Applying Legendre transformation method for Belle II tracking

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12.05.2014, F2F tracking meeting | Pisa

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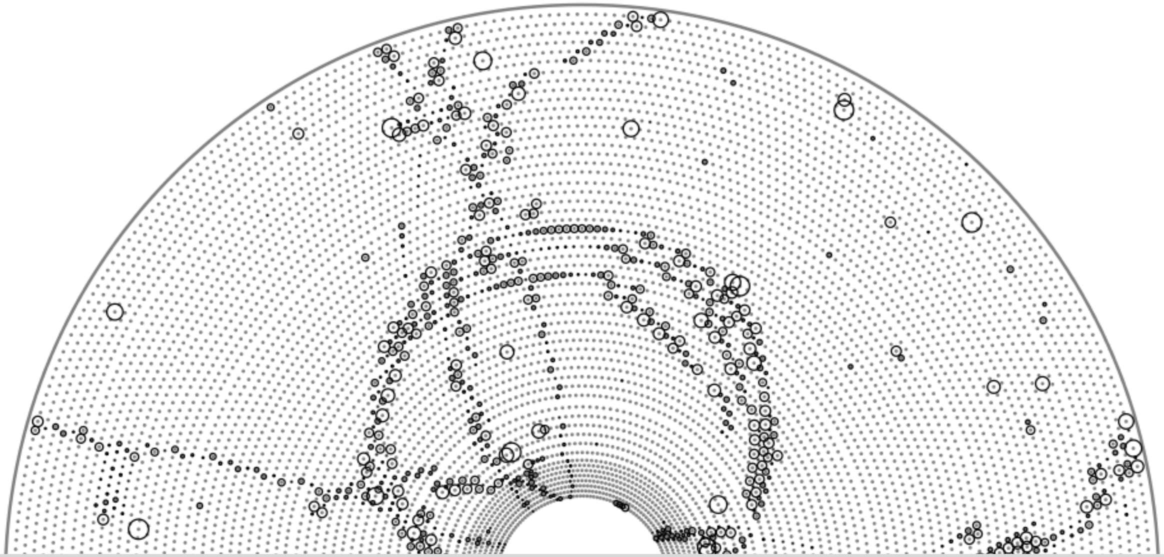
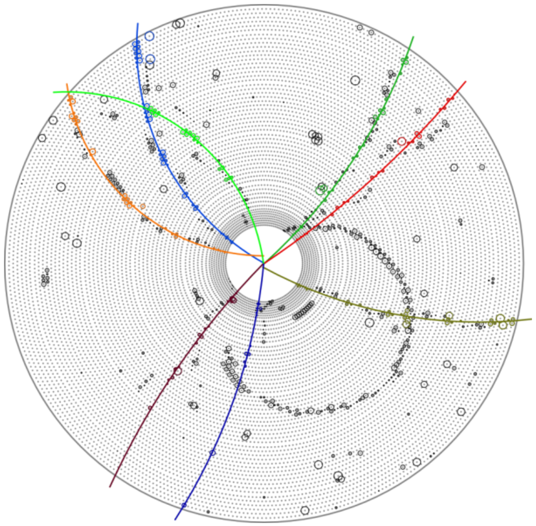


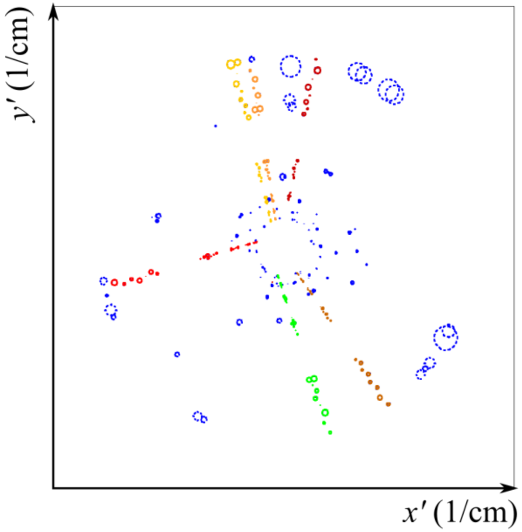
The method

- The main task of track finding is to determine which hits belongs to a common track

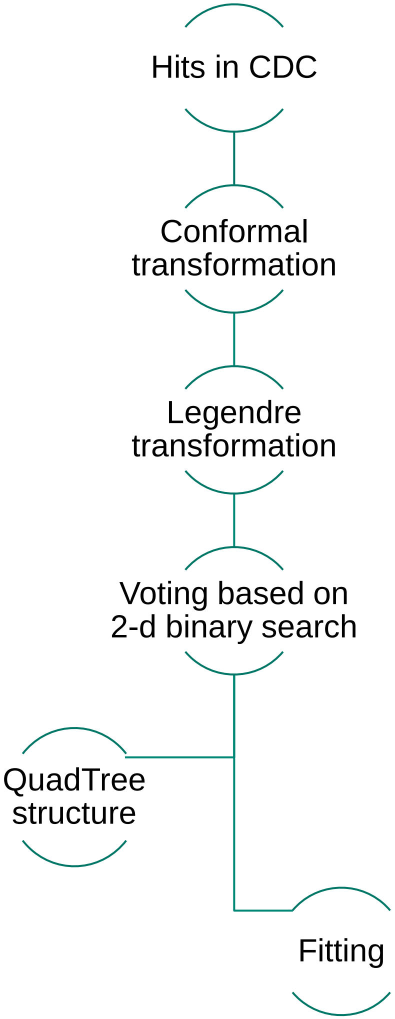
- We present a method of track finding which based on reconstruction of linear hit patterns in conformal space

- Legendre transformation of drift circles allows to build track with higher efficiency than using only position of the wire.





Chain of the method



Hits in CDC

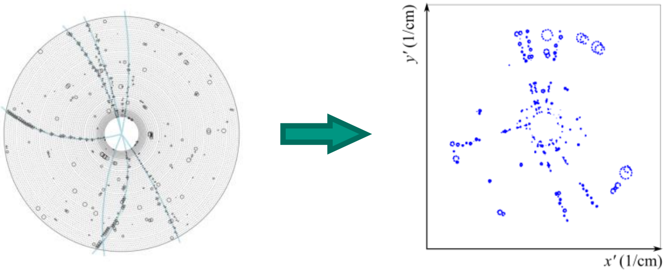
Conformal transformation

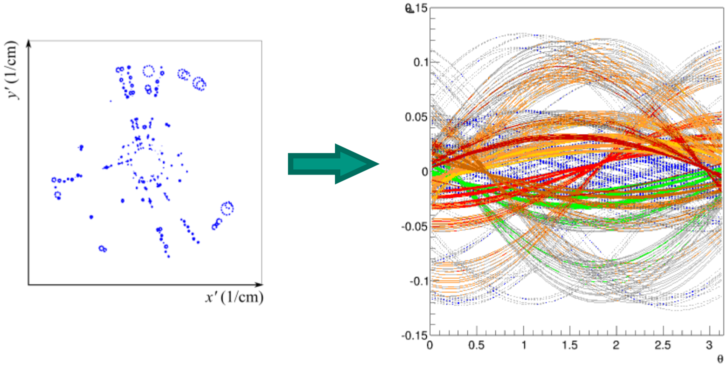
Legendre transformation

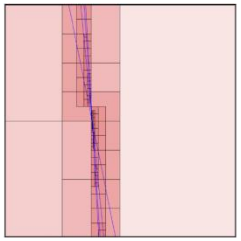
Voting based on 2-d binary search

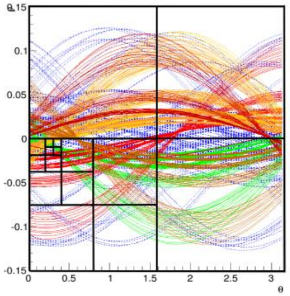
QuadTree structure

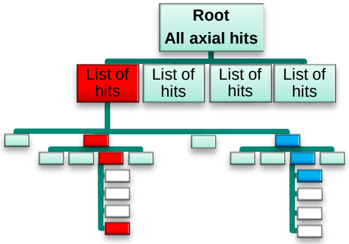
Fitting











Root
All axial hits

List of hits

List of hits

List of hits

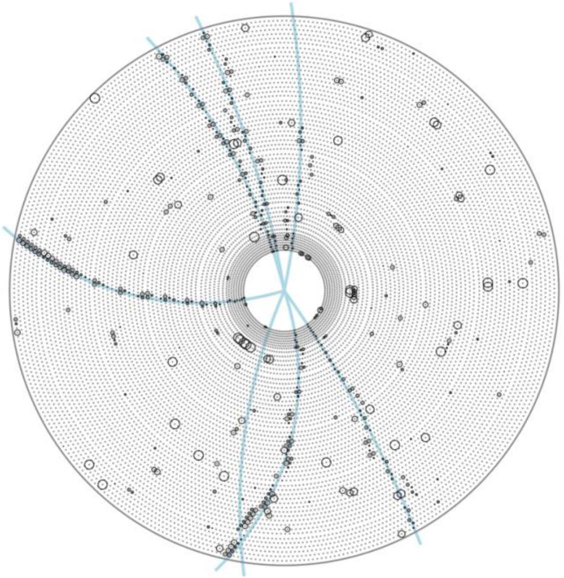
List of hits

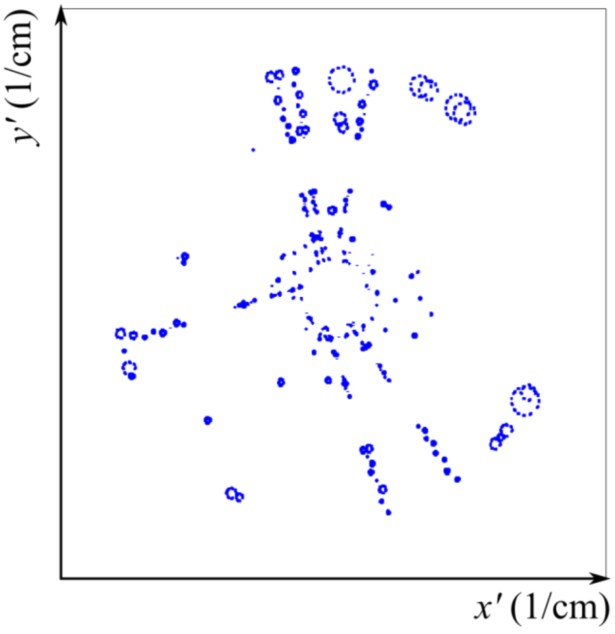
Conformal transformation

- Conformal transformation which transforms circles through origin into lines:

$$x' = \frac{2x}{x^2 + y^2}$$

$$y' = \frac{2y}{x^2 + y^2}$$



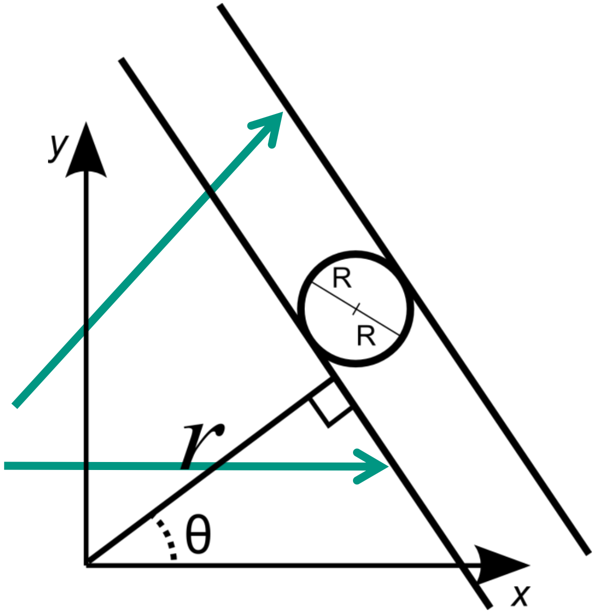


Transformation into Legendre space

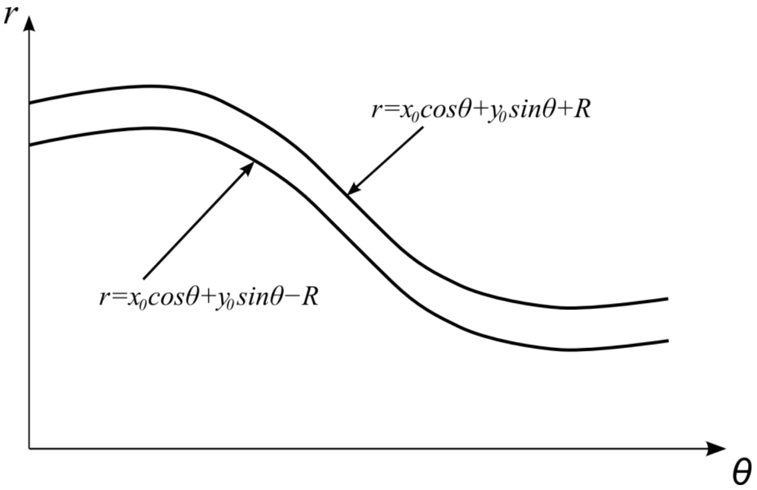
- The method is based on applying Legendre transformation to each drift circle in conformal space
- Legendre transformation of the circle can be written in next form:

$$f(x) \overset{\mathcal{L}}{\leftrightarrow} \begin{cases} r = x_0 \cos\theta + y_0 \sin\theta + R & \text{for concave} \\ r = x_0 \cos\theta + y_0 \sin\theta - R & \text{for convex} \end{cases}$$

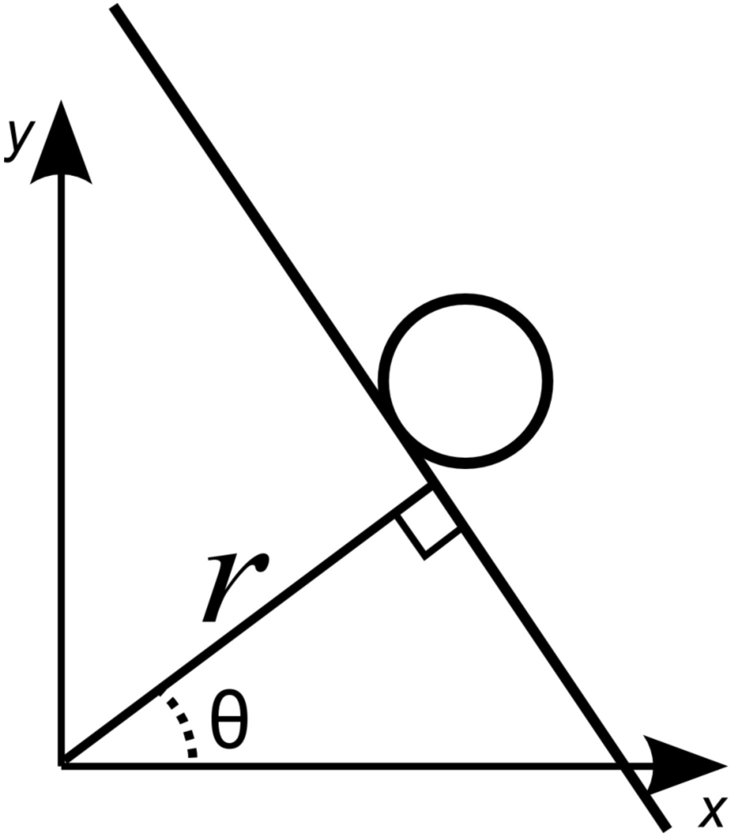
which presents tangents to the circle

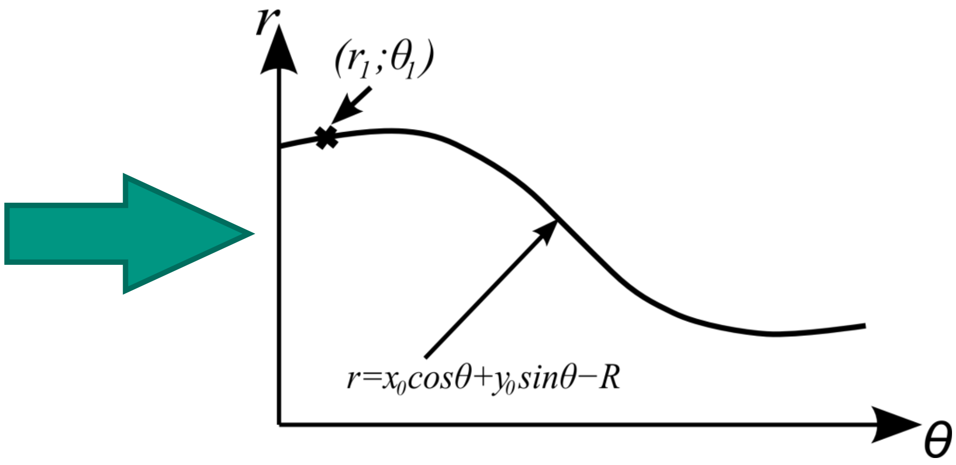


- Representation of the circle in the r, θ Legendre transformation space



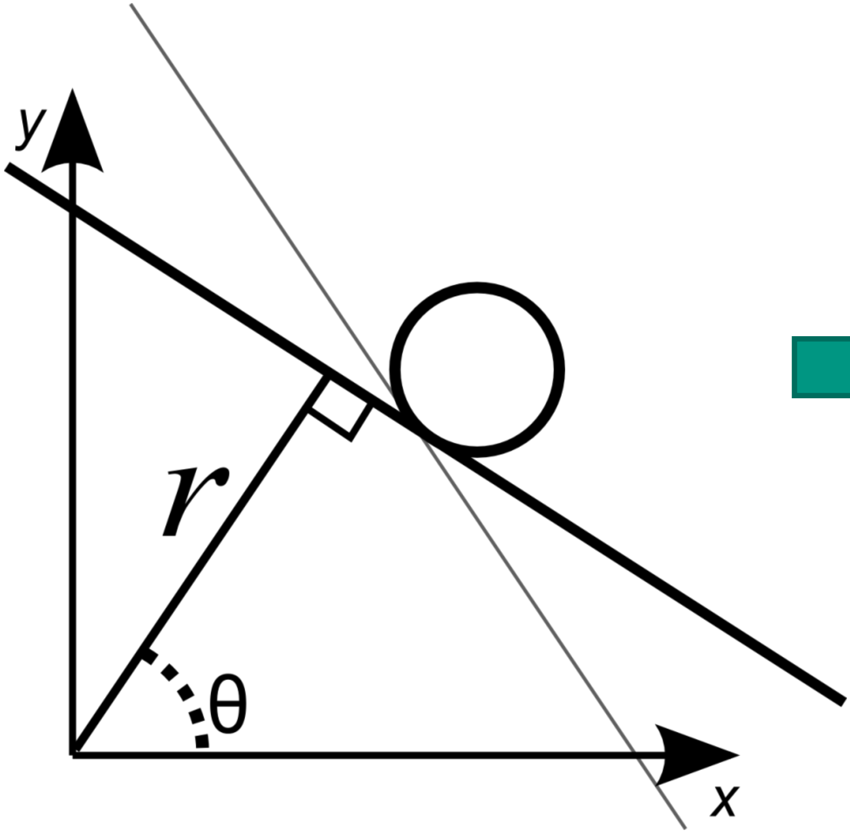
Transformation into Legendre space

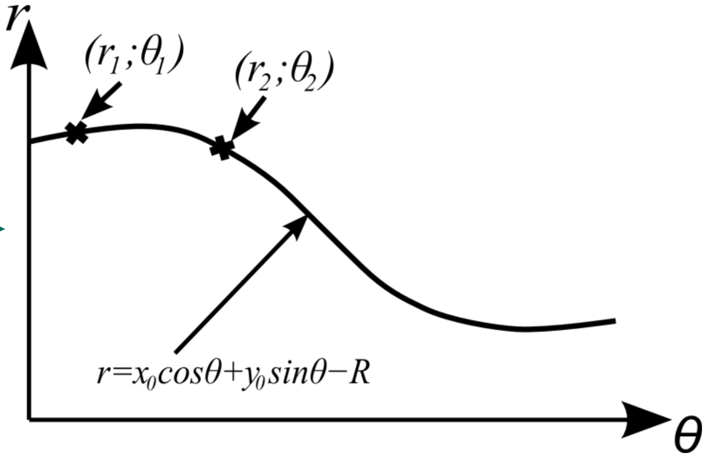




Each tangent has corresponding point in r, θ Legendre transformation space

Transformation into Legendre space





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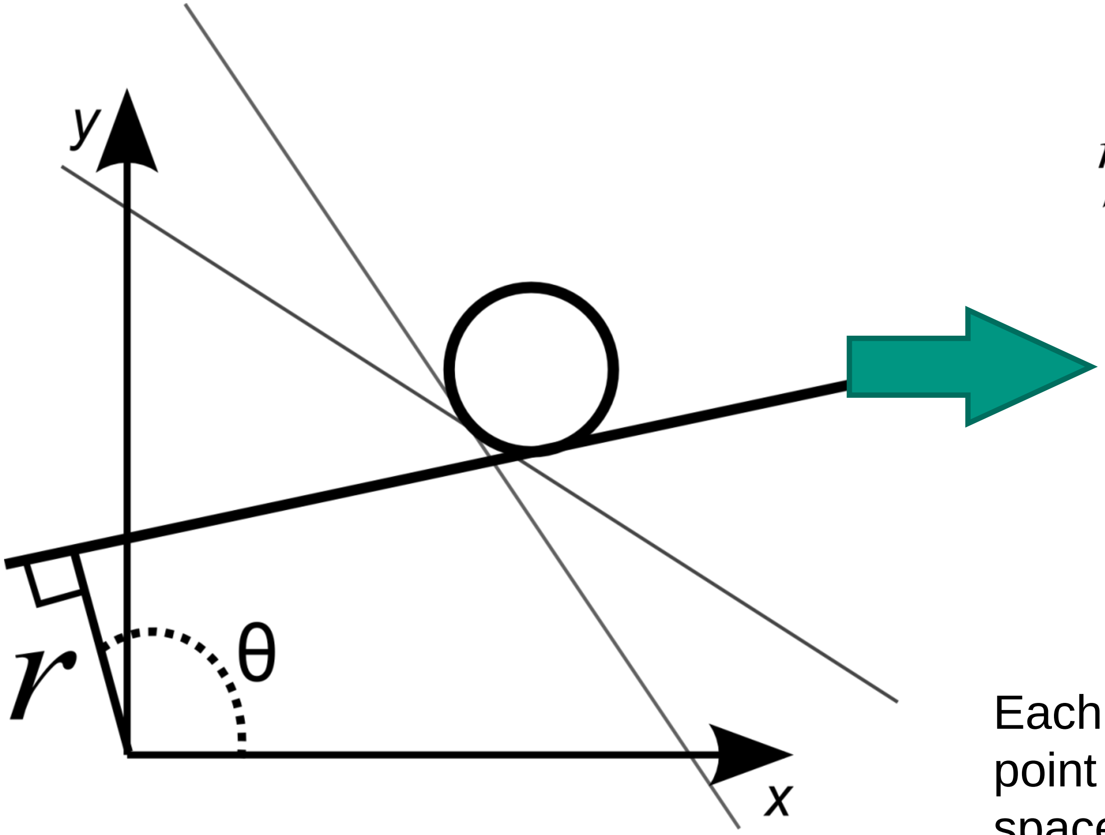
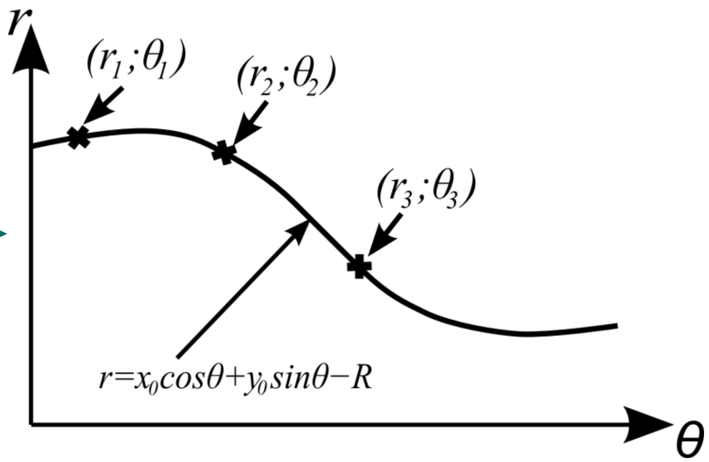
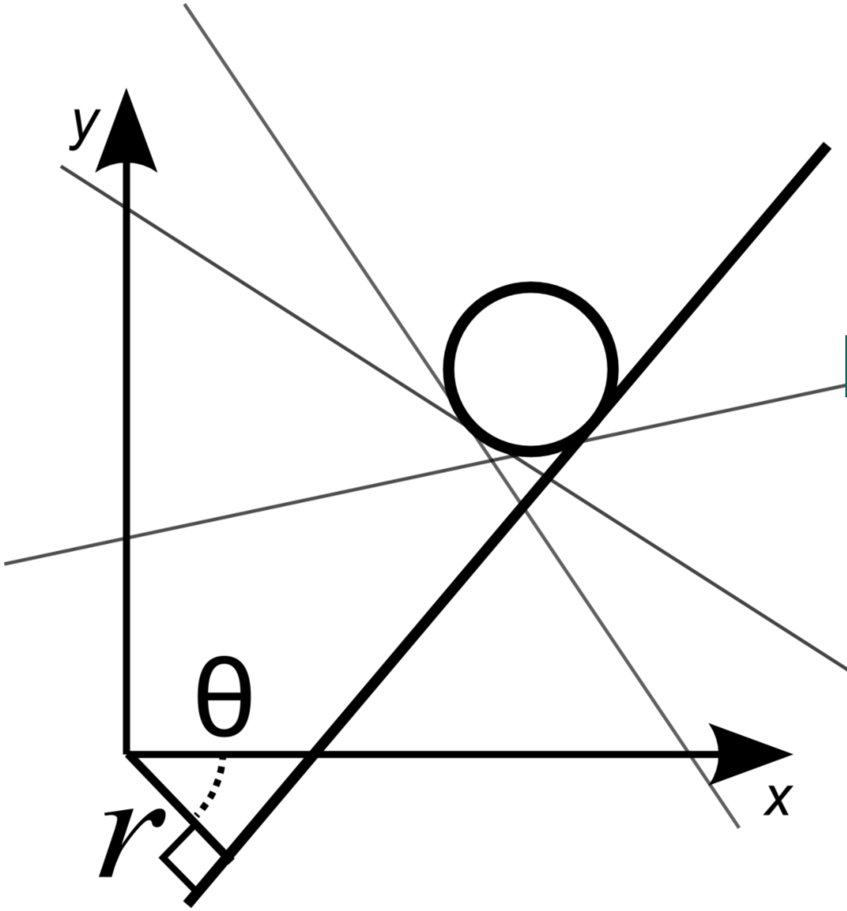
Transformation into Legendre space

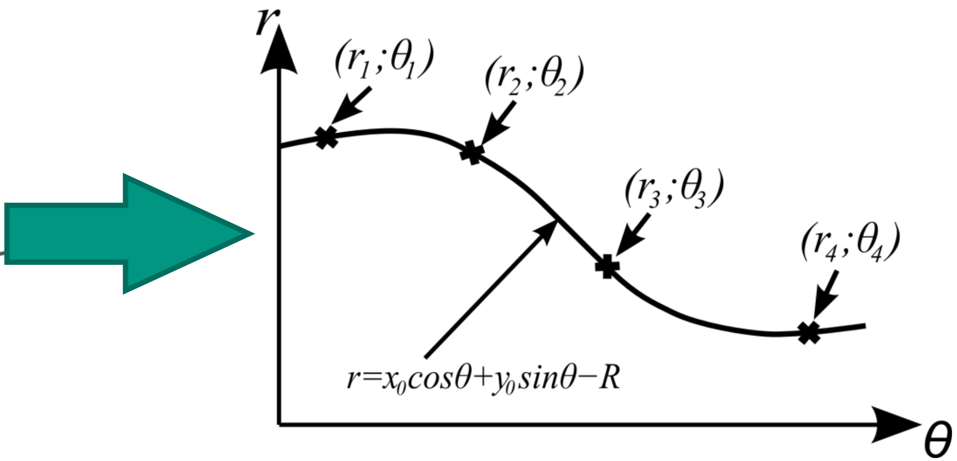




Each tangent has corresponding point in r,θ Legendre transformation space

Transformation into Legendre space

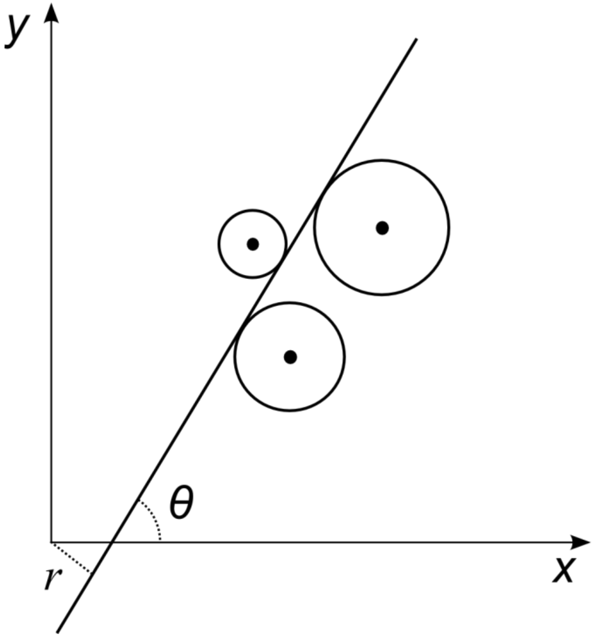


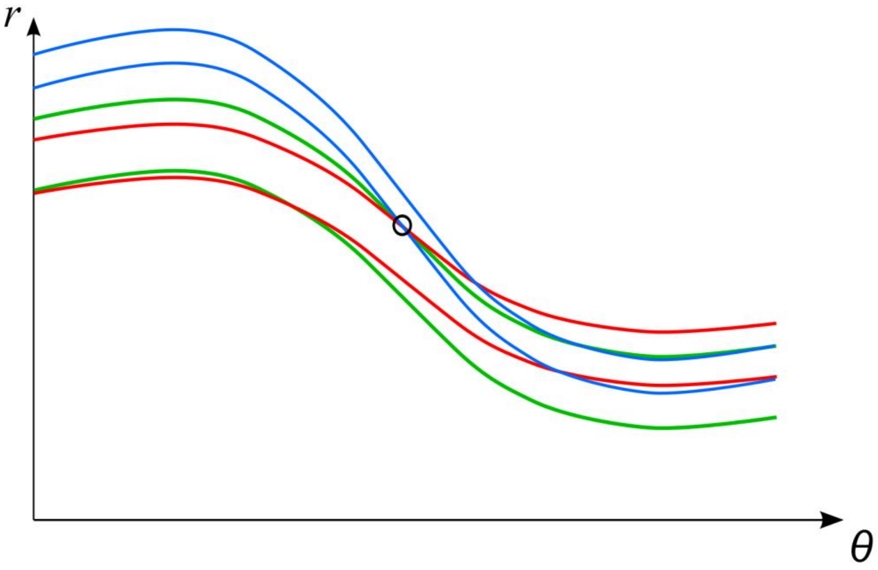


Each tangent has corresponding point in r, θ Legendre transformation space

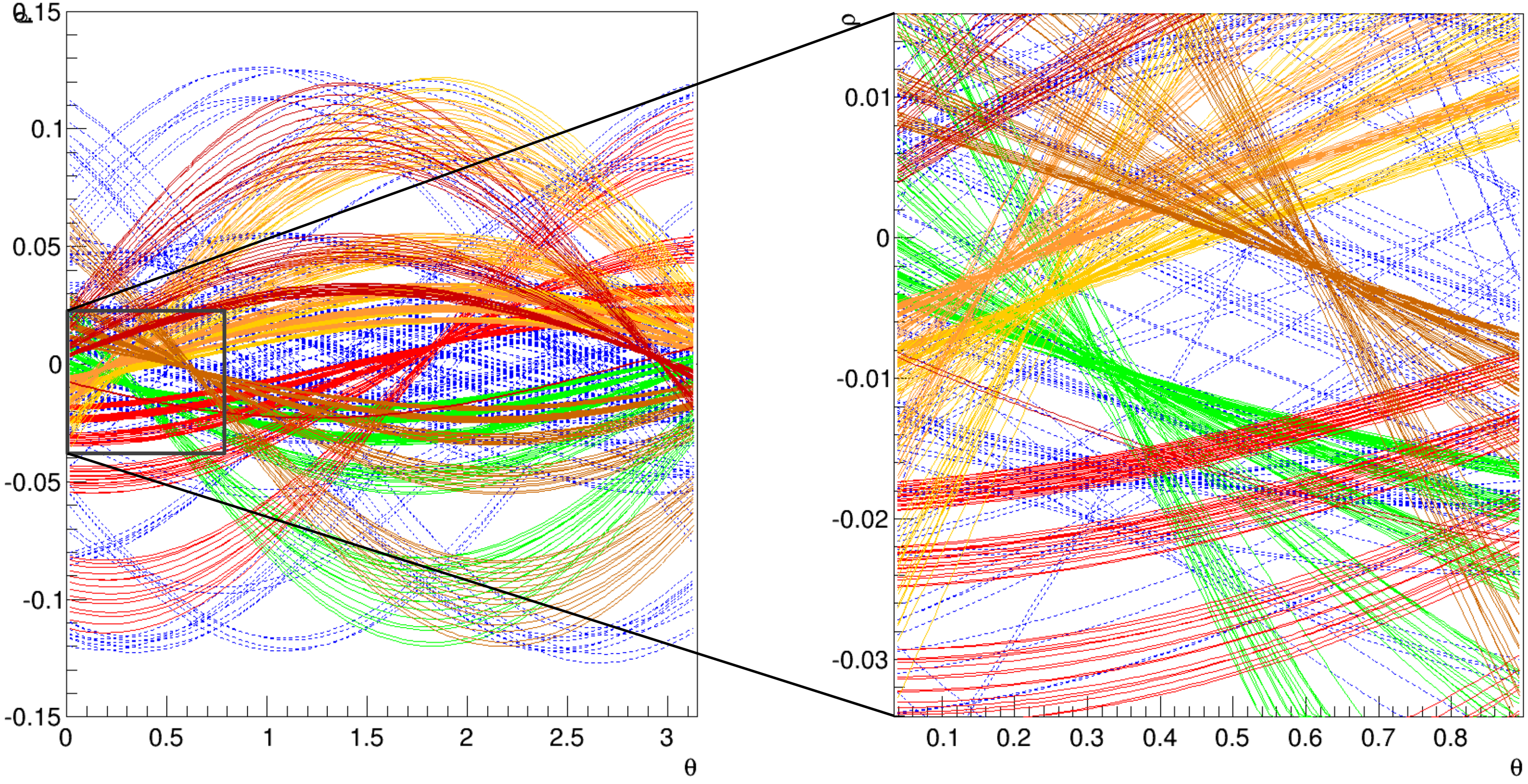
Finding of candidates

- The point of most sinograms intersection in r, θ space represents parameters of the common tangent to each drift circle belonging to the track



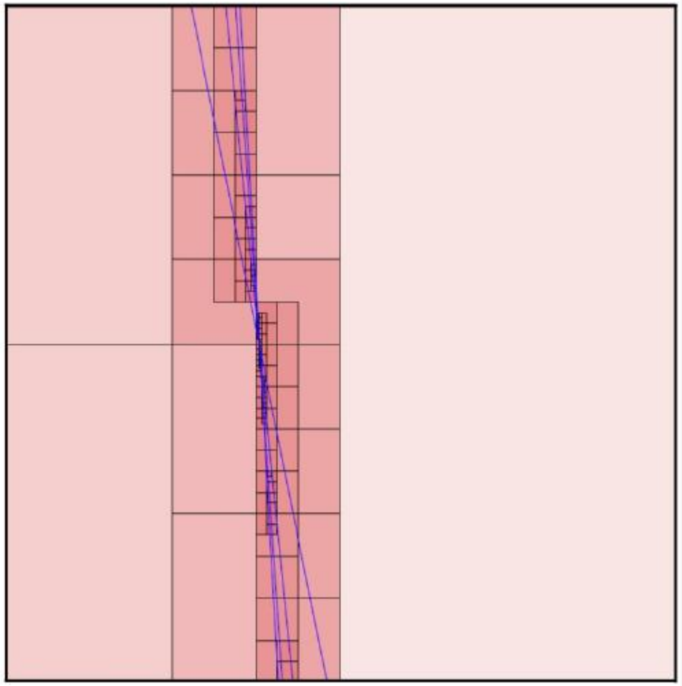


Sinograms of simulated event



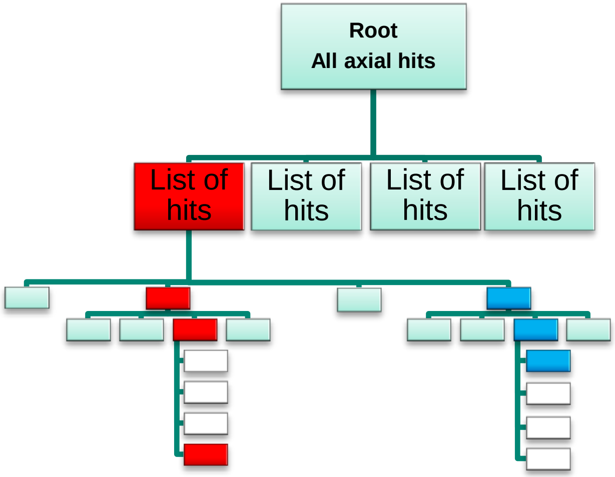
Voting or “How to find the point of most intersections?”

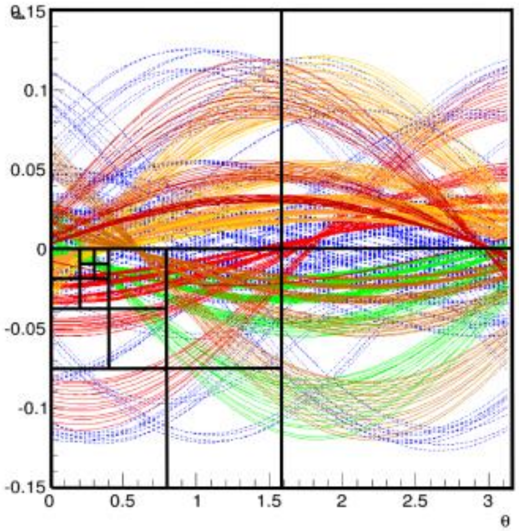
- Let each drift circle to *vote* for a set of possible parameters in Legendre space
  - Bin acquires vote if sinogram of drift circle passes through it
- Voting algorithm (based on 2-D binary search):
  - Split (r, θ) space into 4 bins
  - Accumulate votes in each bin
  - Select bins which passes threshold on number of votes
  - Continue bin splitting and voting until desired (r, θ) resolution reached
- Bin with the most of votes (hits) indicates hits pattern of track candidate



Using QuadTree for voting algorithm

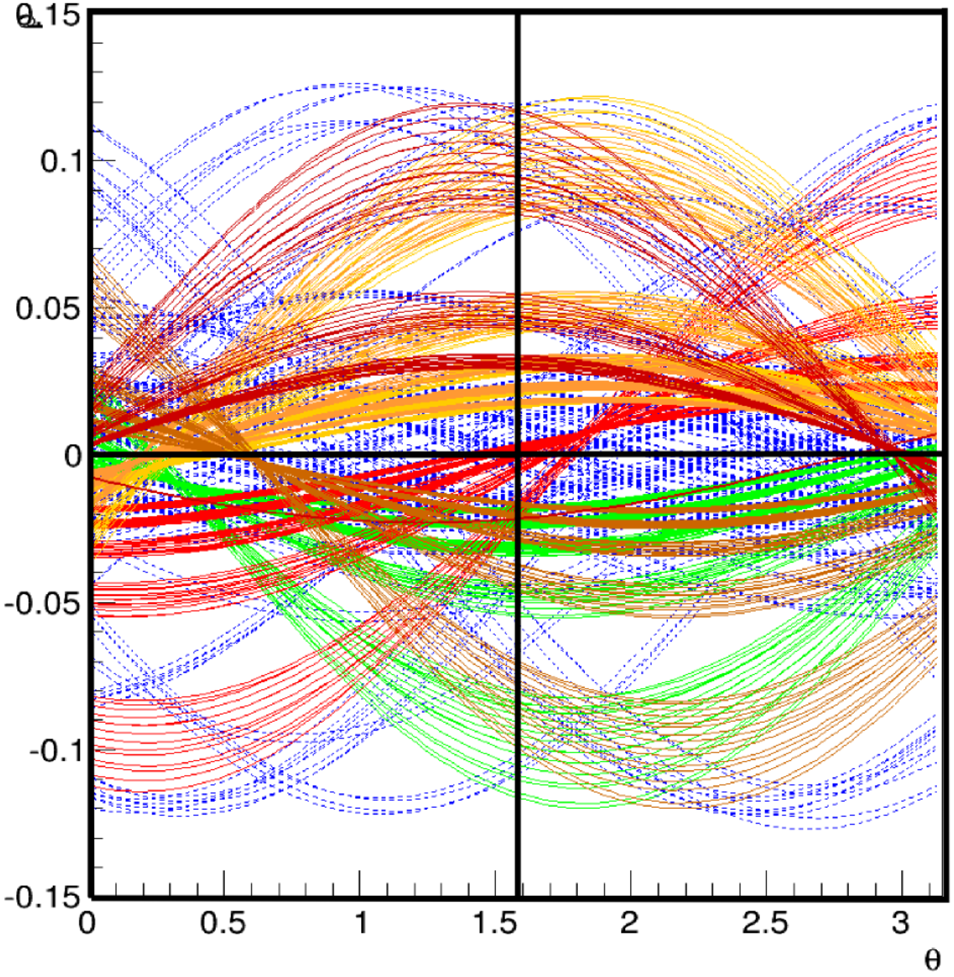
- Nodes has one parent and 4 children (except root and leafs nodes)
- Each node holds hits which gave votes for current bin
- Each node can communicate with children and parent nodes
- Results of voting stored in the tree and used in next iteration of track finding
  - Previously procedure similar to filling the tree was used few times per event


Root
All axial hits
List of hits
List of hits
List of hits
List of hits



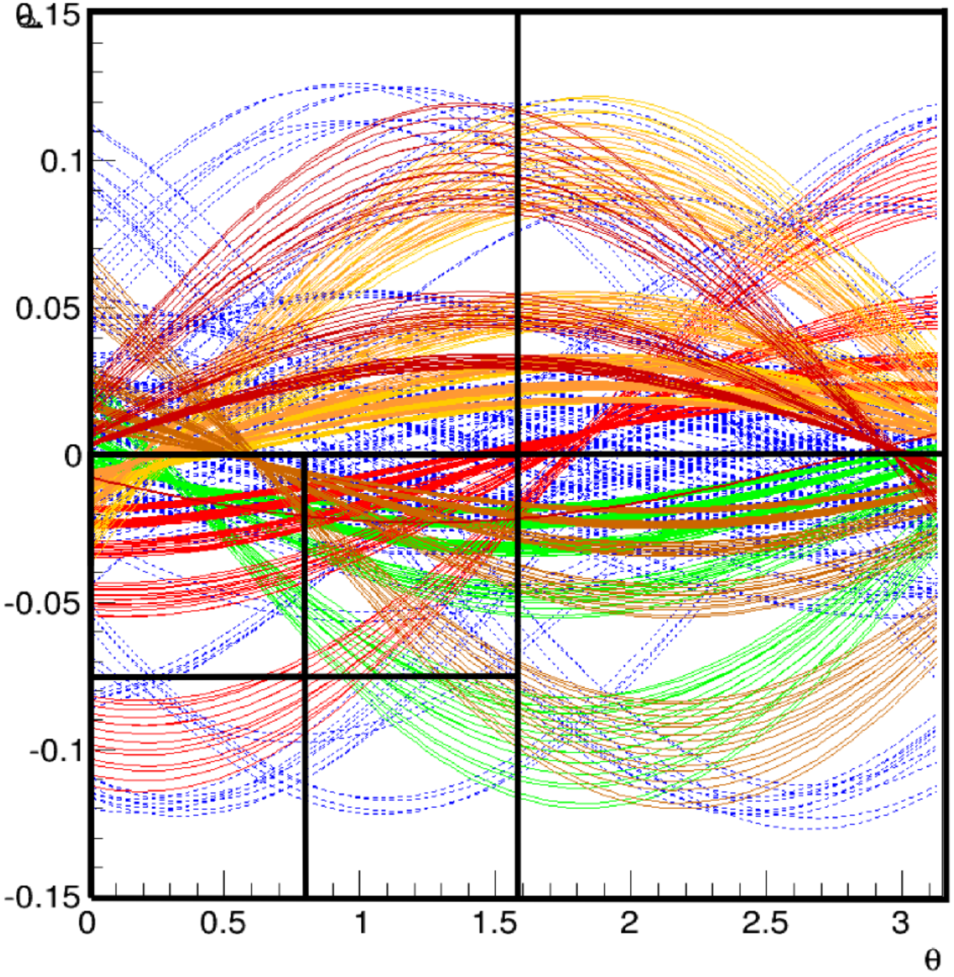
Voting

- **Step 1**



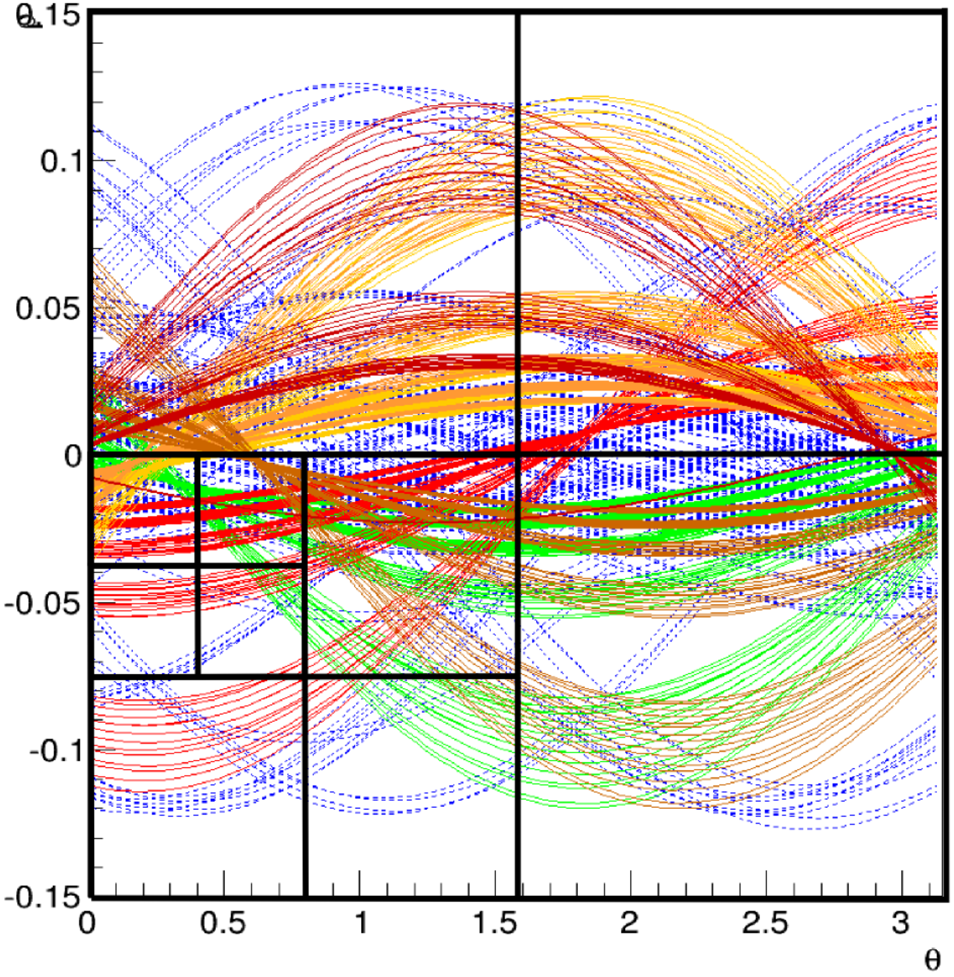
Voting

- **Step 2**



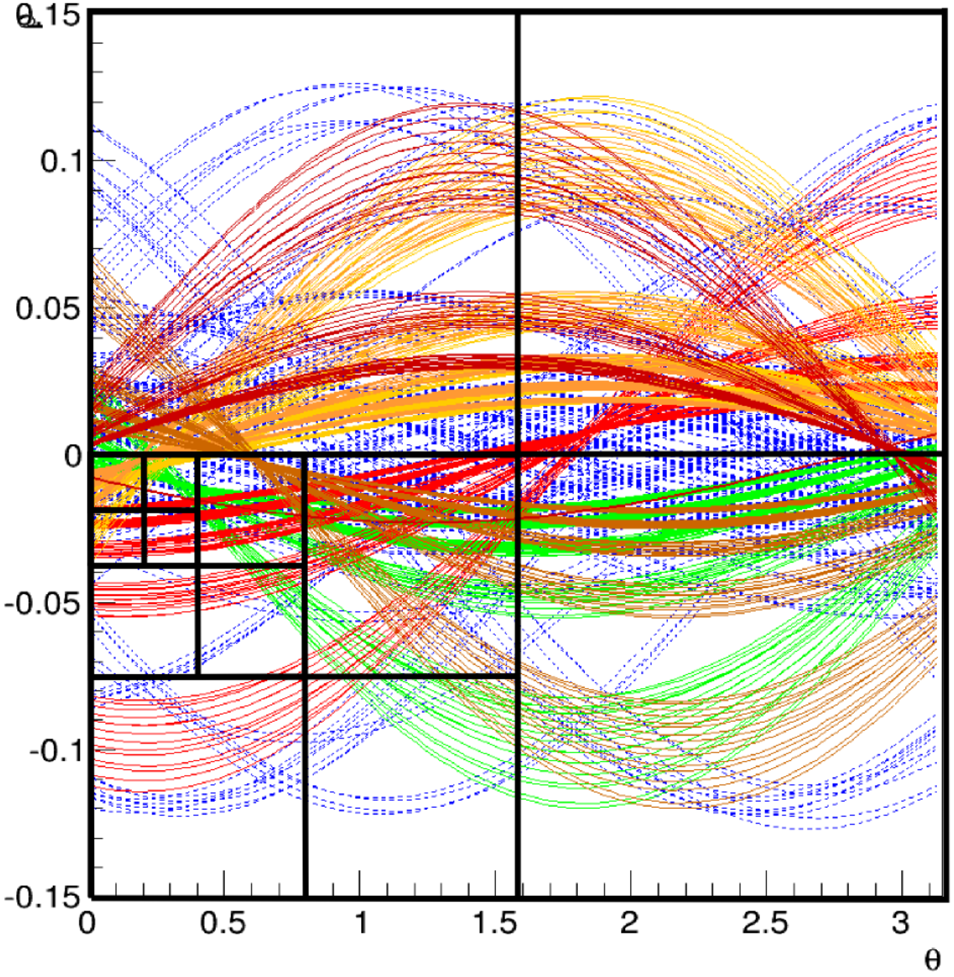
Voting

- **Step 3**



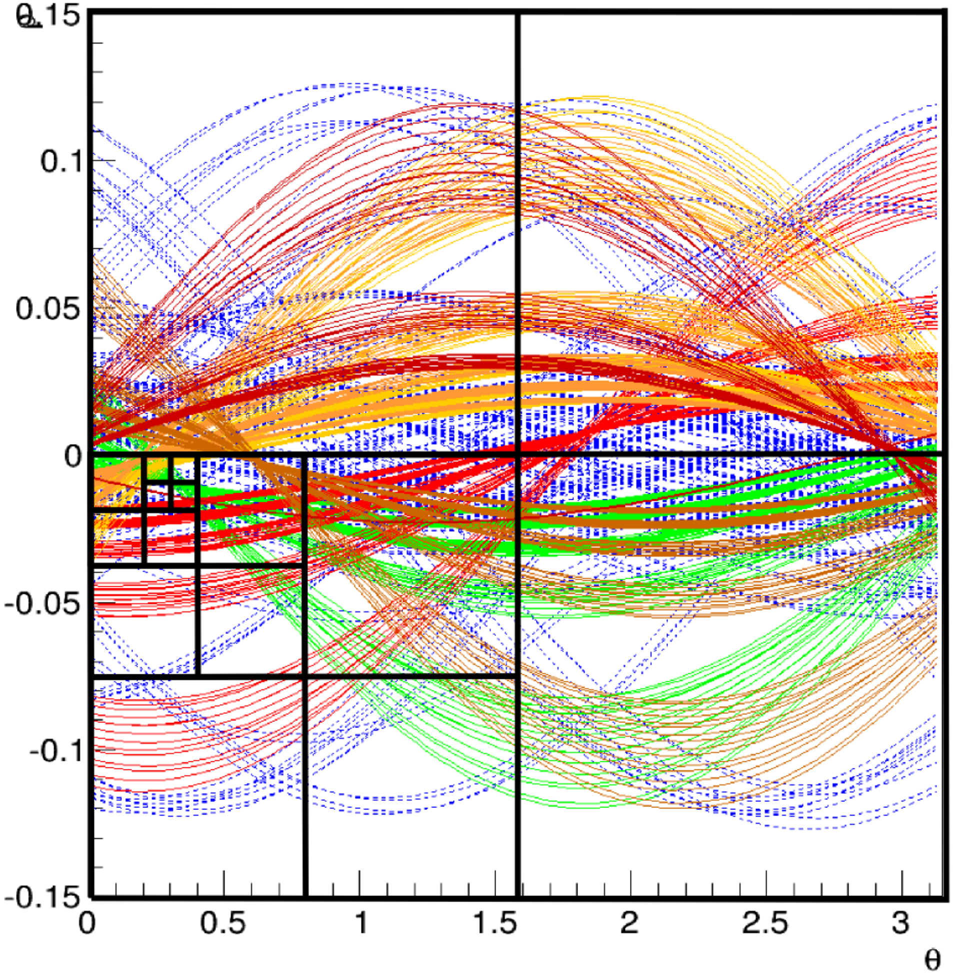
Voting

- **Step 4**



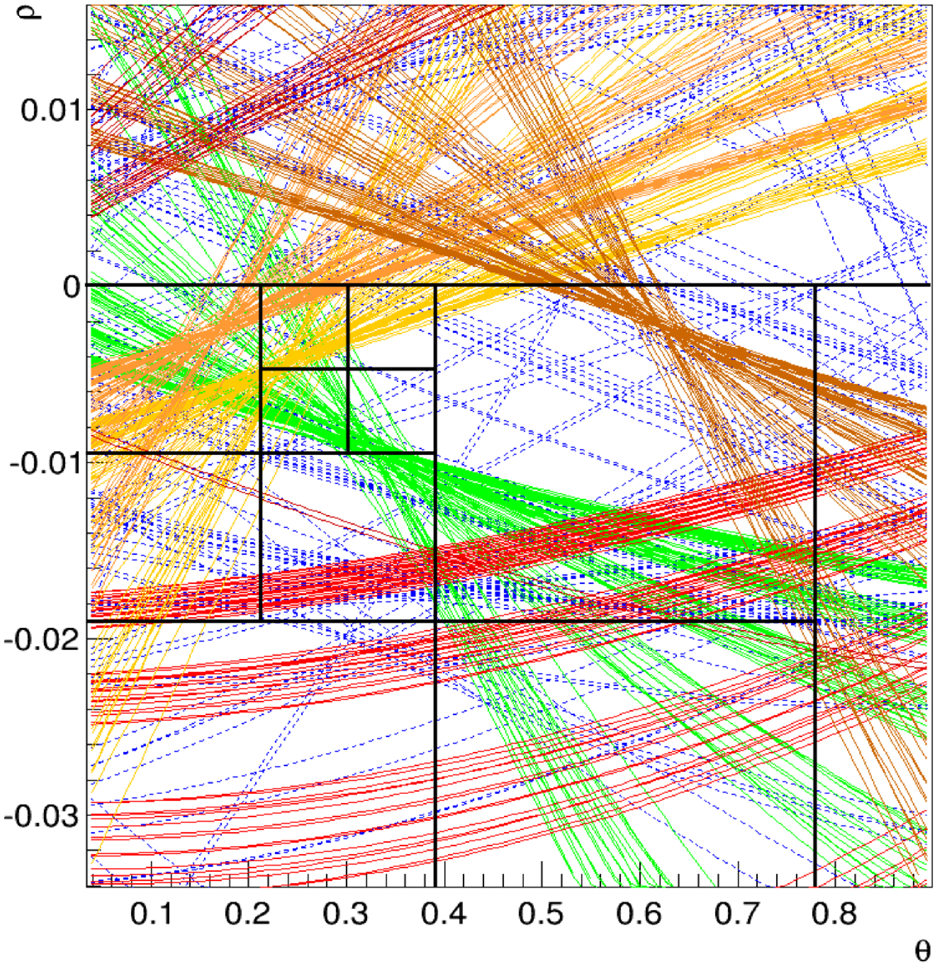
Voting

- **Step 5**



Voting

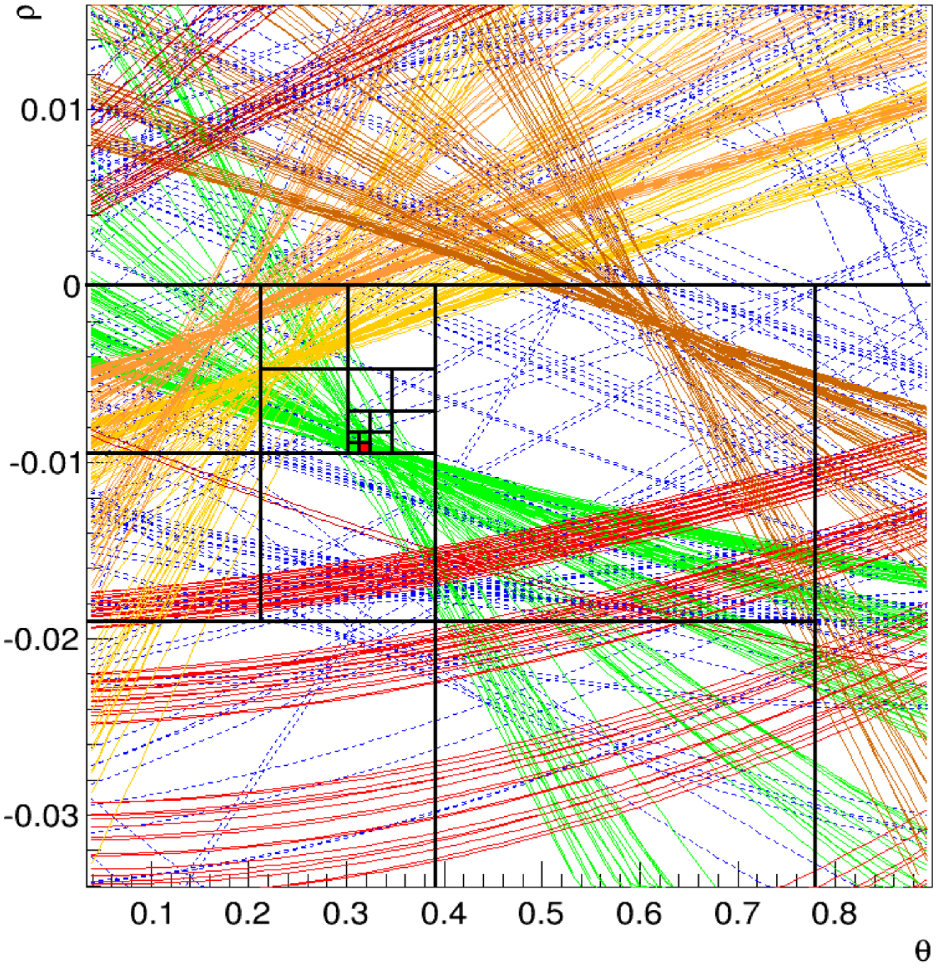
- **Step 6**



Voting

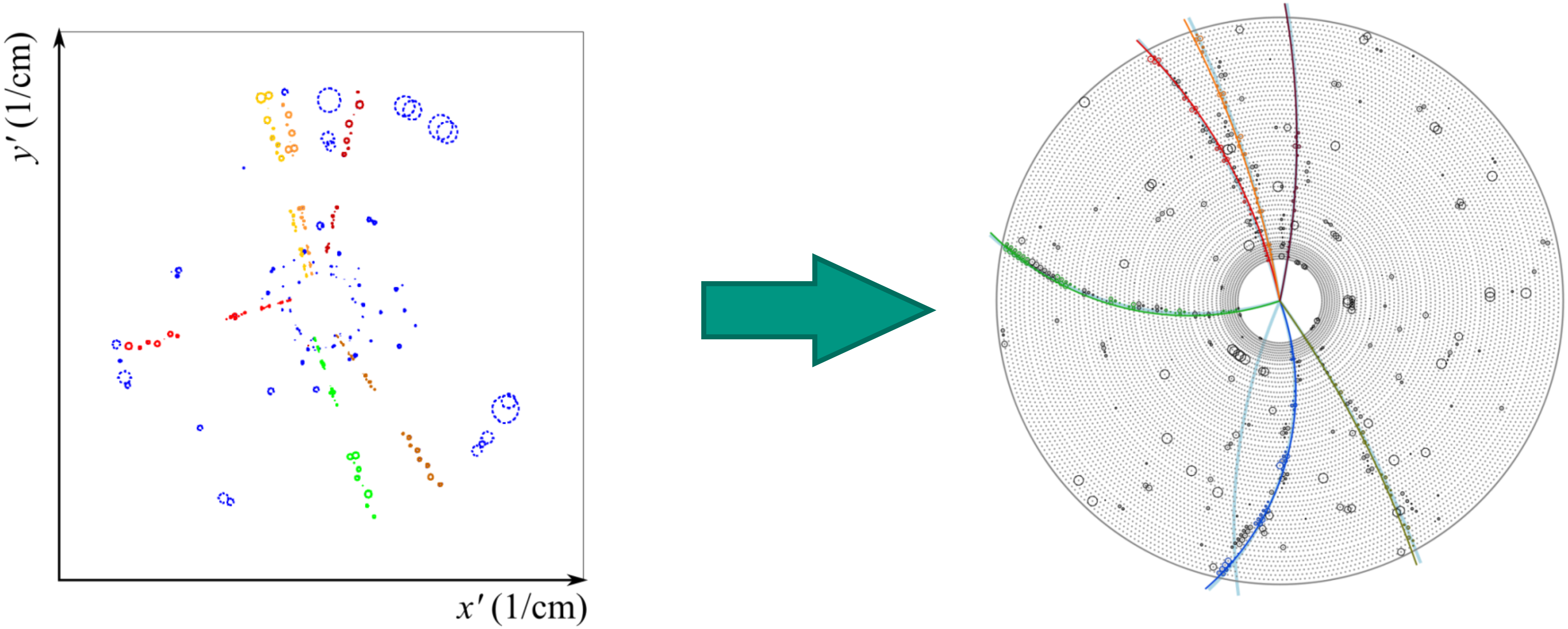
- **Step 10**

Last step, track candidate parameters are defined



Actual results of algorithm

- Simulated event: $B^- \rightarrow D^0(\rightarrow K^- \pi^+)\, \pi^- +$ beam background

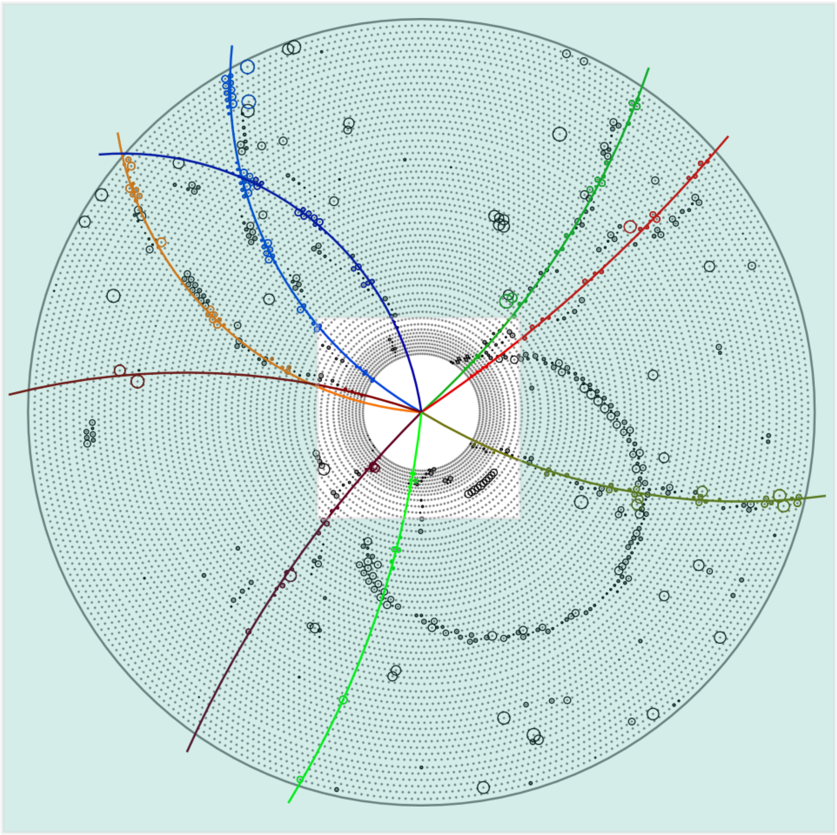


Working only with tracks originating close from IP

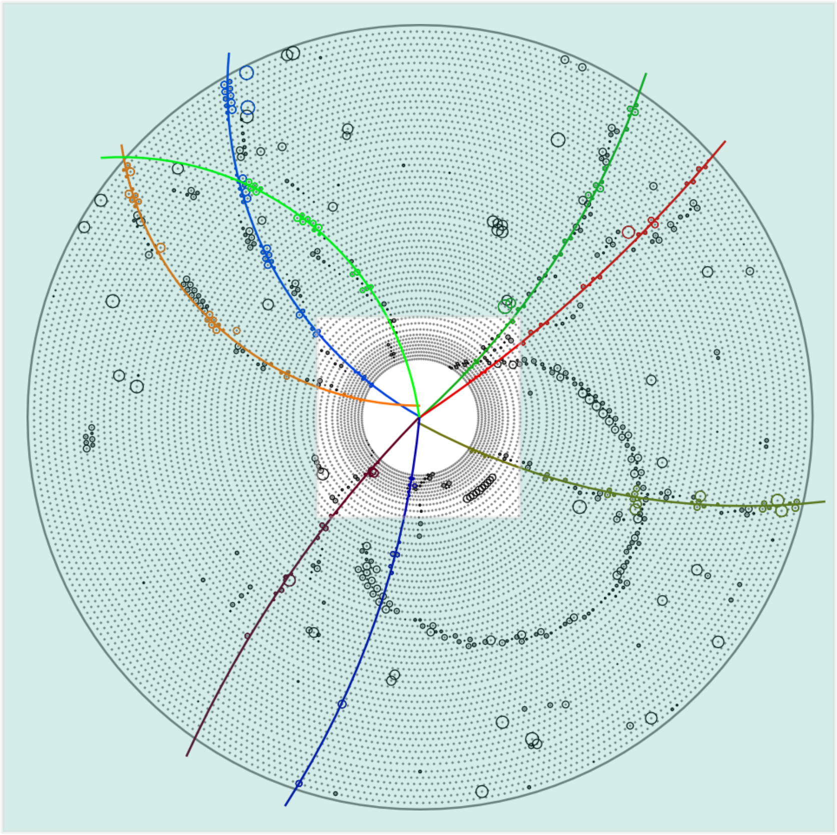
Efficient for finding tracks with high momentum

Track fitting

- Fitting based on approximation of hit patterns by circle.
- Basing on fit result new hits appended to the track



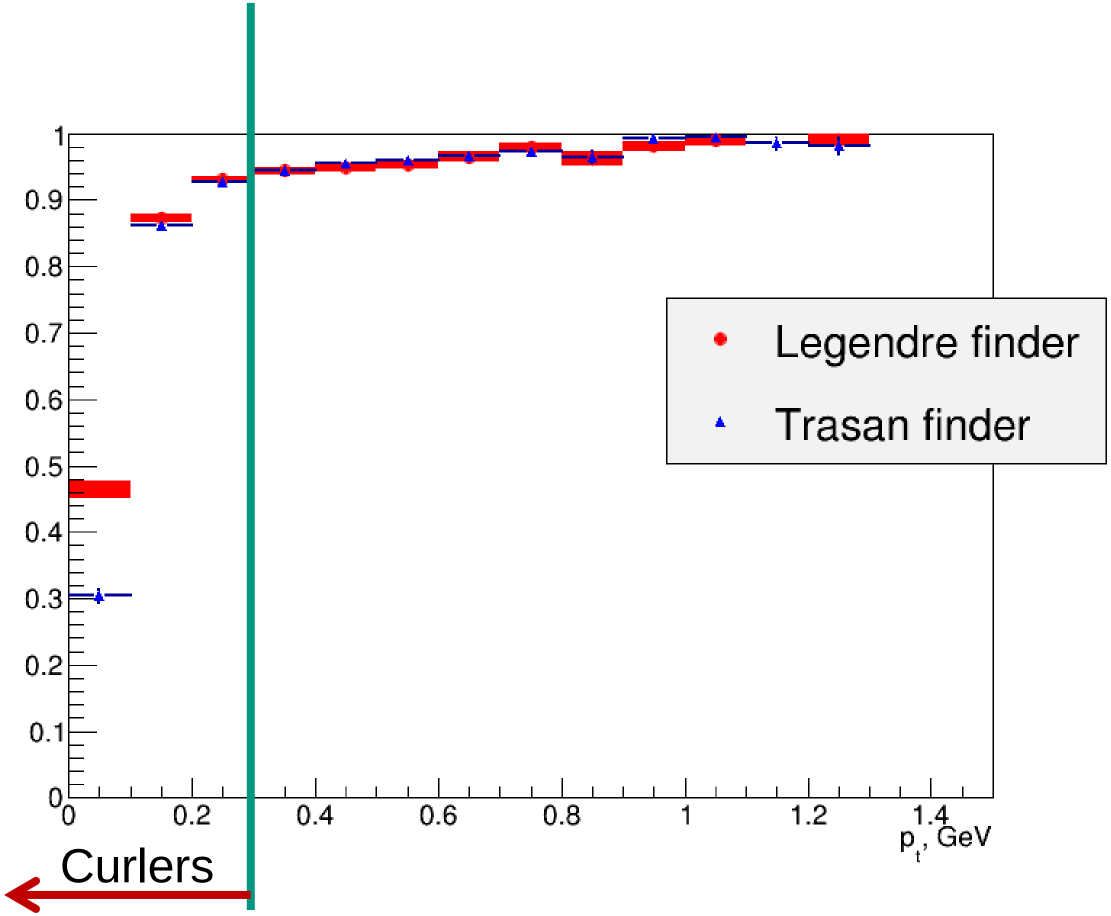
Before



After

Efficiency

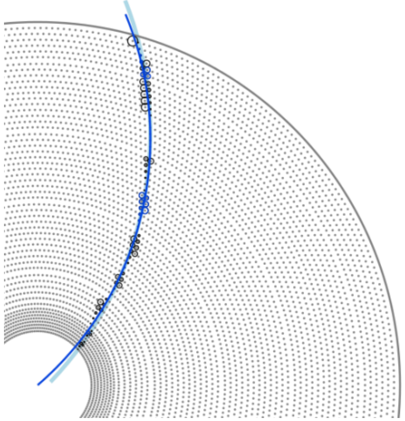
- Trasan vs Legendre finder efficiency (based on pre-generated sample of generic $B\overline{B}$ decays)

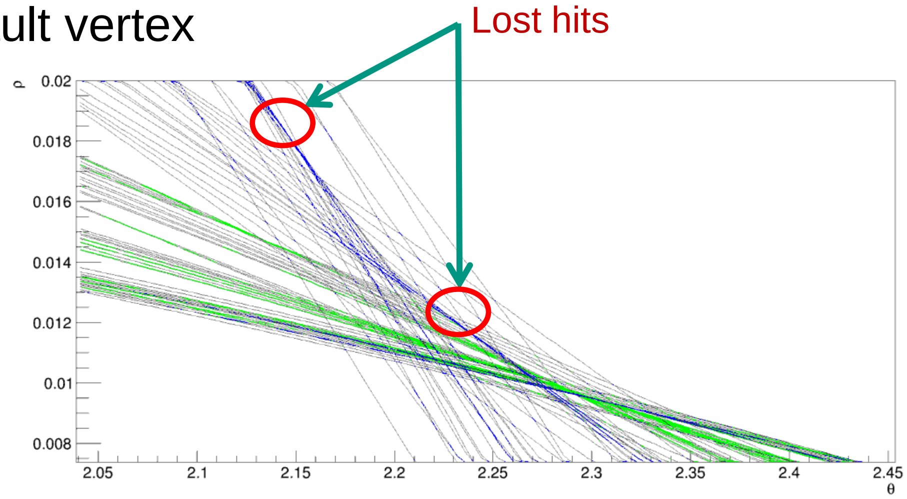


Curlers

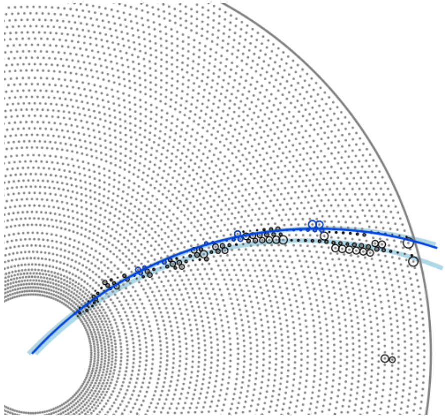
Efficiency improvement

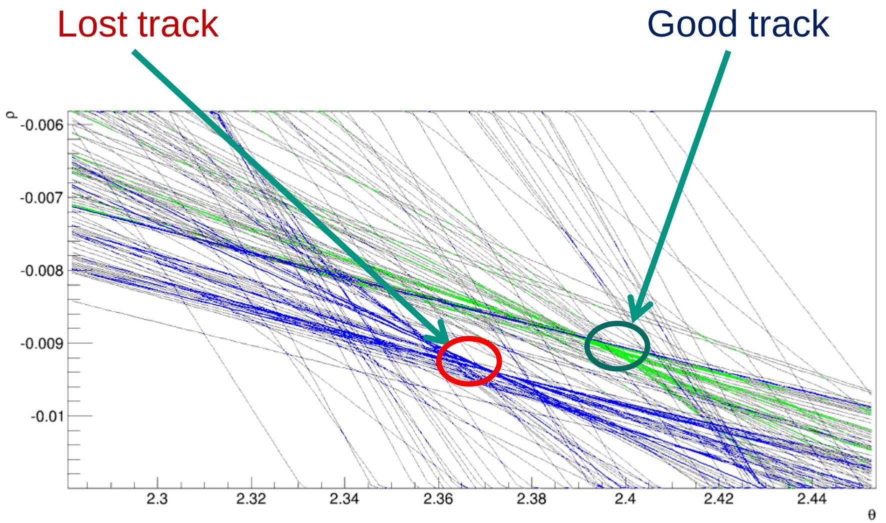
Track coming from non-default vertex




Lost hits

Two overlapping tracks




Lost track

Good track

Efficiency improvement

- Ways of solution:
  - It’s possible to add hits to track using neighboring nodes in QuadTree – can be used in tracks with lot of energy losses and non-IP tracks
  - Applying conformal transformation with respect to some point of found tracklet
    - Equal to shifting coordinates to new selected point
    - In this new transformation track from non-IP will be straight line in conformal space

Conclusion

- The method can perform fast track finding
- Limited to tracks originating from IP
- With using QuadTree structure time of processing decreased by ~20%
  - It gives great opportunities for increasing quality of track finding
- Highly efficient for track finding
- Works quite well with high-pt tracks

Thank you for attention!