

- What tools do we have to solve MDPs with continuous S and A?