Minimal examples of MINLPs

Niloy Saha

1 MINLP with linear objective and non-linear constraint

Let $\mathbb{S}$ denote the set of switches and $\mathbb{C}$ denote the set of controllers. Each controller $j \in \mathbb{C}$ has a capacity μ_j, and a unit cost of using it c_j. Let the binary variable x_{ij} denote whether switch $i \in \mathbb{S}$ is assigned to controller $j \in \mathbb{C}$. The response time of a controller is given as:

$$rt_j = \frac{1}{\mu_j - \sum_{i \in \mathbb{S}} x_{ij}} \quad \forall j \in \mathbb{C} \tag{1}$$

We want to assign switches to controllers so that the overall cost of assignment $\sum_j c_j \sum_i x_{ij}$ is minimized, provided the response time of a controller rt_j is below a threshold θ.

The problem is formulated as an mixed integer non-linear programming (MINLP) problem with a linear objective and non-linear constraint as follows:

$$\begin{aligned}
\text{min} \quad & \sum_j c_j \sum_i x_{ij} && \text{(2a)}\\
\text{subject to} \quad & \sum_{j \in \mathbb{C}} x_{ij} = 1 \quad \forall i \in \mathbb{S} && \text{(2b)}\\
& rt_j \leq \theta \quad \forall j \in \mathbb{C} && \text{(2c)}\\
& x_{ij} \in \{0, 1\} && \text{(2d)}
\end{aligned}$$

Here Equation (2b) ensures each switch is assigned to only one controller, and Equation (2c) specifies the non-linear constraint.

2 MINLP with non-linear objective and linear constraints

Here, we consider the same problem as above, but consider the objective as minimizing the overall response time.

$$
\begin{aligned}
\min \quad & \sum_{j} rt_j && \text{(3a)} \\
\text{subject to} \quad & \sum_{j \in \mathbb{C}} x_{ij} = 1 \quad \forall i \in \mathbb{S} && \text{(3b)} \\
& x_{ij} \in \{0, 1\} && \text{(3c)}
\end{aligned}
$$

Here Equation (3b) ensures each switch is assigned to only one controller, and Equation (3a) specifies the non-linear objective.