Complex Network Theory
Social Network Theory - Community Structure

Lecture delivered by Prof. Niloy Ganguly

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1 Continued... from lecture on March 16,2006.

Consider the graph shown in figure 1 below.

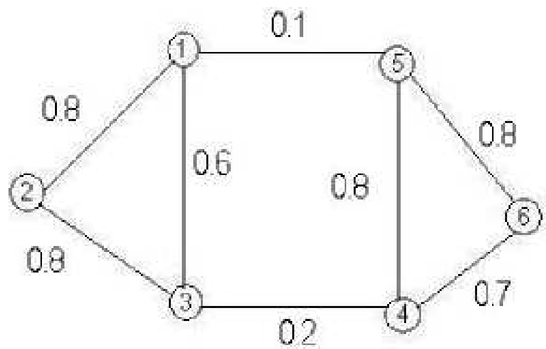

Figure 1: Weighted undirected graph

Diagonal = $\sum$ Edge weights incident

1.5	-0.8	-0.6	0	-0.1	0
-0.8	1.6	-0.8	0	0	0
-0.6	-0.8	1.6	-0.2	0	0
0	0	-0.2	1.7	-0.8	-0.7
-0.1	0	0	-0.8	1.7	-0.8
0	0	0	-0.7	-0.8	1.5

$$\lambda = \begin{pmatrix} 0.0 & 0.4 & 2.2 & 2.3 & 2.5 & 3.0 \end{pmatrix}$$

Eigen Values.

$$\lambda = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \\ -0.4 \\ -0.7 \\ -0.7 \end{pmatrix}$$

Time Complexity for finding 2_{nd} Eigen value will be $\mathbf{O}(\frac{m}{\lambda_3 - \lambda_2})$.
This method is called ***Lanzos Method***, where m is number of edges.
This method will be fast if

- Sparce Matrix
- Distinct difference between 3_{rd} and 2_{nd} Eigen value i.e.$\lambda_3 - \lambda_2$.

2 Kernighan-Lin Algorithm

- Number of elements in a community is fixed.
- Randomly divided
- Benefit Function:
  $Q =$ Number of elements that lies within a group - Number of edge that lies between a group.
- Consider all possible pair and calculate δQ i.e. change in cost function due to the swap.
- Swap the pair which has highest δQ

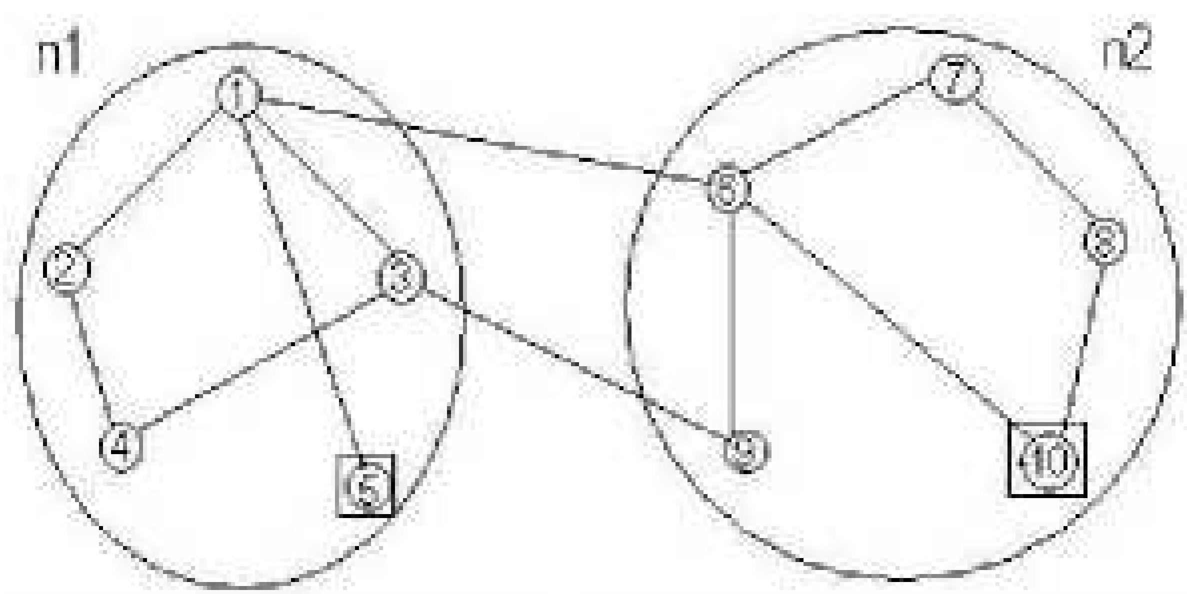

Figure 2: Q = 10 -2 = 8

- Repeate this till all possible pair exhausted.
- Reswapping is prohibited in the future.

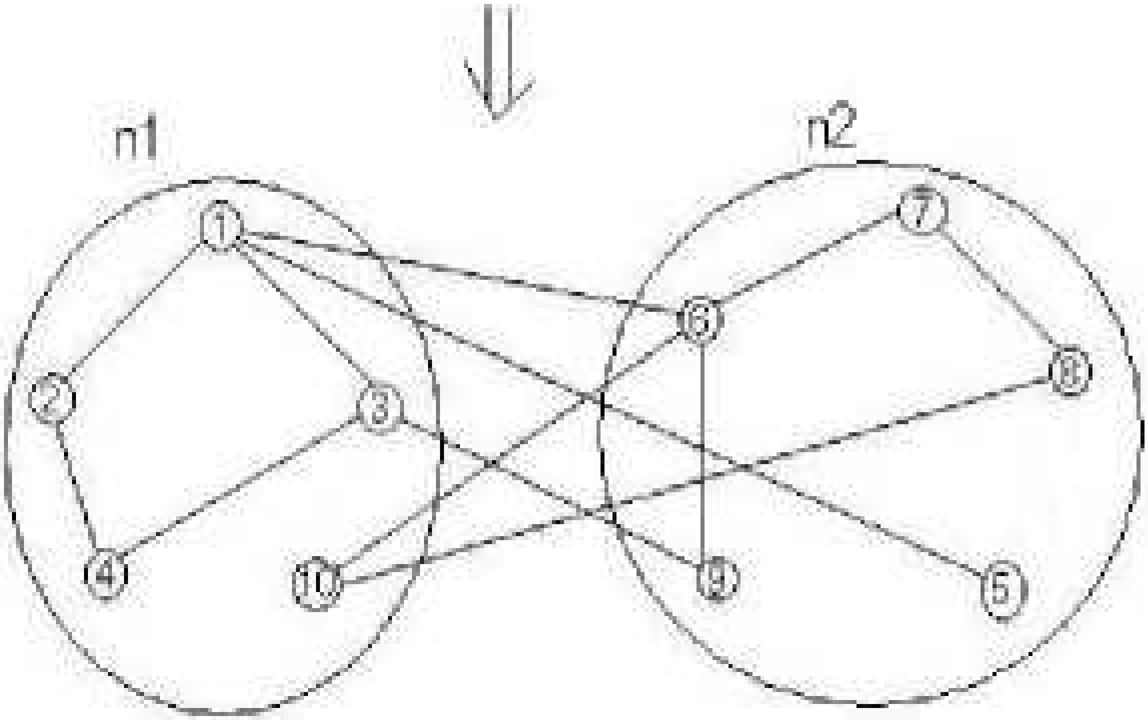

Figure 3: By swapping element 5 of n_1 with element 10 of n_2, we get this graph

- Number of possible pairs = $n_1 \times n_2 = \mathbf{O}(n^2)$
- For calculating δQ we do not consider sign.
- Probe through all the steps and select the step where $\mathbf{Q}$ is highest.

3 Hierarchical Clustering

- Based upon Threshiold weights i.e. increasing number of connection.
- WTS:Formulate or Define weights.
- Weight between two nodes(X,Y)
  - Number of Node independent path between two (X,Y)
  - Minimum number of nodes to be removed to make (X,Y) disconnected.
- Other method can be
  - Number of paths

$$W_{ij} = \sum_{l=0}^{\infty} (\alpha A_{ij})^l$$

Where α is small number.

$$\sum_{l=0}^{\infty} X^l = 1 + X + x^2 + ... + X^{\text{inf}}$$

$$= \frac{1}{1 - X}$$

So

$$W = [1 - \alpha A]^{-1}$$