Mixed Naive Bayes Classifier – Worked Example

Given

- Number of samples: **6**
- Features:
- **x_1**: Binary (0, 1) → Bernoulli Naive Bayes
- **x_2**: Continuous (fractional) → Gaussian Naive Bayes
- Class label:
- **y**: Binary (0, 1)

Dataset

x_1	x_2	y
1	2.1	1
1	1.8	1
0	2.5	1
1	0.9	0
0	1.2	0
0	1.0	0

Step 1: Class Prior Probabilities

Count of each class: - y = 1 → 3 samples - y = 0 → 3 samples

$$P(y = 1) = \frac{3}{6} = 0.5$$

$$P(y = 0) = \frac{3}{6} = 0.5$$

Step 2: Bernoulli Parameters for x_1

For y = 1

Values of x_1: 1, 1, 0

$$P(x_1 = 1|y = 1) = \frac{2}{3}$$

$$P(x_1 = 0|y = 1) = \frac{1}{3}$$

For y = 0

Values of x_1: 1, 0, 0

$$P(x_1 = 1|y = 0) = \frac{1}{3}$$

$$P(x_1 = 0|y = 0) = \frac{2}{3}$$

Step 3: Gaussian Parameters for x_2

Formulas

Mean:

$$\mu = \frac{1}{n}\sum x$$

Variance:

$$\sigma^2 = \frac{1}{n}\sum (x - \mu)^2$$

For y = 1

Values of x_2: 2.1, 1.8, 2.5

$$\mu_1 = \frac{2.1 + 1.8 + 2.5}{3} = 2.13$$

$$\sigma_1^2 \approx 0.082$$

$$x_2|y = 1 \sim \mathcal{N}(2.13, 0.082)$$

For y = 0

Values of x_2: 0.9, 1.2, 1.0

$$\mu_0 = \frac{0.9 + 1.2 + 1.0}{3} = 1.03$$

$$\sigma_0^2 \approx 0.0156$$

$$x_2|y=0 \sim \mathcal{N}(1.03, 0.0156)$$

Final Model Parameters

Priors

$$P(y=1) = 0.5, \quad P(y=0) = 0.5$$

Bernoulli (x_1)

$$P(x_1=1|y=1) = \frac{2}{3}$$

$$P(x_1=1|y=0) = \frac{1}{3}$$

Gaussian (x_2)

$$x_2|y=1 \sim \mathcal{N}(2.13, 0.082)$$

$$x_2|y=0 \sim \mathcal{N}(1.03, 0.0156)$$

Classification Rule

$$P(y=c|x_1,x_2) \propto P(y=c)\,P(x_1|y=c)\,\mathcal{N}(x_2|\mu_c,\sigma_c^2)$$

Predict the class with the higher posterior probability.