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Section 17

Assignment 4

Ans to the Ques 1

x_1 , x_3 are Mutimomial Distributaion

a

	0	1
Y	0.4	0.6

	Y=0	Y=1
$x_1 = 0$	3/4	1/6
$x_1 = 1$	0	2/6
$x_1 = 2$	1/4	3/6

	Y=0	Y=1
$x_3 = 0$	1/4	4/6
$x_3 = 1$	3/4	2/6
~~$x_1 = 2$~~		

X_2 Gaussian.

Y=1

$$\mu = \frac{25.4 + 21.1 + 18.5 + 22.8 + 28.9 + 21.8}{6}$$

$$= 23.083$$

$(x-\mu)^2$

$(25.4 - 23.083)^2 = 5.368$

$(21.1 - 23.083)^2 = 3.932$

$(18.5 - 23.083)^2 = 21.004$

$(22.8 - 23.083)^2 = 0.080$

$(28.9 - 23.083)^2 = 33.837$

$(21.8 - 23.083)^2 = 1.646$

$$\sigma^2 = \frac{\sum (x-\mu)^2}{6}$$

$$= 10.978$$

C|1 $y=1$

$\cancel{P(y=1|x_1=2)} = 2/6$ $\cancel{P(y=1|x_3=1)} = 1$

$\cancel{P(y=1)=0.6}$

$P(x_1=2|y=1)=2/6$ $P(x_3=1|y=1) = \cancel{1}\ 2/6$

$P(y=1)=0.6$ $P(x_2 = \overset{22.2}{\cancel{25.2}} | y=1) = \frac{1}{\sqrt{2\pi\, 10.978}} e^{\frac{-(22.2-23.08)^2}{2\times(10.97)}}$

$S_1 = P(y=1 | x_1=2, x_2=22.2, x_3=1) = 0.116$ #

$y=0$

$P(x_1=2|y=0) = 1/4$

$P(x_3=1|y=0) = 3/4$

$P(y=0) = 0.4$

$P(x_2=22.2|y=1) = \frac{1}{\sqrt{2\pi\, 16.307}} e^{\frac{-(22.2-26.125)}{2\times 16.307}}$

$S_2 = P(y=0|x_1=2, x_2=22.2, x_3=1)$

$= 0.0039$

As $S_1 > S_2$

so $y=1$

$\underline{Y=0}$

$$\mu = \frac{28.5+29.2+19.2+27.6}{4} = 26.125$$

$(x-\mu)^2$

$(28.5-26.125)^2 = 5.64$

$(29.2-26.125)^2 = 9.45$

$(19.2-26.125)^2 = 47.95$

$(27.6-26.125)^2 = 2.176$

$(\sigma^2) = \frac{\Sigma(x-\mu)^2}{4}$

$= 16.307$

$P(x_3=0|y=1) = 0.67$

b) $P(x_1=1|\cancel{x_2}=1) = 2/6$ $\cancel{P(X_3=0|X_3=0)=0.67}$. $p(y=1) = 0.6$

$P(x_2=25.2|y=1)$

$\cancel{P(X=1|X_2=25.2)}$

$$= \frac{1}{\sqrt{2\pi 6^2}} e^{-\frac{1}{26^2}(25.2-23.083)}$$

$= 0.098$

$\therefore P(\cancel{Y}=1|\cancel{x_1}=1, x_2=25.2, x_3=0)$

$= 2/6 \times 0.67 \times 0.098 \times 0.6$

$= 0.0130$

d)

$$P(X_i = K \mid Y = C) = \frac{\text{Count}(K, C) + 1}{\text{Count}(C) + (\text{possible values})}$$

~~Y=1~~

	Y=0	Y=1
$X_1=0$	4/7	2/9
$X_1=1$	1/7	3/9
$X_1=2$	2/7	4/9

	Y=0	Y=1
$X_3=0$	$\frac{1+1}{4+2} = \frac{2}{6}$	5/8
$X_3=1$	4/6	3/8

Ans to the Q2

~~Let W0 constant~~

XOR

$W_0 + W_1(0) + W_2(0) \leq 0 \;;\; W_0 \leq 0$ — (i)

$W_0 + W_1(1) + W_2(0) > 0 \;;\; W_0 + W_1 > 0$ — (ii)

$W_0 + W_1(0) + W_2(1) > 0 \;;\; W_0 + W_2 > 0$ — (iii)

$W_0 + W_1(1) + W_2(1) \leq 0 \;;\; W_0 + W_1 + W_2 \leq 0$ — (iv)

(ii) and (iii)

$(W_0 + W_1) + (W_0 + W_2) > 0 + 0$

$2W_0 + W_1 + W_2 > 0$ — (v)

$2W_0 + (-W_0) > 0$ [From (iv)]

$W_0 > 0$ $\boxed{\text{Contradiction}}$

as (i)

Ans to the ques 3

a//

$$a_0 = X$$

$$a_1 = \sigma(W_0^1 \cdot a_0)$$

$$a_2 = ReLU(0, z) ; z_2 = W_1^2 a_1 + W_3^2 a_3$$

$$a_3 = \sigma(W_0^3 \cdot a_0)$$

$$a_4 = \sigma(W_1^4 a_1 + W_2^4 a_2 + W_3^4 a_3) = \bar{y} \text{ (Ans)}$$

b//

$$J = \frac{1}{2}(y - \bar{y})^2 = \frac{1}{2}(y - a^4)^2$$

$$\frac{\delta J}{\delta W_3^4} = \frac{\delta J}{\delta a^4} \times \frac{\delta a^4}{\delta z^4} \times \frac{\delta z^4}{\delta W_3^4}$$

$$\boxed{\frac{\delta J}{\delta W_3^4} = -(y - a^4)\, a^4(1 - a^4)\, a^3.} \text{ Ans}$$

$$\frac{\delta J}{\delta W_3^2} = \frac{\delta J}{\delta a^4} \cdot \frac{\delta a^4}{\delta z^4} \cdot \frac{\delta z^4}{\delta a^2} \cdot \frac{\delta a^2}{\delta z^2} \cdot \frac{\delta z^2}{\delta W_3^2}$$

$$\boxed{\frac{\delta J}{\delta W_3^2} = (-y + a^4) \cdot a^4(1 - a^4) \cdot W_2^4 \cdot (1_{z^2 > 0}) \cdot a^3} \text{ Ans}$$

$$\frac{\delta J}{\delta W_0^1} = \frac{\delta J}{\delta a^4} \cdot \frac{\delta a^4}{\delta z^4} \cdot \frac{\delta z^4}{\delta a^1} \cdot \frac{\delta a^1}{\delta z^1} \cdot \frac{\delta z^1}{\delta W_0^1}$$

$$\boxed{\frac{\delta J}{\delta W_0^1} = -(y - a^4)\, a^4(1 - a^4) \cdot W_1^4\, a^1(1 - a^1) \cdot a_0} \text{ Ans}$$

$$\frac{\delta a^4}{\delta z^4} = \frac{\delta \sigma(z^4)}{\delta z^4}$$

$$= \frac{\delta\left(\frac{1}{1 + e^{-z^4}}\right)}{\delta z^4}$$

$$= (1 + e^{-z})^{-2} \cdot e^{-z^4}$$

$$= \frac{e^{-z^4}}{(1 + e^{-z})^2}$$

$$= \frac{1}{(1 + e^{-z})} \cdot \frac{e^{-z}}{(1 + e^{-z})}$$

$$= \frac{1}{1 + e^{-z^4}} \cdot \frac{1 + e^{-z^4} - 1}{1 + e^{-z^4}}$$

$$= a^4 \cdot (1 - a^4)$$

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$x = 0.7$

$$a_1 = \sigma(1 \times 0.7) = \frac{1}{1+e^{-0.7}} = 0.668$$

$$a_3 = \sigma(2 \times 0.7) = \frac{1}{1+e^{-1.4}} = 0.802$$

$$a_2 = \text{ReLU}(1.5 \times 0.668 + 1 \times 0.802)$$

$$= 1.804$$

$W_0^1 = 1$

$W_0^3 = 2$

$W_1^2 = 1.5$

$W_3^2 = 1$

$W_1^4 = 2.5$

$W_2^4 = 1.2$

$W_3^4 = 0.8$

$$a_4 = \sigma(2.5 \times 0.668 + 1.2 \times 1.804 + 0.8 \times 0.802)$$

$$a_4 = \frac{1}{1+e^{-4.4764}} = 0.988 \text{ (Ans)}$$

$\underline{x = 1.5}$

$$a_1 = \frac{1}{1+e^{-(1 \times 1.5)}} = 0.817$$

$$a_3 = \frac{1}{1+e^{-(2 \times 1.5)}} = 0.952$$

$$a_2 = \text{ReLU}(1.5 \times 0.817 + 1 \times 0.952)$$

$$= 2.1775$$

$$a^4 = \frac{1}{1+e^{-(2.5 \times 0.817 + 1.2 \times 2.1775 + 0.8 \times 0.952)}}$$

$$= 0.995 \text{ (Ans)}$$

$\underline{\underline{d}}$ $x = 0$ $y = \cancel{0.1} = 0.98$ $\eta = 0.1$

$$W_3^4 = W_3^4 - 0.1 \frac{\delta J}{\delta W_3^4}$$

$$= 0.8 - 0.1\left(-(0.98 - \cancel{0.9}\ 0.958)0.958\ (1 - 0.9589)0.5\right)$$

$$W_3^4 = 0.800083$$

$a_0 = x = 0$

$$a_1 = \frac{1}{1 + e^{-1\times 0}} = 0.5$$

$$a_3 = \frac{1}{1 + e^{-2\times 0}} = 0.5$$

$$a_2 = (1\times 0.5) + (1.5\times 0.5)$$

$$= 1.25$$

$$a_4 = \frac{1}{1 + e^{-(\cancel{1\times 0.5}\ 2.5\times 0.5 + 1.2\times 1.25 + 0.8\times 0.5)}}$$

$$= 0.9589$$

$$W_3^2 = W_3^2 - 0.1 \frac{\delta J}{\delta W_3^2}$$

$$= 1 - 0.1\left((-0.98 + 0.9589)\times 0.9589 \times (1 - 0.9589)\times 1.2 \times 1 \times 0.5\right)$$

$$= 1.0000996$$

$$W_0^1 = W_0^1 - 0.1 \frac{\delta J}{\delta W_0^1}$$

$$= 1 - 0 = 1$$

$x = -0.5;\ y = 0.89;\ \eta = 0.1$

$$W_3^4 = 0.800083 - 0.1\big((0.9643-0.89)0.9643\ (1-0.9643)0.731\big)$$

$$= 0.79989$$

$$W_3^2 = \cancel{W_3^2}\ 1.0000996 - 0.1\big((0.9643-0.89)\times 0.9643\times (1-0.9643)\times 1.2\times 1\times 0.731\big)$$

$$= 0.999871$$

$$W_0^1 = 1 - 0.1\big((0.9643-0.89)\times (0.9643)\times (1-0.9643)\times 2.5\times 0.3775(1-0.3775)\times -0.5\big)$$

$$= 1.000075$$

$a_0 = -0.5$

$$a_1 = \frac{1}{1+e^{-(1\times -0.5)}} = 0.3775$$

$$a_3 = \frac{1}{1+e^{-(-2\times -0.5)}} = 0.73$$

$$a_2 = (1\times 0.3775) + (1.5\times 0.731)$$

$$= 1.474$$

$$a^4 = 0.9643$$

$X = 1.2 \quad Y = 0.52$

$$0.79989$$

$$W_3^4 = \cancel{0.800083} - 0.1\big((0.9947 - 0.52) \times 0.9947 \times (1 - .9947) \times 0.968\big)$$

$$W_3^4 = 0.799\cancel{766}\,647$$

$$W_3^2 = 0.999871 - 0.1\big((0.9947 - 0.52) \times 0.9947 \times (1 - 0.9947) \times 1.2 \times 1 \times 0.968\big)$$

$$= 0.99958$$

$a_0 = 1.2$

$$a_1 = \frac{1}{1 + e^{-(1.000075 \times 1.2)}} = 0.7686$$

$$a_3 = \frac{1}{1 + e^{-(2 \times 1.2)}} = 0.968$$

$$z_4 = (1.5 \times .7686 + 1.0000996 \times 0.968)$$

$$= 2.1209$$

$$a_4 = \frac{1}{1 + e^{-z_4}} = 0.9947$$

$$W_0^1 = 1.000075 - 0.1\big((0.9947 - 0.52) \times 0.9947 \times (1 - 0.9947) \times 2.5 \times 0.7686 (1 - 0.7686) \times 1.2\big)$$

$$= 0.99941472$$

Epoch 1 Done

Epoch 2

Answer to the Ques 4

a)

	A	B	C
18-30	25	15	10
31-~~40~~ 50	30	5	5
51+	5	10	15
	$P(A)=\frac{60}{120}$ $=0.5$	$P(B)=\frac{30}{120}$ 0.25	$P(C)=\frac{30}{120}$ $=0.25$

$(50+40+30)$

$n=120$

$$H(S) = -(0.5\log_2 0.5 + 0.25\log_2 0.25 + 0.25\log_2 0.25)$$

$$= 1.5$$

b)

18-30

$P(A)=\frac{25}{50}=0.5$

$P(B)=\frac{15}{50}=0.3$

$P(C)=\frac{10}{50}=0.2$

$H_1=-(0.5\log_2 0.5+0.3\log_2 0.3+0.2\log_2 0.2)$

$=1.485$

31-50

$P(A)=\frac{30}{40}=0.75$

$P(B)=\frac{5}{40}=0.125$

$P(C)=\frac{5}{40}=0.125$

$H_2=-(0.75\log_2 0.75+0.125\log_2 0.125+0.125\log_2 0.125)$

$=1.061$

51+

$P(A)=\frac{5}{30}=0.167$

$P(B)=\frac{10}{30}=0.33$

$P(C)=\frac{15}{30}=0.5$

$H_3=-(0.167\log_2 0.167+0.33\log_2 0.33-0.5\log_2 0.5)$

$=1.459$

$$H(S/Age)=\frac{50}{120}\times 1.485+\frac{40}{1\cancel{40}}\times 1.061+\frac{30}{120}\times 1.459$$

$$=1.3375$$

c||

$$I(S, Age) = H(S) - H(S|Age)$$

$$= 1.5 - 1.3375$$

$$= 0.1625$$

d||

	A	B	C
low	10	40	10
Medium	20	5	5
High	30	0	0

~~$P(A) = \frac{60}{120} = 0.5$~~ ~~$P(B) = \frac{}{120}$~~

$60 + 30 + 30$

$= 120$

low

$P(A) = \frac{10}{60} = 1/6$

$P(B) = \frac{40}{60} = 2/3$

$P(C) = \frac{10}{60} = 1/6$

Mid.

$P(A) = \frac{20}{30} = 2/3$

$P(B) = 5/30 = 1/6$

$P(C) = \frac{5}{30} = 1/6$

High

$P(A) = \frac{30}{30} = 1$

$P(B) = 0$

$P(C) = 0$

$H(High) = -(1 \log_2 1 + 0 + 0)$

$= 0$

~~$H(S/Interest) = \frac{60}{120} \times$~~

$H(low) = (1/6 \log_2 1/6 + \frac{2}{3} \log_2 2/3 + 1/6 \log_2 1/6)$

$= 1.2516$

$H(Mid) = -(2/3 \log_2 2/3 + 1/6 \log_2 1/6 + 1/6 \log_2 1/6)$

$= 1.2516$

$$H(S/Interest) = \frac{60}{120}(1.2516) + \frac{30}{120}(1.2516) + \frac{30}{120} \times 0$$

$$= 0.9387$$

$I(S, \text{Interest}) = 1.5 - .9387$
$= 0.5613$

e) As $I(S, \text{Inten}) > I(S, \text{Age})$
Interest is the Root Node