POMDP Value Iteration $|S| \sim 10\text{-}20$

SARSOP $|S| \sim 10{,}000$

POMDP Formulation Approximations

POMDP Computational Complexity

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Sad facts

- Infinite horizon POMDPs are *undecidable*

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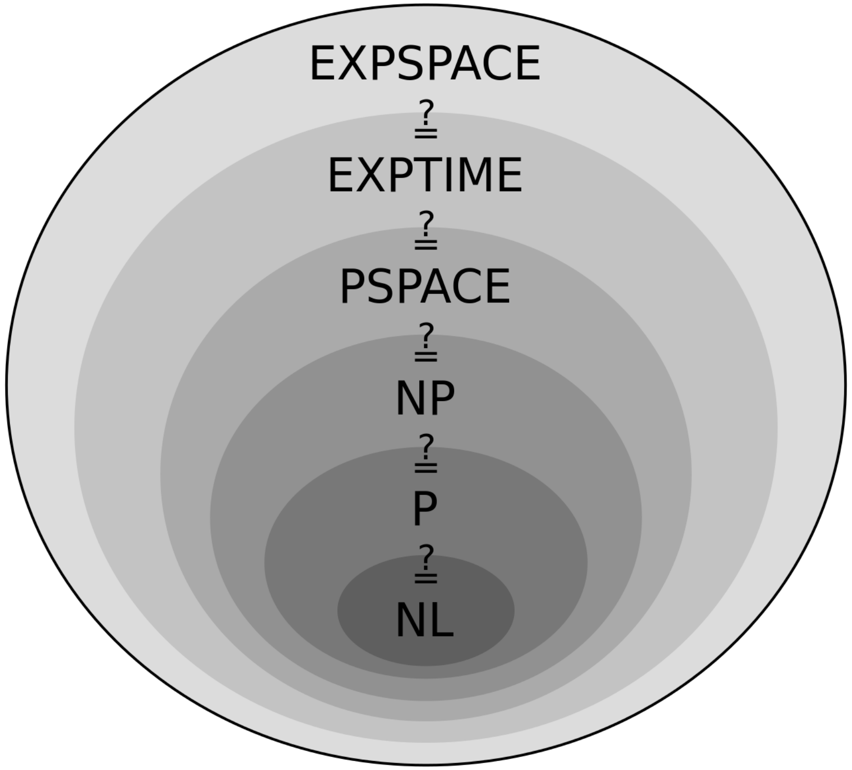

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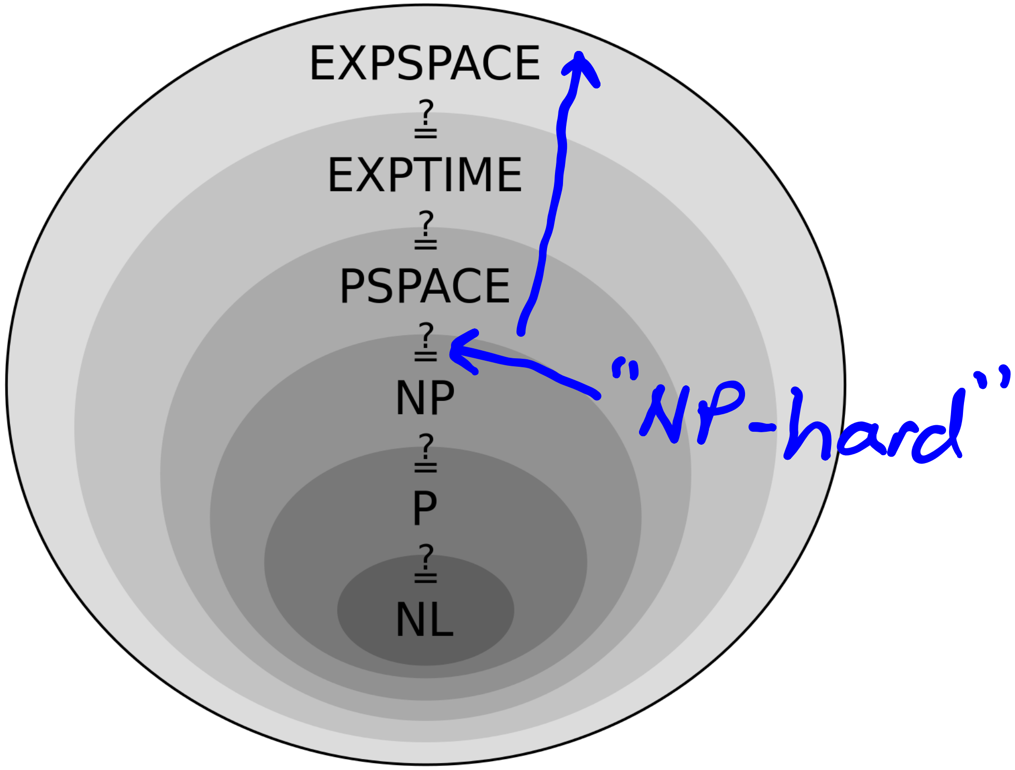

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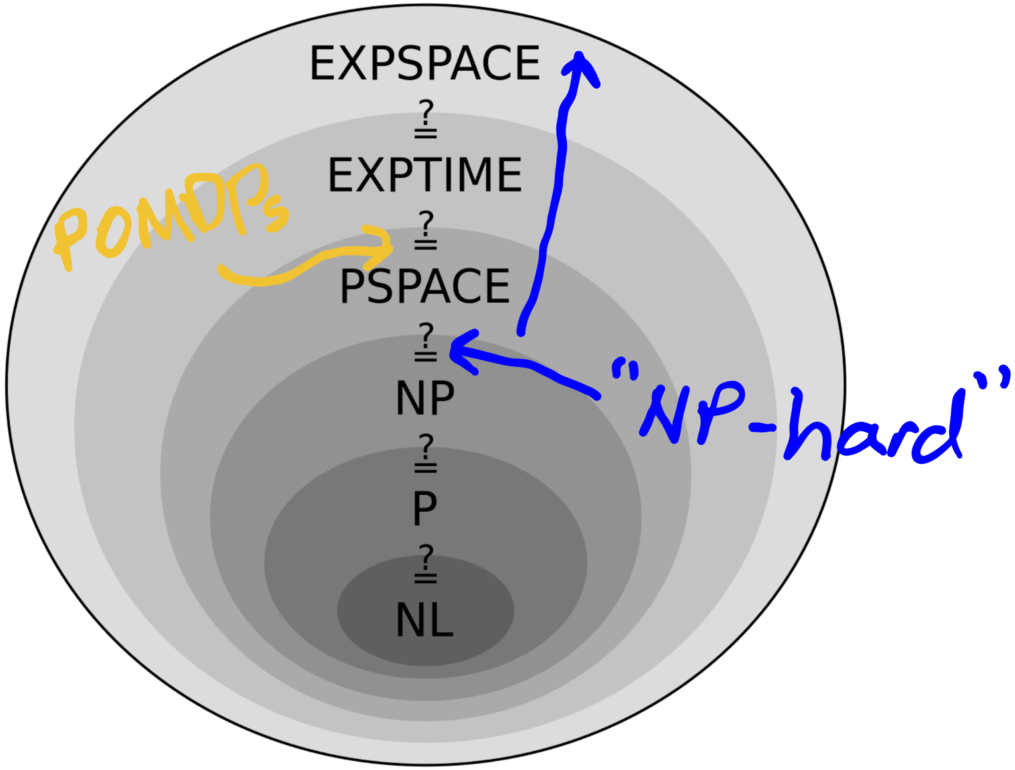

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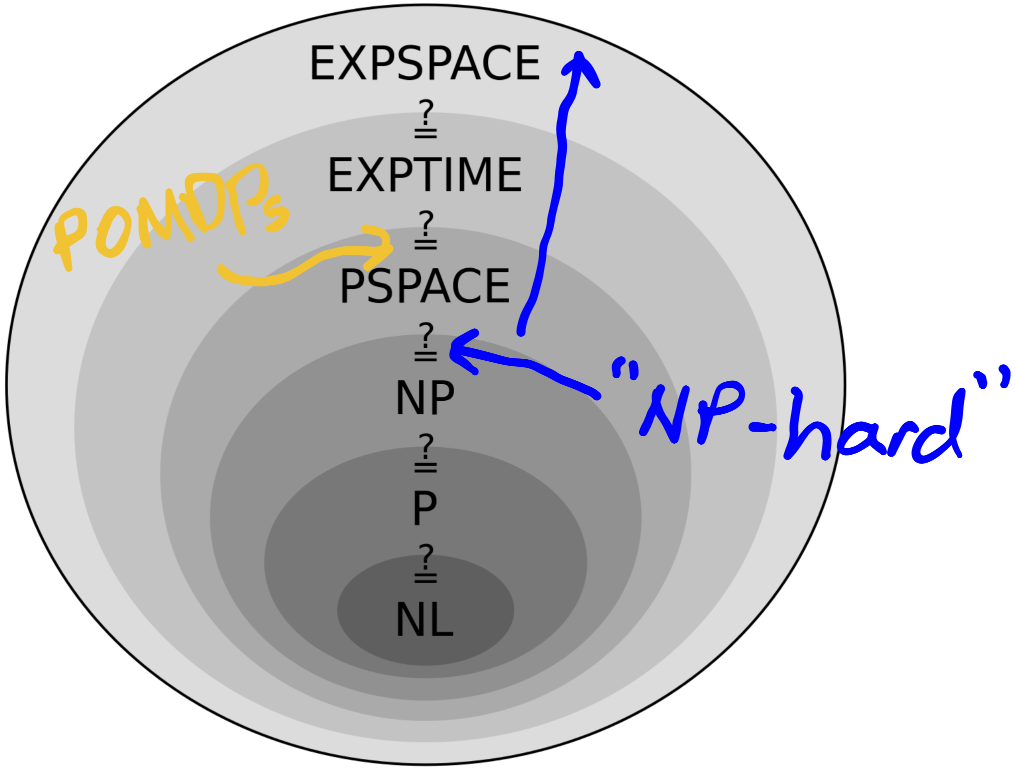

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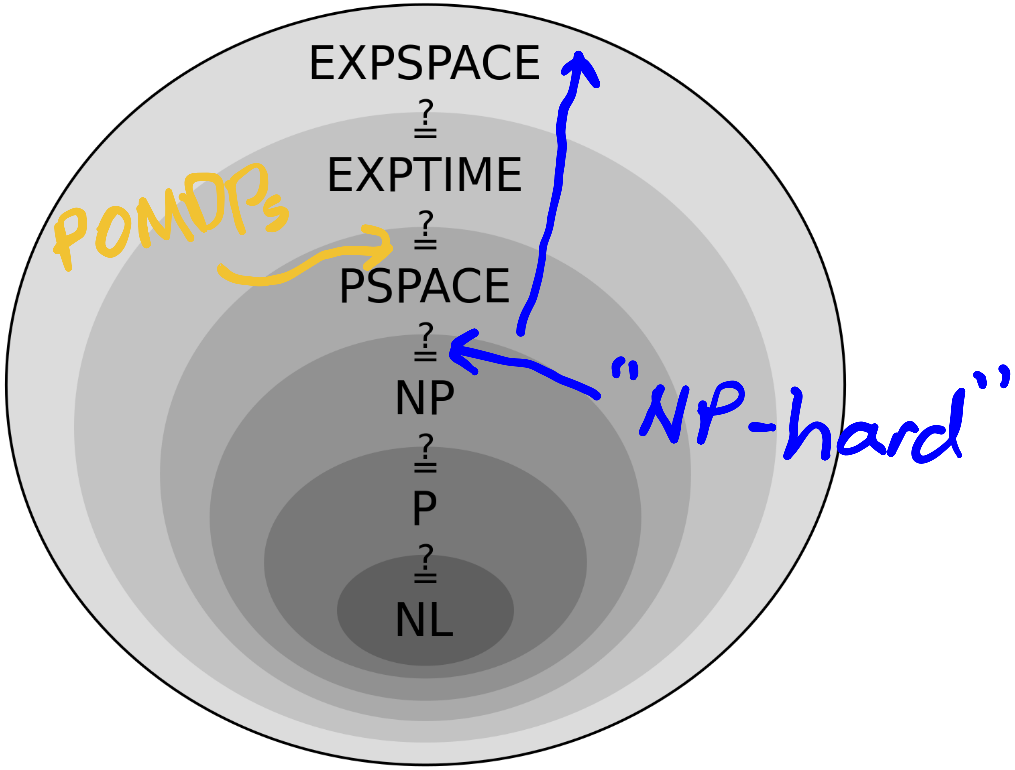
    (we think)



Approximate POMDP Solutions

Approximate POMDP Solutions

Numerical Approximations

(approximately solve original problem)

Approximate POMDP Solutions

Numerical Approximations

(approximately solve original problem)

Offline

Approximate POMDP Solutions

$\bar{V} - \underline{V} = \varepsilon$

Numerical Approximations

(approximately solve original problem)

Offline

~~Last week~~

Today

scalable to $|S| \sim 10{,}000$

Approximate POMDP Solutions

Numerical Approximations

(approximately solve original problem)

Offline

Last week

Online

Approximate POMDP Solutions

Numerical Approximations

(approximately solve original problem)

Offline

Last week

Online

Thursday

Approximate POMDP Solutions

Numerical Approximations

(approximately solve original problem)

Offline

Last week

Online

Thursday

Formulation Approximations

(solve a slightly different problem)

Approximate POMDP Solutions

Numerical Approximations

(approximately solve original problem)

Offline

Last week

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Thursday

Formulation Approximations

(solve a slightly different problem)

Today!

POMDP Objective

POMDP Objective

$$\pi^* = \underset{\pi : B \to A}{\operatorname{argmax}} \ \mathrm{E}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(b_t))\right]$$

POMDP Objective

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$$b' = \tau(b, a, o)$$

Certainty Equivalent

POMDP Objective

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$$\pi_{\mathrm{CE}}(b) = \pi_s(\underset{s\sim b}{\mathrm{E}}[s])$$

$$b' = \tau(b, a, o)$$

Certainty Equivalent

MDP LQR

Optimal for LQG

LQG POMDP

$$T(\mathbf{s}' \mid \mathbf{s}, \mathbf{a}) = \mathcal{N}(\mathbf{s}' \mid \mathbf{T}_s\mathbf{s} + \mathbf{T}_a\mathbf{a}, \boldsymbol{\Sigma}_s)$$

Linear Dynamics

Gaussian Process Noise

$$O(\mathbf{o} \mid \mathbf{s}') = \mathcal{N}(\mathbf{o} \mid \mathbf{O}_s\mathbf{s}', \boldsymbol{\Sigma}_o)$$

Gaussian Observation Noise

mean

covariance

$$b(\mathbf{s}) = \mathcal{N}(\mathbf{s} \mid \boldsymbol{\mu}_b, \boldsymbol{\Sigma}_b)$$

$$\boldsymbol{\mu}_p \leftarrow \mathbf{T}_s\boldsymbol{\mu}_b + \mathbf{T}_a\mathbf{a}$$

$$\boldsymbol{\Sigma}_p \leftarrow \mathbf{T}_s\boldsymbol{\Sigma}_b\mathbf{T}_s^\top + \boldsymbol{\Sigma}_s$$

$$\mathbf{K} \leftarrow \boldsymbol{\Sigma}_p\mathbf{O}_s^\top\left(\mathbf{O}_s\boldsymbol{\Sigma}_p\mathbf{O}_s^\top + \boldsymbol{\Sigma}_o\right)^{-1}$$

$$\boldsymbol{\mu}_b \leftarrow \boldsymbol{\mu}_p + \mathbf{K}\left(\mathbf{o} - \mathbf{O}_s\boldsymbol{\mu}_p\right)$$

$$\boldsymbol{\Sigma}_b \leftarrow (\mathbf{I} - \mathbf{K}\mathbf{O}_s)\boldsymbol{\Sigma}_p$$

Bayesian Update

Kalman Filter

$$\pi^*(b) = -K_{LQR}\,\mu_b$$

QMDP

POMDP Objective

$$\pi^* = \underset{\pi:B\to A}{\operatorname{argmax}} \operatorname{E}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, \pi(b_t))\right]$$

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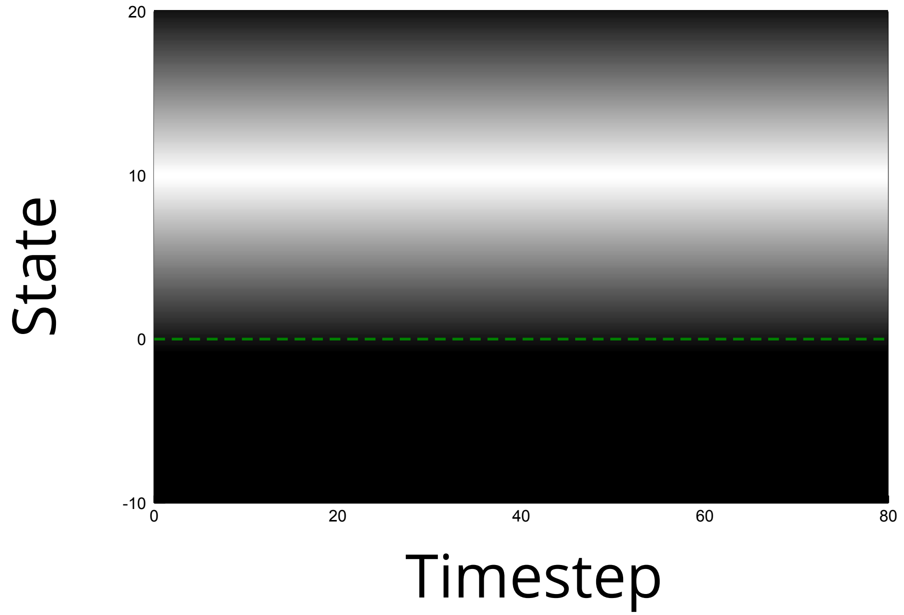
solve underlying MDP to get QMDP

$$\pi_{\mathrm{QMDP}}(b) = \underset{a\in A}{\operatorname{argmax}} \; \underset{s\sim b}{\mathrm{E}} \left[\underline{Q_{\mathrm{MDP}}(s, a)}\right]$$

$$b' = \tau(b, a, o)$$

Example: Tiger POMDP with Waiting

POMDP Example: Light-Dark



$$\mathcal{S} = \mathbb{Z} \qquad \mathcal{O} = \mathbb{R}$$

$$s' = s + a \qquad o \sim \mathcal{N}(s, s - 10)$$

$$\mathcal{A} = \{-10, -1, 0, 1, 10\}$$

$$R(s,a) = \begin{cases} 100 & \text{if } a = 0, s = 0 \\ -100 & \text{if } a = 0, s \neq 0 \\ -1 & \text{otherwise} \end{cases}$$

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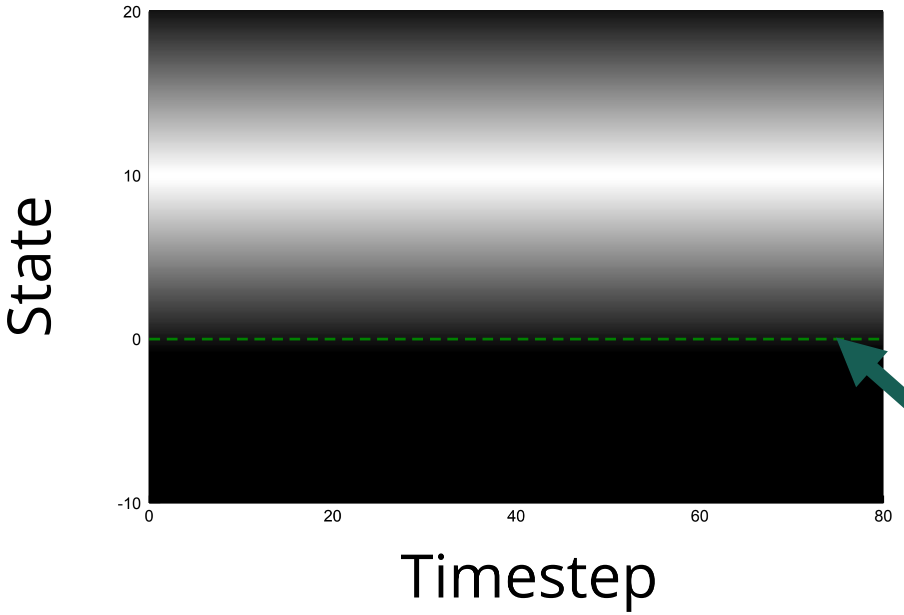

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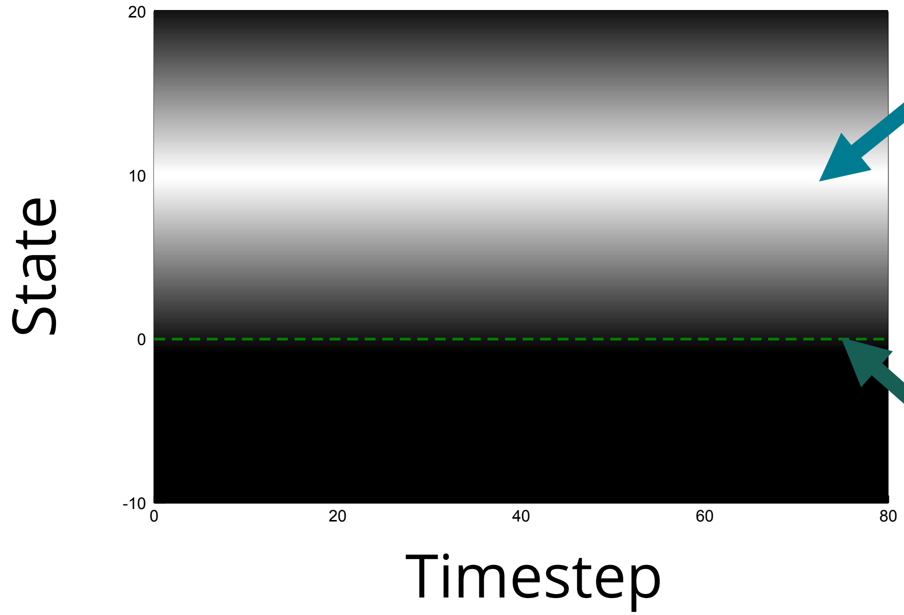
Goal: $a = 0$ at $s = 0$

POMDP Example: Light-Dark

Accurate Observations



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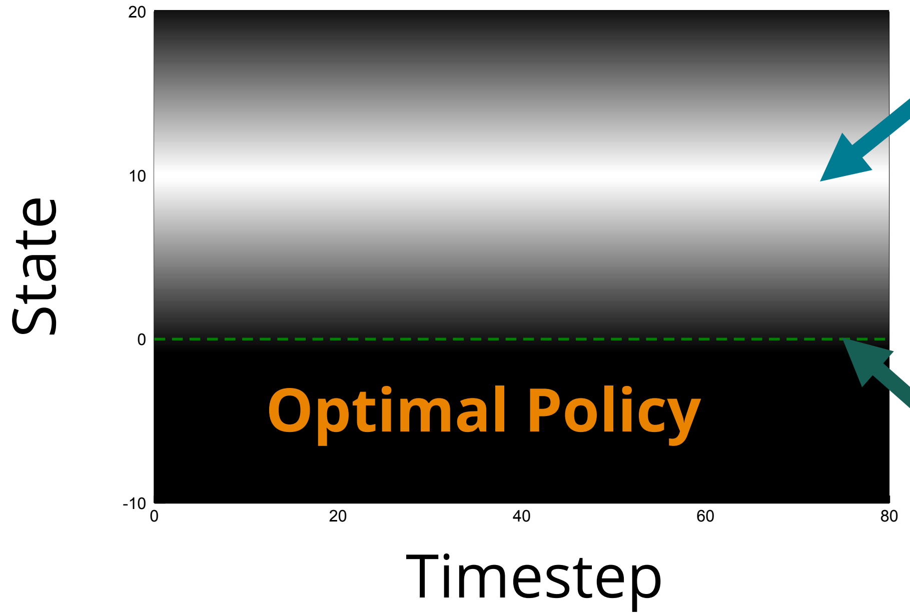

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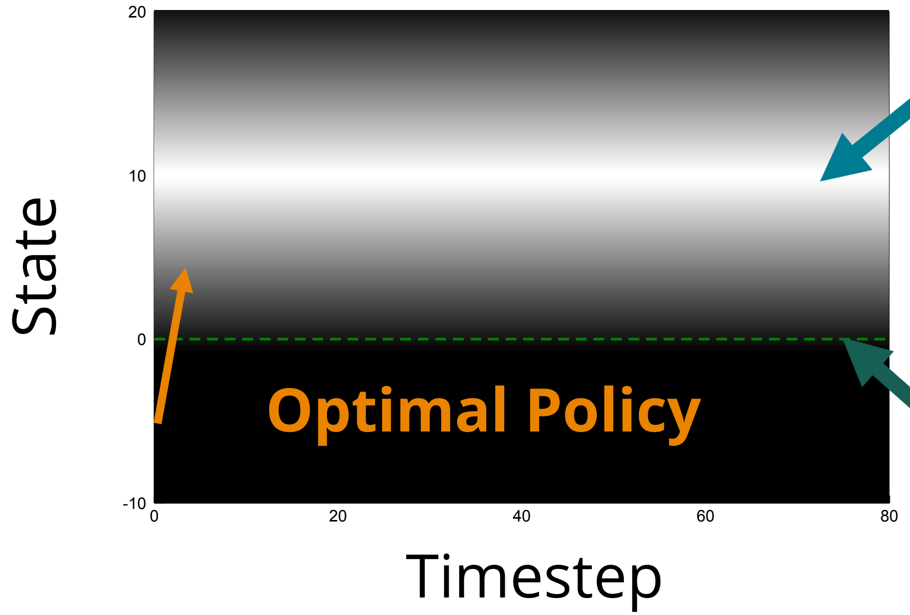

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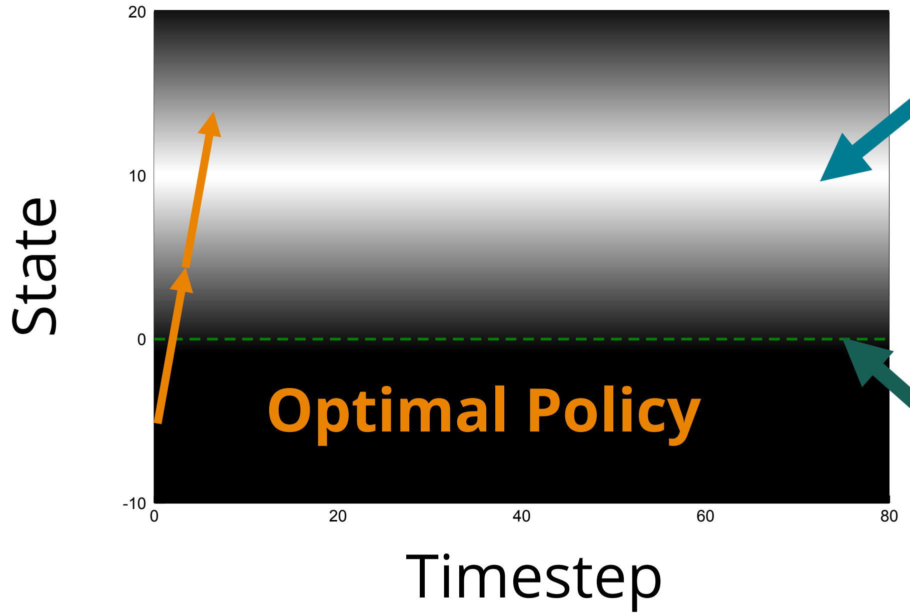

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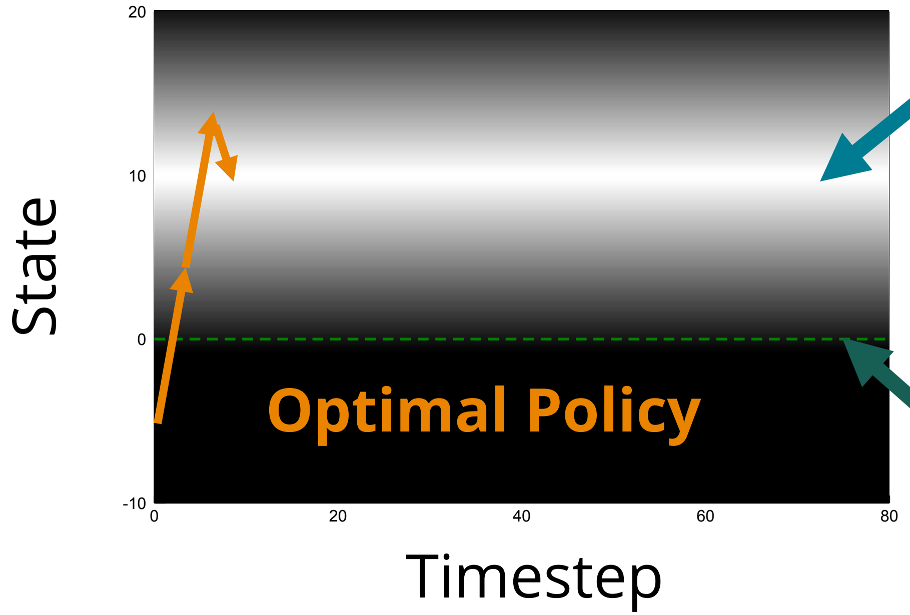

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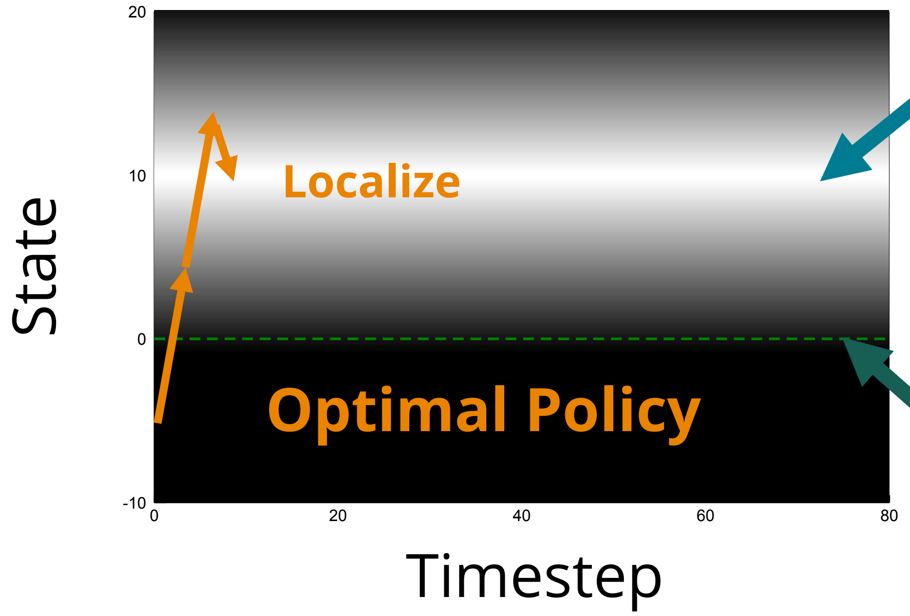

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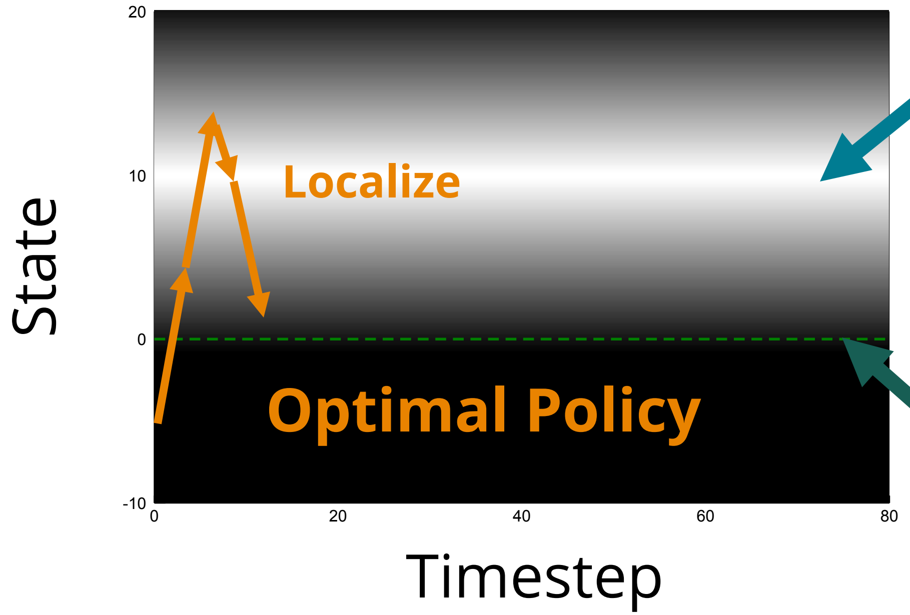

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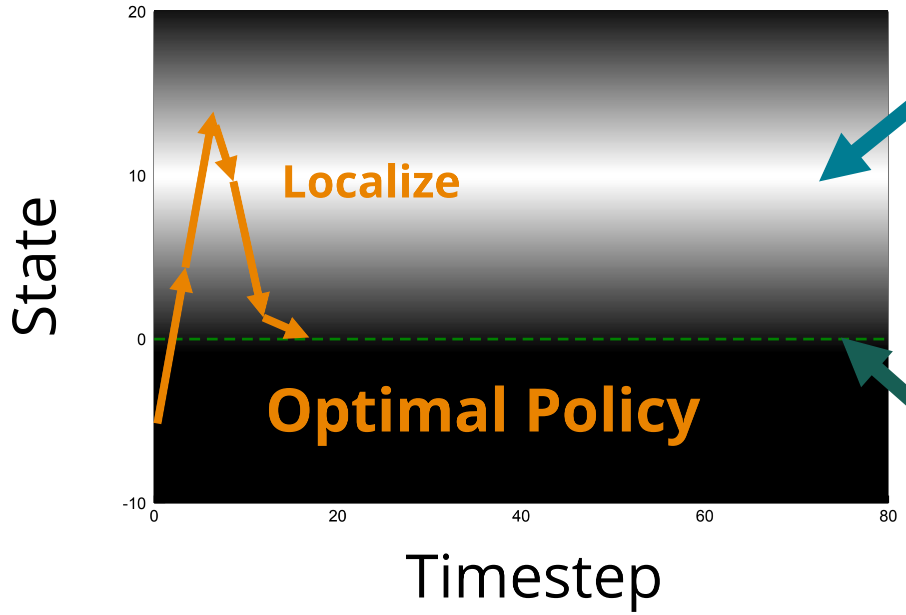

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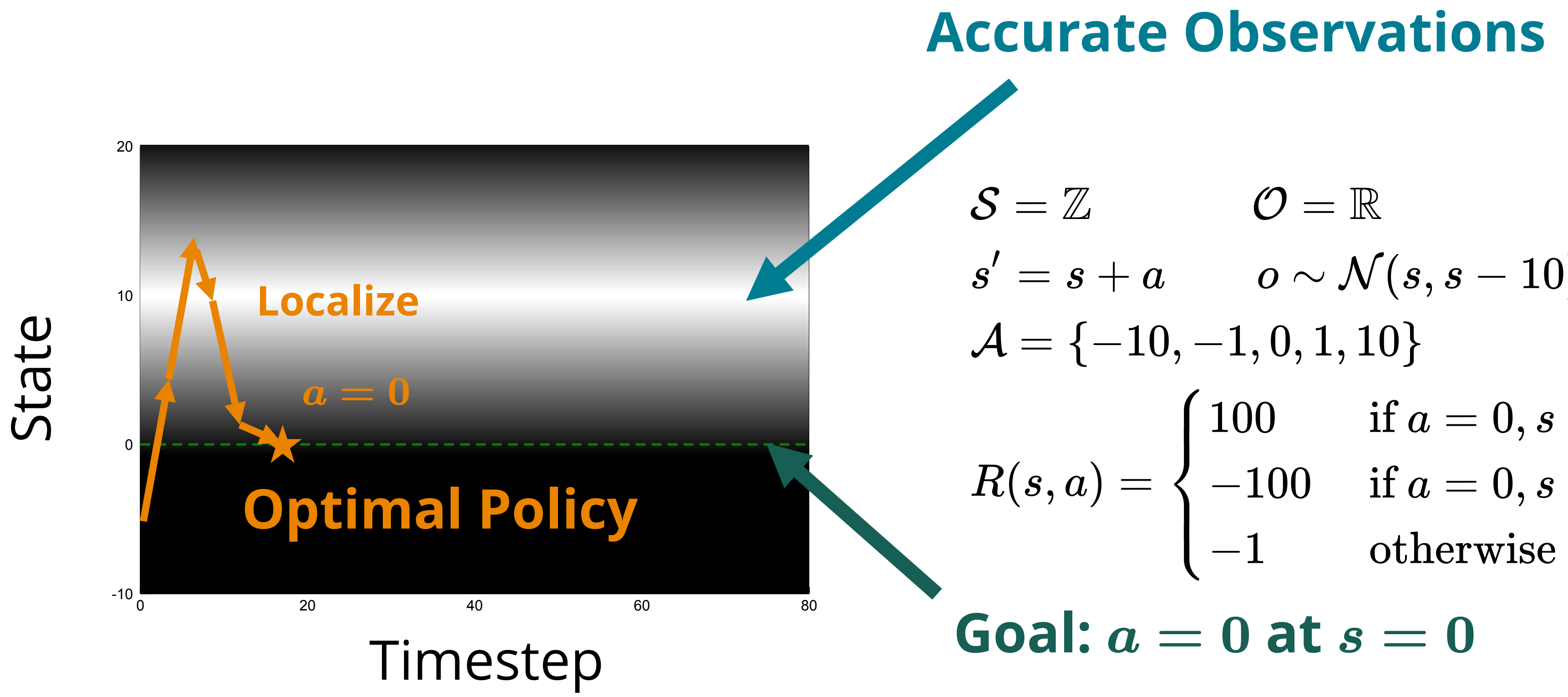

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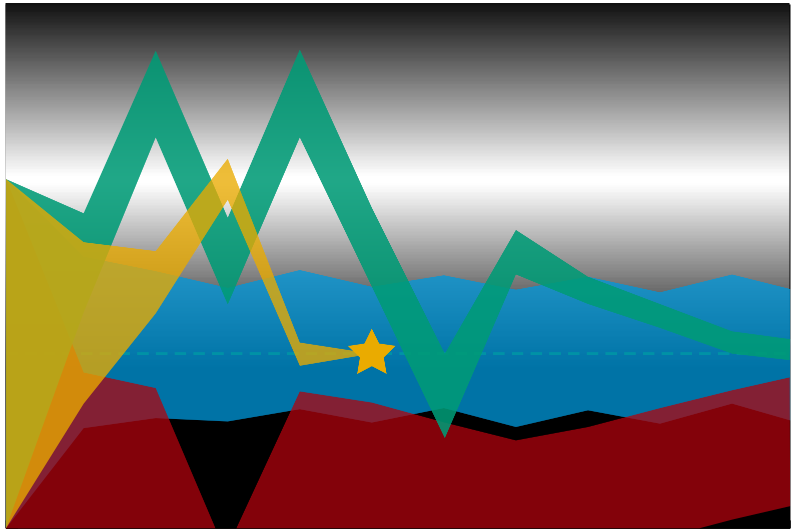
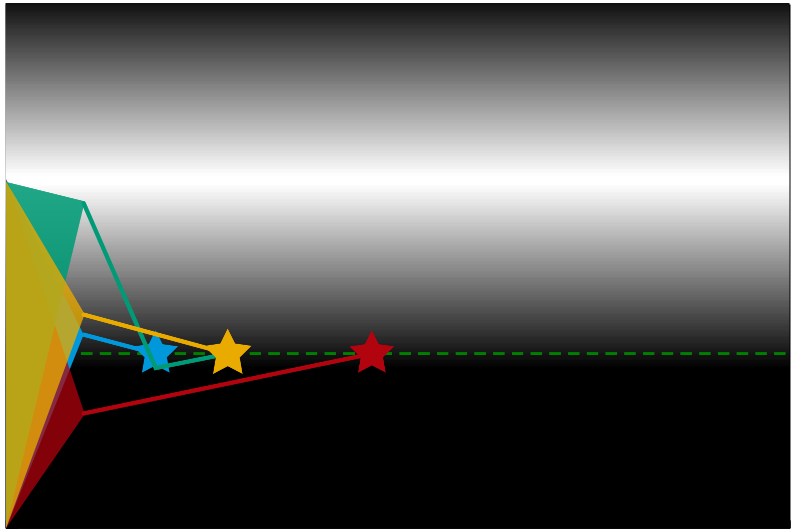
POMDP Solution



QMDP

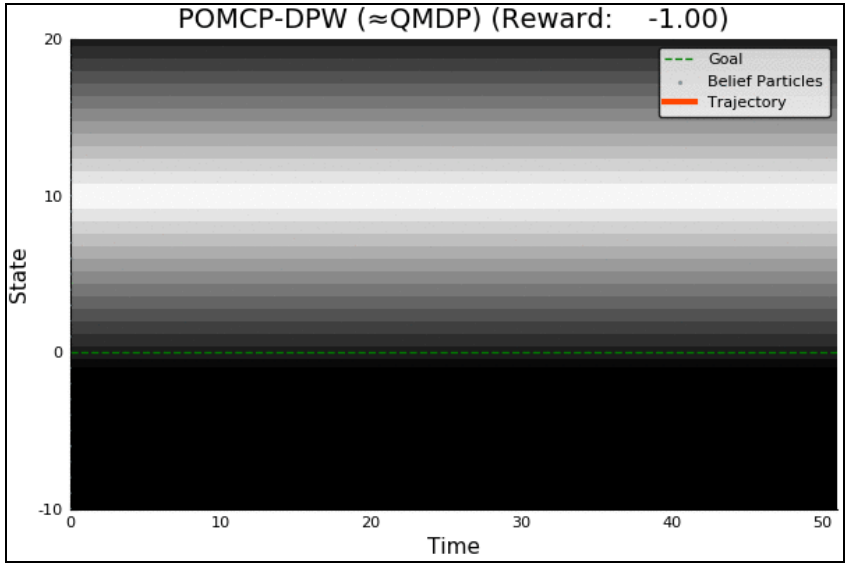


Same as **full observability** on the next step

Information Gathering

QMDP

Full POMDP



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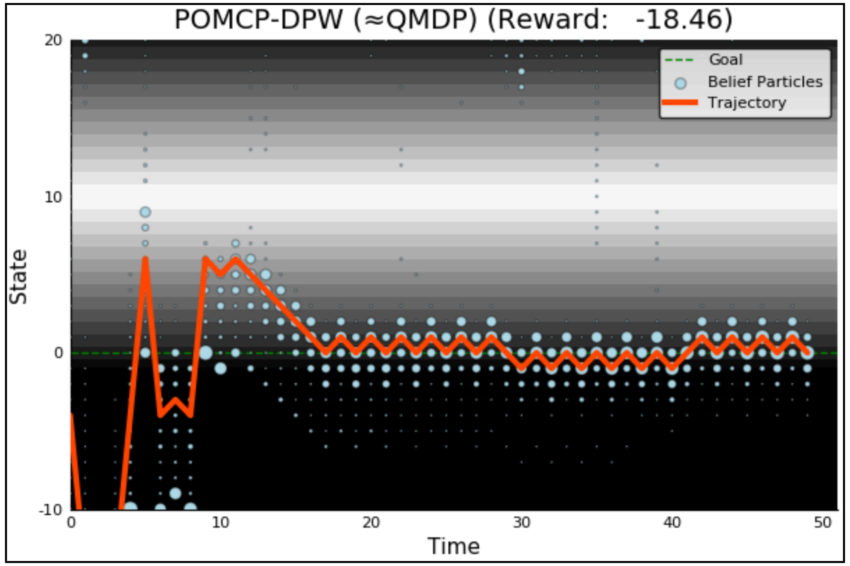

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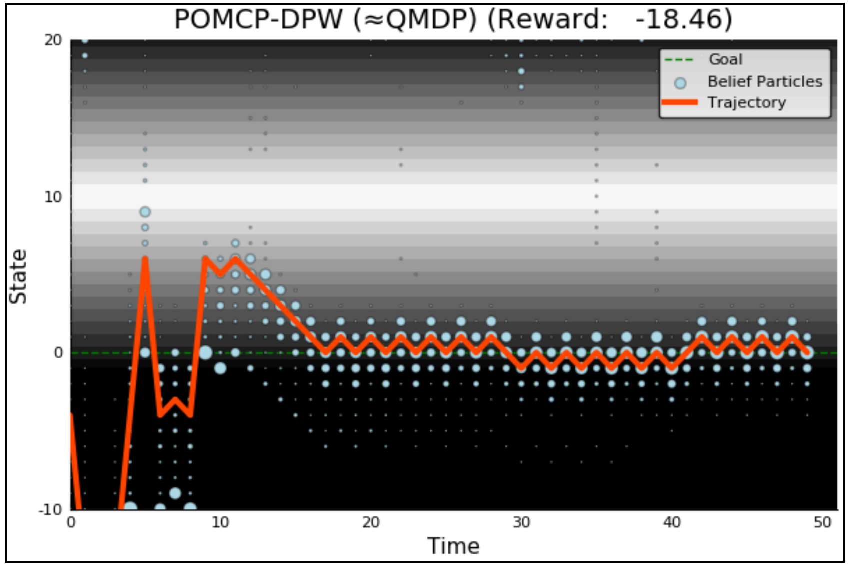
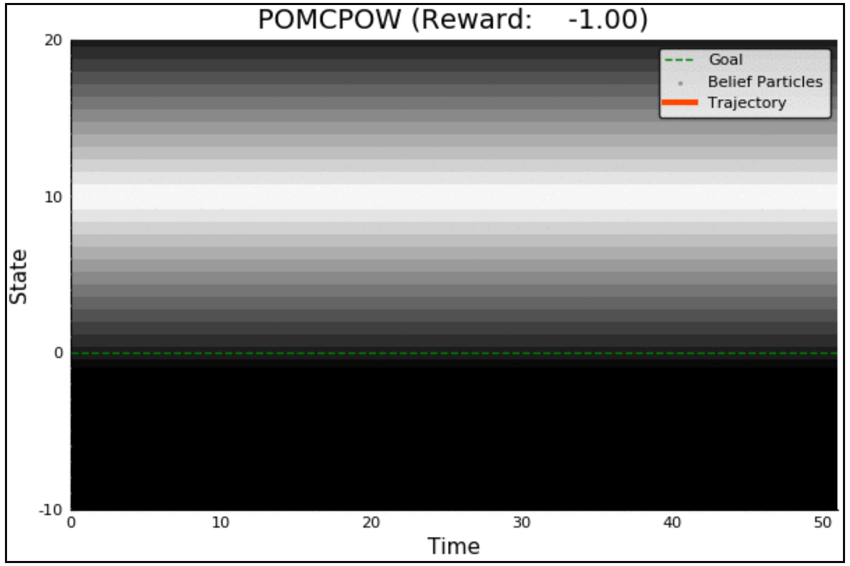
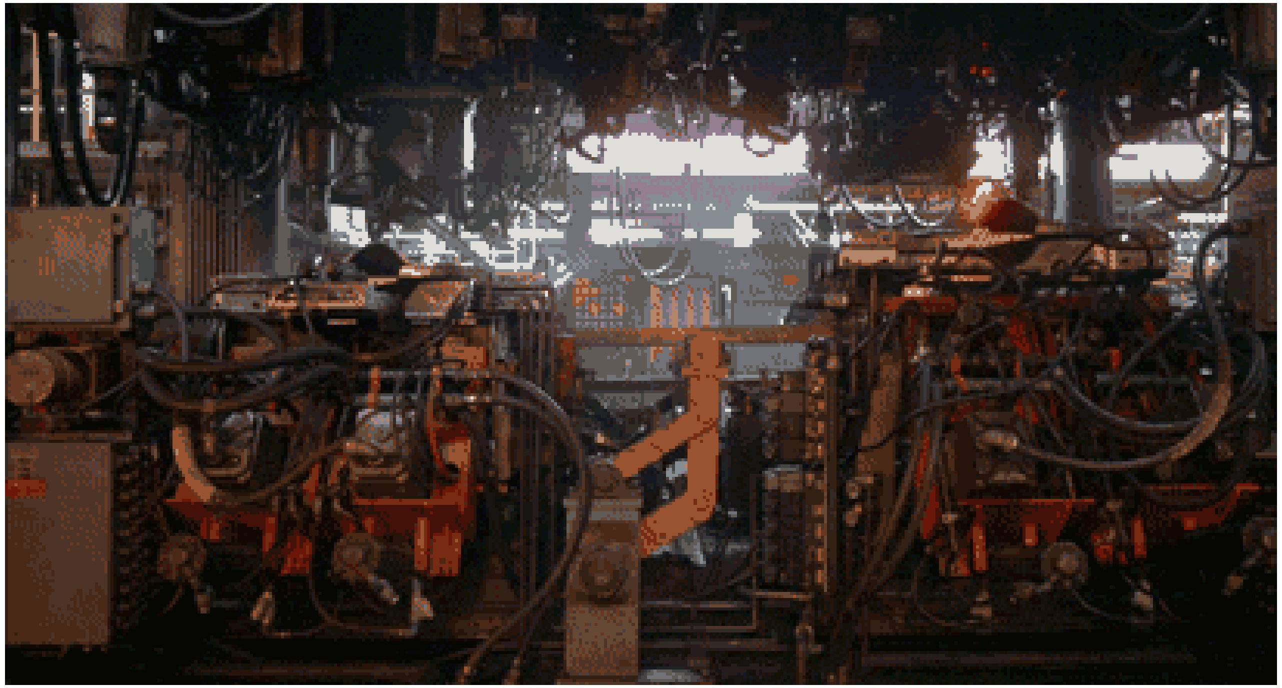




QMDP

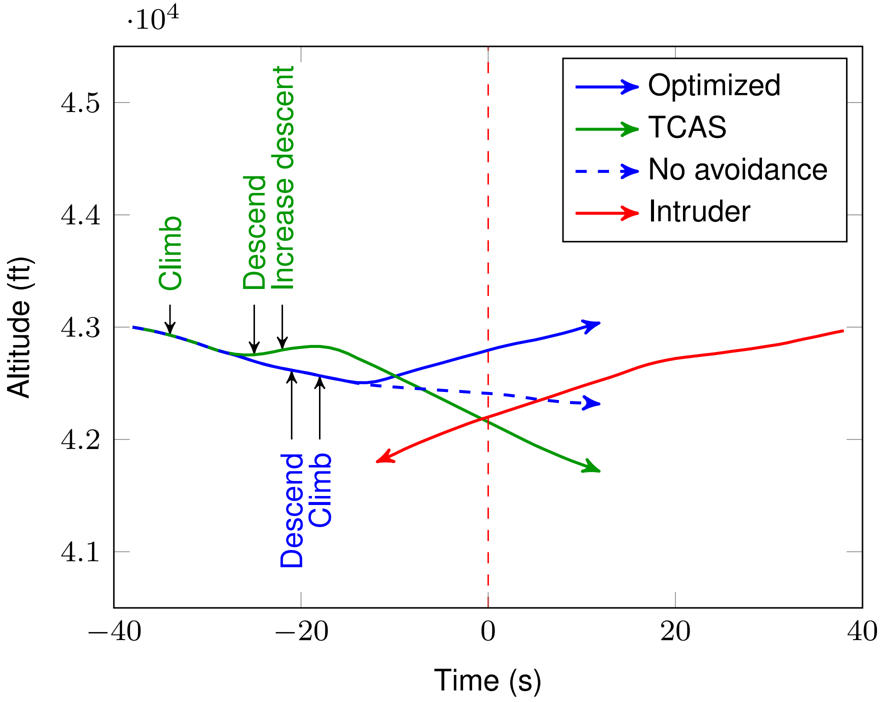
INDUSTRIAL GRADE



QMDP

ACAS X

[Kochenderfer, 2011]



Hindsight Optimization

POMDP Objective

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$$b' = \tau(b, a, o)$$

FIB

POMDP Objective

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k-Markov

POMDP Objective

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Open Loop

POMDP Objective

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