Report Assignment III - Trajectory Control

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1 Minimum Time Trajectory

Given q_0, q_f, $\dot{q}_{max}^{joint}$ and $\ddot{q}_{max}^{joint}$, the fastest trajectory for a single joint will either be of triangular (1) or of trapezodial (2) profile.

$$\dot{q}_{max}^{joint} \leq \sqrt{\ddot{q}_{max}^{joint} * (q_f - q_0)} \tag{1}$$

$$\dot{q}_{max}^{joint} > \sqrt{\ddot{q}_{max}^{joint} * (q_f - q_0)} \tag{2}$$

1.1 Triangular Trajectory

To find the time t_b, where the joint will stop accelerating and start deaccelerating we solve:

$$t_b = \sqrt{\frac{q_f - q_0}{\dot{q}}_{max}} \tag{3}$$

From t_b we can find the final time:

$$t_f = 2t_b \tag{4}$$

The angle at a specific point in time $t_0 \leq t \leq t_f$ is given by:

$$q(t) = \begin{cases} q_0 + \frac{1}{2}\ddot{q}_{max}t^2 & \text{if } t_0 \leq t \leq t_b \\ q_f + \frac{1}{2}\ddot{q}_{max}(t - t_f)^2 & \text{if } t_b \leq t \leq t_f \end{cases} \tag{5}$$

1.2 Trapeziodial Trajectory

In contrast to a triangular trajectory, the joint will move with $\dot{q}_{max}$ for a time span τ. As here it is guaranteed to hit $\dot{q}_{max}$, t_b will be found:

$$t_b = \frac{\dot{q}_{max}}{\ddot{q}_{max}} \tag{6}$$

We find T, the point where the joint will start to deaccelerate:

$$T = \frac{q_f - q_f}{\dot{q}_{max}} \tag{7}$$

As a trapez is symmetrical, we get the final time as:

$$t_f = T + t_b \tag{8}$$

The time span of maximum velocity is given by:

$$\tau = T - t_b \tag{9}$$

Finally, the angle at a specific point in time $t_0 \leq t \leq t_f$ is given by:

$$q(t) = \begin{cases} q_0 + \frac{1}{2}\ddot{q}_{max}t^2 & \text{if } t_0 \leq t \leq t_b \\ q_0 + \frac{1}{2}t_{max}^2 + \dot{q}_{max}(t - t_b) & \text{if } t_b \leq t \leq T \\ q_f + \frac{1}{2}\ddot{q}_{max}(t - t_f)^2 & \text{if } T \leq t \leq t_f \end{cases} \tag{10}$$

2 Synchronization

2.1 Abstract

As most robots consist of more than one joint, the motion of the n joints must be synchronized, so that all joints finish thir motion at the same time. To archieve this, the acceleration and velocities may need to be adjusted.

2.2 Prodecure

First of all the trajectories of all joint moving independently must be calculated to obtain their rise time t_b and their τ. As the robot is only as fast as the slowest part in each aspect, we get the new parameters as:

$$t_b^{new} = max(t_b^i), 1 \leq i \leq n \tag{11}$$

$$\tau_{new} = max(\tau_i), 1 \leq i \leq n \tag{12}$$

The total motion time follows as:

$$t_f^{new} = 2t_b^{new} + \tau_{new} \tag{13}$$

For each joint calculate their new velocities and accelerations as:

$$\dot{q}_{max}^{new} = \frac{q_f - q_0}{t_f - t_b} \tag{14}$$

$$\ddot{q}_{max}^{new} = \frac{\dot{q}_{new}}{t_b} \tag{15}$$

3 Numerical Optimization

3.1 Abstact

As robots excecute tasks in cycles with a specific frequency, the commands for accelerate and deaccelerate should be adapted to the frequency on instructions. This may increase the time, the robot needs to perform a move, but ensures, that everything will be executed correctly. The frequency f defines the time, between to commands.

3.2 Procedure

The new t_b will be to next cycle. We calculate:

$$t_b^{num} = (\lfloor \frac{T - \tau}{f} \rfloor + 1) * f \tag{16}$$

To calculate the new τ we use:

$$\tau_{num} = (\lfloor \frac{\tau}{f} \rfloor + 1) * f \tag{17}$$

Note, that T and τ in (16), (17) are used from the non-optimized trajectory.

From this equations we can find:

$$T_{num} = t_n^{num} + \tau_{num} \tag{18}$$

And

$$t_f^{num} = 2t_b^{num} + T_{num} \tag{19}$$

As velocities and acceleration will most likely change, we calculate the new values as:

$$\dot{q}_{num}^{max} = \frac{t_f - t_0}{T_{new}} \tag{20}$$

And

$$\ddot{q}_{num}^{max} = \frac{\dot{q}_{num}^{max}}{t_b^{new}} \tag{21}$$

Finding the angle at a specfic point in time $t_0 \leq t \leq t_f^{num}$ is almost the same as in a "normal" trapeziodial trajectory:

$$q(t) = \begin{cases} q_0 + \frac{1}{2}\ddot{q}_{max}^{num} t^2 & \text{if } t_0 \leq t \leq t_b^{num} \\ q_0 + \frac{1}{2}t_{max} num^2 + \dot{q}_{max}^{num}(t - t_b^{num}) & \text{if } t_b 6^{num} \leq t \leq T^{num} \\ q_f + \frac{1}{2}\ddot{q}_{max}^{num}(t - t_f^{num})^2 & \text{if } T^{num} \leq t \leq t_f^{num} \end{cases} \tag{22}$$

4 3 Point Trajectory

4.1 Abstract

Here the robot will move from a point A to a point C passing through point B. In this case the robot will come to a stop at point B ($\dot{q}_{t_B} = 0$), even though in practical use it is always better to keep a steady velocity throughout the trajectory. Furthermore only one joint will be moving to bring the robot from A to C.

4.2 Computuation

4.2.1 Constrains

$$q(t_A) = \theta_A \tag{23}$$

$$q(t_C) = \theta_C \tag{24}$$

$$\dot{q}(t_A) = 0 \tag{25}$$

$$\dot{q}(t_C) = 0 \tag{26}$$

4.2.2 Symbolic equations

Since we have 4 constrains, they can only be satisfied by at least a 3^{rd} order polynomial. We get:

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{27}$$

$$\dot{q}(t) = a_1 + 2a_2 t + a_3 t^2 \tag{28}$$

$$\ddot{q}(t) = 2a_2 + 6a_3 t \tag{29}$$

Solving these eqations from point A to B and then from B to C will get us the trajectory from A to C via B.

4.3 Examplary procedure

In this example, we move joint 1 of the robot from $q_0 = 2°$ to $q_f = -10°$ via $q_b = 15°$. The needed time should be $t_{AB} = t_{BC} = 2s$. This example will also be in the submitted code.

4.3.1 A - B

Solving for t = 0, gives:

$$q(0) = q_0 = 2 = a_0 + a_1(0) + a_2(0^2) + a_3(0^3) \tag{30}$$

$$\dot{q}(0) = 0 = a_1 + 2a_2(0) + 3a_3(0^2) \tag{31}$$

We get $a_0 = 2$ and $a_1 = 0$. To get a_3 and a_4 we solve for t=2:

$$q(2) = q_b = 15 = 2 + a_2(2^2) + a_3(2^3) \tag{32}$$

$$\dot{q}(2) = 0 = 2a_2(2) + 3a_3(2^2) \tag{33}$$

We get $a_2 = 9.75$ and $a_3 = -3.25$

4.3.2 B - C

Applying the same calculations for the trajectory from B to C we get $\tilde{a}_0 = 15$, $\tilde{a}_1 = 0$, $\tilde{a}_2 = -18.75$, $\tilde{a}_3 = 6.25$

4.3.3 Final Trajectory

Putting all of this toghether we get the following trajectory for $0 \leq t \leq 2$:

$$q(t) = 2 + 9.75t^2 - 3.25t^3 \tag{34}$$

$$\dot{q}(t) = 19.5t - 9.75t^2 \tag{35}$$

$$\ddot{q}(t) = 19.5 - 19.5t \tag{36}$$

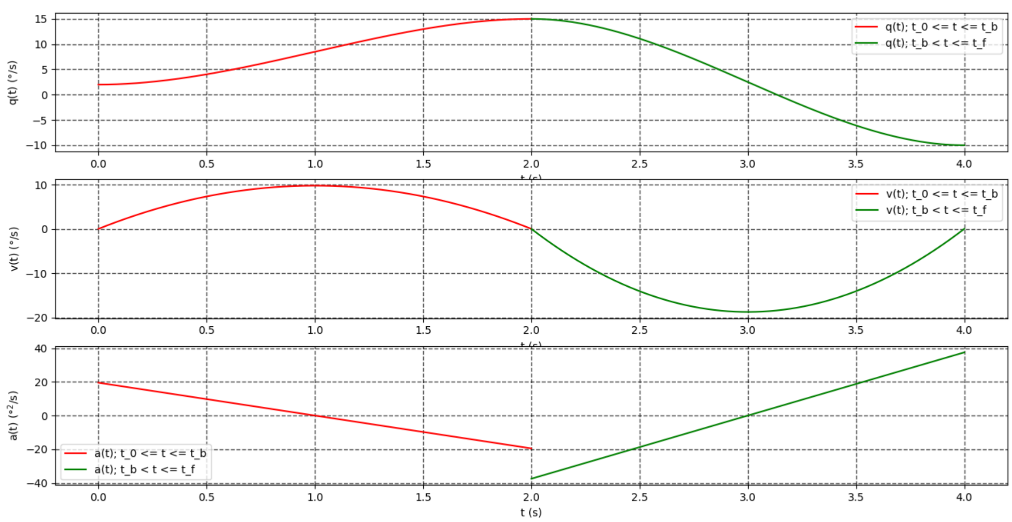
And for $2 < t \leq 4$:

$$q(t) = 15 + -18.75(t - t_b)^2 + 6.25(t - t_b)^3 \tag{37}$$

$$\dot{q}(t) = -37.5(t - t_b) + 18.75(t - t_b)^2 \tag{38}$$

$$\ddot{q}(t) = -37.5 + 37.5(t - t_b) \tag{39}$$

This will result in the following plot:



5 Code Annotations

As my code does not follow the provided code skeleton for every method, I will provide a brief overview of the methods. At the bottom of the submitted Python file, you will find test-dummies which can be used to display the solution.

5.1 Trajectory Time

This method is equivalent to the skeleton.

5.2 Time Sync

This method will calculate $\dot{q}_{max}^{new}$ and $\ddot{q}_{max}^{new}$ for each joint and will return a updated q params list as well as calculate, update and return the new time parameters. Parameters are a list of all q parameters.

5.3 Numerical Sync

This method will firstly synchronize all trajecories using time sync. After that new time parameters will be calcualted depending on the frequency of instructions. With the new time parameters, the $\dot{q}_{max}^{num}$ and $\ddot{q}_{max}^{num}$ are calculated. The method will return the new time parameters as well as a list of updated q parameters for each joint. Parameters are a list of all q parameters as well as the frequency of instructions (if no frequency is given the default of $\frac{1}{100}$ will be used).

5.4 Trapeziodal Trajectory

This method is equivalent to the skeleton.

5.5 Plot Trajectory

This method plots a TRIANGULAR or TRAPEZODIAL trajecory. It will get all relevant points to plot from trapeziodal trajectory. Note, that this method will only display $q(t), \dot{q}$ and $\ddot{q}$ for a single joint. This method returns nothing and takes the q - and t parameters as parameters.

5.6 3p Trajectory

This method will calculate a trajectory of a joint between 2 points by passing another point on the way. It will automatically plot the trajectory automatically and will return nothing. Parameters are: The q parameters of the joint, the value of the position to be passed in degrees, as well as t_0, t_b and t_f