Applying Legendre transformation method for Belle II tracking

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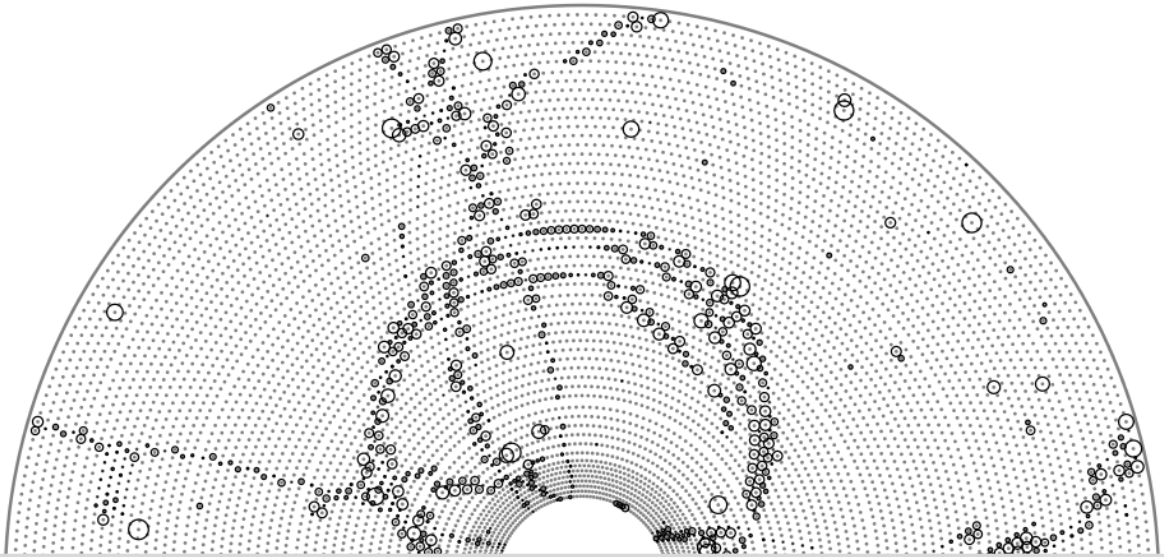
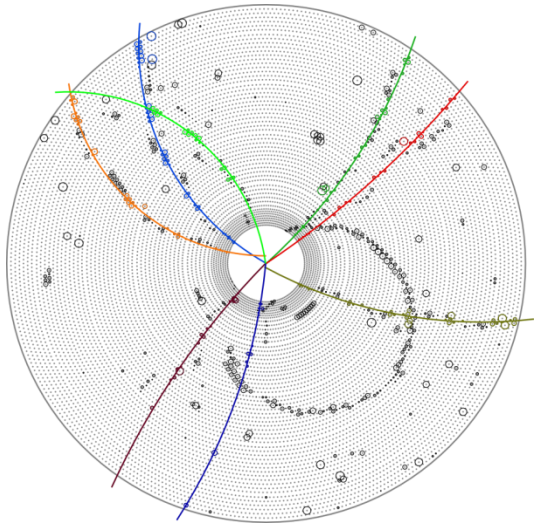
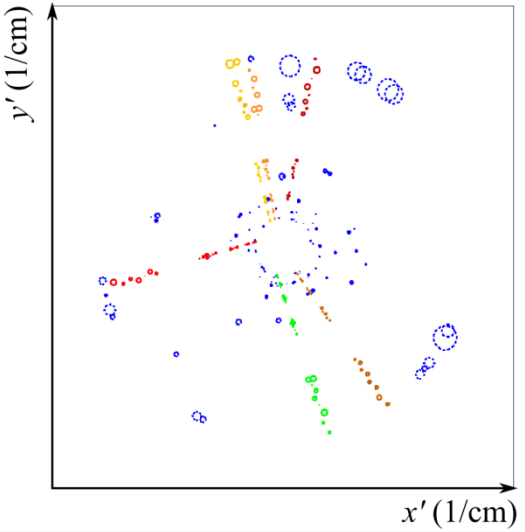


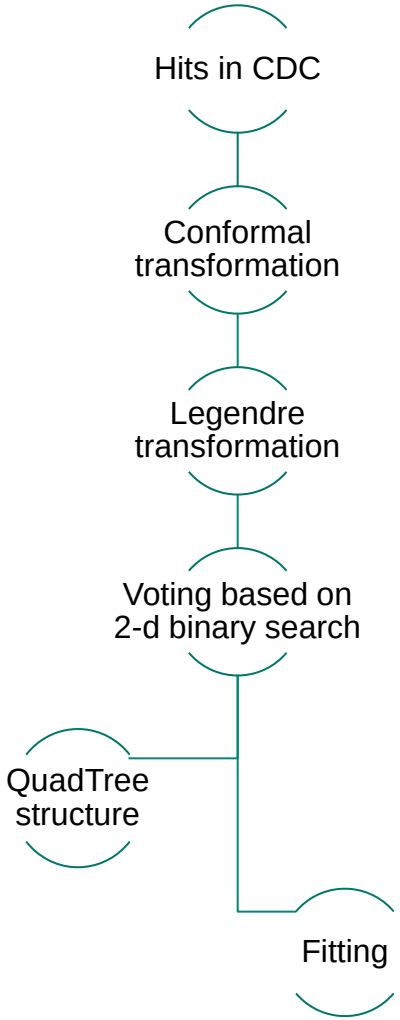
The method

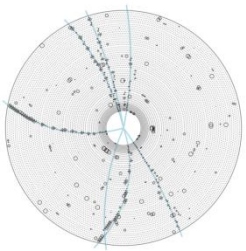
- The main task of track finding is to determine which hits belongs to a common track
- We present a method of track finding which based on reconstruction of linear hit patterns in conformal space
- Legendre transformation of drift circles allows to build track with higher efficiency than using only position of the wire.

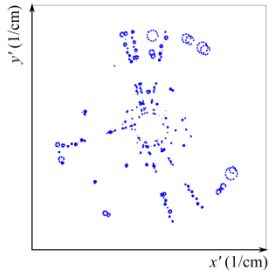


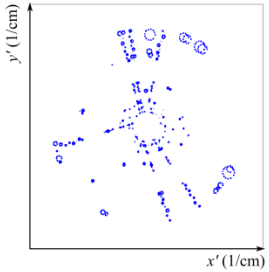


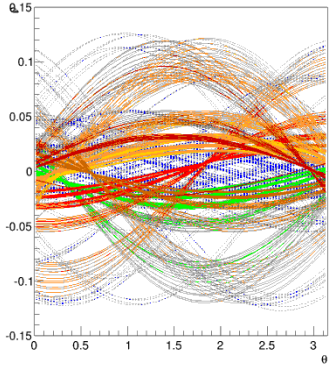
Chain of the method

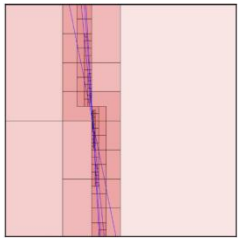


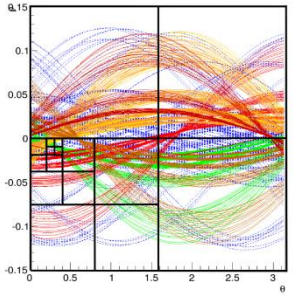


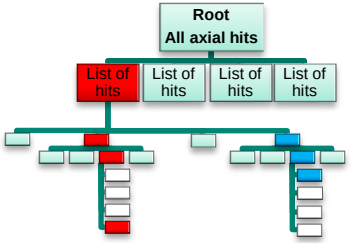










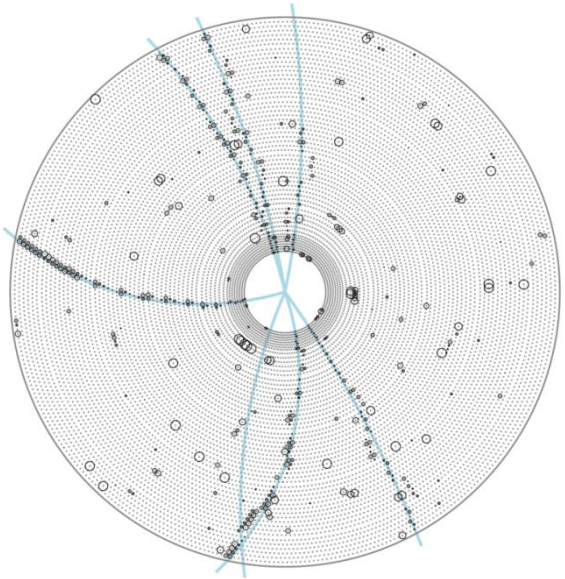


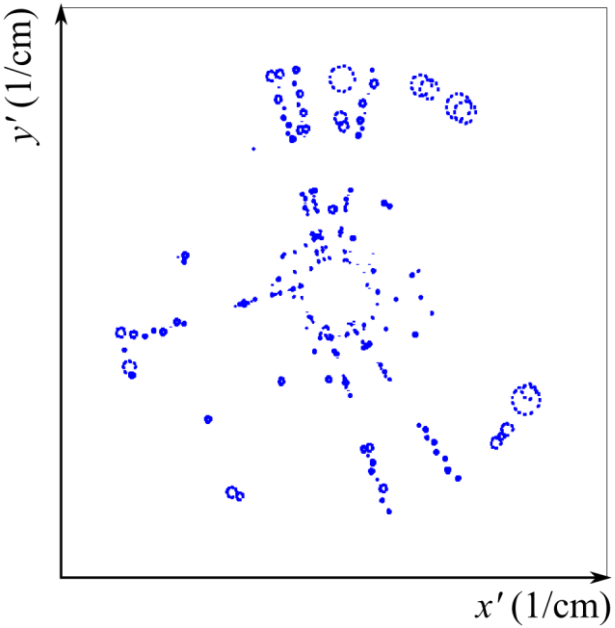
Conformal transformation

- Conformal transformation which transforms circles through origin into lines:

$$x' = \frac{2x}{x^2 + y^2}$$

$$y' = \frac{2y}{x^2 + y^2}$$



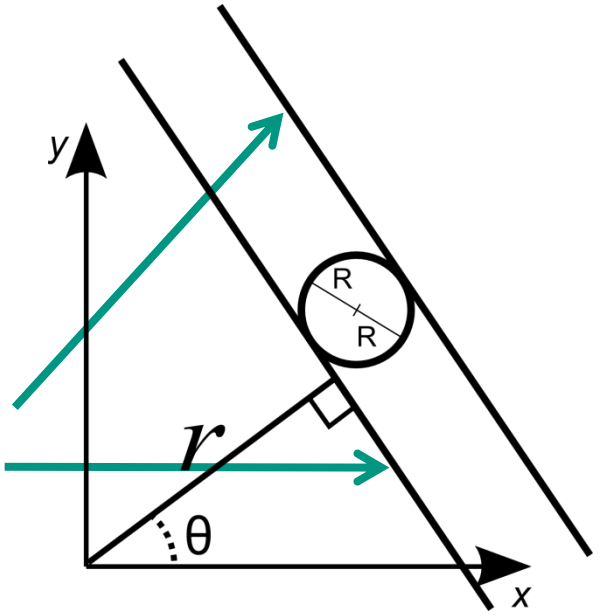


Transformation into Legendre space

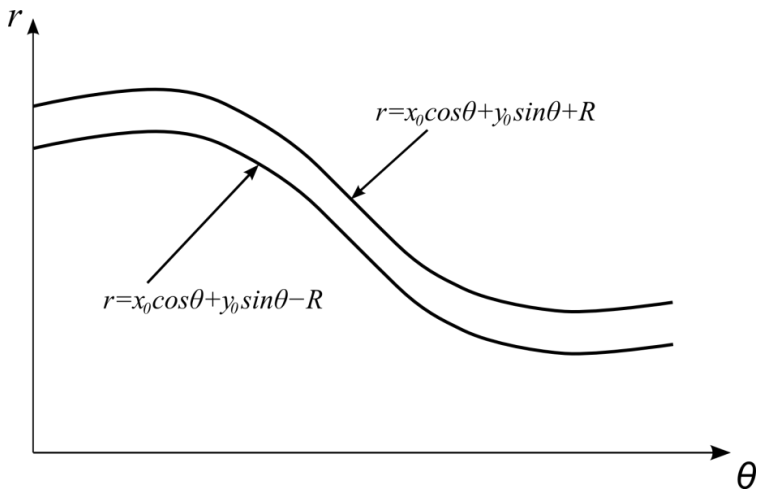
- The method is based on applying Legendre transformation to each drift circle in conformal space
- Legendre transformation of the circle can be written in next form:

$$f(x) \overset{\mathcal{L}}{\leftrightarrow} \begin{cases} r = x_0 \cos\theta + y_0 \sin\theta + R & \text{for concave} \\ r = x_0 \cos\theta + y_0 \sin\theta - R & \text{for convex} \end{cases}$$

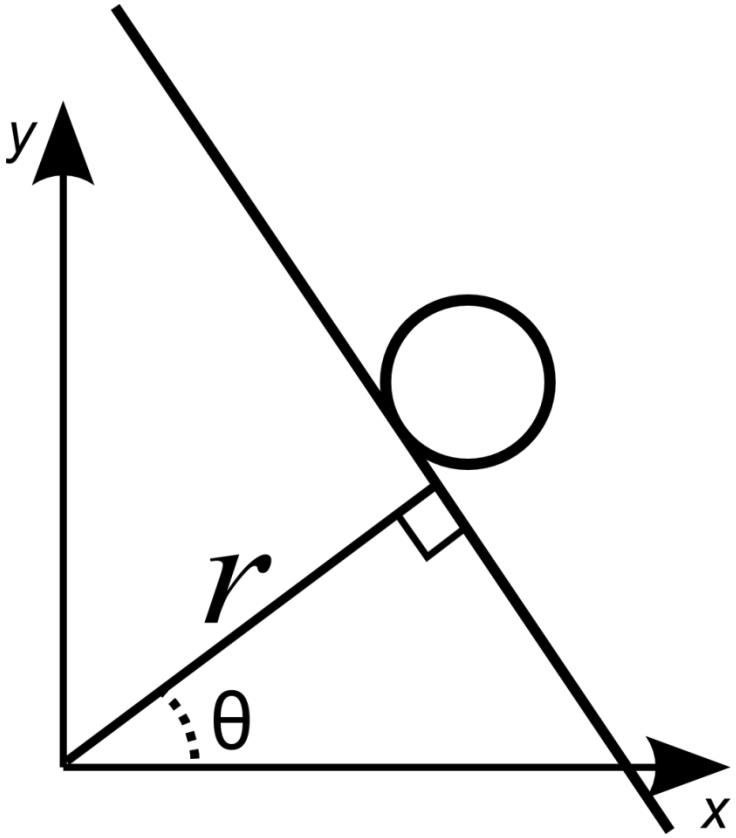
which presents tangents to the circle

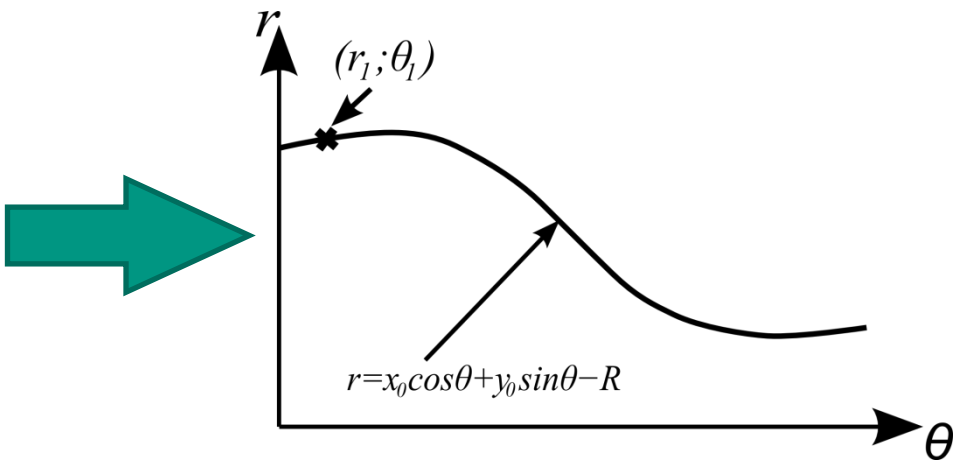


- Representation of the circle in the r, θ Legendre transformation space



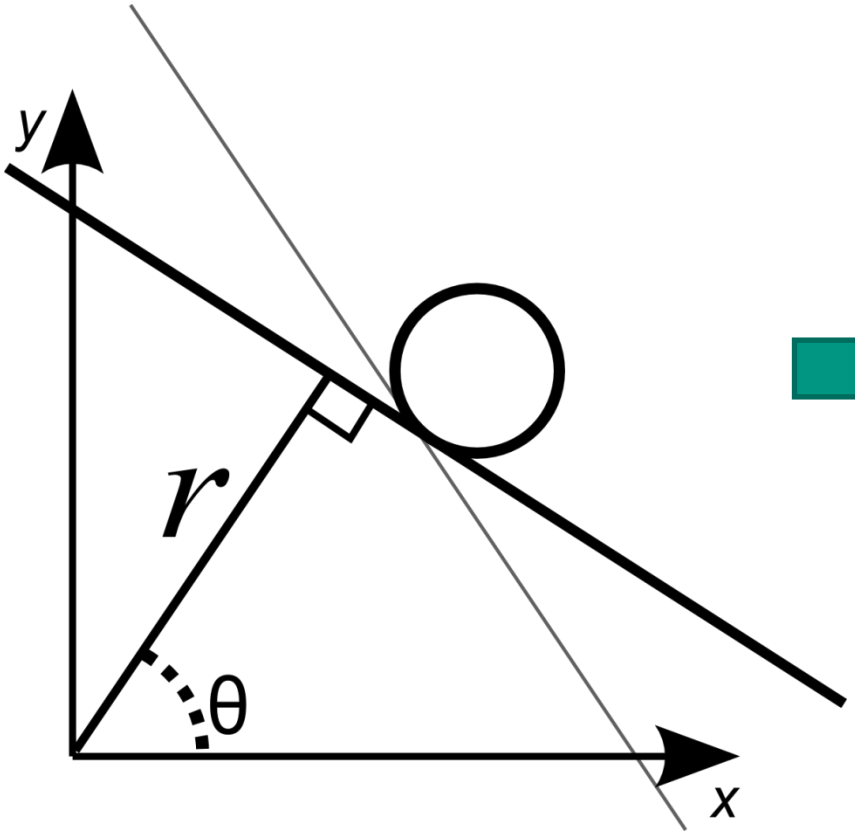
Transformation into Legendre space

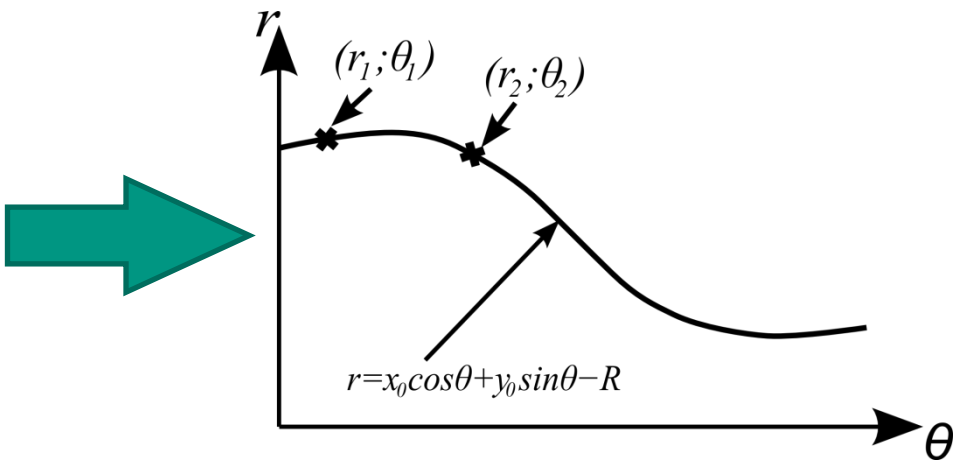




Each tangent has corresponding point in r, θ Legendre transformation space

Transformation into Legendre space





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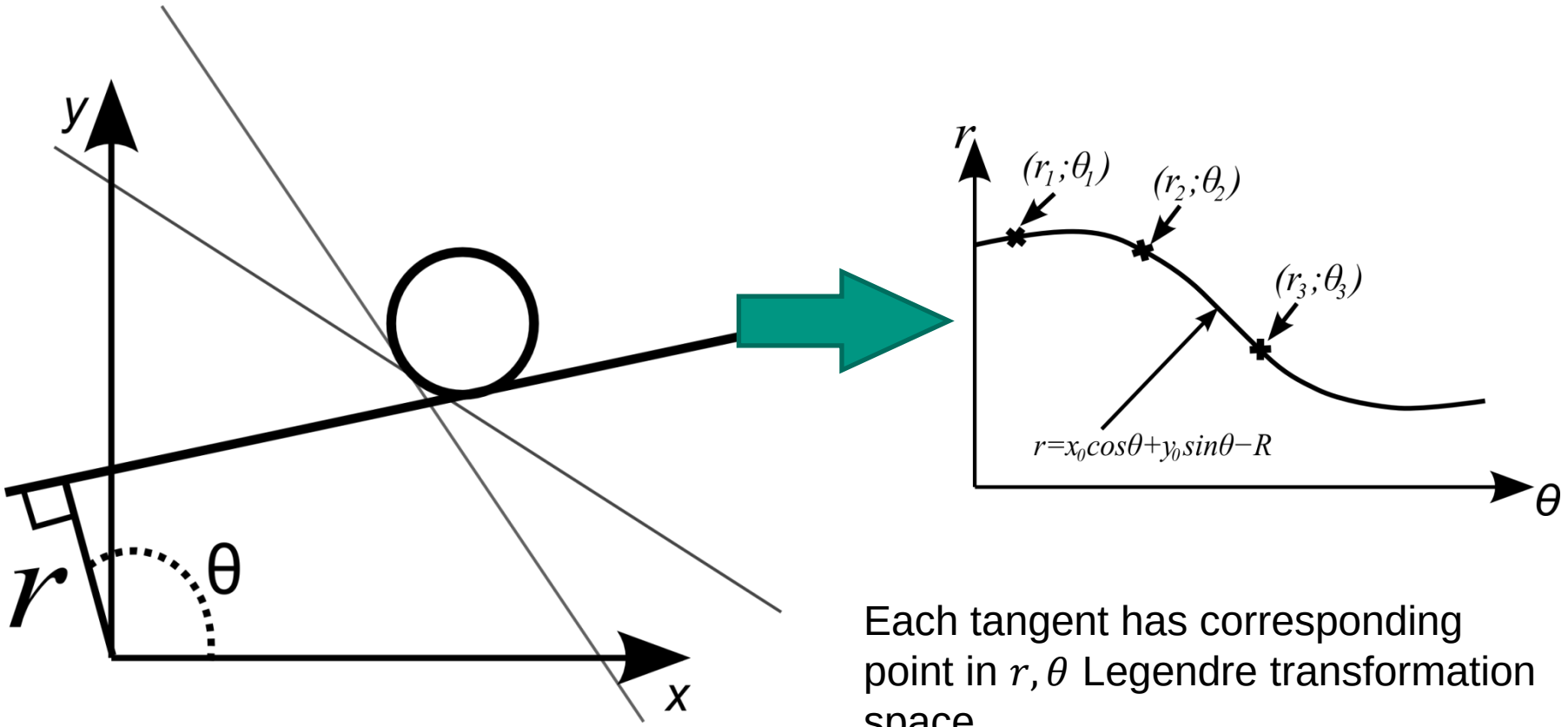
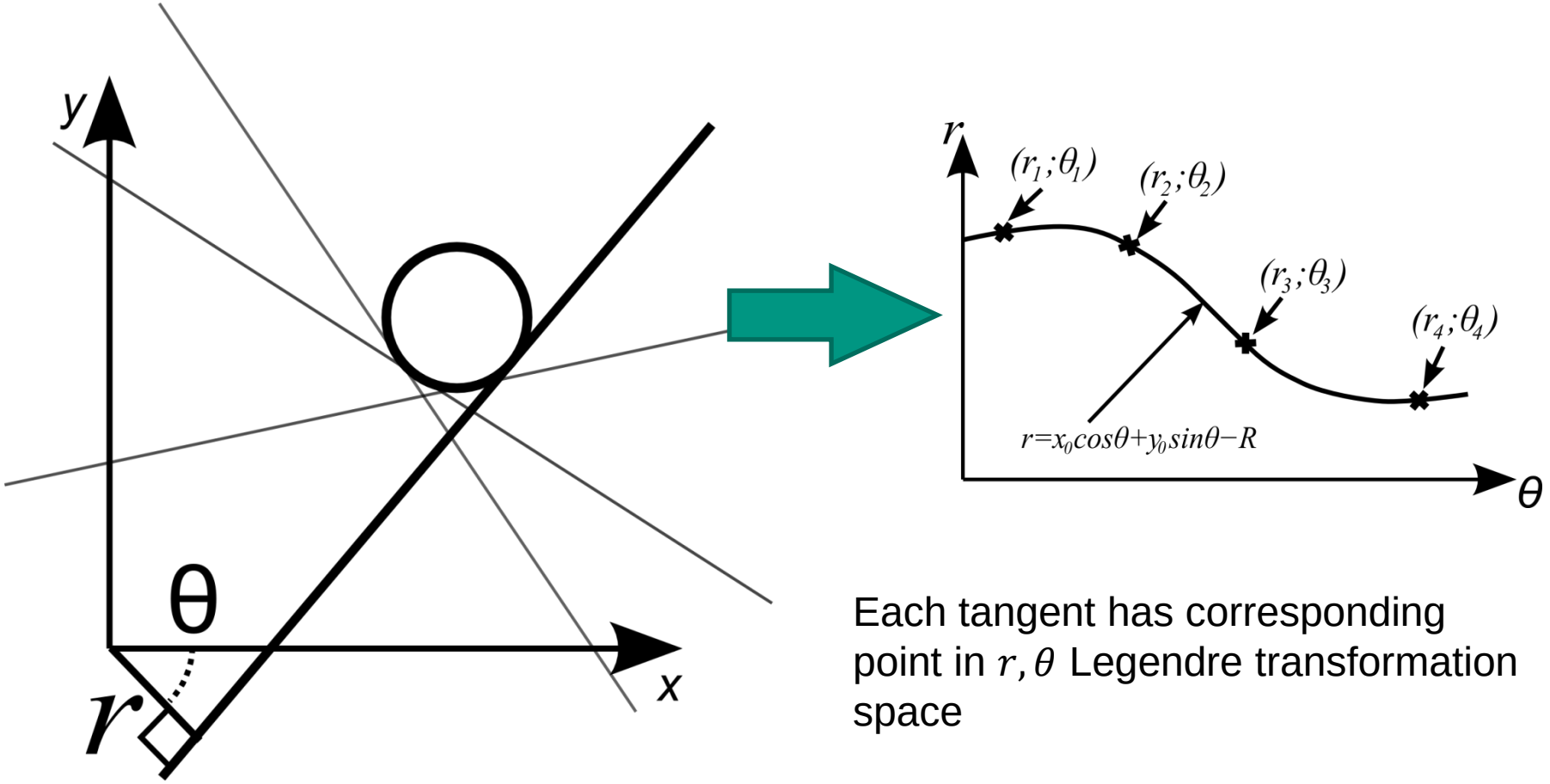


Each tangent has corresponding point in r, θ Legendre transformation space

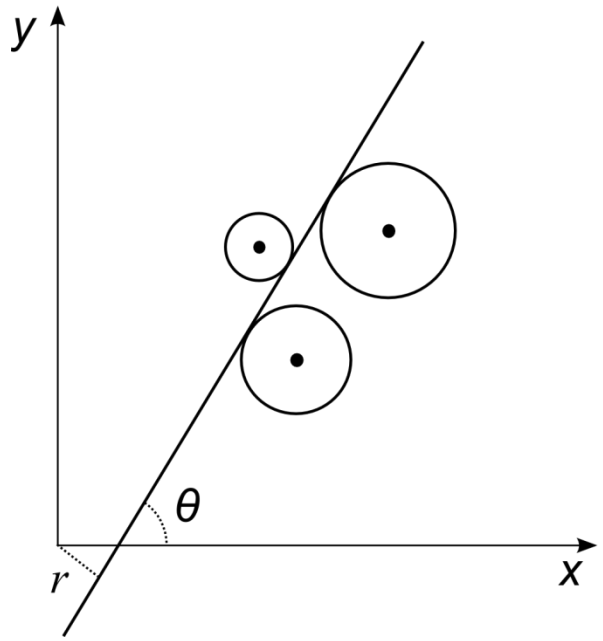
Transformation into Legendre space

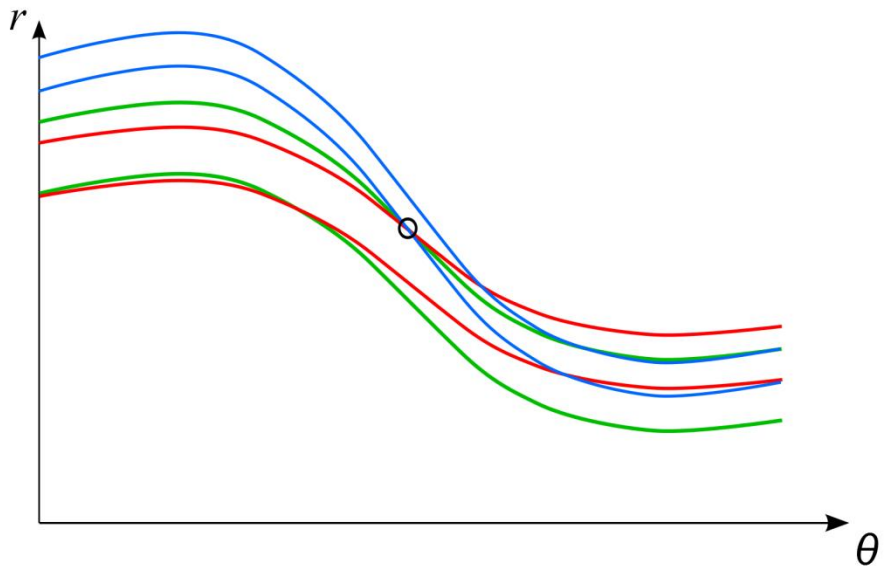


Each tangent has corresponding point in r, θ Legendre transformation space

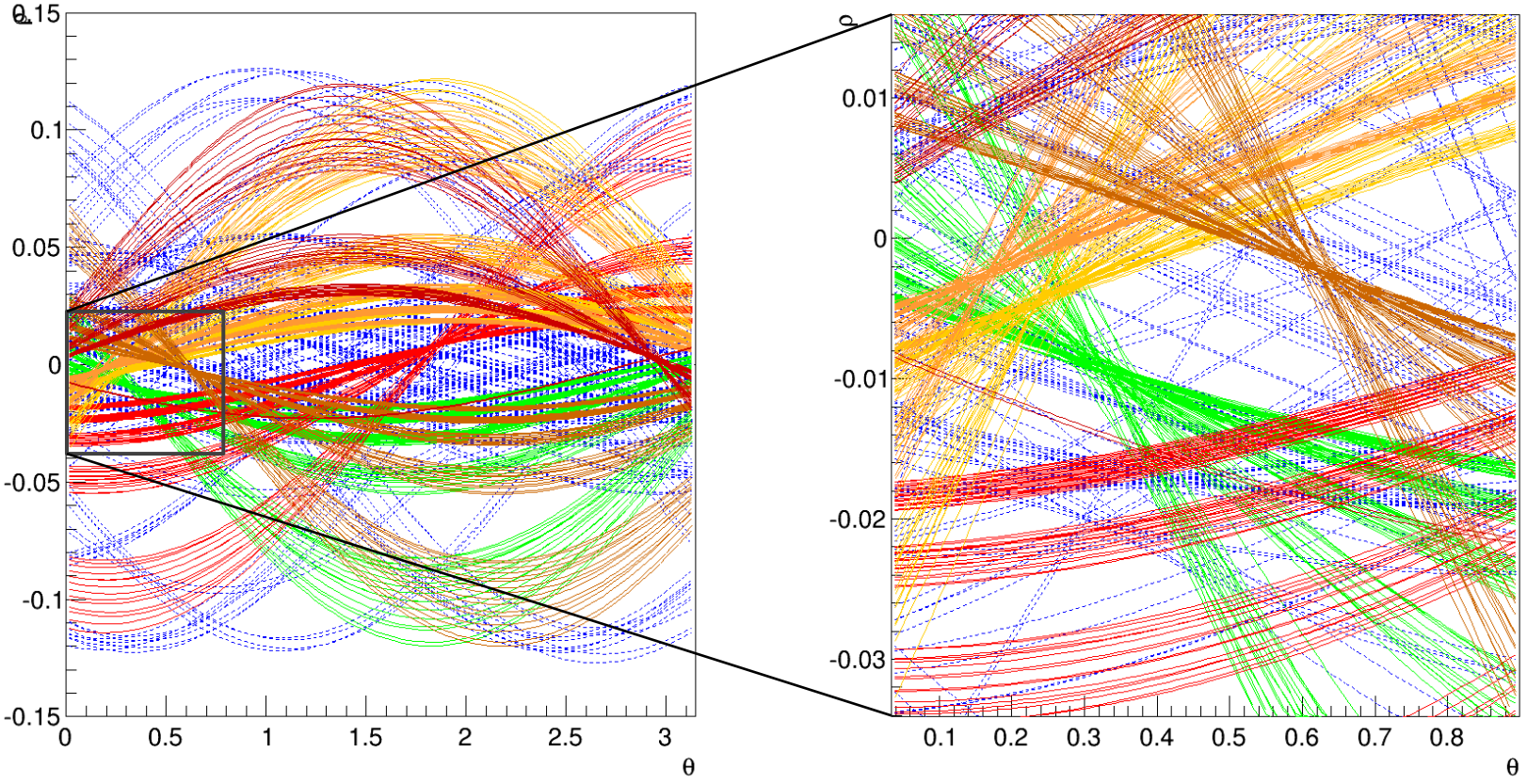
Finding of candidates

- The point of most sinograms intersection in r, θ space represents parameters of the common tangent to each drift circle belonging to the track



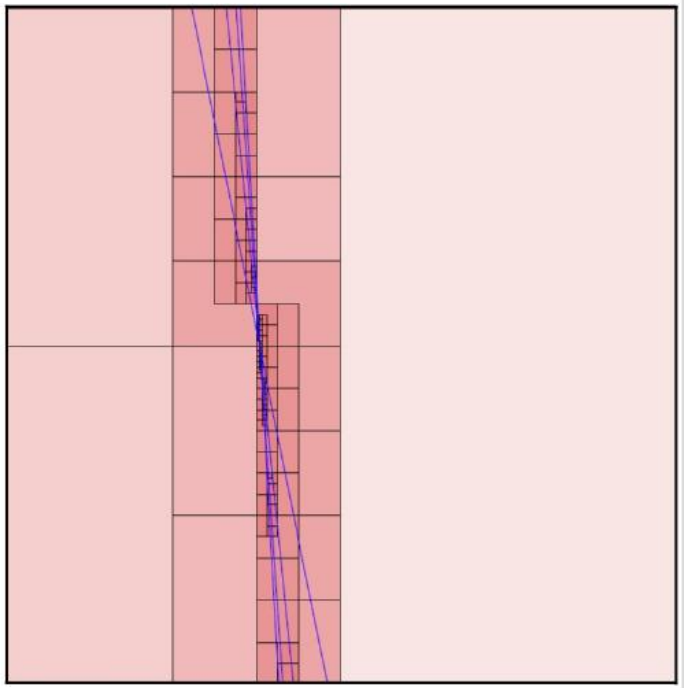


Sinograms of simulated event



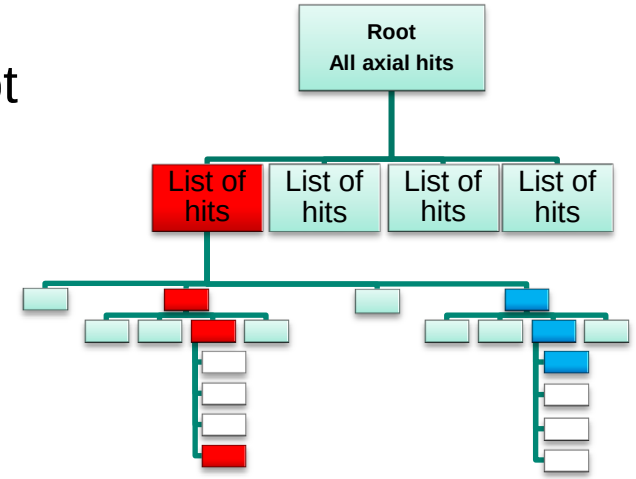
Voting or “How to find the point of most intersections?”

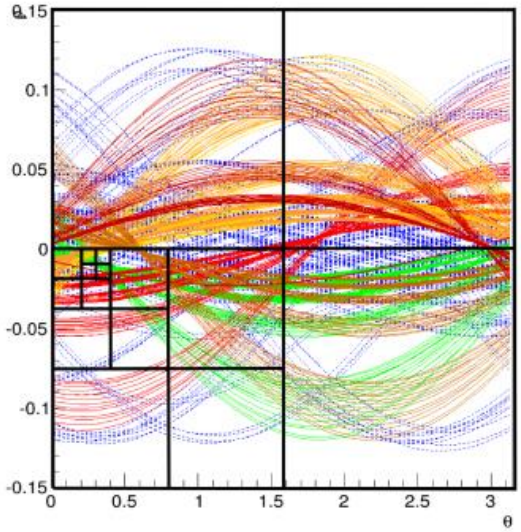
- Let each drift circle to *vote* for a set of possible parameters in Legendre space
  - Bin acquires vote if sinogram of drift circle passes through it
- Voting algorithm (based on 2-D binary search):
  - Split (r, θ) space into 4 bins
  - Accumulate votes in each bin
  - Select bins which passes threshold on number of votes
  - Continue bin splitting and voting until desired (r, θ) resolution reached
- Bin with the most of votes (hits) indicates hits pattern of track candidate



Using QuadTree for voting algorithm

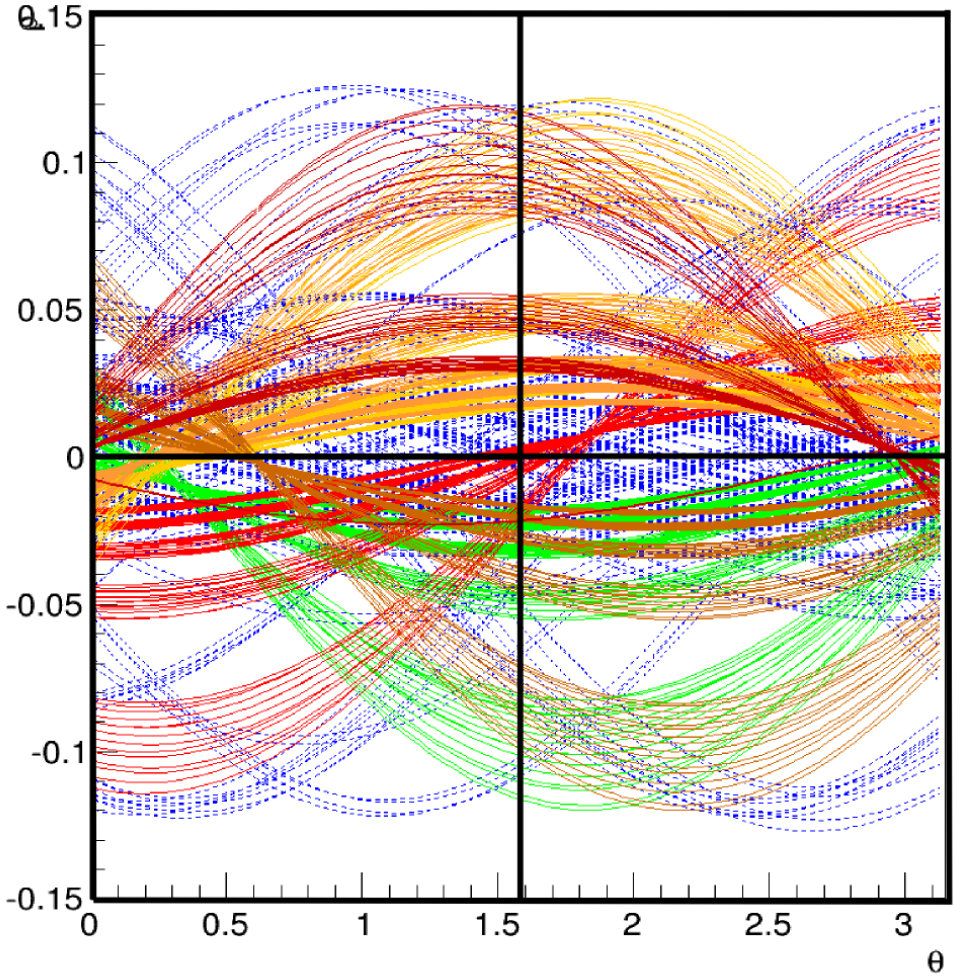
- Nodes has one parent and 4 children (except root and leafs nodes)
- Each node holds hits which gave votes for current bin
- Each node can communicate with children and parent nodes
- Results of voting stored in the tree and used in next iteration of track finding





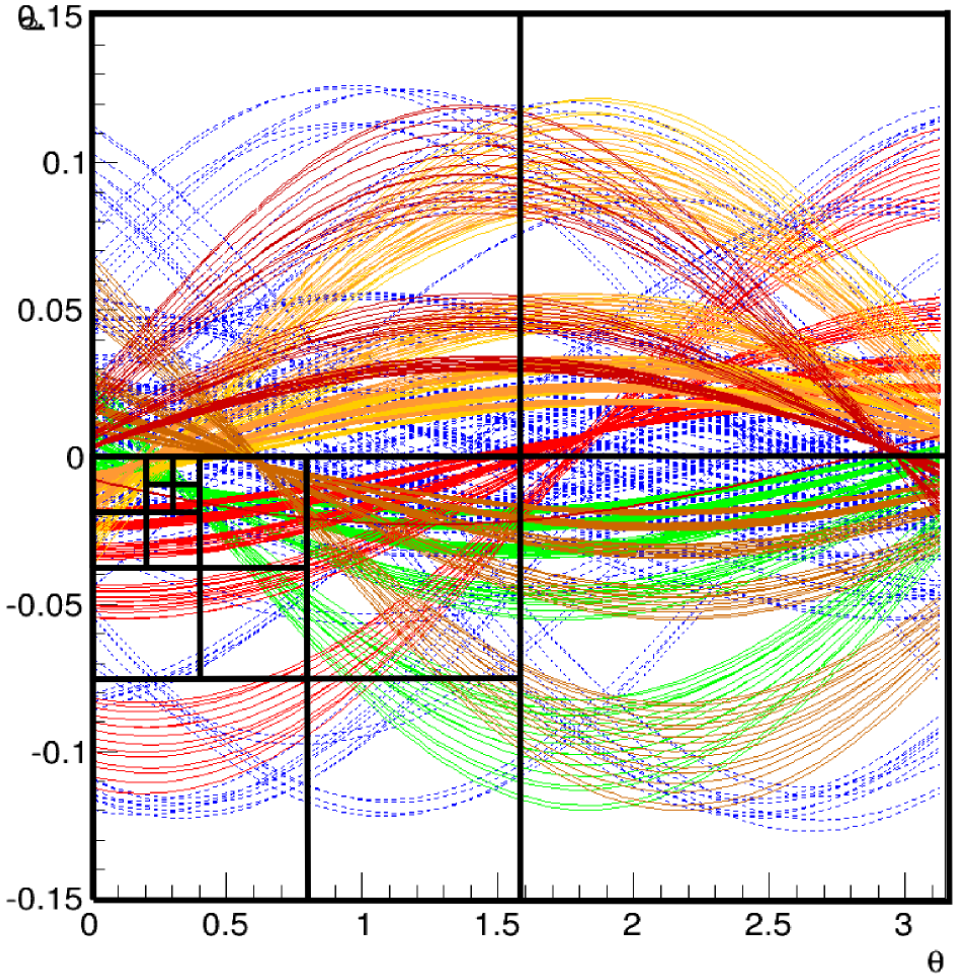
Voting

- **Step 1**



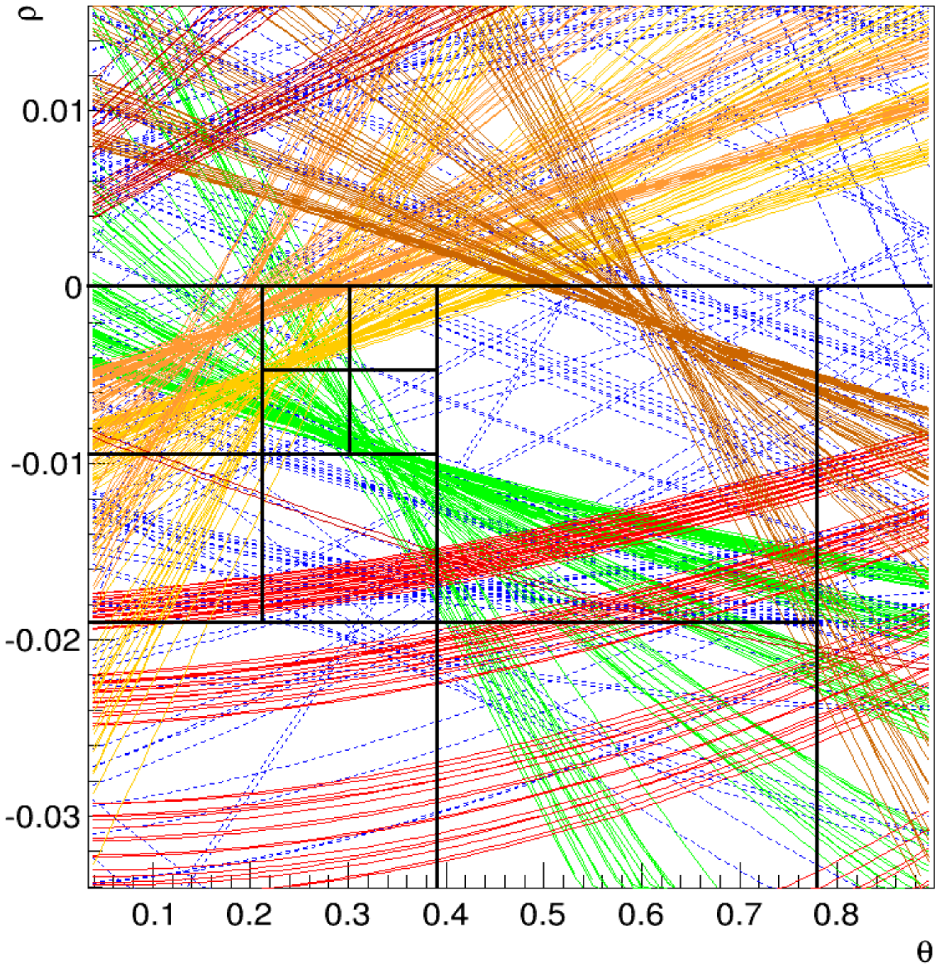
Voting

- **Step 5**



Voting

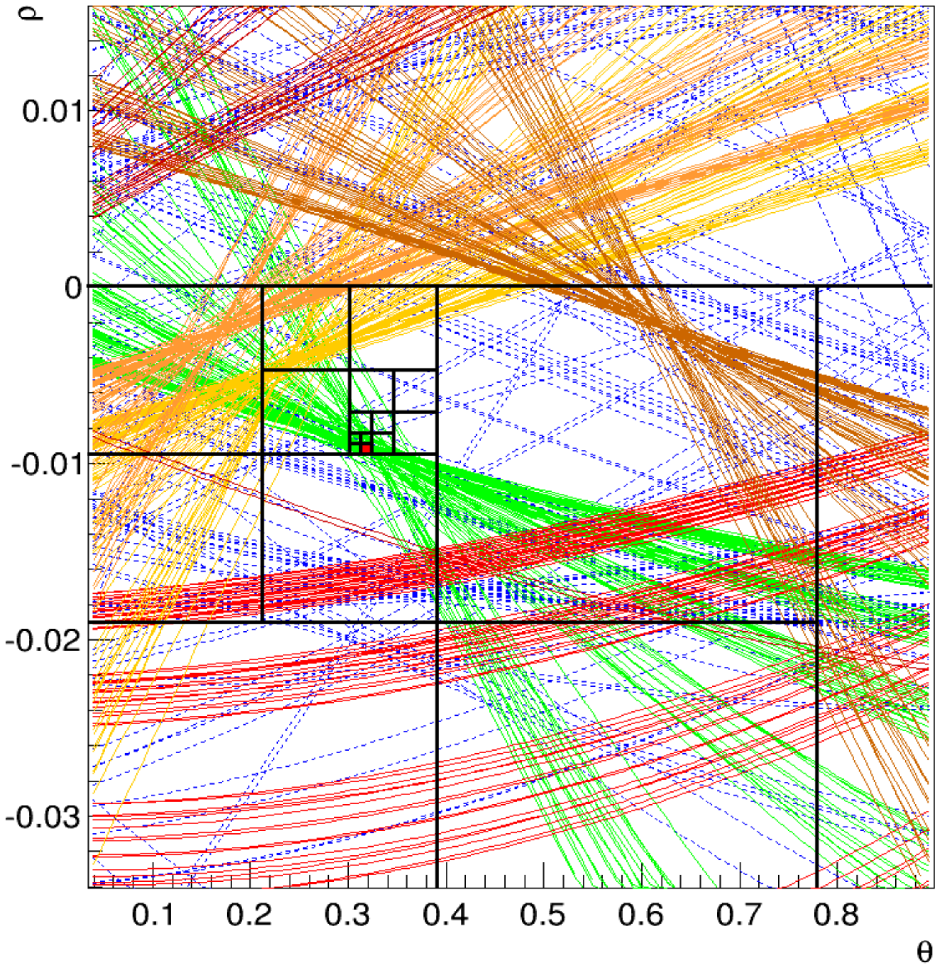
- **Step 6**



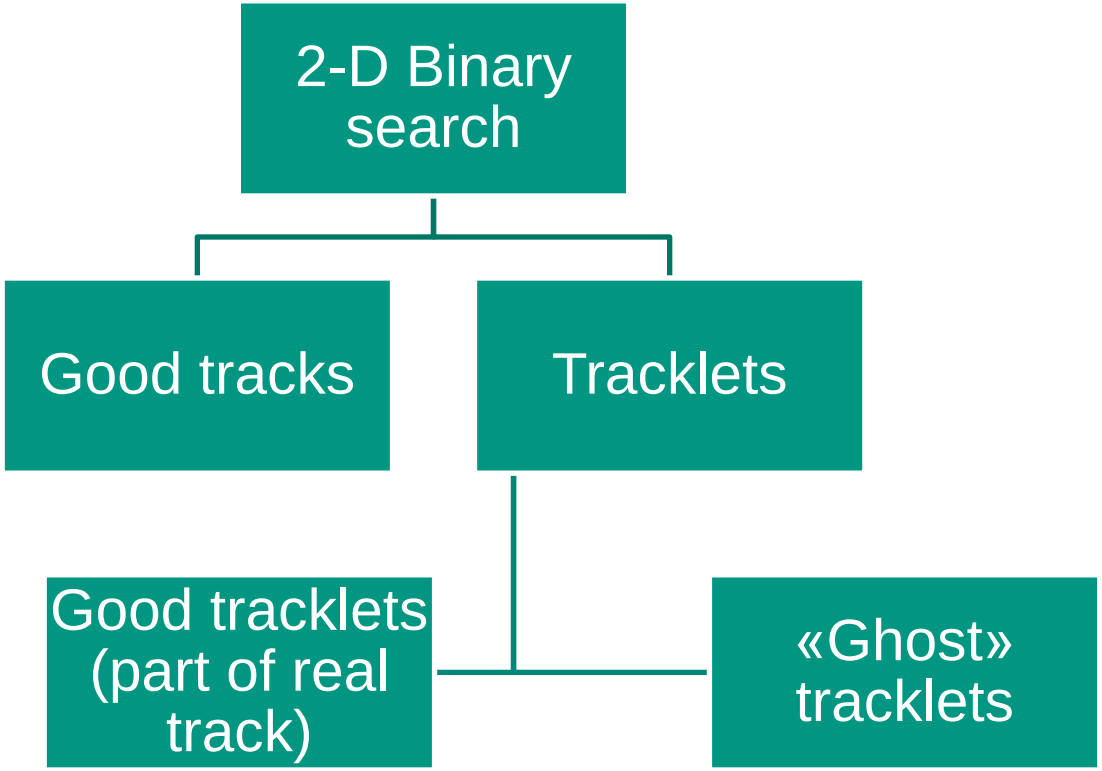
Voting

- **Step 10**

Last step, track candidate parameters are defined



Results of Legendre finding



Tracks processing

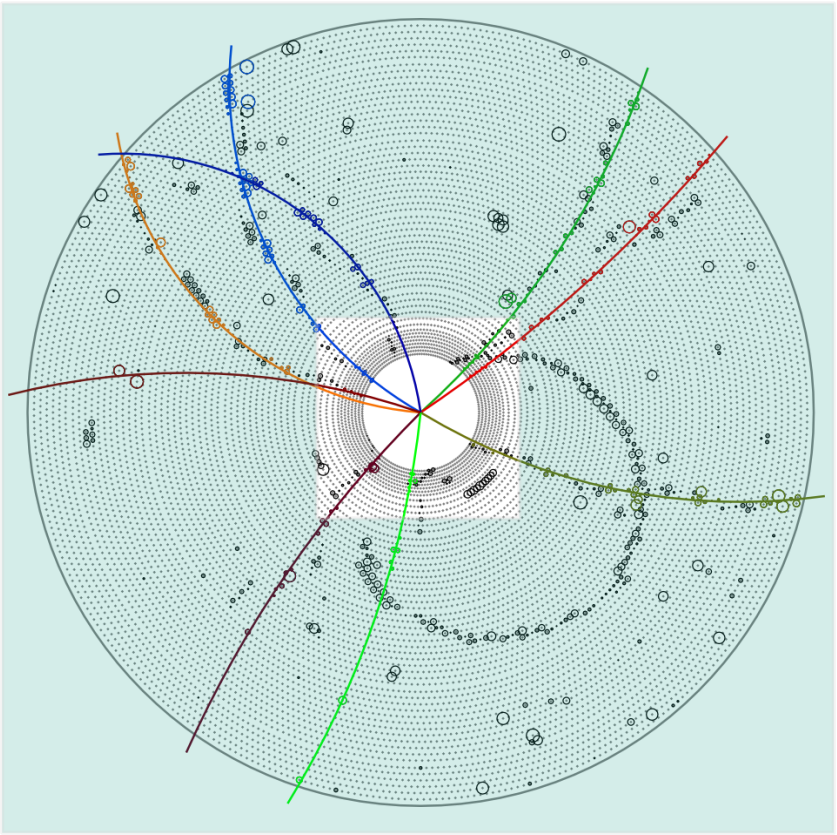
- After each successful candidate finding:
  - circular fitting procedure applied;
  - hit appending;

- After finishing pattern recognition with 2-D binary search track merging applied:
  - Based on combination of pairs of tracks ;
  - If two tracks successfully fitted together ($\chi^2/ndf < 1$) tracks are merged into one track

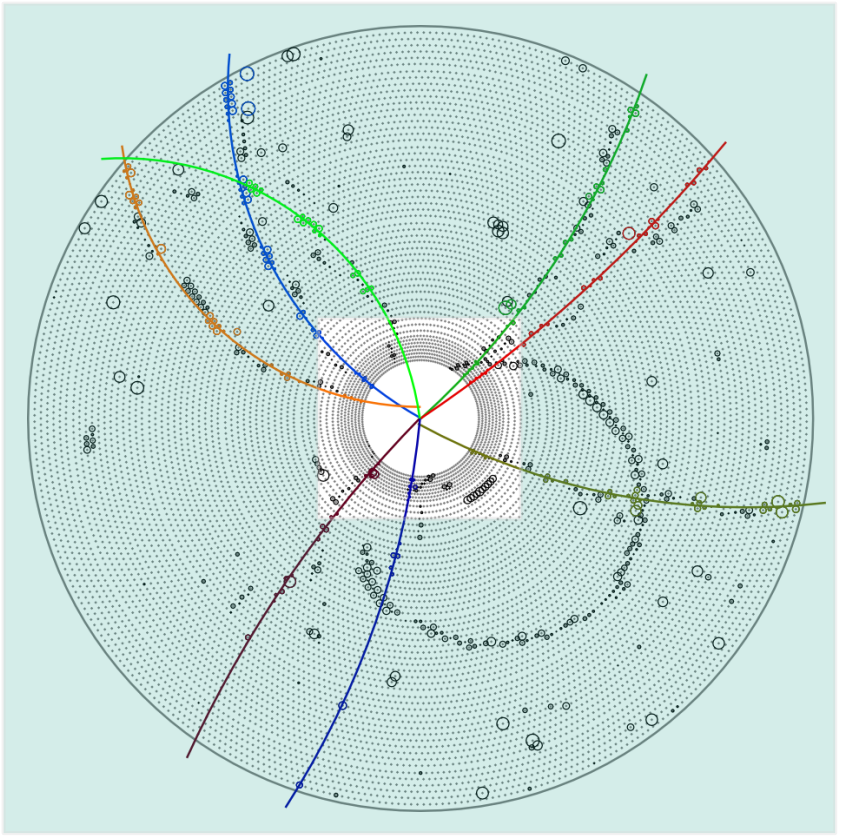
- Next step: trying to append unused hits to tracks and hits reassignment basing on distance to track

Track fitting

- Fitting based on approximation of hit patterns by circle.
- Basing on fit result new hits appended to the track



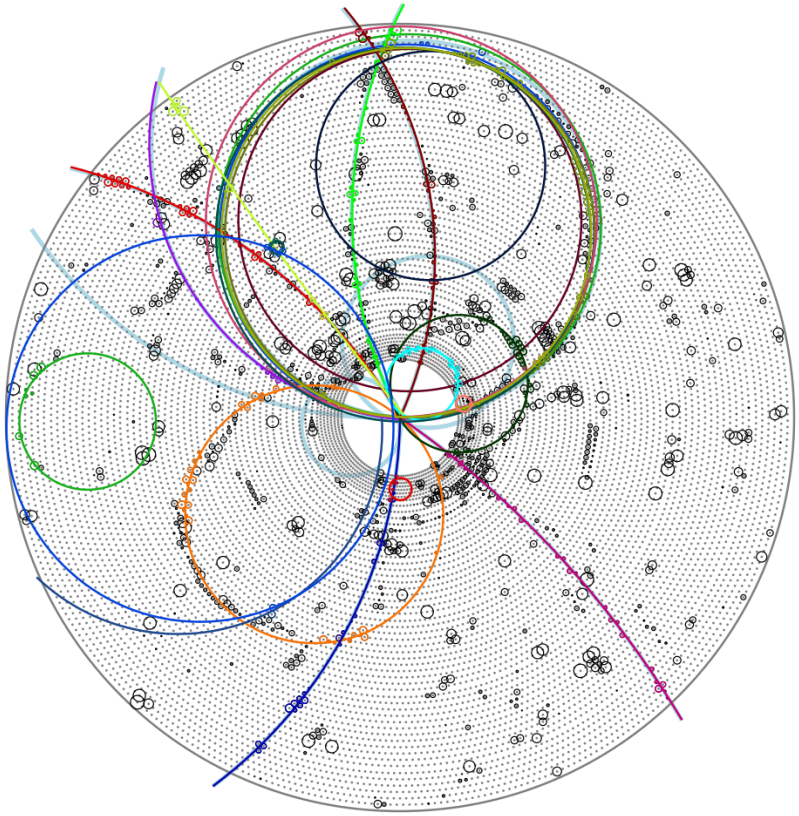
Before



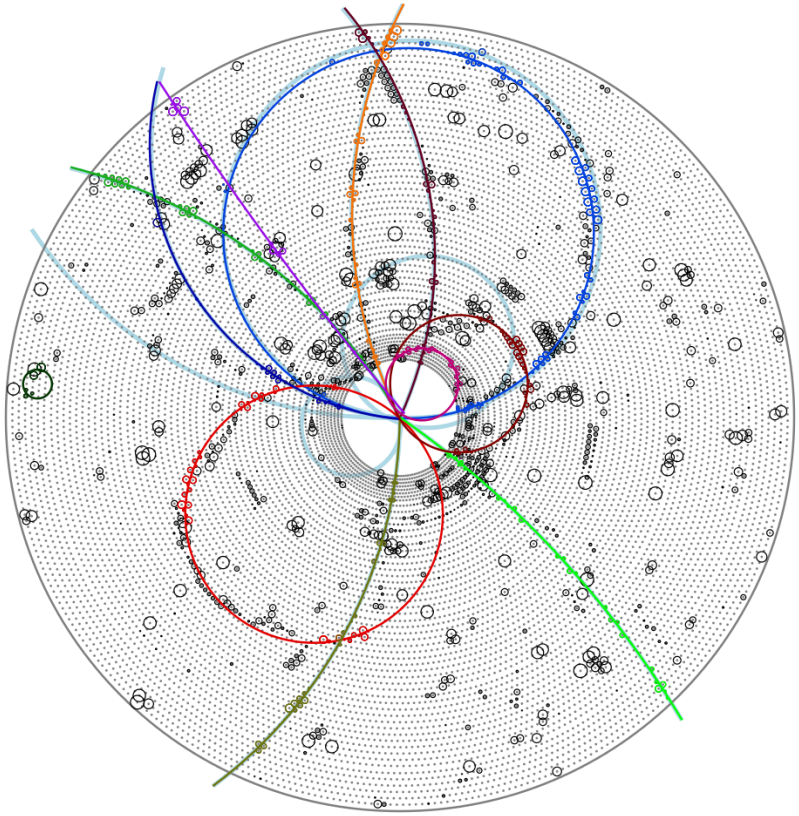
After

Track merging

- Pair of tracks fitted together by circular fit
- If $\chi^2/ndf < 1$ tracks are merged into one track



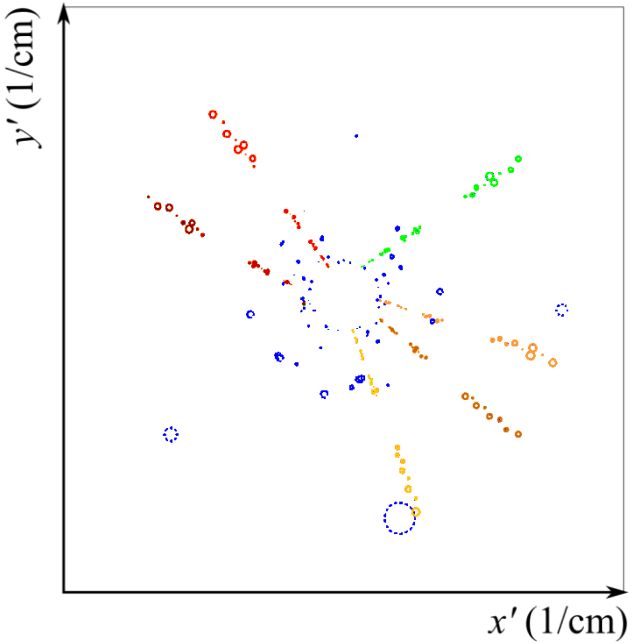
Before

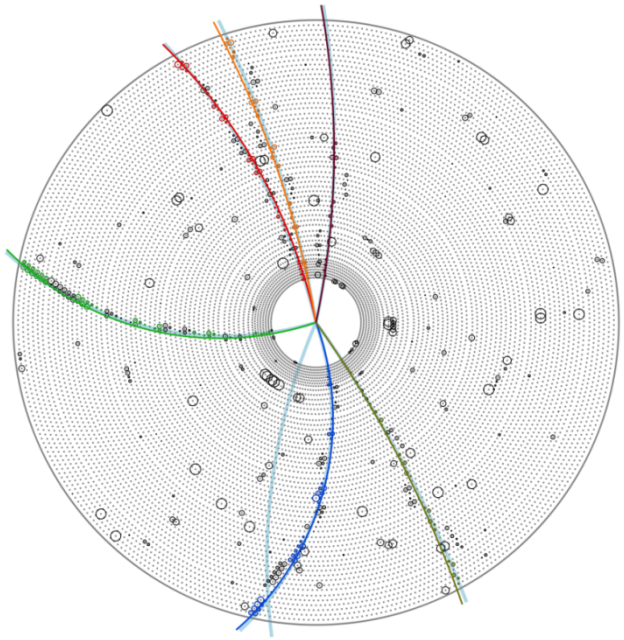


After

Actual results of algorithm

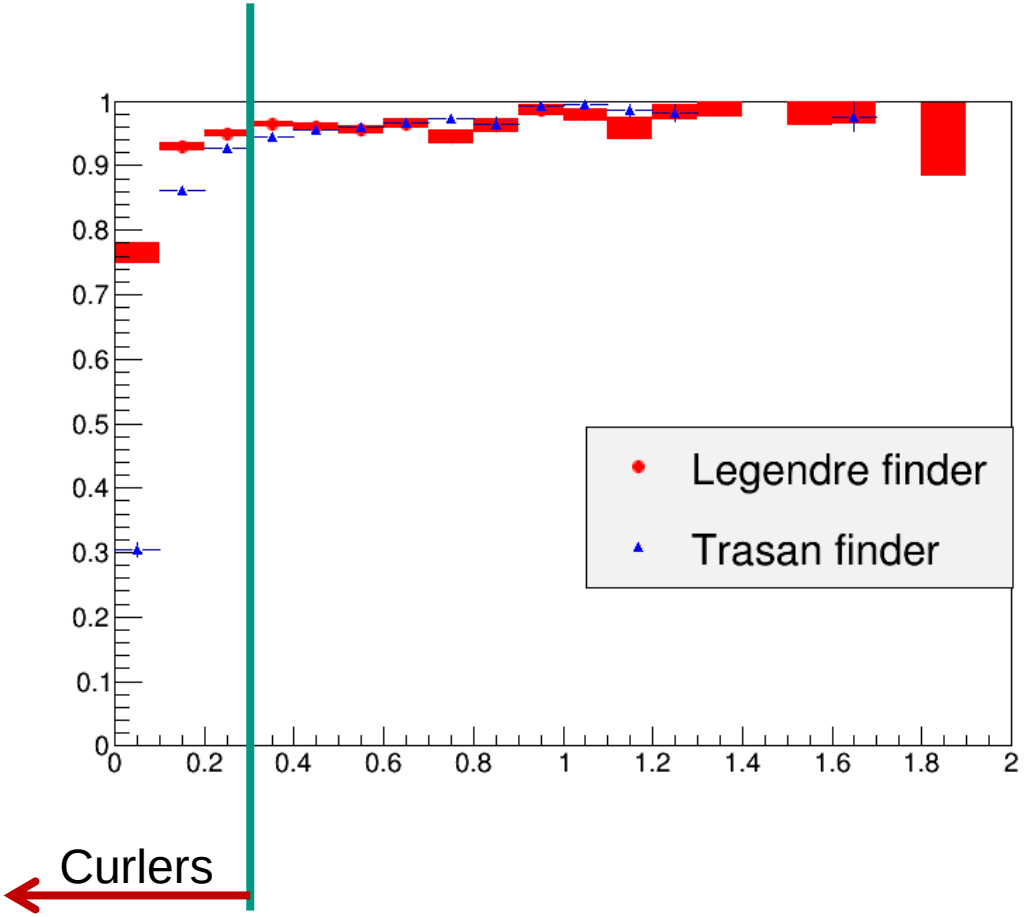
- Simulated event: $B^- \to D^0(\to K^- \pi^+)\, \pi^- + \text{beam background}$





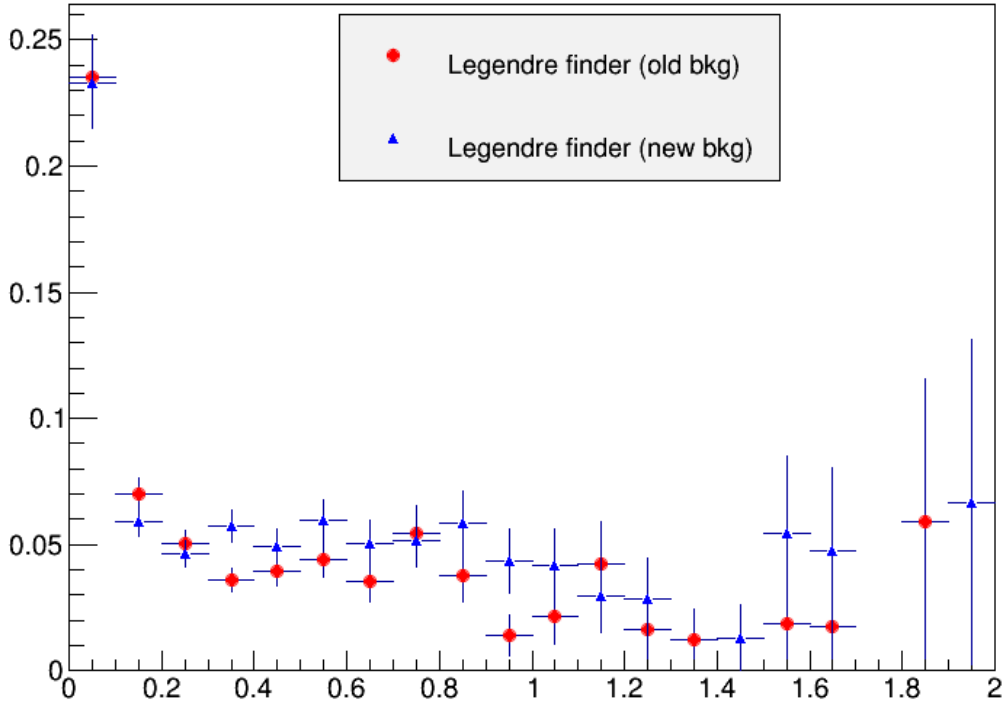
Efficiency

- Trasan vs Legendre finder efficiency (based on pre-generated sample of generic $B\overline{B}$ decays)



Inefficiency: dependence on background sample

- Comparison of inefficiency depending on background sample
  - Old sample: summer 2013 (Coulomb+RBB+Touschek)
  - New sample: recent simulation (Coulomb+RBB+Touschek+twoPhoton)
- Changing in inefficiency has visible effect



Conclusion

- The method can perform fast track finding
- Limited to tracks originating from IP
- Highly efficient for track finding
- Works well with high-pt tracks

Thank you for attention!