Legendre Segment Finder Status

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Muon Week 11/6/07

Outline

- Legendre Transform

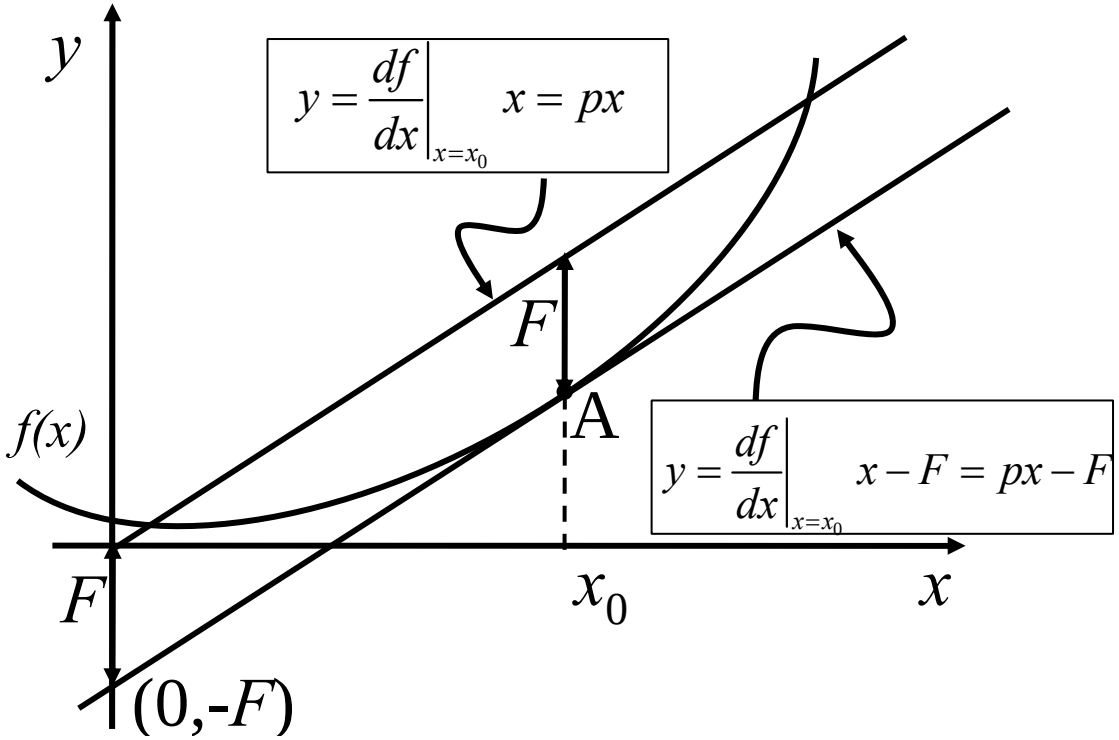
- Legendre Segment Finder in Athena
  (very preliminary…)

Legendre Transform (LT) of Convex Functions

$$\frac{\mathrm{d}^2 f}{\mathrm{d}^2 x} > 0 \quad \text{convex function}$$

$$f(x) \overset{LT}{\longleftrightarrow} F(p) \quad \text{Legendre transform pairs}$$

$$F(p) = \sup_x \left[px - f(x) \right] = -\inf_x \left[f(x) - px \right]$$



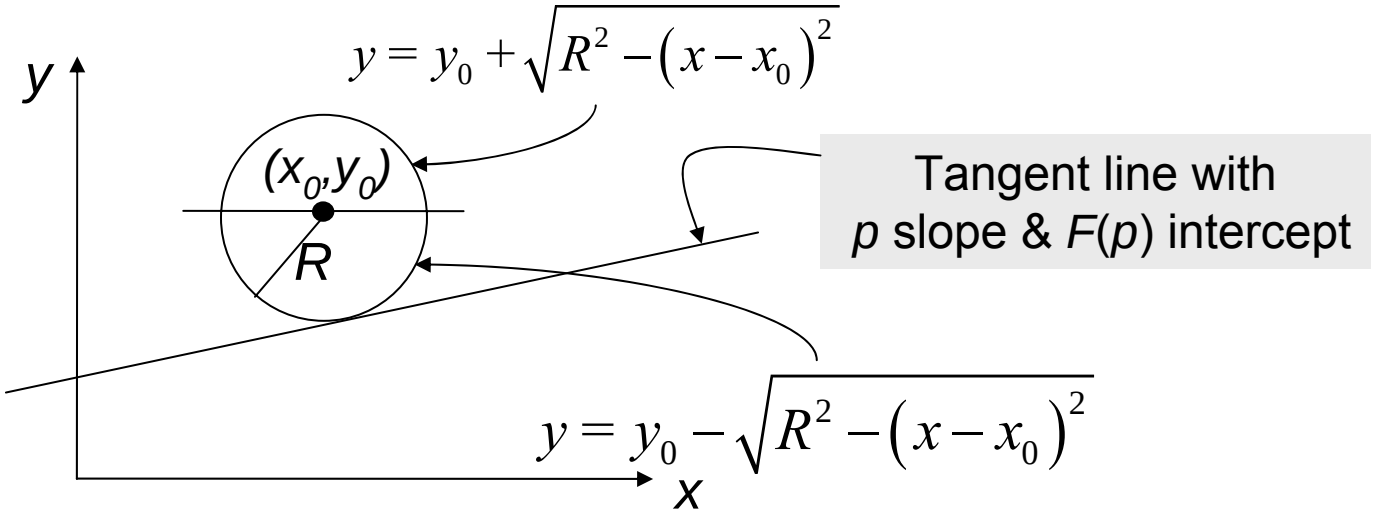
Calculate LT at a point x_0:

$$p = \left.\frac{df}{dx}\right|_{x=x_0} \Rightarrow x_0 = X(p)$$

$$F(p) = px_0 - f(x_0)$$

$$= pX(p) - f\left(X(p)\right)$$

Legendre Transform (LT) of a Circle (1)



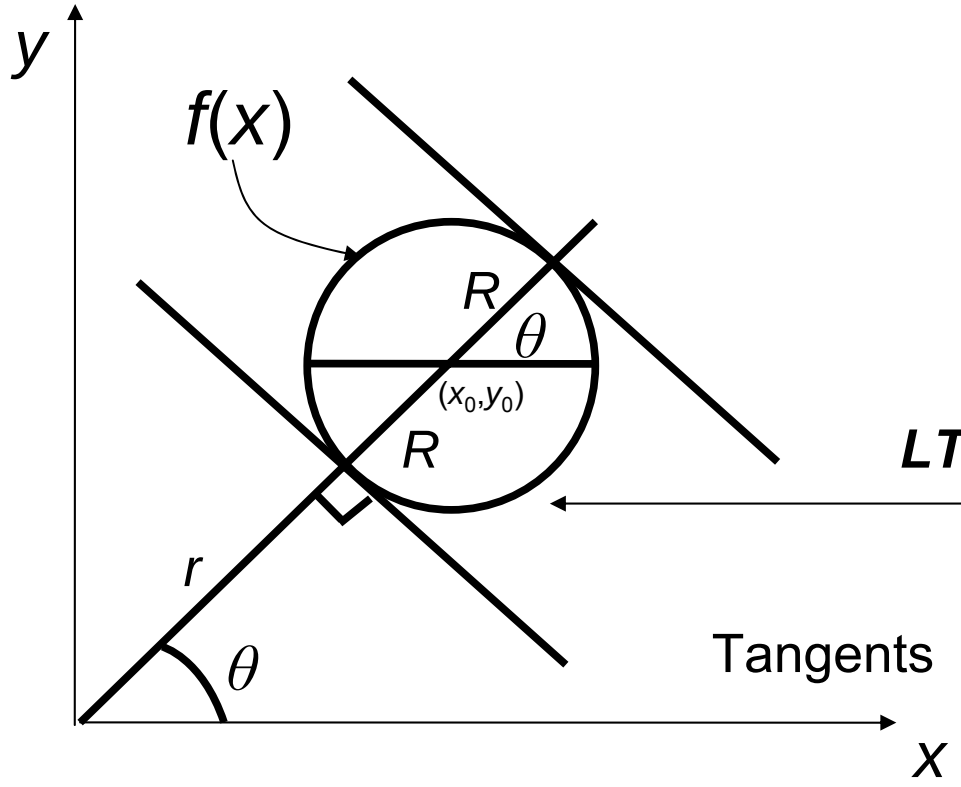
$$f(x) = \begin{cases} y_0 + \sqrt{R^2 - (x - x_0)^2} & \text{concave part} \\ y_0 - \sqrt{R^2 - (x - x_0)^2} & \text{convex part} \end{cases}$$

intercept ↓ slope

$$F(p) = \begin{cases} +y_0 - x_0 p + R\sqrt{p^2 + 1} & \text{concave part} \\ -y_0 + x_0 p + R\sqrt{p^2 + 1} & \text{convex part} \end{cases}$$

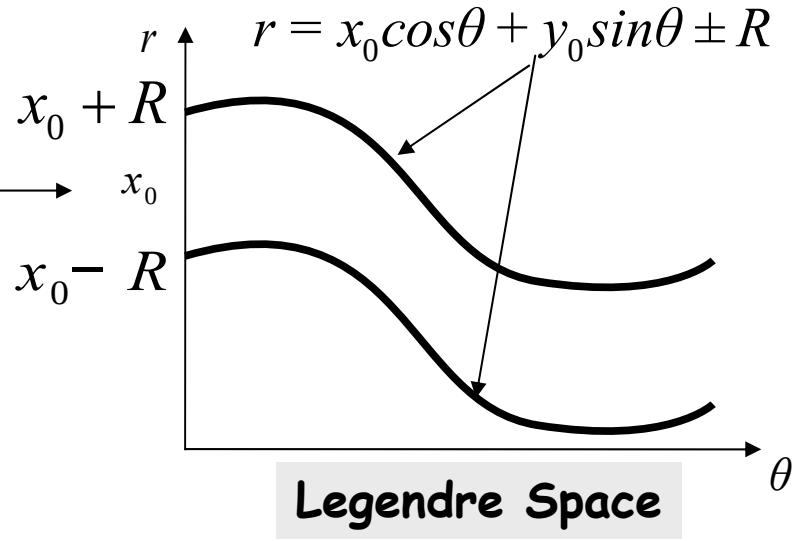
a circle becomes a hyperbola!

Legendre Transform (LT) of a Circle (2)



Canonical form of a line:

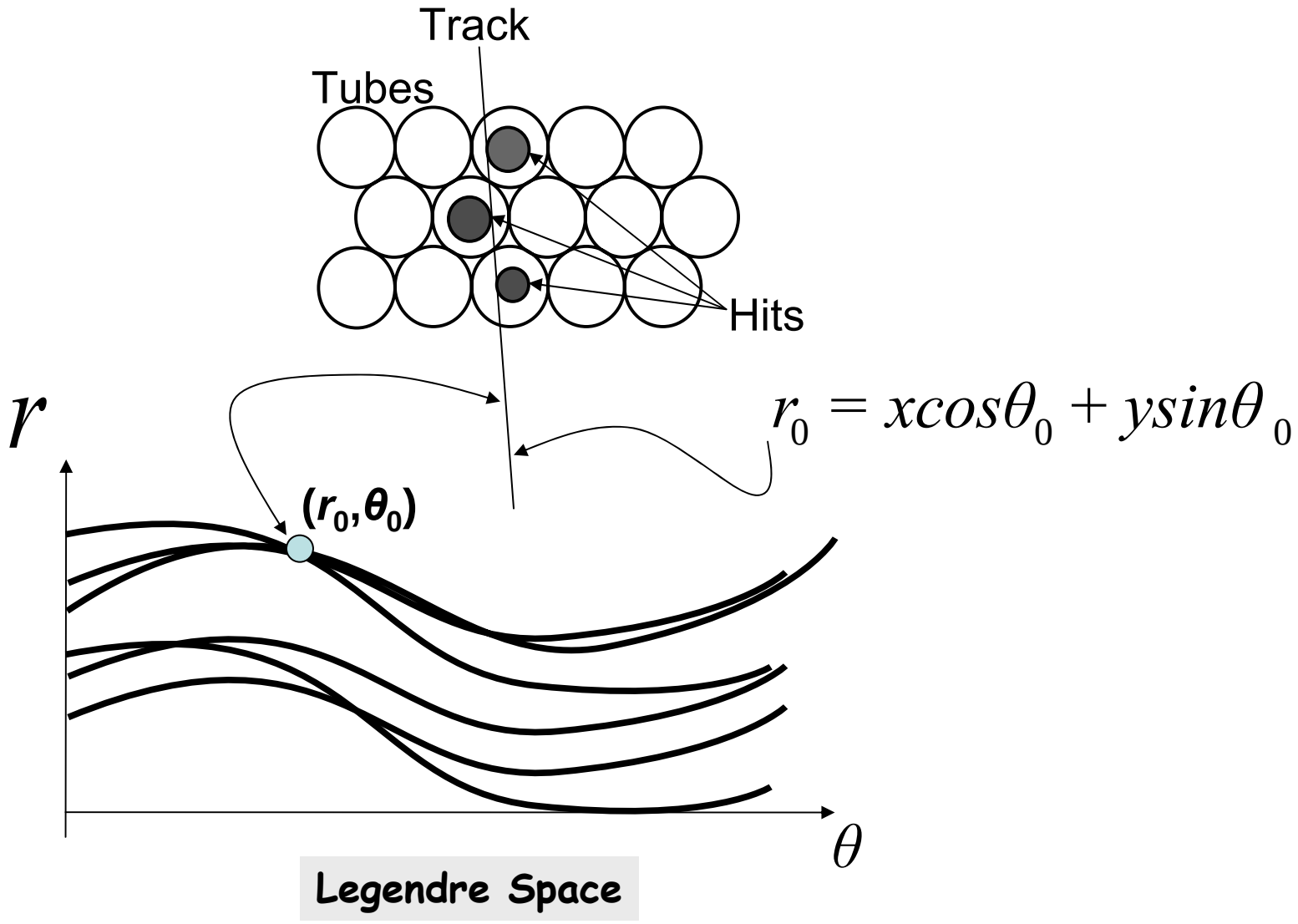
$$r = x\cos\theta + y\sin\theta$$



Legendre Space

$$f(x) \xleftrightarrow{LT} F(p) = \begin{cases} r = x_0\cos\theta + y_0\cos\theta + R \\ r = x_0\cos\theta + y_0\cos\theta - R \end{cases}$$

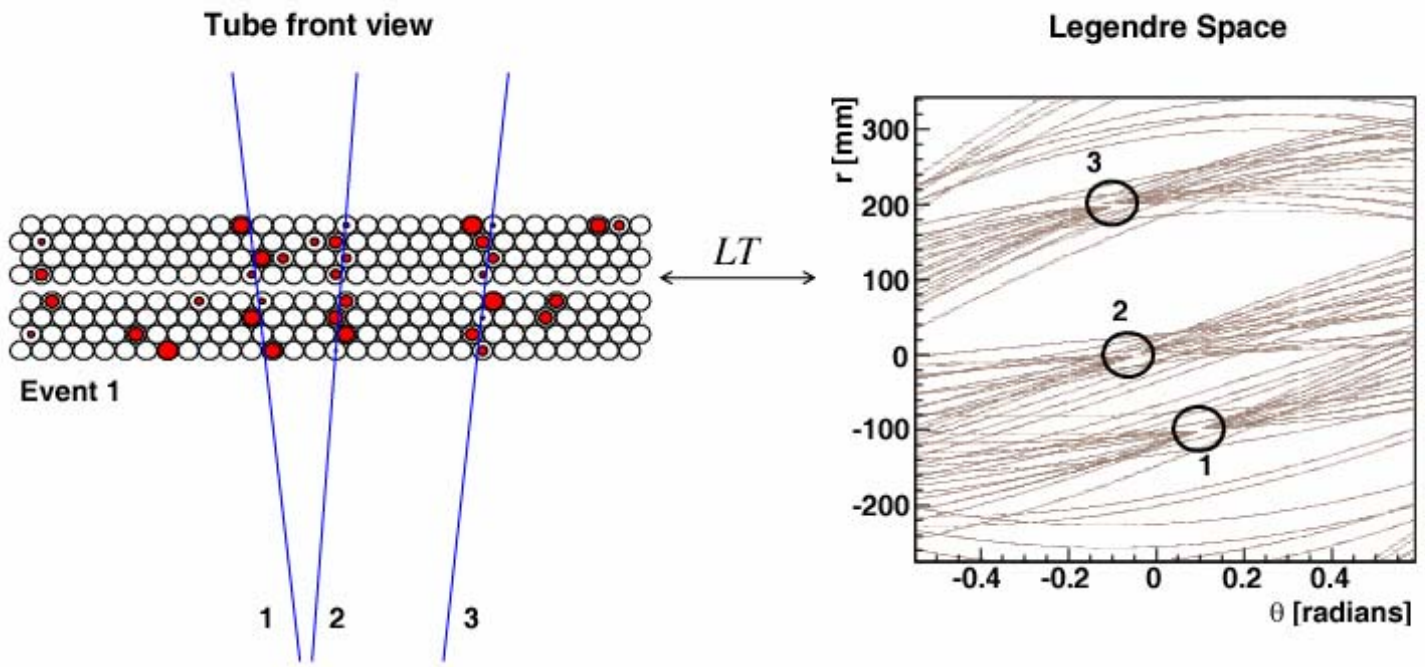
Legendre Transform for Tracking



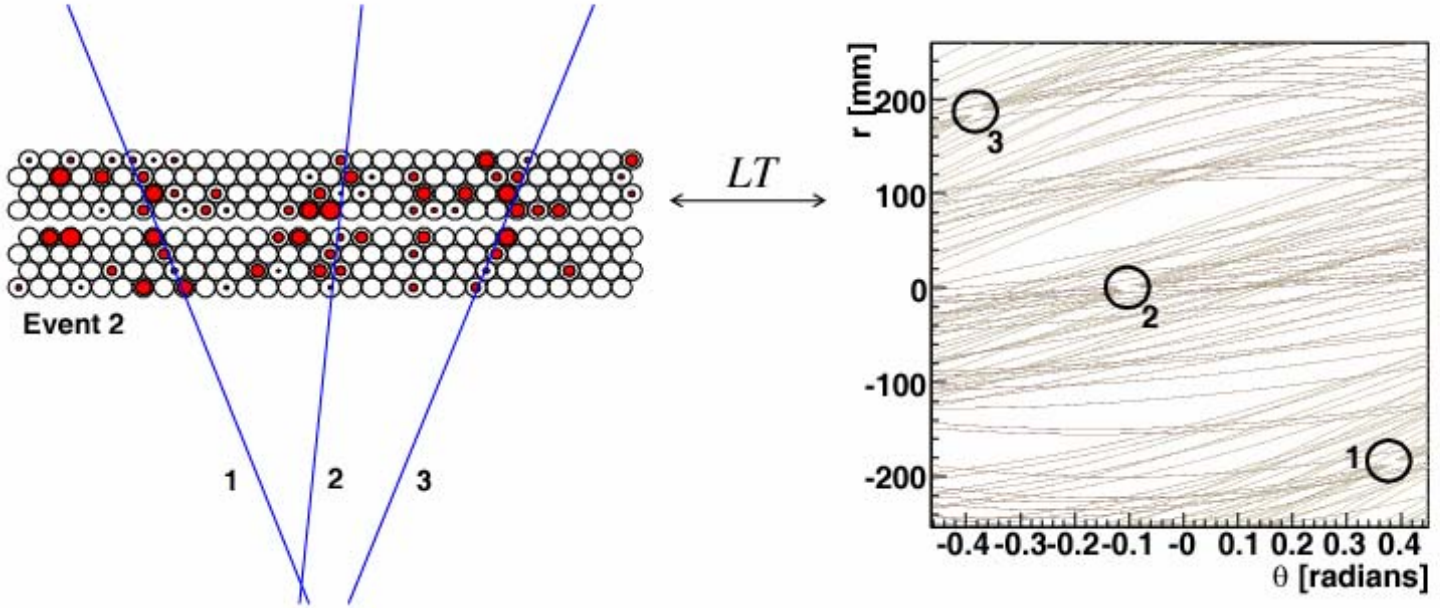
- To each circle corresponds a couple of sinograms in the Legendre Space.
- The point with the maximum intensity defines the common tangent of all circles.

Performance of Legendre Algorithm - Multi Track Events

with 50% noise level



with 200% noise level



Study the Legendre method in the Athena framework (Thanks Niels!)

NTULegendreSegments

DCMathSegmentMaker

MoMu

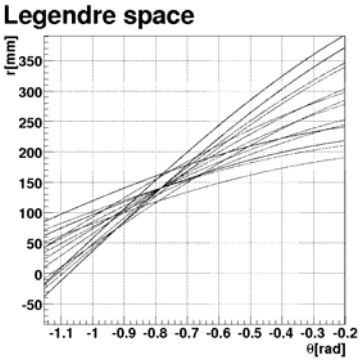
NTULegendre::SegmentFinder

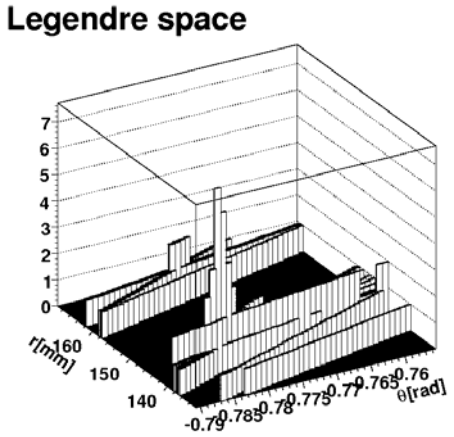
TrkDriftCircleMath::SegmentFinder

transform

Clustering-
Extraction of lines

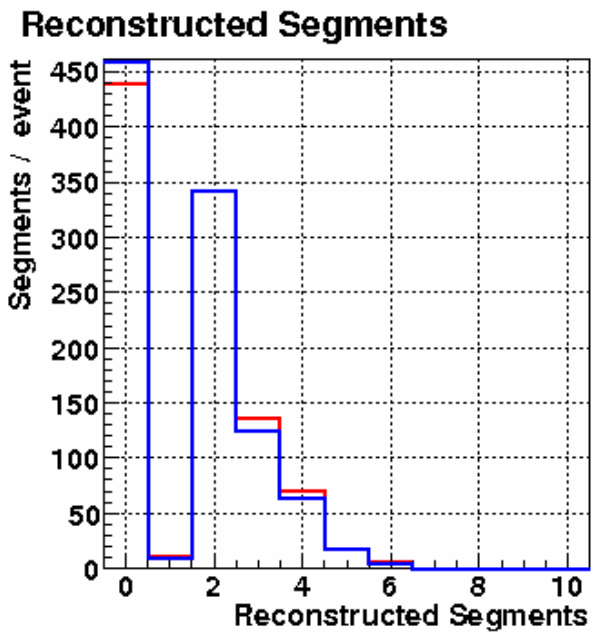
Segment Cleaning
(as in TrkDriftCircle::SegmentFinder

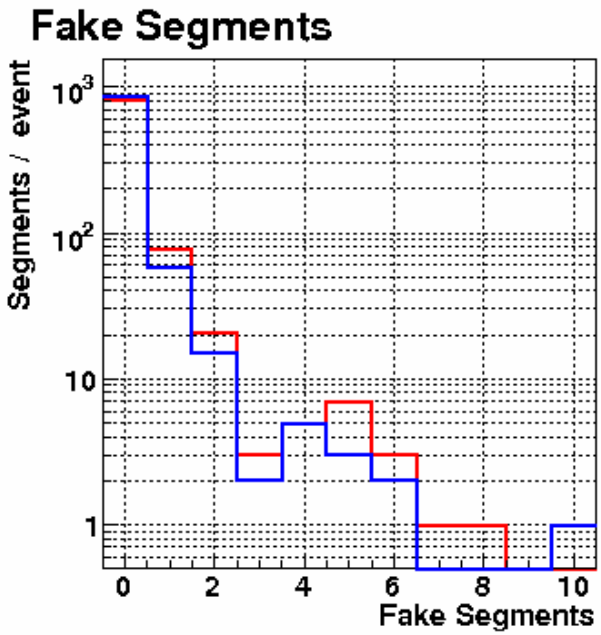




Legendre-MoMu Comparison

2 GeV



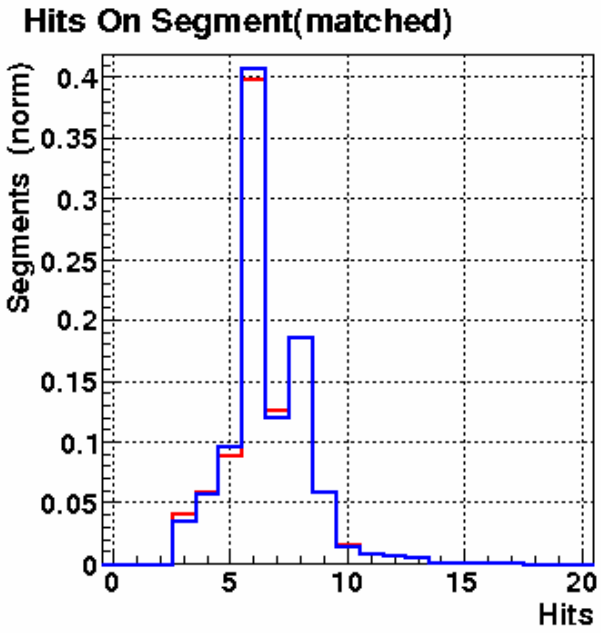


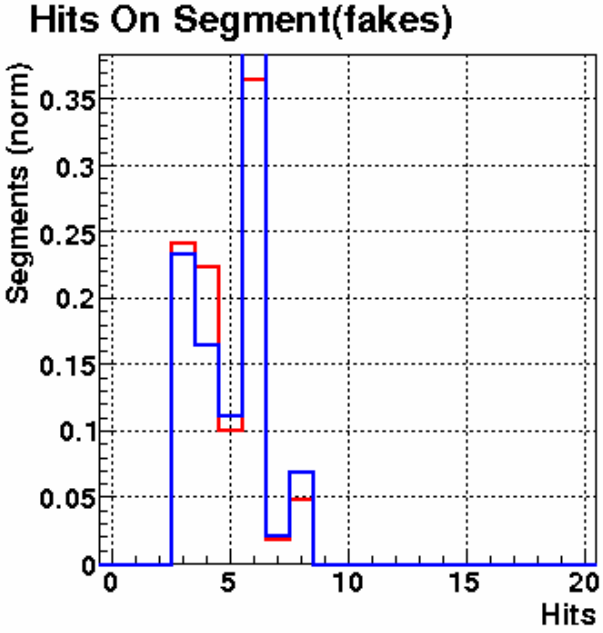
MoMu

Legendre

eff: 72.1% / 69.3%

fake: 11.1% / 8.2%

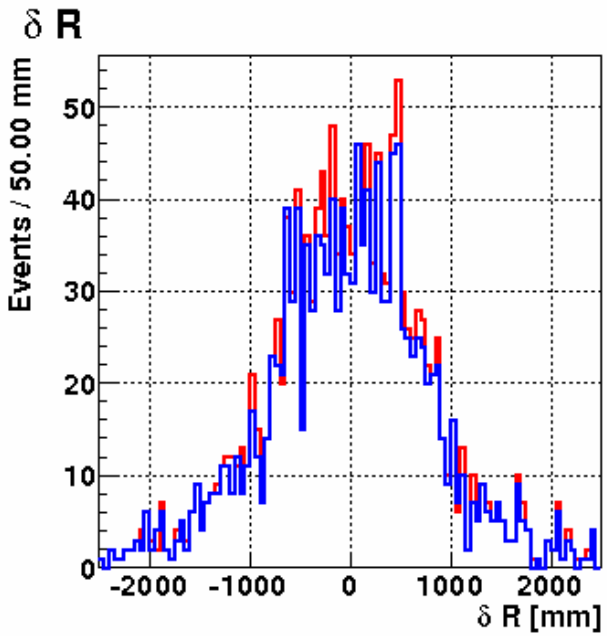


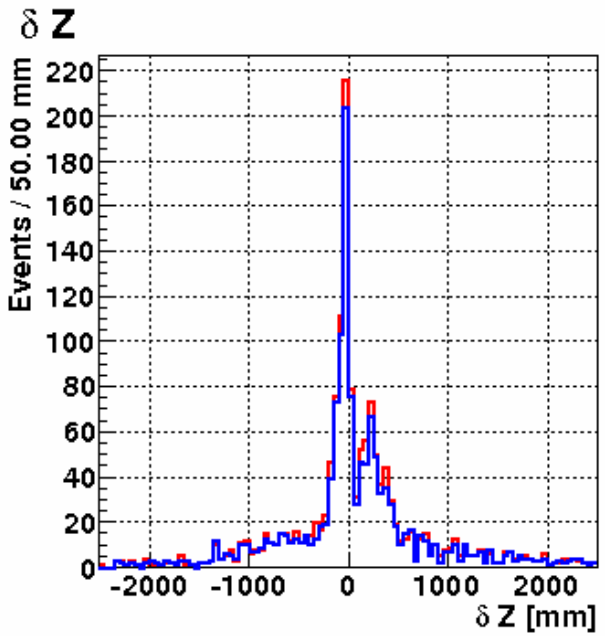


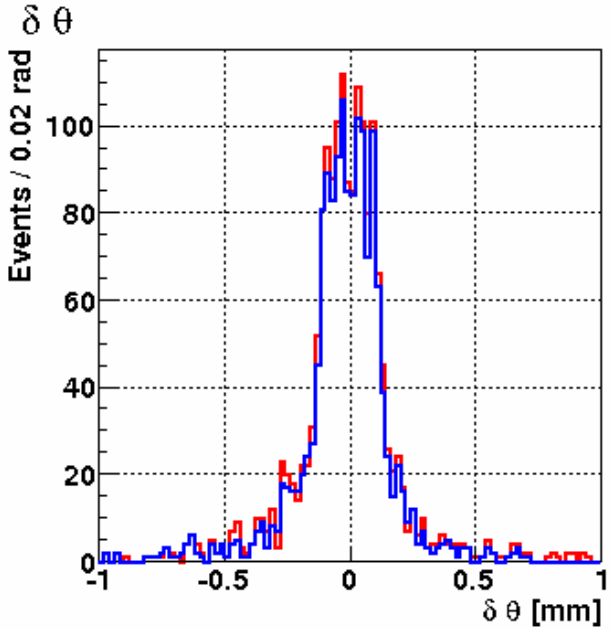
$$\text{eff} = \frac{\text{rec. segm.}}{\text{sim. segm.}}$$

$$\text{fake} = \frac{\text{fake segm.}}{\text{rec. segm.}}$$

Legendre-MoMu Comparison







2 GeV

MoMu

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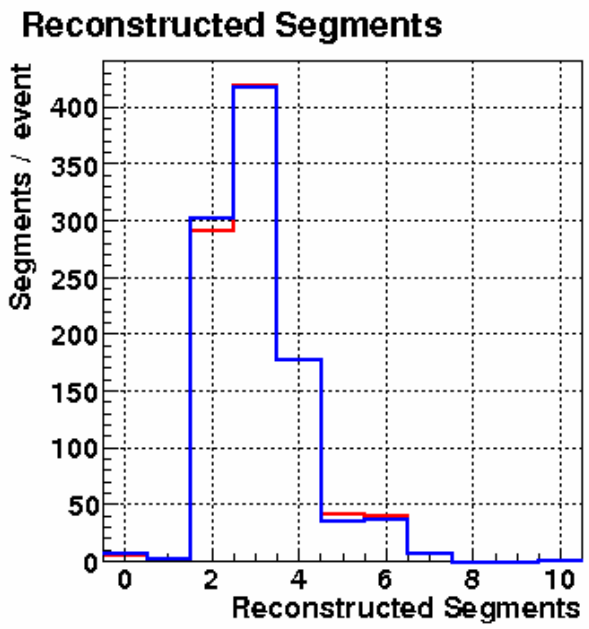
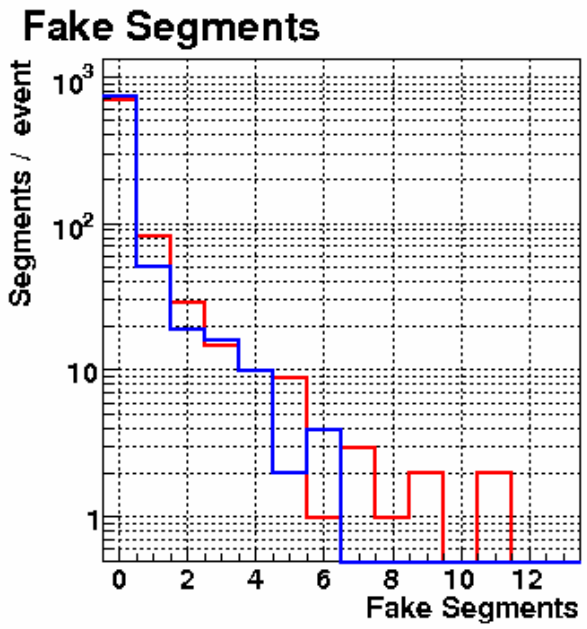
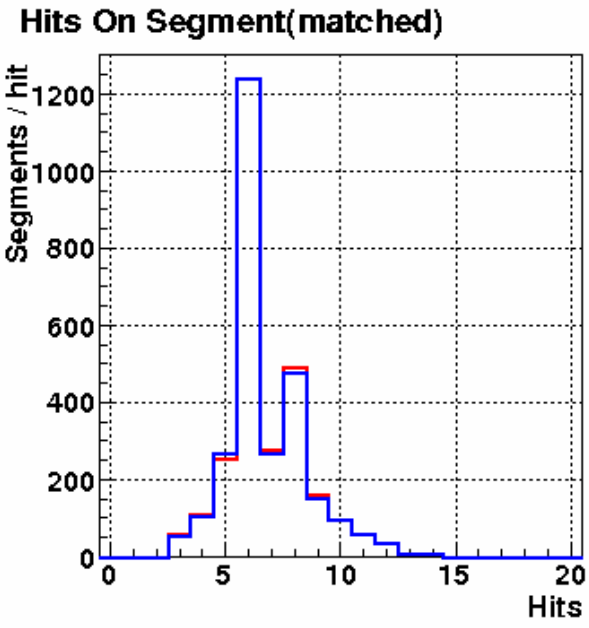
Fake: 11.1% / 8.2%

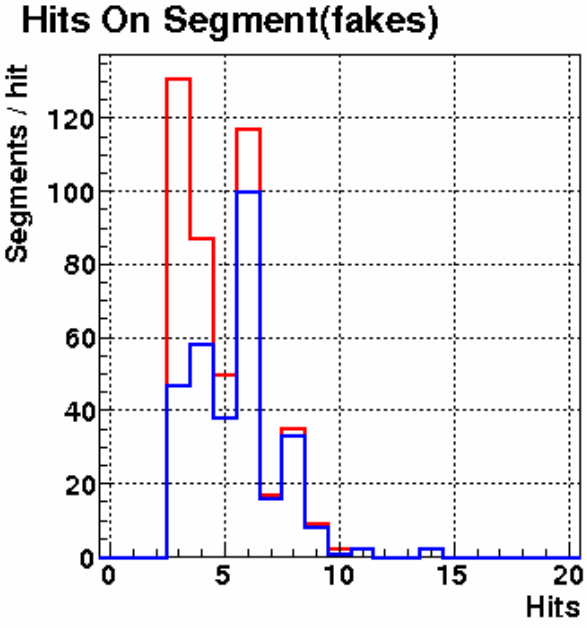
Legendre-MoMu Comparison

20 GeV









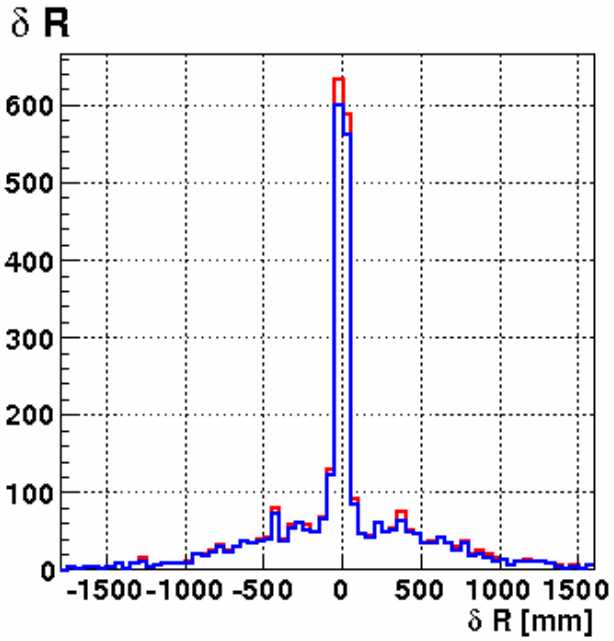
MoMu

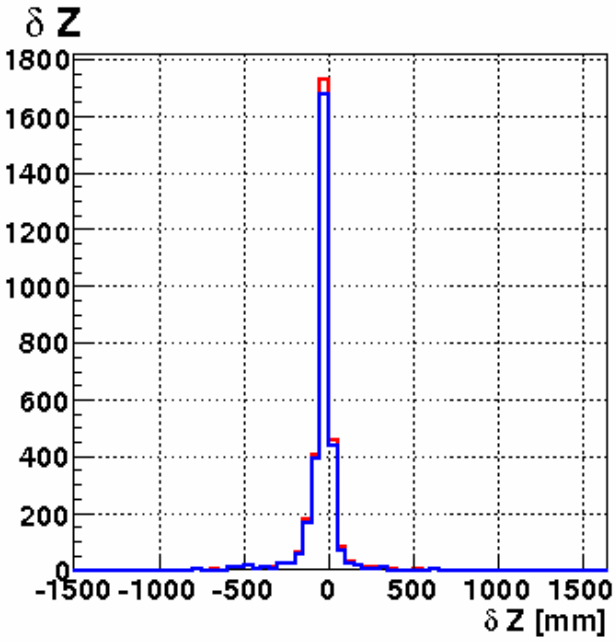
Legendre

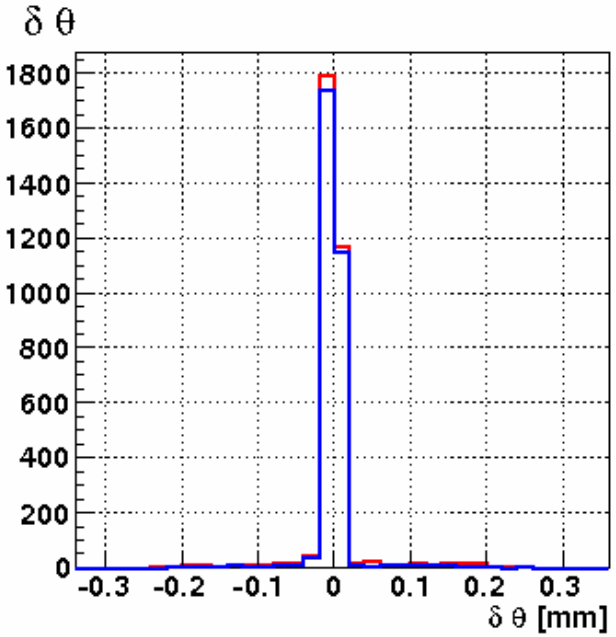
Eff: 97.6% / 96.3%

Fake: 12.7% / 7.8%

Legendre-MoMu Comparison







20 GeV

MoMu

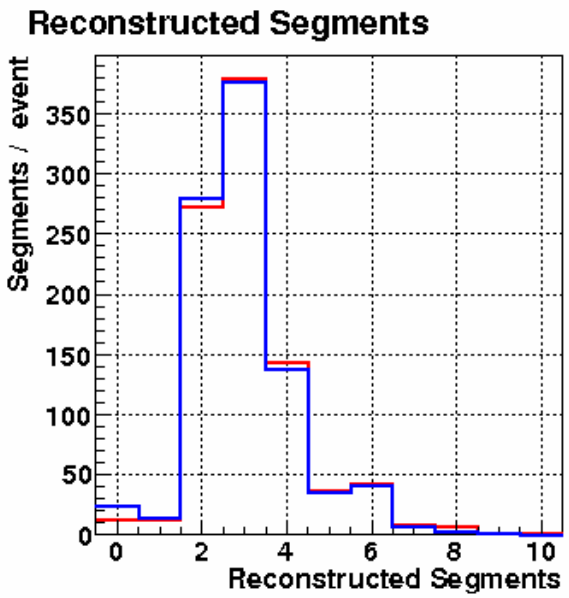
Legendre

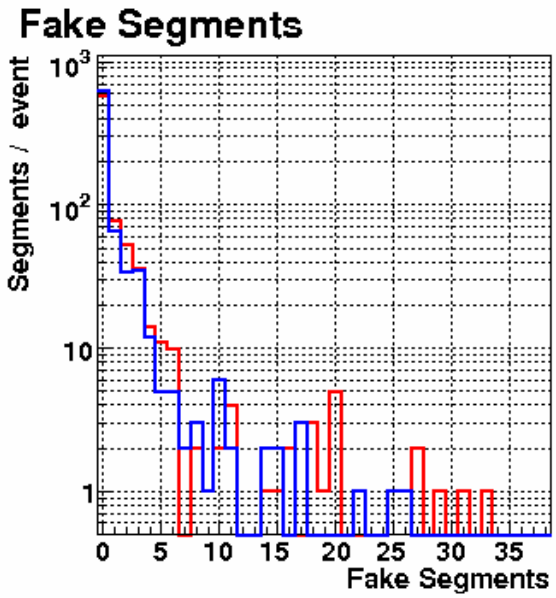
Eff: 97.6% / 96.3%

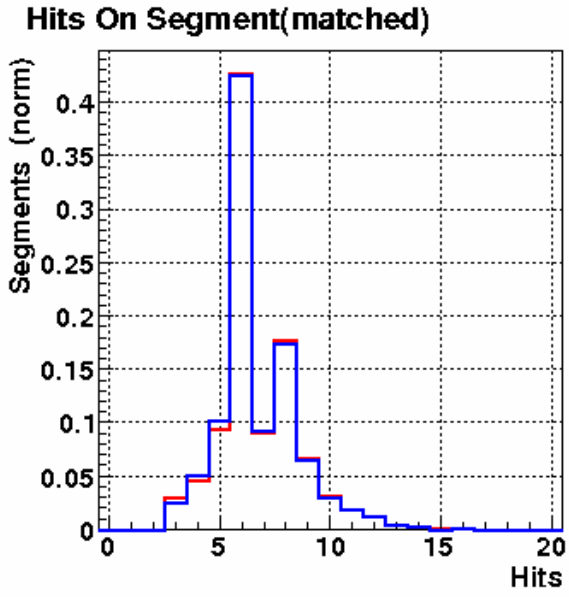
Fake: 12.7% / 7.8%

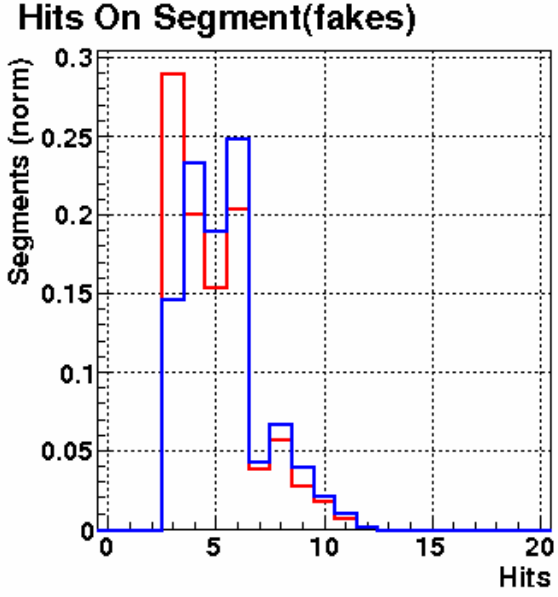
Legendre-MoMu Comparison

100 GeV









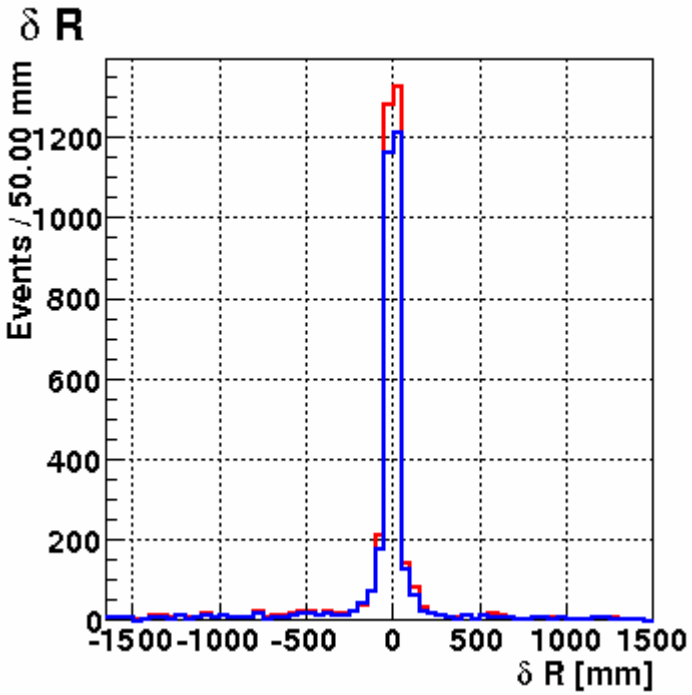
MoMu

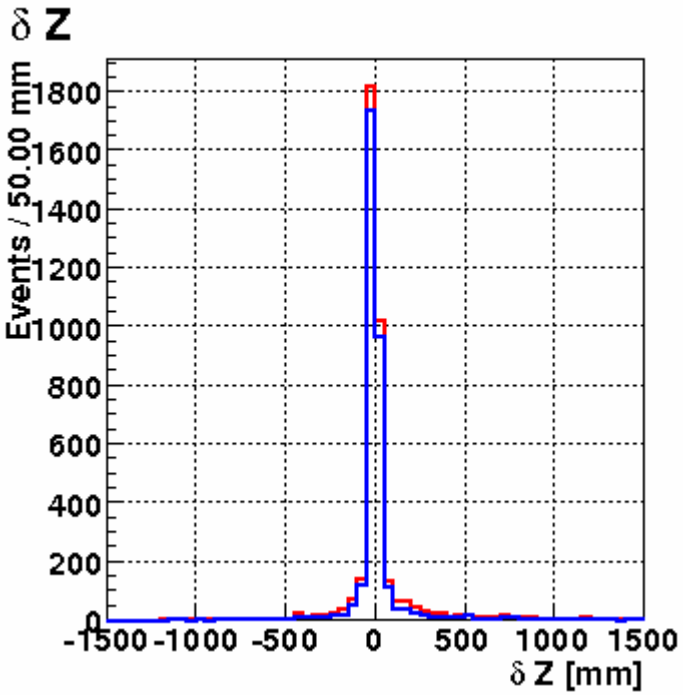
Legendre

Eff: 97.3% / 94.5%

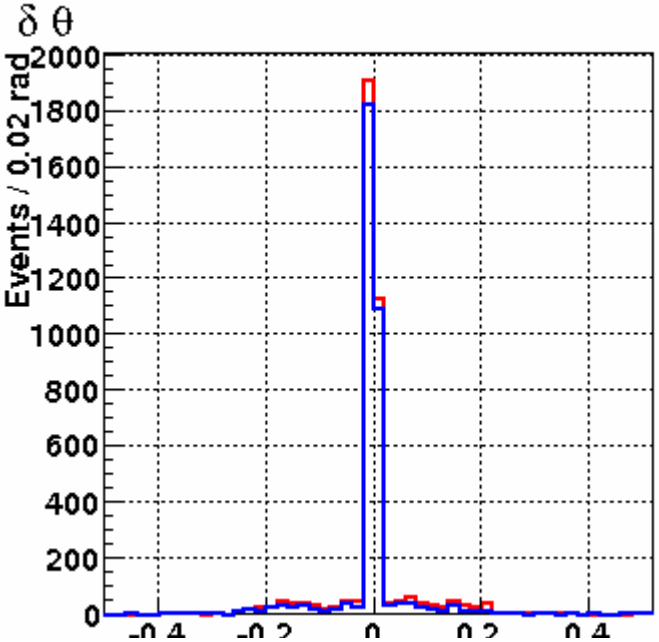
Fake: 52.3% / 37.1%

Legendre-MoMu Comparison





100 GeV



MoMu

Legendre

Eff: 97.3% / 94.5%

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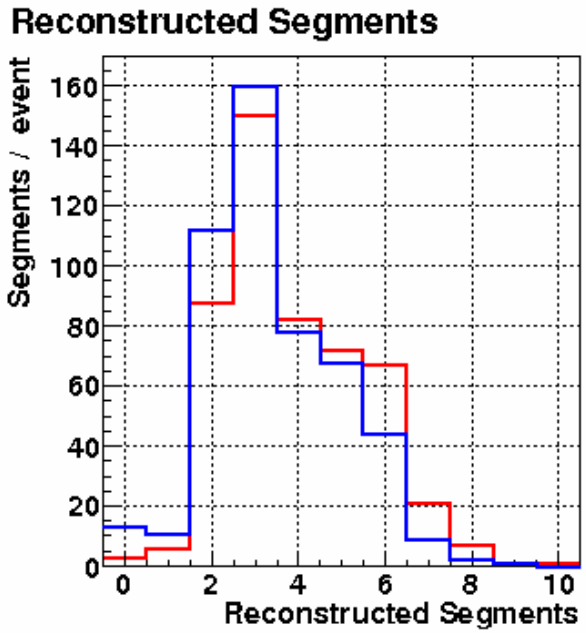
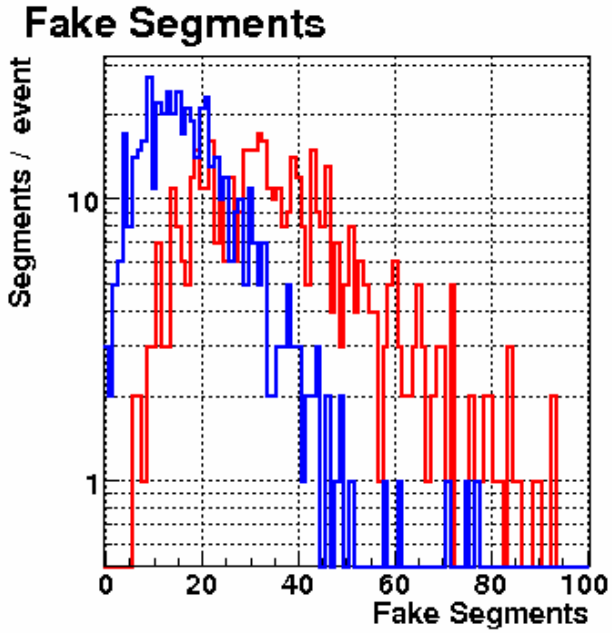
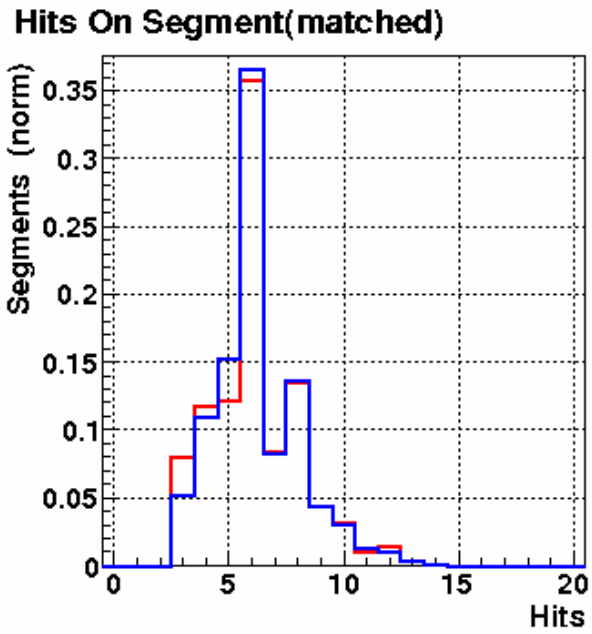
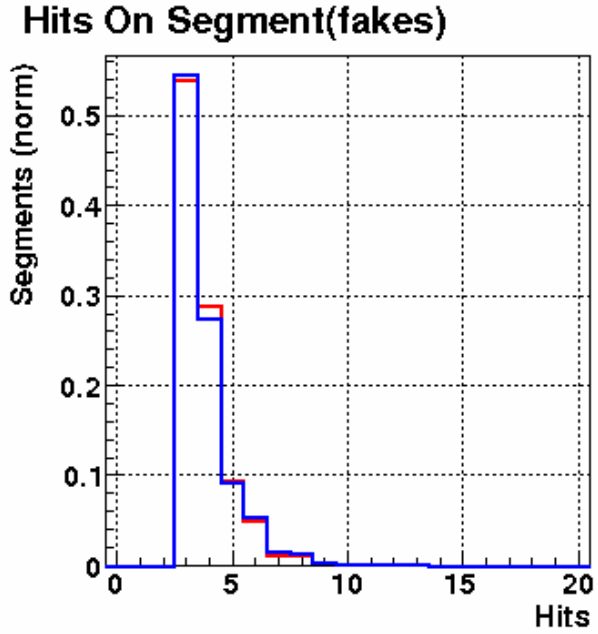
Legendre-MoMu Comparison





100 GeV
w/ bkg
Safetyfactor=5



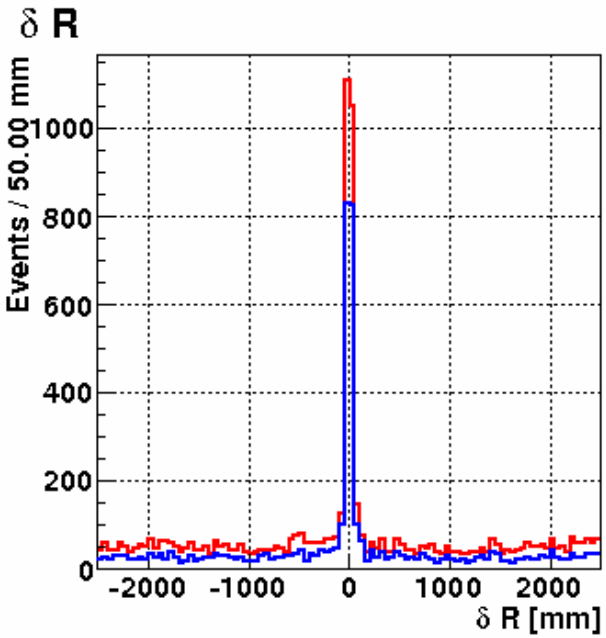


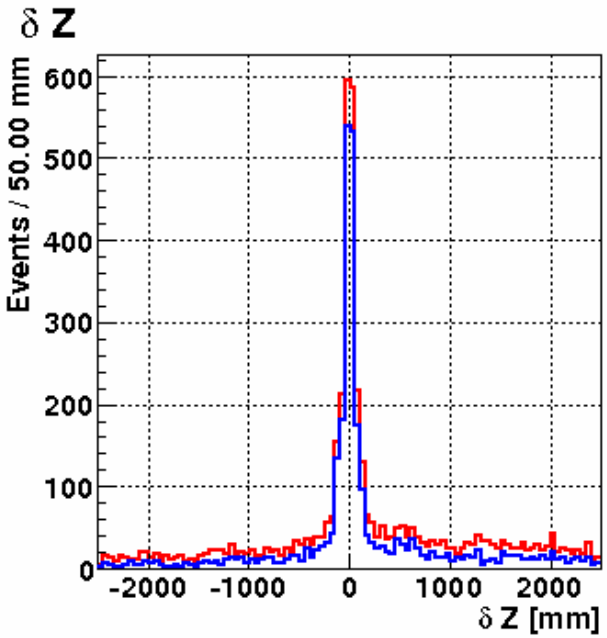
MoMu

Legendre

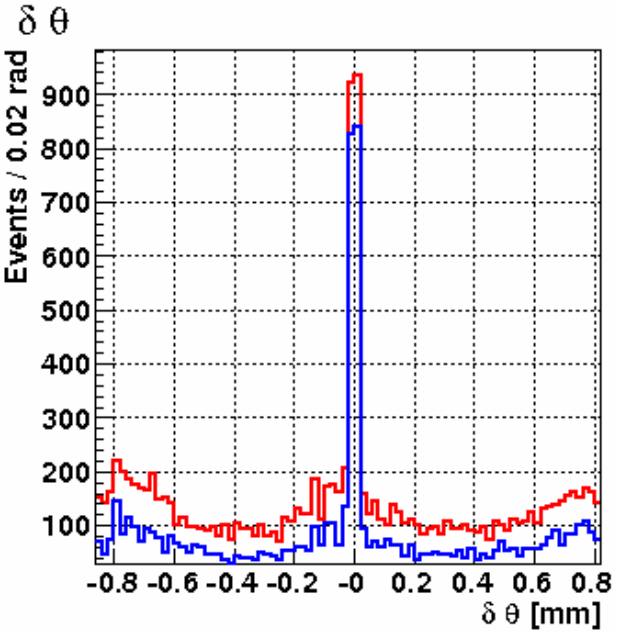
Fake: 941% / 525%

Legendre-MoMu Comparison





100 GeV
w/ bkg
Safetyfactor=5



Fake: 941% / 525%

Timing & Memory studies