Using Complex Shifts in Rayleigh Quotient Iteration to Compute Close Eigenvalues

Nils Friess

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UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

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Symmetric Eigenvalue Problem

We want to compute one eigenpair $(\lambda, \mathbf{v})$ of a real symmetric $n \times n$ matrix $\boldsymbol{A}$

$$\boldsymbol{A}\mathbf{v} = \lambda \mathbf{v} \,.$$

Assumptions

(A1) $\boldsymbol{A}$ is large and sparse.

(A2) A good approximation of $\mathbf{v}$ is available.

(A3) λ lies in the interior of the spectrum.

(A4) The eigenvalues around λ are closely spaced.

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Power method

Algorithm: Power method

Input: Nonzero unit vector $\mathbf{x}^{(0)}$

for $k = 1, 2, \ldots$ **until convergence do**

  $\mathbf{x}^{(k)} \leftarrow \boldsymbol{A}\mathbf{x}^{(k-1)}$

  Normalise $\mathbf{x}^{(k)}$

Converges to $\mathbf{v}_n$ under mild assumptions.

Converges linearly with rate

$$\rho = \frac{\lambda_{n-1}}{\lambda_n}$$

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Shifted Inverse Iteration

Algorithm: Shifted Inverse Iteration

Input: Nonzero unit vector $\mathbf{x}^{(0)}$

for $k = 1, 2, \ldots$ **until convergence do**

Solve $(\boldsymbol{A} - \mu \boldsymbol{I})\mathbf{x}^{(k)} = \mathbf{x}^{(k-1)}$

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