

Title

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Master thesis  
submitted by

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Editing time: November 2014 – November 2015



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# 1. Experimental Setup

In the following sections the experimental setup for this thesis - the Belle II detector and the SuperKEKB collider - are described as far as it is necessary for the tracking software. Detailed information can be found elsewhere [1].

## 1.1 SuperKEKB and the Belle II Detector

The future Belle II is currently being build at the KEK high energy research facility in Tsukuba, Japan. It is a general purpose  $4\pi$  detector for high energy particle experiments. A scheme of the whole detector with the planned measuring devices can be found in figure 1.1.

The Belle II detector is used for measuring the electron positron collisions induced by the SuperKEKB collider. As its predecessor KEKB its center of mass energy is tuned in such a way that it lays near the  $\Upsilon(4S)$  resonance. This beam energy is chosen that way because a  $\Upsilon(4S)$  particle decays in nearly all cases into a pair of B-mesons. The daughter particles of these B mesons open a window to a very wide window of physical studies like CP violation or B-meson spectroscopy. The energy of the colliding electrons an positrons is asymmetric which leads to a slightly asymmetric collision as well. This fact is exploited in measuring the decay length of the two formed B-Mesons - one of the main ingredients for quantifying the CP violation in the B-Meson system. The usage of electrons and their antiparticles for collisions leads to very clean events compared to other collision partners like protons or even gold atoms. Clean means in this context a reduced number of created decay products (only 11 charged tracks on average) and more or less no pile up (only one collision occurring at the same time) are expected. These circumstances can be used for high precision measurements on the formed particles.

## 1.2 The Tracking Detectors

As every general purpose high energy particle physics detector the Belle II detector is composed of several single measuring devices - each one with a different task. For collecting as many information on the physical process in the collision as many infromation on the created decay products are needed. These include

- (1) Identification of the created particles (for example by their mass),
- (2) Measurement of the kinematic properties (like momentum and energy) of the charged particles,
- (3) Resolution of their origin position - also called their vertex.

The tracking detectors described in this thesis are mainly for recording the momenta of the particles as well as their vertex position. Further more special applications are identification of the particle type by their energy loss.

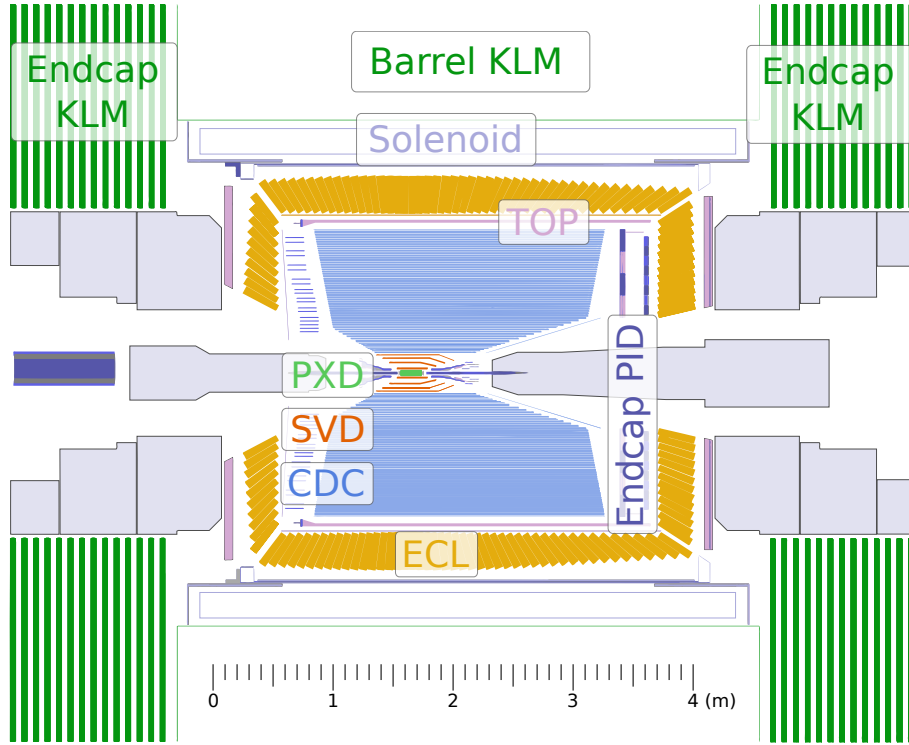


Figure 1.1: Scheme showing the planned Belle II detector in top view. The single measuring devices are shown with their names. Some more information on the tracking devices can be found in the text. Taken from [2].

The measuring principle of the momentum and also the vertex is to let the charge particles fly through a sensor region with space-resolved measurement and obtain their full path through the detector - a so called track - by combining the different location information - the so called hits. By applying a magnetic field - in Belle II it has a nominal field strength of  $B = 1.5 \text{ T}$  parallel to the beam axis - the  $z$  axis in the coordinate system of the detector - the charged particles are curled. By measuring the curvature of the curled tracks an estimation of the momentum in the plane perpendicular to the beam axis - the  $r$ - $\phi$ -plane - can be made. The remaining  $z$ -direction parallel to the beam axis can be computed with the angle of the track to the beam axis.

There exist many different ways to measure the position of a charge particle. Each of them exploits the charge of the particles and their interaction with materials which makes these techniques unusable for neutral particles. In the following two of the used ways in Belle II are described in more detail.

### 1.2.1 CDC

CDC is an acronym for Central Drift Chamber and is the largest tracking detector in Belle 2. The CDC is a typical gas chamber detector with 14336 sense wires arranged in 56 layer and 9 superlayers. These wires are strained mostly parallel to the beam axis in a cylindrical tank flooded with gas. The gas is a mixture of 50 % helium and 50 % ethane. For more information on gas chamber detectors see for example [3]. Each charged particle produced in the collision of the electron and positron interacts with the gas and can ionize the helium atoms. The energy deposit of a charged particle is given by the Bethe formula [4]

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{4\pi n z^2}{m_e c^2 \beta^2} \cdot \left( \frac{e^2}{4\pi\epsilon_0} \right)^2 \left[ \ln \left( \frac{2m_e c^2 \beta^2}{I(1 - \beta^2)} \right) - \beta^2 \right] \quad (1.1)$$



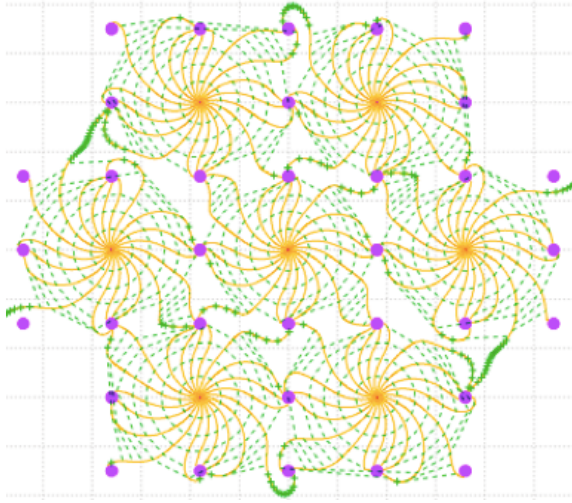


Figure 1.2: Simulation of the electron field in an extract of the wires in the central drift chamber. The violet circles depict the field wires whereas the red small points are the sense wires. The yellow paths are examples for the drift paths of ionized electrons. This simulation includes the distortion by the magnetic field. Taken from [5].

which describes the average energy per travel length. This energy is transferred to the shell electrons of the helium gas atoms. As the ionization energy of helium is very low (about 20 eV) compared to the typical particle energies (up to several 100 eV) the helium atoms release their electrons very easily. These free electrons get accelerated in the electric field induced between the sense wires and field wires with a high negative voltage attached. An instructive simulation of the typical electronic field distribution between 7 sense wires and 34 field wires can be found in figure 1.2. As the electric field near the sense wires increases with  $\propto 1/r$  the free electrons gain more and more energy so that they can ionize helium atoms by themselves as well. This whole flood of free charge can be detected by the electronics attaches to the sense wires and produce a sharp peak there. The more immobile helium atomic cores get absorbed later by the field wires. Because the free electrons have to drift through the gas the measured peak has a delay up to 500 ns for the largest drift cells to the initial charged particle passage. With analysis the time and location information of each measured peak the path of the charged particle through the CDC can be reconstructed. A typical event display of the simulated passage of one charge pion with 1 GeV can be seen in figure ???. Each wire and each drift cell is almost rotationally symmetric. Therefore there is no way in obtaining information on the direction of the electron flood hitting the sense wire. Therefore each hit produced a so called drift circle rather than a single hit point. These drift circled can also be seen in the figure.

If all CDC wires were strained parallel to the beam axis there were no way in measuring the angle between the charged particles and the beam axis. That is the reason why some of the wires are installed with a small tilting angle. These wires are called stereo wires in contrast to the axial wires without tilting angle. A measured hit on the stereo wires does not give information neither on the position along the beam axis nor in the layer parallel to it. Only the information collected among all axial and stereo hits lead to a correct estimation of the momentum in all directions. A sketch of some axial and stereo wires sharing the same distance from the beam axis - also called a layer of wires - are shown in 1.4.

### 1.2.2 VXD

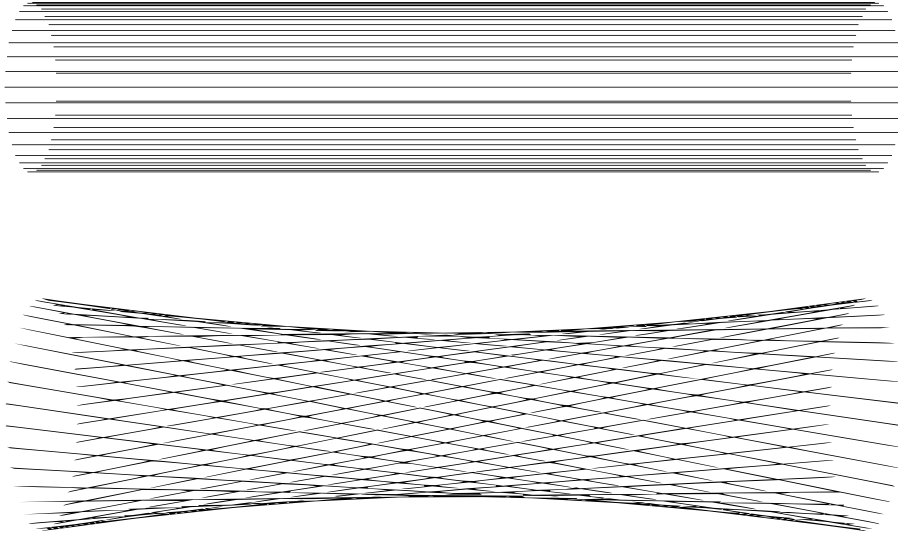


Figure 1.3: Drawing sketching the axial (top) and stereo wires in the CDC. The skewing against the beamline of the stereo (bottom) wires is exaggerated. Taken from [6].

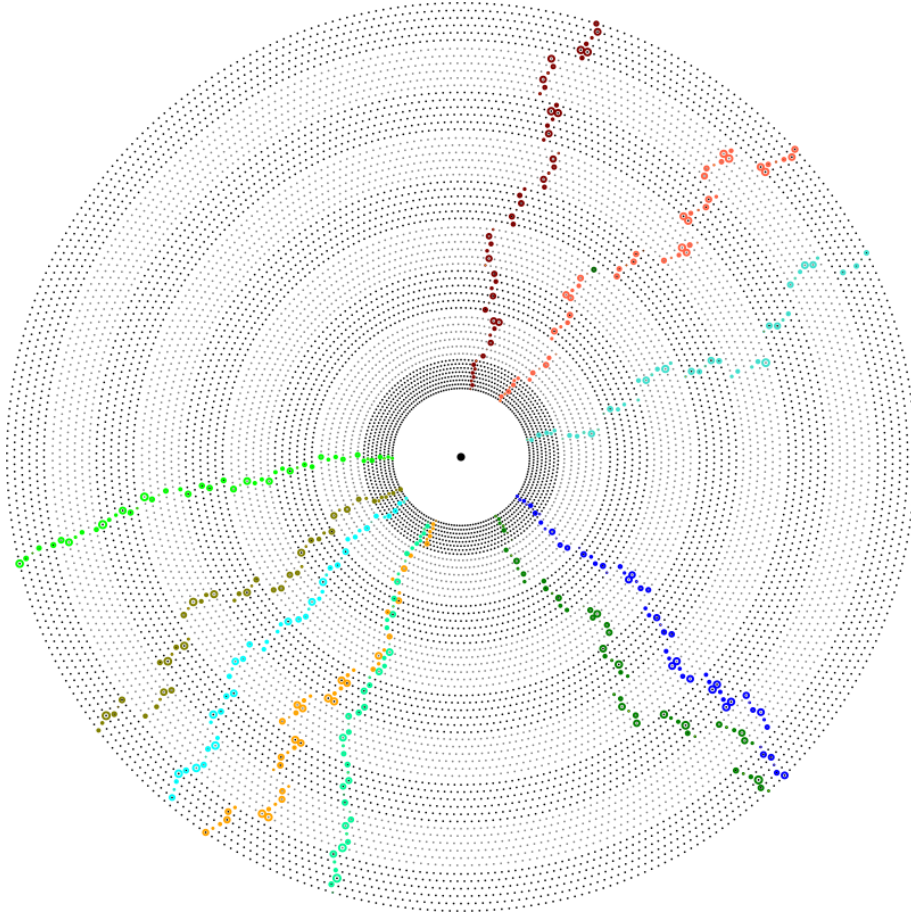


Figure 1.4: Simulated event of the passage of  $10 \pi^\pm$  with approximately 10 GeV through the CDC shown in a plane perpendicular to the beam axis. The interaction point is drawn as a black circle in the center. Each particle is colored differently to distinguish between them. Only the CDC detector is shown for better visibility. The wires are shown as gray (stereo) or black (axial) points. The hidden wires are depicted with their drift circles. The appearing disruption in the circular paths of the particle arise due to the stereo and axial wires.

## 2. Track Finder Theory and Multivariate Classification

Before explaining the implemented changes to the track finder for the Belle II experiment in more detail, the principles of the Belle II software framework are explained briefly. More information can be found elsewhere [1]. Afterwards common figures of merit for all track finders are explained and discussed and the working principles of the already implemented track finders are illustrated.

### 2.1 The Belle II Analysis Framework (`basf2`)

For simulation, data acquisition, data processing and analysis of the Belle II experiment the Belle Analysis Software Framework 2 (`basf2`) is used. Although - guided by its name - it seems to be build on top of the old software framework used for the Belle experiment it is a complete rewrite of the software using modern programming principles in the coding languages C++ [7] and Python 2.7 [8]. Together with external programming libraries like ROOT [9] or EvtGen [10] that are already on the market this framework builds the base for every software written for the experiment.

The software is divided into several packages - each serving a single purpose or summarizing code for a single detector. Examples for the packages are CDC, SVD or the tracking packaged which is described in more detail in later chapters.

Each usage of the Belle Analysis Software Framework - if it is either a simulation, a reconstruction or an analysis does not matter - consists of processing one or more so called *paths* build with *modules*. These modules perform a dedicated small task like simulating the hard scattering event (the so called `EvtGen` module), writing out data to a root file (the module is called `RootOutput`) or performing a track reconstruction (for example with the module `TrackFinderCDCAutomaton`). The presence, the order and the parameters of the modules are determined in *steering* files written with python. The modules itself can be written in C++ or python.

In these steering files a path is created, filled and passed to the framework which handles loading the corresponding C++ libraries and calling the modules for every event that should be processed. An example of a small steering file for track finding can be found in listing 2.1. Caused by this extremely modular structure not only parallel processing but also debugging of intermediate steps can be performed much easier.

Because many modules need the data produces by other modules before there is a need for intermodular communication. This communication is performed within the framework with the help of the data store. This class as a wrapper around a collection of named `TClonesArrays` from the ROOT library [11] which can store lists of instances of nearly arbitrary C++ classes. It is used widely in the framework to store all sorts of things like

missing

Figure 2.1: The visualization of the intermodular communication while processing the path implemented with the steering file described in listing 2.1.

More text

the hit information produced by the particles in the simulation or the found tracks after the track finding modules. The modules have read and write access to every so called store array in the data store. A visualization of the data flow between the modules created with the steering file in 2.1 can be found in figure 2.1. The data store can be written to or read from disk using ROOTs own serialization mechanism together with data member dictionaries for the C++ classes created by the C++ interpreter of ROOT called CINT [12].

missing

Listing 2.1: Python steering file to create a typical basf2 path. After loading the needed python libraries the path is created and filled with the modules. In the end this path is processed and for each event the modules are executed in the given order and with their given parameters. For more information on the used modules see their documentations.

## 2.2 Working Principle of the implemented CDC Track Finder in basf2

One part of this work was the improvement and further development of the track finder modules for the CDC tracking detector. Therefore the working principles of the two track finder for this detector are described here briefly. For more information on the first track finder - the legendre track finder - see [13]. More information on the second described track finder - the automaton track finder - can be found in [6].

The general purpose of a track finder algorithm is to partition all measured wire hits into exclusive sets of hits that may come from the same charged particle passing through the detector. It does so by using several assumptions on the charged particle producing the wire hits like the form of their trajectory and therefore the possible patterns of the hits. After fitting a mathematical model of a trajectory to these hits one can gain information on the momentum or the vertex position of this particle. There are several different approaches to find the correct sets of hits which are in the following called tracks.

The reason to have two track finders for the CDC is their different ansatz. The legendre track finder is a so called global track finder whereas the automaton track finder is a local one. A global algorithm uses the information of all wire hits simultaneously. The legendre track finder does this by applying a mathematical projection to the wire positions which should in principle project all hits belonging to the same track onto the same coordinates. A local algorithm however tries to use neighboring wire hits to construct clusters of hits. These clusters are then enlarged by using neighborhood relations until a full track can be found. In the following these two principles are described in more detail.

### 2.2.1 The Legendre Track Finder

The principle of using the legendre transformation for tracking algorithms in high energy particle experiments was first described by Alexopoulos [14]. It uses an extended version of the hough transformation introduced by Paul V.C. Hough in 1962 [15]. The algorithm uses

the fact that each trajectory in the  $r$ - $\phi$ -plane of the detector can be described by a circle - assuming no energy loss - because of the applied magnetic field. In a first approximation one can also assume that each particles comes from the interaction point which is valid for the bigger part of the decay products. Therefore the trajectory in the  $r$ - $\phi$  direction can be described by two parameters: the radius  $R$  of the circle and the angle  $\theta$  between an arbitrary but fixed axis and the tangent to the circle at the interaction point.<sup>1</sup>

Simplified the idea is to calculate each trajectory that could have possibly created one of the axial hits and draw them all in a 2d histogram with the trajectory parameters  $R$  and  $\theta$  as the coordinate axes. As there are only a small number of correct trajectory which are however responsible for a great number of hits there are is a small parameter set which appears very often in the histogram. These parameters can then be used to create tracks.

For applying this algorithm the  $x$  and  $y$  coordinate pair of every axial wire hit together with the drift length  $d$  is transformed by the function

$$x' = \frac{2x}{x^2 + y^2 - d^2} \quad y' = \frac{2y}{x^2 + y^2 - d^2} \quad d' = \frac{2R}{x^2 + y^2 - d^2}$$

$$R = x' \cos(\theta) + y' \sin(\theta) \pm d'$$

into the legendre space as it can be seen in figure 2.2. In the first step of the transformation, the wire hits are transformed to the inverted plane. With this transformation each circular trajectory through the interaction point is mapped onto a line. The two trajectory parameters  $R$  and  $\theta$  are now functions of the slope and the axis interception of this line.

After that each drift circle is transformed into a pair of sinusoidal functions - also called sinograms. This function is constructed in this way to use the fact that each trajectory of a charged particle responsible for a wire hit must touch the drift circle tangentially. Each point on the constructed sinusoidal functions correspond to one possible trajectory of a particle which could have created such a hit. There are two sinusoidal functions because the osculation point can be on the far or the near side of the drift circle - the trajectory circle can circumscribe the drift circle or not.

With using the information of a single hit one ends up with an infinite number of trajectory hypothesis. But as a charged particle passed many drift cells until it leaves the CDC detector - in some cases up to 100 hits - several wire hits are created with the same trajectory parameters. As these same parameters correspond to the same point in the legendre space, the sinusoidal functions of the wire hits intersect in this point as it can also be seen in figure 2.2. The task of the legendre algorithm is now to do the transformation of the hit coordinates and find those intersections.

Imperfections due to energy loss and material effects make the sinusoidal function not interact in one single point but rather in smeared area. To copy with this problem but still find the intersections with a good performance a peak search in the legendre space is applied. The legendre space is divided into small bins. For each bin the number of sinusoidal functions passing this area is counted. The bin with the highest weight is assumed to be the bin with the highest number of sinusoidal intersections. From the wire hits contributing to this bin a new track is created and the search is repeated until a threshold in the bin entry is undercut. As the legendre space is mostly empty this procedure can be further improved in performance by refining the bin deviation from very coarse bin sizes to finer ones only for those bins which have a certain amount of sinusoidal functions in them. Because these

<sup>1</sup>When dropping the last assumption of tracks coming from the origin one has to introduce another parameter - often called  $d_0$  - which describes the minimal distance from the circle to the origin in the  $r$ - $\phi$  plane. It is easy to generalize the described algorithm to three dimensions. It has to be analyzed further if this third dimension can lead to an efficiency gain.

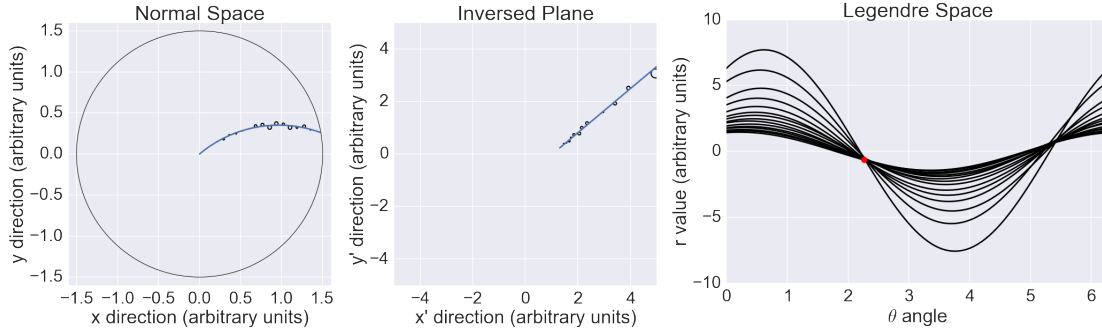


Figure 2.2: Transformation of some wire hits belonging to the same charge particle (left side) to the inverted plane (middle) and to the legendre space with the sinuoidal functions (right). As described in the text, the circular trajectory is first transformed into lines and then into intersecting sinograms. Each sinogram includes all possible trajectory parameters which would have touched the hit from which the function was created. The intersection corresponds to the parameters of the trajectory and are marked with a red circle. For better visibility only the wrong half of the sinograms is colored gray.

gray

bins are divided into 4 subbins the concept is also called a quad tree search. The whole search is depicted in figure 2.3.

After finding the possible hit subsets for the track candidates a post processing procedure consisting of hit reassignment, hit deletion and track merging is applied to account for energy losses, trajectories not coming from the interaction point and finding inefficiencies. This post processing is described in more detail further down.

### 2.2.1.1 Stereo Hit Finding

The before mentioned legendre track finding algorithm does only work for axial hits as a precise position in  $x$  and  $y$  is needed for calculating the position in the legendre space. Therefore it can not be applied for hits coming from stereo wires as they have a certain range of possible  $x$ - $y$ -coordinates. But as soon as a trajectory with the axial hits is found, each stereo hit can be reconstructed to match the trajectory in such a way that its drift circle touches the trajectory circle of the candidate. As the stereo wire can be best approximated by a single line in 3d space the  $z$  position is now also fixed. As it is not possible to gain information whether the drift circle is includes in the trajectory circle or not - the same reason for the two sinograms before - each stereo hit leads to two slightly different reconstructed  $z$  positions.

An ideal trajectory in the  $z$ - $r$  plane resembles a straight line analogous to the circle in the  $r$ - $\phi$  plane and can be described by two parameters also. This time these parameters are the slope  $\tan \lambda^2$  and the distance  $z_0$  on the  $z$ -axis to the interaction point. The plane spanned by these two parameters is analogous to the legendre space in the axial case. As before each point corresponds to a certain trajectory. By using the two reconstructed  $z$  information from each stereo hit a function in this plane can be drawn which includes every possible trajectory that would have passed the stereo hit. For axial hits this function was given by the two sinograms. For stereo hits it is a straight line in the  $\tan \lambda$ - $z_0$  plane:

$$z_0 = z_{\text{rec}} - \tan \lambda \cdot r_{\text{rec}}$$

<sup>2</sup>It is described by the angle  $\lambda$  for consistency with the so called helix parameters

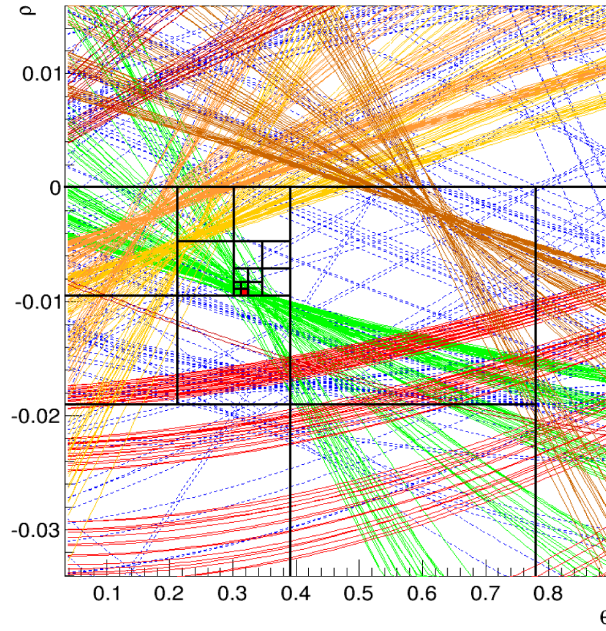


Figure 2.3: Depiction of a quad the search to find the bin with the most interceptions of sinograms shown in red here. Only the first round of the search is shown for better visibility. Each color shows sinograms that belong to the same charged particle. Taken from [16].

missing

Figure 2.4: Stereo hit finder algorithm shown for a single track. The track trajectory in the  $r$ - $\phi$  plane leads to only two possible  $z$  positions for each stereo hit (left side). In principle each stereo hit could have been created by an infinite number of tracks. Some of them are shown in color in the center. They all correspond to a single point in the  $\tan \lambda$ - $z_0$  plane which is shown on the right side together with the correct trajectory parameters as a black dot.

with the reconstructed  $z$  position  $z_{\text{rec}}$  and the reconstructed radius  $r_{\text{rec}}$ . The whole process can be seen in figure 2.4. Again, a quad tree search is applied to search for the point with the highest number of interceptions which corresponds to the trajectory parameters compatible with the largest number of hits. This time only the single highest trajectory is stored as one single trajectory in the  $r$ - $\phi$  plane can only have one single trajectory in the  $r$ - $z$  plane. The whole algorithm is repeated for all other found axial-only trajectories.

As the energy loss has more or less no influence on the  $z$  motion because of the much higher momentum in this direction and a non-zero distance to the interaction point is already accounted for, a post processing is not needed here. See the chapter on the SegmentTrackCombiner for a description how to cope with the remaining inefficiencies.

### 2.2.2 The Automaton Track Finder

Clusterizer, Automaton-Principle

## 2.3 The used Figures Of Merit

For testing and developing and also for later usage in the experiment setup we need to compile numbers from the implemented track finder algorithms to show how well they work. There are three different classes of figures of merit to describe a track finder. All three classes listed here are described in more detail below. The three classes are:

- the efficiency (like the hit efficiency or the finding efficiency, also split up for different particle types, momentum regions or areas in the detector)
- the error rate (like the clone or fake rate)
- the computational performance (like timing and memory consumption)

The last class should be quite clear and is measured by the basf2-own measuring algorithms. As the tracking is part of the online reconstruction and may be once adopted for the high level trigger, the timing performance is very important.

The other two classes can best be described with the algorithm how they get computed. It starts with a full Monte Carlo simulation of generic BB events with the full detector simulation afterwards. The created hits can then be parted into distinct sets - each describing a simulated tracks, called MCTrackCand (they are called track candidates to fit the naming convention of the track finder). Only those tracks are kept as a MCTrackCand, that have at least 3 hits in the CDC - otherwise they can not be fitted and even if there were a chance to find them via track finding, they could not be used for physics. After that, the simulated hits with the stripped MC information are used for track finding with the method to be evaluated. This can be done easily as the simulated hits are transformed in a format that is similar to the format that will be used later for the data coming from the experiment. After the track finder has produced a list of track candidates also, we can use the saved MC information to match tracks from the track finder to tracks from the mc algorithm (also called MC track finder) by counting the number of hits they share. The different cases are depicted and described in table 2.1.

The finding efficiency now describes the rate of MCTrackCands which are labeled matched by their total amount. Building this ratio can also be done for bins in various variables, like perpendicular momentum ( $p_T$ ), angle in the curling plane ( $\phi$ ), number of tracks per event (multiplicity) and many more. A perfect track finder would have a finding efficiency of 100 %. In most of the cases, the finding efficiency drops for tracks in a certain region of these variables - like for low momentum tracks. The hit efficiency is the mean of the ratios between the number of hits in the MCTrackCands matched to a non-fake track candidate to the number of hits in total to this track. Here also a perfect track finder would have a hit efficiency of 100 %. As in most of the cases the track finder loses some outlying hits the hit efficiency drops. The fake and clone rates are just the number of track candidates labeled as fake or clone by the matching algorithm divided by the number of found tracks in total. A perfect track finder would have both number set to 0 %. A high fake rate is caused by a track finder with too loose cuts when putting together single pieces of tracks to a big track or by one which picks up background hits often. A track finder with a high clone rate on the other hand has too harsh cuts and splits up tracks into more than one piece.

## 2.4 Track Fitting

The track finding algorithms have the task to partition the measured hits into sets with each set forming a single track candidate. After that the purpose of a track fitting algorithm is to fit a model for the trajectory to the measured information of the hits to gain the



	<p>There is a one to one connection between a MCTrackCand and a track from the track finder. The MCTrackCand is labeled found and the other track is labeled matched.</p>
	<p>The MCTrackCand is found twice. The track from the track finder with the higher percentage (the green one in this example) is labeled matched, the other one cloned. The MC-TrackCand is nevertheless labeled found.</p>
	<p>The track from the track finder is created with hits from many different MCTrackCands. As none of the corresponding hit ratios exceeds 66 %, the track is called fake. There is no precise reason why the number 66% was chosen. The hit ratios of the MC-TrackCands itself do not play any role here. TODO: Is found for MC possible?</p>
	<p>The found track does not describe any of the MCTrackCands well (or well enough) - but is made out of background hits. This track is also called a fake.</p>

Table 2.1: This tabular shows the four different cases for the matching between tracks found by the track finder (on the left side of the pictures) and MCTrackCands (shown on the right side). The different colors differentiate between different tracks. The connection between tracks shows that these two tracks share hits. The two percentages on the arrows are the percentages of hits they share in respect to the total number of hits in the MCTrackCand/track candidate from the track finder.

particles properties like the momentum or the vertex position. The model used for the fit can be very easy without taken into account material effects or energy loss. The track fitting algorithms implemented in **basf2** however try take care of the interaction of the particles with material correctly by using the same algorithms in fitting that were used for simulating the detector geometry. As the simulation itself depends strongly on the track parameters this implies resimulating the particle with every fitting step. This procedure of stepping through the measurements iteratively fits perfectly well to the Kalman fitter algorithm used as the main procedure.

Kalman beschreiben, DAF beschreiben

The fitting procedures are implemented into **basf2** with the external library **genfit** [17] - a library for generic and experiment independent track fitting. Only a small interface for accessing the **genfit** procedures from the **basf2** code is implemented. See the chapter on the vxd momentum estimation for more information on this topic.

## 2.5 Multivariate Classification

When doing track finding - especially in post processing procedures - many decisions must be made: whether a hit belongs to a track, two tracks should be merged or a track should be dismissed as fake etc. In the context of inefficiencies and fakes due to background hits these decisions can depend on many input variables. Additionally there is the need to have a smooth transition between the two corner cases dismiss all and accept all to allow for subsequent optimization. This problem of deciding between two different possibilities is called classification in statistics and there are many ways to deal with such problems (see for example [18] or [19]). For most of the classification tasks in the track finding package for the CDC detector a classification algorithm known as *boosted decision trees* (BDT) is used. In the following the mean features are described. See [20] for more information on the implementation.

Classification, BDTs

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