Erratum for Figure 5 of Biswas et al.

Niloy Biswas & Pierre E. Jacob

February 1, 2022

There was a bug in the code used to produce Figure 5 of Biswas et al. [2019]. The choice of step-size for the MALA and ULA chains was not set in the code as it was described in the paper. The corresponding script ula_versus_mala/ula_versus_mala.R has now been corrected. Below we provide details and an updated Figure 5.

1 Choice of step-size in previous code

- Langevin Monte Carlo methods are based on a proposal of the form

$$Y = X + \frac{1}{2}h^2 \nabla \log \pi(X) + hZ, \tag{1}$$

  or equivalently

$$Y = X + \frac{1}{2}\epsilon \nabla \log \pi(X) + \sqrt{\epsilon} Z, \tag{2}$$

  with $\epsilon = h^2$. In other words, the term in front of the gradient should be commensurate to the variance of the noise term. Our previous code for Figure 5 implemented this proposal correctly following the parameterization in (2). In our previous code, `ula_stepsize` and `mala_stepsize` corresponded to $\epsilon = h^2$ above.

- For this example, the optimal scaling for MALA corresponds to $h \sim d^{-1/6}$ or $h^2 \sim d^{-1/3}$. However we got it wrong in the implementation: we chose $\epsilon = h^2 \sim d^{-1/6}$, leading to a decaying acceptance rate for MALA as dimension grew, i.e. a sub-optimal tuning of MALA.

2 Updated code and experiments

We have updated the script ula_versus_mala/ula_versus_mala.R. In the updated script, `ula_stepsize` and `mala_stepsize` correspond to h in (1), and we choose $h \sim d^{-1/6}$.

2.1 Choice of step-size

We experiment with step-sizes of the form $h = Cd^{-1/6}$ for various C and d. We obtain Figure 1, confirming that the MALA acceptance rate is stable with d.

2.2 Upper bounds on mixing times

With the corrected step-sizes, we can again estimate upper bounds on $t_{\text{mix}}(\delta)$, say with $\delta = 0.25$. We obtain Figure 2, based on 50 independent meetings for each configuration (dimension, step-size, algorithm). The upper bounds associated with ULA tend to be smaller. However they provide information on the convergence of ULA to its limiting distribution, which is not π. And, they are simply upper bounds, so the actual mixing times could be ordered differently.

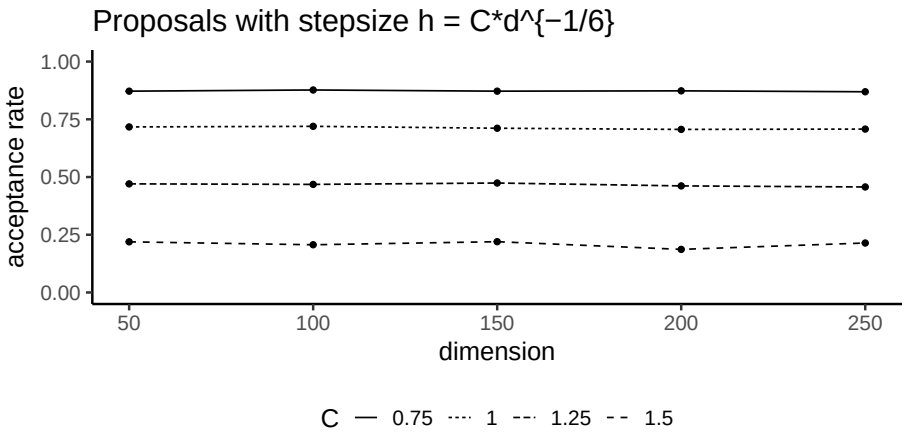

Figure 1: Acceptance rate of MALA for various C and d, with `mala_stepsize` $h = Cd^{-1/6}$.

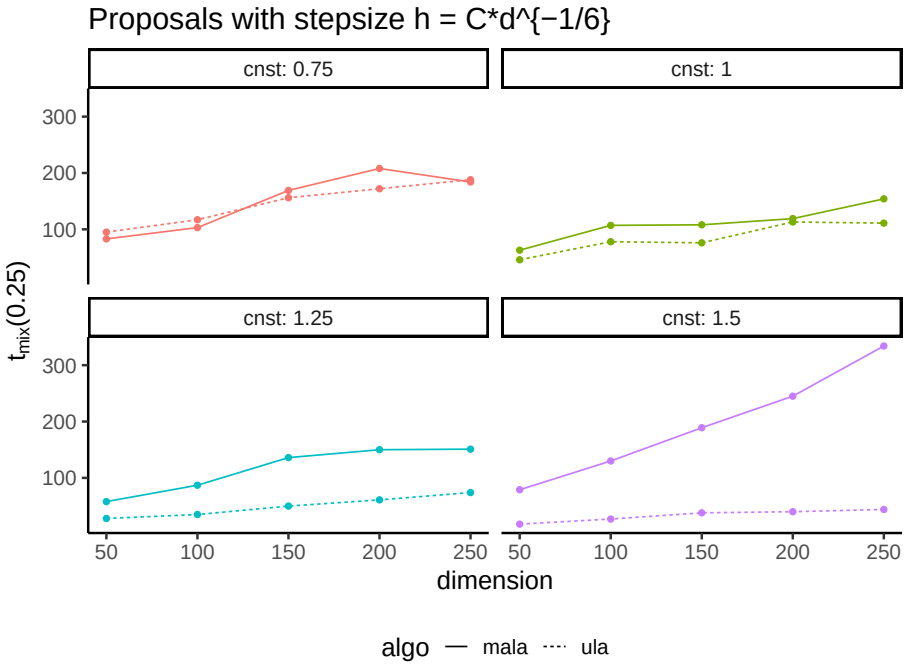

Figure 2: Upper bounds on the mixing time, for various dimensions and various step-sizes of the form $Cd^{-1/6}$, for ULA and for MALA.

References

N. Biswas, P. E. Jacob, and P. Vanetti. Estimating convergence of markov chains with l-lag couplings. pages 7389–7399, 2019. URL http://papers.nips.cc/paper/8958-estimating-convergence-of-markov-chains-with-l-lag-couplings.pdf. 1