R&D Misallocation and the Growth Slowdown*

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Abstract

This paper identifies worsening R&D allocative efficiency as a potential driver of declining US economic growth. Within a simple endogenous growth framework, I develop a closed-form solution of the growth rate that can be decomposed into a frontier growth-rate, only achievable with the growth-maximizing resource allocation, and an allocative efficiency measure, measuring the gap between realized and frontier growth. Combining model with data on the innovation activity of US firms I estimate that allocative efficiency declined significantly from 1975 to 2014. Comparing the 1975-94 period to the 2005-14 period, I find that declining allocative efficiency can account for a 25%-40% lower economic growth in the latter period, which can explain the entire concurrent decline in economic growth as documented in the literature. I discuss potential drivers of declining allocative efficiency including waning federal support of R&D, institutional and technological change, and increasing labor market power over inventors.

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1 Introduction

The rate of productivity growth has declined in the past 20 years. From 1975 to 1994 productivity grew at a pace of 0.7 p.p. per year, compared to an annual from rate of 0.5 p.p. for the 2005-2020 period.[1] Through the lens of workhorse growth models, this decline could be driven by two factors: resources allocation to research and development (R&D) and the economies efficiency in translating these resources into economic growth:

$$\text{Growth } (\downarrow) = \text{R\&D Resources } (-/\uparrow) \times \text{R\&D Productivity } (\downarrow)$$

Empirically, the US has invested a relative stable share of its output into R&D, implying that declining growth is driven by declining aggregate R&D productivity (Bloom et al., 2020).

In this paper, I provide evidence suggesting that a significant fraction of declining aggregate R&D productivity due to rising resource misallocation in the R&D sector, explaining the majority of the growth slowdown. My evidence suggests that some firms conduct too much, while others conduct too little R&D, and increasingly so. I hence argue that declining aggregate R&D productivity is not only driven by declining R&D productivity at the micro level, as argued for example in Bloom et al. (2020) and Olmstead-Rumsey (2022), but also by increasing misallocation of R&D resources across innovative firms.

At the core of the paper is a simple decomposition of the economic growth rate into a frontier growth rate, only achievable by optimal R&D resource allocation, and an allocative efficiency adjustment term measuring the impact of frictions on economic growth. The closed form solution for the latter allows me to treat the model as an accounting framework for economic growth that can be directly applied to data on the R&D sector.

I derive these formulae in a simple endogenous growth model nesting workhorse models in the literature (Romer, 1990; Aghion and Howitt, 1992; Acemoglu and Cao, 2015). In the model, a fixed mass of innovative firms with potentially different *R&D productivity* hire inventors to maximize the private value created from innovation. I introduce private frictions flexibly by allowing for exogenous *R&D wedges* in firms' first-order conditions. Finally, growth occurs as a by-product of innovation, however, I allow for a potential gap between the private value created from innovation and its growth impact, which I will refer to as *impact-value wedge*, that captures the concerns about externalities emphasized in the growth literature.

[1] I calculate these number using the TFP growth measures from Comin et al. (2022) . I annualize the quarterly growth rate as $g_{ann} = (1 + g_{qrt})^4 - 1$.

Within this framework, I derive a closed form solution of the economic growth rate depending on the joint distribution of R&D productivities, R&D wedges, and impact-value wedges. The formula can be decomposed into a frontier growth rate, which reflects the growth-maximizing resource allocation, and an allocative efficiency adjustment term. In absence of heterogeneity in impact-value wedges, allocative efficiency is low if there is significant variation in R&D wedges as this pushes the distribution of relative R&D efforts away from their growth-maximizing optimum. With heterogeneity in impact-value wedges the same insight applies to adjusted R&D wedges, which take into-account the gap between private incentives and growth impact of innovation. One interpretation of this result is a recipe for optimal subsidies, where the planner should choose subsidies, which are equivalent to R&D wedges, to offset the dispersion in incentive alignment. These formulae thus give us a tool to understand the impact of resource allocation on economic growth.

Next, I measure the relevant parameters using data on patents and R&D expenditure of US listed firms from 1975 to 2014. I measure the private value created from R&D using patent valuations and construct a proxy for the growth-impact of an innovation using citation measures. Together with information on R&D expenditures, these data allow me to measure the relevant model parameters through the lens of the model and, thus, estimate allocative efficiency over time.

Combining model and data, I estimate that allocative efficiency has declined remarkably over time. Comparing the 1975-94 period to the 2005-14 period, I find that declining allocative efficiency can explain essentially the entire decline in economic growth documented in the literature, driven by rising heterogeneity in R&D and impact-value wedges. Inspecting the year-by-year estimates of allocative efficiency, I find a relatively steady decline. Interestingly, the implied frontier growth rate is slightly increasing over time, which echoes the findings in Brynjolfsson et al. (2019), who argue that technology progress is picking up, but that the economy fails to take advantage of them.

Finally, the average level of allocative efficiency suggests a large gap between the realized and frontier growth rate. My main specification suggests that the US is only achieving 60% of its frontier growth rate, which similar the findings in Hsieh and Klenow (2009), who found large cost of resource misallocation in the production sector. Naturally, it remains an open question to which degree the measured misallocation is preventable. I shed some light on this question in the companion paper Lehr (2022b), which investigates the link of R&D wedges to financial frictions and labor market power in the market for inventors. The evidence therein suggests that the documented private frictions are partly due to monopsony power, while investment and financial frictions appear to play little to no role. These findings suggest a potential role for policy to tap into the growth potential and move the innovation sector closer to the frontier again.

Literature. This paper contributes to three strands of the literature. First, a growing literature documents the recent decline in the economic growth rate and investigates its origins.[2] Similar to Bloom et al. (2020) and Olmstead-Rumsey (2022), I argue that declining aggregate R&D productivity is at the heart of the decline, however, I attribute this change to rising R&D misallocation instead of declining R&D productivity at the micro-level, which is similar to the perspective advanced in Aghion et al. (2022b) and de Ridder (2021).

Second, I provide new findings potentially linked to the drivers of aggregate R&D resources allocation and productivity.[3] My framework allows for a closed-form decomposition of the economic growth rate and clarifies how private frictions and externalities shape economic growth through their impact on the allocation of R&D resources across firms. My estimates suggest that these frictions matter quantitatively. These findings are closely connected to de Ridder (2021) and Aghion et al. (2022a), who argue that heterogeneity in growth externalities across firms lead to quantitatively important R&D misallocation.[4] My framework is able to speak to these findings, but takes a more holistic, and admittedly abstract, approach by quantifying the importance of misallocation in the R&D sector due to private frictions and externalities jointly for US growth.

Third, the documented R&D return dispersion speaks to the literature on factor misallocation.[5] Focusing on R&D instead of static production factors such as capital and labor introduces a dynamic component linking factor return heterogeneity to the growth rate instead of static production efficiency. Nonetheless, my framework allows for a closed-form solution and direct link to the data as developed in Hsieh and Klenow (2009) for the production sector. My estimates suggest that misallocation is not only large in the production

[2]See e.g. Gordon (2016); Syverson (2017); Bloom et al. (2020); Akcigit and Ates (2021); de Ridder (2021); Aghion et al. (2022b,a); Olmstead-Rumsey (2022); Liu et al. (2022)

[3]For related literature, see e.g. Romer (1990); Aghion and Howitt (1992); Acemoglu and Cao (2015); Acemoglu et al. (2018); Akcigit and Kerr (2018a); Peters (2020); de Ridder (2021); Aghion et al. (2022b); Terry (2022).

[4]Closely related, Akcigit et al. (2022) investigate optimal policy in a model with heterogeneous externalities and private frictions using a mechanism design approach.

[5]See Restuccia and Rogerson (2008); Hsieh and Klenow (2009); Midrigan and Xu (2014); David et al. (2016); David and Venkateswaran (2019); David et al. (2021) for more recent advances.

sector, but also in the R&D sector.

Fourth, my measure of R&D frictions builds on recent advances in the literature on measuring innovation. [6] I quantify the private frictions using the patent valuation measure developed in Kogan et al. (2017) and use the text-based patent-impact measure developed in Kelly et al. (2021) to estimate public-private incentive misalignment. I contribute to the literature by developing a mapping from data to core economic parameters in a simple economic growth model that provide important insights on resource allocation in the R&D sector. My findings highlight the importance of understanding the empirical distribution of R&D resources for the study of economic growth and development of innovation policy.

Organization. The remainder of this paper is structured as follows. Section 2 introduces the theory and develops the main formulas. Section 3 introduces the data and discusses measurement of core model parameters. Section 4 combines data and theory to investigate the role of resource allocation for economic growth. I conclude in Section 5.

2 Theory

In this section I introduce a simple model of endogenous growth that nests workhorse models in the literature. Within this model I derive a closed form solution for the growth rate, which can be decomposed in a frontier growth rate and an allocation efficiency term.

2.1 Model Setup

Time is infinite, discrete, and indexed by t. There is a unit mass of workers in the economy.

Static production. Aggregate output per capita in the economy is given by

$$Y_t = A_t(1 - L), \tag{1}$$

where A_t is aggregate productivity and $1-L$ is the production labor input. Note that A_t can encompasses technological efficiency as well as static production frictions such as markups.[7]

[6] For earlier literature, see Lerner (1995); Hall et al. (2001, 2010).

[7] For example, Peters (2020) derives a formulation in a similar setup that further decomposes production efficiency into a term depending on markup heterogeneity and a technological efficiency term. My results are compatible with this interpretation as long as markup heterogeneity is constant and independent of other frictions.

I will focus on its evolution and assume that changes over time are driven by changes in technological efficiency only.

Firms. There is a unit mass of innovative firms indexed by i. Firms assign value V_{it} to successful innovations, which I will take as given.[8] Firms hire R&D input ℓ_{it}, which could be a composite good, at input price W_t to achieve innovation arrival rate z_{it}:

$$z_{it} = \varphi_{it} \ell_{it}^{\frac{1}{1+\phi}}. \tag{2}$$

Finally, firms are subject to first-order condition wedge Δ_{it} such that their equilibrium labor choice ℓ_{it}^* satisfies

$$\frac{\partial z_{it}}{\partial \ell_{it}}\Big|_{\ell_{it}=\ell_{it}^*} V_{it} = (1 + \Delta_{it}) \times W_t. \tag{3}$$

Note that the left-hand side is simply the marginal benefit of research input, while the right-hand side is the marginal cost adjusted for the R&D wedge. If $\Delta_{it} = 0$, we recover the frictionless benchmark in which firms equalize marginal benefit to marginal cost. Otherwise, firms' choices are distorted relative to the benchmark with larger level of Δ_{it} implying too little demand for R&D resources and vice versa. There are many potential interpretations of Δ_{it} including adjustment frictions, financial frictions, labor market power, and R&D subsidies. Importantly, these frictions need to affect the firms' R&D input choices.

Factor markets. I assume that the input factor is fixed at the aggregate level:

$$L = \int_0^1 \ell_{it} di. \tag{4}$$

This assumption captures that research talent is scarce and supplied relatively inelastically (Goolsbee, 2003; Wilson, 2009; Akcigit et al., 2017). Mechanically, this assumption focuses the attention on the allocation within the R&D sector.[9]

Growth. I assume that the value created by a firm is linked to its productivity impact via impact-value wedge ζ_{it}, which acts as an exchange rate between value received by the firm and the growth impact of an innovation. Firms with a large impact-value wedge produce

[8]In workhorse endogenous growth models, this value is linked to discounted future profits and innovation opportunities. I derive explicit formulation under a range of models in Appendix D.

[9]Another way to think about this assumption is that policy already fixed the optimal amount of R&D workers in the economy and that the following results thus only concern the allocation of talent across firms and not across the production and research sector.

more growth per dollar of private value created and vice versa. The growth rate is given by

$$g_t \equiv \frac{\dot{A}_t}{A_t} = \int_0^1 \zeta_{it} \cdot z_{it} \cdot V_{it} \cdot di. \tag{5}$$

Note that ζ_{it} has a prominent role in the endogenous growth literature as it determines the degree to which firms' incentives are aligned with a growth-maximizing planner. Variation therein can arise e.g. because some firms are better at taking advantage of their inventions or due to heterogeneity in the innovation quality and knowledge externalities .[10]

Closing the model. I will focus exclusively on innovation sector in this economy and thus forgo an explicit household sector. Accordingly, I will use a simplified equilibrium definition.

Definition 1. *A Growth Equilibrium is a sequence* $\{\{V_{it}, \varphi_{it}, \Delta_{it}, \zeta_{it}, \ell_{it}\}_{i\in[0,1]}, W_t, g_t\}_{t=0,\ldots,\infty}$ *satisfying equations (2)-(5).*

2.2 Results

With the model in place, we can derive the main result of this section in Proposition 1.

Proposition 1. *Under equations (2)-(5), we can express the economic growth rate as the product of two terms:*

$$g_t = \tilde{g}_t \times \Xi_t, \tag{6}$$

where $\tilde{g}_t$ *measures the growth frontier, i.e. the growth-rate in absence of frictions and* Ξ_t *measures the allocative efficiency, i.e. the fraction of maximal growth-rate that the economy is actually achieving. The formula for the latter is given by*

$$\Xi_t = \frac{\int_0^1 \tilde{\omega}_{it} \left((1+\Delta_{it}) \cdot \zeta_{it}\right)^{-\frac{1}{\phi}} di}{\left(\int_0^1 \tilde{\omega}_{it} \left((1+\Delta_{it}) \cdot \zeta_{it}\right)^{-\frac{1+\phi}{\phi}} di\right)^{\frac{1}{1+\phi}}} \quad \text{with} \quad \tilde{\omega}_{it} = \frac{(\gamma_{it}\zeta_{it})^{\frac{1+\phi}{\phi}}}{\int_0^1 (\gamma_{jt}\zeta_{jt})^{\frac{1+\phi}{\phi}} dj}, \tag{7}$$

where $\gamma_{it} \equiv \varphi_{it} V_{it}$.

The proposition tells us that we can derive a single summary statistic for the impact of first-order condition frictions on the growth rate in this economy. To develop some intuition for this result, I will first introduce a simplified version in the next Corollary.

[10] For the former, see e.g. de Ridder (2021); Mezzanotti (2021); Aghion et al. (2022b,a). For the latter, see e.g. Akcigit and Kerr (2018b); Akcigit and Ates (2021).

Corollary 1. *Let the Impact-Value wedge be constant across firms, i.e. $\zeta_{it} = \zeta_t$, then allocative efficiency is given by*

$$\Xi_t = \frac{\int_0^1 \omega_{it} \left(1 + \Delta_{it}\right)^{-\frac{1}{\phi}} di}{\left(\int_0^1 \omega_{it} \left(1 + \Delta_{it}\right)^{-\frac{1+\phi}{\phi}} di\right)^{\frac{1}{1+\phi}}} \quad \textit{with} \quad \omega_{it} = \frac{\gamma_{it}^{\frac{1+\phi}{\phi}}}{\int_0^1 \gamma_{jt}^{\frac{1+\phi}{\phi}} dj}. \tag{8}$$

Allocative efficiency Ξ_t is independent of the common level of $1+\Delta_{it}$ with $\Xi_t = 1$ if $\Delta_{it} = \Delta_t$. Furthermore, $\Xi_t < 1$ if $\Delta_{it} \neq \Delta_{jt}$ for some i and j with $\omega_{it}, \omega_{jt} > 0$.

Corollary 1 reveals that under common incentive alignment, allocative efficiency depends only on the appropriately weighted distribution of R&D wedges. Two results stand out. First, in absence of any dispersion in the R&D wedge, we achieve maximal efficiency, regardless of the level of Δ. The latter is a direct implication of a fixed mass of R&D resources, which requires that factor price movements offset common levels of frictions. Second, dispersion in the R&D wedge shifts the allocation of R&D resources across firms away from their privately efficient distribution. This harms growth as firm and planner incentives are equally aligned across firms. Corollary 2 further establishes that the link between dispersion in the R&D wedges and allocative efficiency is monotonic up to a second order approximation.

Corollary 2. *If $\zeta_{it} = \zeta_t$, then, up to a second order approximation, Ξ_t is given by*

$$\Xi_t \approx \exp\left(-\frac{1}{2\phi} \cdot \sigma_\omega^2(\ln(1 + \Delta_{it}))\right), \tag{9}$$

where $\sigma_\omega^2(\cdot)$ is the weighted variance with observation weights $\{\omega_{it}\}$.

Returning to the main formula in Proposition 1 we can see that the results coincide except for an adjustment for differential incentive alignment across firms, such that dispersion in the adjusted R&D wedge $(1 + \Delta_{it})/\zeta_{it}$ reduces growth due to resource misallocation. The adjustment effectively translates private frictions into growth units, which can enhance or dampen their dispersion. For example, consider a severely financially constrained firm, which is excellent at translating its mediocre inventions into large market value. Financial constraints limit the firm's R&D, which implies a large R&D wedge, however, the firm's excessive ability to profit from innovation would have otherwise led it to engage in too much innovation. R&D and Impact-Value wedge thus might partially offset each other, which would lead one to over-estimate their impact on growth if considered separately. Naturally, the reverse can occur. One example might be startups, which are both financially constrained and less able to take advantage of their ideas than established firms.

This result also implies that there is efficient dispersion in Δ_{it}. In particular, we achieve the frontier growth-rate if $1+\Delta_{it} \propto 1/\zeta_{it}$. In this case, private frictions directly offset the incentive misalignment such that firms on net have the correct incentives. As I show in Appendix A.4, a growth-maximizing planner directly imposes this result via R&D subsidies. Corollary 3 further clarifies this result by showing that allocative efficiency is declining in the variance of the adjusted R&D wedge.

Corollary 3. *Up to a second order approximation, Ξ_t is given by*

$$\Xi_t \approx \exp\left(-\frac{1}{2\phi}\cdot\sigma^2_\omega(\ln(1+\Delta_{it})+\ln\zeta_{it})\right), \tag{10}$$

where $\sigma^2_\omega(\cdot)$ is the weighted variance with observation weights $\{\omega_{it}\}$.

We can decompose dispersion in the adjusted wedge further into:

$$\sigma^2_\omega(\ln(1+\Delta_{it})+\ln\zeta_{it}) = \sigma^2_\omega(\ln(1+\Delta_{it}))+\sigma^2_\omega(\ln\zeta_{it})+2\sigma_\omega((1+\Delta_{it}),\ln\zeta_{it}). \tag{11}$$

Thus, variation in the R&D wedge has a negative impact on growth as long as the correlation between the R&D and Impact-Value wedge is sufficiently low. Combined these results give us tools to investigate the importance of resource allocation across firms for economic growth. Furthermore, since the result does not require any notion of constant growth rate across time, they also allow us to directly investigate the importance of changing frictions to the evolution of the growth rate. What remains is to measure $\{\phi, \{\gamma_{it}, 1+\Delta_{it}, \zeta_{it}\}\}$.

3 Data and Measurement

This section introduces the data and discusses measurement of the model parameters.

3.1 Data

I estimate the level and evolution of allocative efficiency using data on US listed firms. I choose this sample as there is sufficient data available to measure the underlying model primitives to directly apply the formulas developed above. My approach to measurement requires three pieces of information: R&D expenditure, value created from R&D and growth impact of R&D.

I obtain annual firm-level R&D expenditure directly from WRDS Compustat, which collects the information from mandatory filings. In addition, this data reports information on industry classification of the firm, which will take advantage of as well.

I will rely on patents to measure both the private value created from R&D as well as their growth impact. Patents are arguably the most direct measure of R&D output available to researchers. They capture an invention that the issuing patent office, in my case the US Patent and Trademark Office (USPTO), deemed both new and useful. A patent grants the owner exclusive rights to the use of the invention described therein, which gives firms strong incentives to patent their inventions.[11]

I use patent valuation estimates from Kogan et al. (2017) to measure the private value created from innovation. An advantage patent valuations is that they directly capture the private value of an invention, which is directly linked to firms' incentives to innovate. In contrast, other patent-based measures of innovation such as (quality-adjusted) patent counts only capture the quantity of innovation, but not its value to the firm (Lerner, 1995; Kogan et al., 2017; Kelly et al., 2021). These concepts can diverge e.g. due to externalities or because some firms are better equipped than other to take advantage of an invention.[12]

I use forward-citations as my primary measure of patent growth impact motivated by a large literature arguing in favor of this interpretation (Lerner, 1995; Bloom et al., 2013; Akcigit and Kerr, 2018b). At the patent-level, I construct forward-citations, i.e. citations received, by the patent within the first 5 years since the patent grant using the combined citations files of USPTO Patentsview, Kogan et al. (2017), and Berkes (2016). I then normalize this measure by the average forward-citations within an application year to make the measure comparable across years as in Kogan et al. (2017).

I aggregate citations and patent valuation up the firm-level using the exclusive mapping between firms and patent developed in Kogan et al. (2017). Patents are recorded in their application year to reflect the timing of innovation. The final dataset thus has annual observations of total R&D expenditure, patent valuations, and forward-citations.

[11] Note that not all inventions are patented and, thus, patent valuation remain an imperfect measure (Cohen and Klepper, 1996)

[12] See e.g. Akcigit and Kerr (2018a); de Ridder (2021); Aghion et al. (2022b)

3.2 Measurement

Measurement turns out to be relatively straight-forward. First, there is a strong consensus in the literature on setting $\phi = 1$ (Acemoglu et al., 2018; Akcigit and Kerr, 2018a). This value implies an elasticity of R&D expenditure to unit cost around 1.

Second, we can measure the first order condition wedge up to a constant factor directly from the R&D return, i.e. the ratio of value created from R&D divided by its cost:

$$\frac{z_{it}V_{it}}{W_t \ell_{it}} = (1+\phi) \times (1+\Delta_{it}), \tag{12}$$

which I implement using 5-year windows with a 1-year lag between R&D expenditure and patent valuations:

$$\widehat{1+\Delta_{it}} \equiv \frac{1}{1+\phi} \times \frac{\sum_{s=0}^{4} \texttt{Patent Valuations}_{it+s}}{\sum_{s=0}^{4} \texttt{R\&D Expenditure}_{it-1+s}}. \tag{13}$$

Note that the relevant formulas are HD(0) in $1+\Delta_{it}$ such that the factor $\frac{1}{1+\phi}$ has no bearing on the aggregate measures. I restrict the sample to observations with at least 50 patents to create a measure of expected returns as required by the model.

Third, we can measure R&D productivity from the firms' first order conditions, which can be rearranged to

$$\gamma_{it} = (1+\Delta_{it}) \times (W_t \ell_{it})^{\frac{\phi}{1+\phi}} \times W_t^{-\frac{1}{\phi}}. \tag{14}$$

Note, again, that the formula for allocative efficiency is independent of the scale of γ_{it} such that I can drop the final wage intercept without loss of generality as it is common across all firms. I thus measure R&D efficiency as

$$\hat{\gamma}_{it} \equiv \widehat{1+\Delta_{it}} \times \left(\sum_{s=0}^{4} \texttt{R\&D Expenditure}_{it-1+s} \right)^{\frac{1}{1+\phi}}. \tag{15}$$

Finally, I measure the impact-value wedge as the ratio of patent citations to valuations. This measurement is accurate if citations measure the growth impact of an invention up to a constant factor, which is broadly in line with the interpretation in the existing literature.

$$\hat{\zeta}_{it} \equiv \frac{\sum_{s=0}^{4} \texttt{Patent Citations}_{it+s}}{\sum_{s=0}^{4} \texttt{Patent Valuations}_{it-1+s}}. \tag{16}$$

One potentially concern with the measurement approach developed above is unrelated industry heterogeneity. For the R&D wedge, differences in the scale elasticity ϕ across industries are a potential source of variation in the return on R&D that is independent of the R&D wedge. For the impact-value wedge, a more imminent threat is heterogeneity in citation conventions that affect the relative frequency of citations across industries even if growth impacts are comparable.[13] I will address both concerns by residualizing the respective parameters with respect to industry×year fixed effects.

Finally, following Proposition 1 I estimate allocative efficiency as

$$\hat{\Xi}_{t,adjusted} = \frac{\int_0^1 \hat{\omega}_{it} \left((\widehat{1+\Delta_{it}}) \cdot \hat{\zeta}_{it} \right)^{-\frac{1}{\phi}} di}{\left(\int_0^1 \hat{\omega}_{it} \left((\widehat{1+\Delta_{it}}) \cdot \hat{\zeta}_{it} \right)^{-\frac{1+\phi}{\phi}} di \right)^{\frac{1}{1+\phi}}} \quad \text{with} \quad \hat{\omega}_{it} = \frac{(\hat{\zeta}_{it}\hat{\gamma}_{it})^{\frac{1+\phi}{\phi}}}{\int_0^1 (\hat{\zeta}_{it}\hat{\gamma}_{jt})^{\frac{1+\phi}{\phi}} dj}. \tag{17}$$

Where useful I provide an alternative estimate assuming a constant impact-value wedge:[14]

$$\hat{\Xi}_{t,simple} = \frac{\int_0^1 \hat{\omega}_{it} (\widehat{1+\Delta_{it}})^{-\frac{1}{\phi}} di}{\left(\int_0^1 \hat{\omega}_{it} (\widehat{1+\Delta_{it}})^{-\frac{1+\phi}{\phi}} di \right)^{\frac{1}{1+\phi}}} \quad \text{with} \quad \hat{\omega}_{it} = \frac{\hat{\gamma}_{it}^{\frac{1+\phi}{\phi}}}{\int_0^1 \hat{\gamma}_{jt}^{\frac{1+\phi}{\phi}} dj}. \tag{18}$$

I collapse the annual estimates across periods using simple means. We can interpret them as the fractions of potential growth the economy actually achieved during the period.

4 Allocative Efficiency and its Evolution

In this section I combine theory and data to investigate the level and evolution of allocative efficiency in the US.

[13] For example, in some technology classes such as computer processors invention might be more cumulative over time, implying a large citation counts for important inventions in absolute terms. On the other hand, in other technology classes such as drugs, invention might be more independent on average, implying lower absolute citation count regardless of the respective growth impact.

[14] Under joint normality with non-constant impact-value wedge, this is also an estimate of the impact of private frictions only as long as they are independently distribution from the former.

4.1 Average Allocative Efficiency

The first column in Table 1 reports estimates of allocative efficiency for the full sample. The adjusted measure reports surprisingly low allocative efficiency suggesting that the US only achieved c. 60% of its growth potential over the period. Against a realized growth-rate of 1.5%, the estimate suggests a frontier growth-rate of 2.5%. The model thus estimates that US productivity would have been around 50% larger at the end of the sample if the US had achieved its frontier growth rate. The estimates are moderated by ignoring variation in the impact-value wedge, however, even the simple measure suggests that resource misallocation decreased US growth by around 0.6 p.p. per year.

Table 1: Estimating Allocative Efficiency

Measure	Period		
	1975-2014	1975-1994	2005-2014
Simple	0.704	0.785	0.592
	(0.013)	(0.019)	(0.018)
Adjusted	0.583	0.704	0.409
	(0.017)	(0.012)	(0.058)

Note: Estimates of allocative efficiency using Proposition 1. See text for variable definition. Allocative efficiency measures are first constructed at the annual measure and then averaged over the relevant sample using the geometric mean. The simple measure assumes a constant Impact-Value wedge. The adjusted measure uses citations to measure the growth impact of patents. Standard errors are reported in paranthesis and constructed via the Delta-method.

These growth-rate cost have large welfare implications. In Appendix A.4 I show how one can translate the growth cost into consumption-equivalent welfare cost, which measure the permanent change in the consumption level that yields to equivalent welfare gains as achieving the frontier growth-rate. For the adjusted (simple) measure, I estimate welfare cost around 70% (40%). For comparison, Hsieh and Klenow (2009) estimate welfare cost of capital misallocation for the US around 40%, while Berger et al. (2022) estimate that US output would be 21% larger in absence of monopsony power in the production sector.

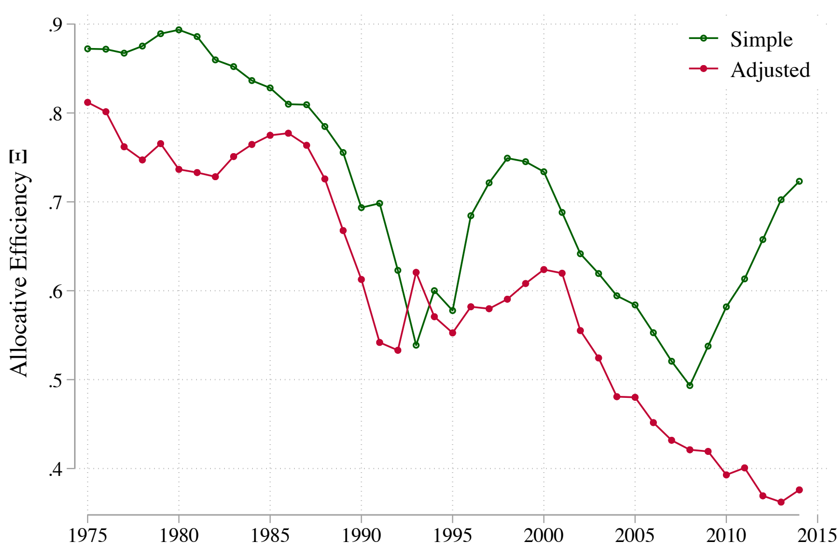
It goes without saying that not all R&D misallocation might be preventable. For example, Asker et al. (2014) argue capital misallocation is partly driven by technological constraints and adjustment frictions in particular. Similar arguments might apply in the case of R&D

misallocation. For example, in the companion paper Lehr (2022b) I argue that this misallocation is partly driven by friction in the market for inventors, which a policy maker might not be able to fully address.

4.2 The Evolution of Allocative Efficiency

Figure 1 plots the annual estimates of allocative efficiency and reveals a relatively steady decline across both measures. The most notable difference is that the simple measure experiences a strong rebound after 2008, while the adjusted measure continues its decline. Furthermore, the simple measure estimates a temporary improvement during the dot-com boom that is absent in the adjusted measure.

Figure 1: Allocative efficiency has declined steadily since 1980



Notes: This figure plots annual estimates of allocative efficiency using the formulas developed in Corollary 1 and Proposition 1. The simple measure assumes a constant Impact-Value wedge across firms, in contrast to the adjusted measure.

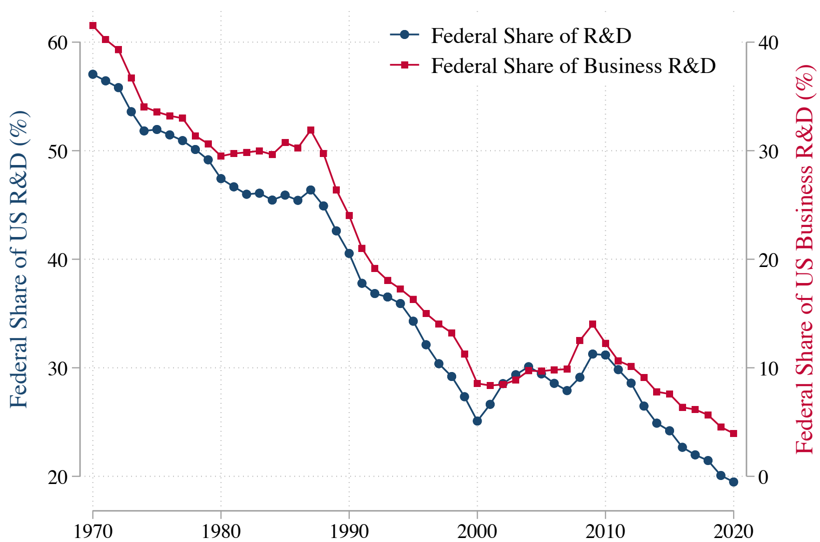
The magnitudes of the decline are remarkable. The simple and adjusted measure start out at around 80% for the first decade from 1975 to 1984 followed by a strong decline toward c. 60% in 1995. The overall decline hold on for both measures until around 2008 at which point they are at or below 50%. In the final decade, there is a strong divergence.[15] While the simple measure rebounds to above 70% in 2014, the adjusted measure continues its decline to ultimately less than 40%. Taking simple averages, column (2) and (3) in Table 1 suggest

[15]This divergence arises as the correlation between the two wedges becomes less negative in the final years. I documented this in greater detail in Appendix C.4.

that allocative efficiency declined 20-30 percentage points when comparing the 1975-94 and 2005-2014 period. All else equal, these estimates suggest that rising misallocation should have led a decline in the annual growth-rate by 25%-40%, which is on the same order of magnitude as the observed decline in economic growth or larger.

The strong decline throughout the sample raises the question as to which forces are driving the observed pattern. A range of alternative explanations are possible. First, it is well documented that federal involvement in R&D decreased significantly during the sample period. As shown in Figure 2, the share of federal in total R&D expenditure declined from 45% in 1975-94 to 29% in 2005-14. Even more important, the share of business R&D funded by the federal government declined from 28% in the early period to 11% in the late period. It is possible that this decline reduced the governments ability to guide private R&D towards high impact innovation, especially those whose benefits are not as easily captured by innovating firms.[16] Alternatively, these research funds might have alleviated potential financial constraints or given firms access to a larger pool of R&D talent.

Figure 2: Federal Support of R&D Has Decline Since 1975



Notes: Author's calculations based on NSF National Pattern.

Second, the growing dispersion in the impact-value wedge could be due to changing institutional environment or technology. A growing literature is arguing the patent system itself and changes therein have deteriorated US innovation infrastructure due to litigation risk,

[16]One example here could be so-called fundamental research, which lays the groundwork for future innovation, but does not always immediately lead to new products. Akcigit et al. (2020) estimate that such investments play an important role in the innovation environment.

blocking of follow on innovation, and giving firms incentives to patent even minor inventions (Jaffe and Lerner, 2007; Kim, 2017; Mezzanotti, 2021). These changes directly impact the gap between the private value and growth impact of an innovation and, thus, could have contributed to rising dispersion in the impact-value wedge. On the technology side, de Ridder (2021) and Aghion et al. (2022b) argue that the IT revolution has created a new class of firm that is able to profit more from its innovation to due static productivity advantages linked to IT investments and data. These differences naturally would also be reflected in the impact-value wedge.

Finally, another possibility is that the superstar firm phenomena and the associated rise in concentration has increased firm's market power in the market for inventors. Labor market power can increase misallocation as firms artificially lower inventor demand to keep wages low (Autor et al., 2020; Seegmiller, 2021; Berger et al., 2022; Yeh et al., 2022).[17] In Lehr (2022a) I provide evidence that firms indeed have market power in the market for inventors and that this is linked to R&D wedges. I propose a model that links market power to firm size as in Card et al. (2018) with the implication that rising dispersion in R&D activity indeed leads to lower allocative efficiency.

4.3 Discussion

Measurement error. An important challenge when measuring the underlying model parameter is measurement error, which would naturally inflate the dispersion in (adjusted) R&D wedges and thus decrease measured allocative efficiency. I propose and implement a parametric approach to estimating the importance of measurement error in Appendix C.1. Perhaps surprisingly, I find that it contributes a small fraction of the overall variance in measured R&D wedges. My largest estimate suggests a contribution of 2.6%.

Magnitudes. The level of estimated allocative efficiency is surprisingly low a first, especially for the adjusted measure. Comparison with the literature, however, puts this into context. Using a similar research design, Hsieh and Klenow (2009) estimate that US output could be 40% larger in absence of capital misallocation in the production sector, which is

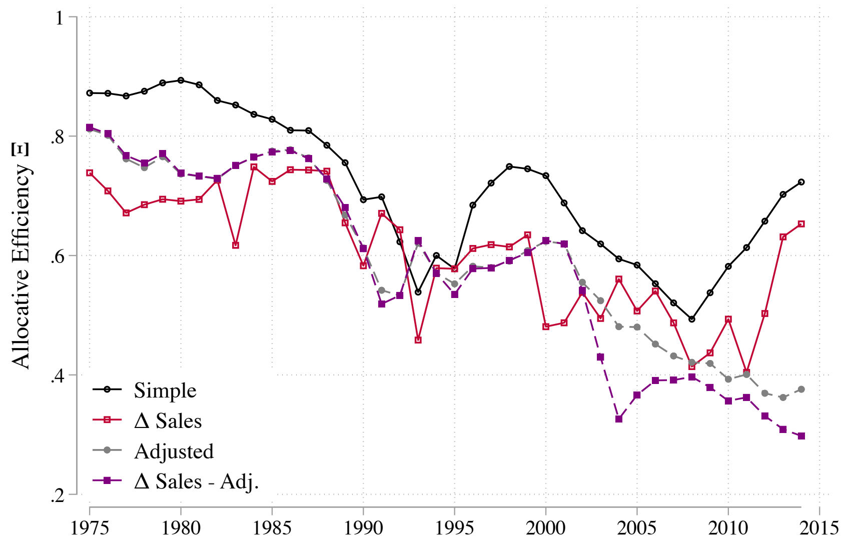
[17] Berger et al. (2022) propose a model according to those lines, however, they find that local concentration, which is the relevant statistic for firms' market power in their model, has decreased for workers. Arguably the market for inventors is more structured by human capital specificity rather than geography, opening the possibility that the associated trends work in the opposite direction. Note also that R&D expenditure and patent valuations have not experienced the same rising concentration as documented in Autor et al. (2020). Appendix Figure C.4 shows that the Top 20 share of R&D expenditure and patent valuations has remained approximately constant in my sample.

close to my unadjusted estimate. The gap to the full estimate is then attributable to measured misalignment of innovation incentives, which other papers have argued to be significant as well (de Ridder, 2021; Aghion et al., 2022a). Nonetheless, healthy skepticism might be useful when considering the level of measured misallocation. That being said, the evolution of measured misallocation is even more surprising as my estimate suggest that allocative efficiency has essentially halved during my sample. I discuss below how we can explain the less dramatic decline in economic growth through the lens of the model as well as how alternative measurement choices affect the picture. The estimated magnitudes suggest two alternative interpretations: either, misallocation is a much larger problem than previously recognized by the literature and requires significantly more attention by the literature, or, our models are a surprisingly bad description of the world and more research is required to bring model and data closer together.

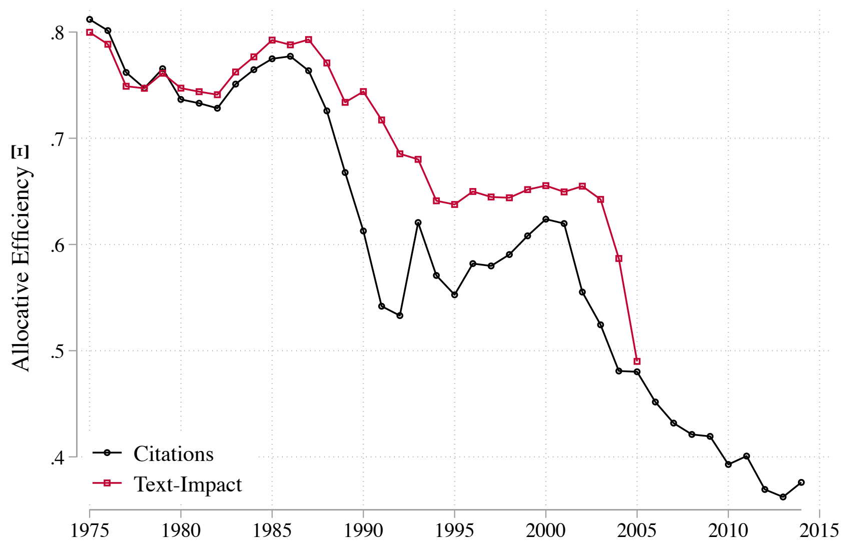
Measurement choices. I make two important measurement assumptions when estimating simple and adjusted allocative efficiency. First, I measure the output created from R&D using patent valuations. One potential concern with this measure is that it might not capture all the value created as some inventions are not patented. Following Bloom et al. (2020) I construct an alternative measure of R&D output from cumulative, non-negative sales changes, which might be a more comprehensive measure, although likely less precise. Panel (a) in Figure 3 shows that the results under this alternative measure are remarkable similar to my main specification. In both cases, allocative efficiency declines monotonically over the sample, however, the levels for the alternative measure are somewhat lower. The finding are thus exactly what we would expect if the alternative measure captured the same dynamics, but with additional measurement error.

A second important measurement assumption was using patent citations as a proxy for the growth impact. I consider the text-based patent impact measure developed in Kelly et al. (2021) as an alternative measure and construct the associated estimates for allocative efficiency. Panel (b) in Figure 3 reports the results and confirms highly similar trends using both measure. The estimate are tightly aligned in the early and late sample with slight divergence around 1990. Thus, the main conclusion are robust to this alternative measure of a patent's growth impact.

Figure 3: Allocative Efficiency Under Alternative Measurement



(a) Private Value Created



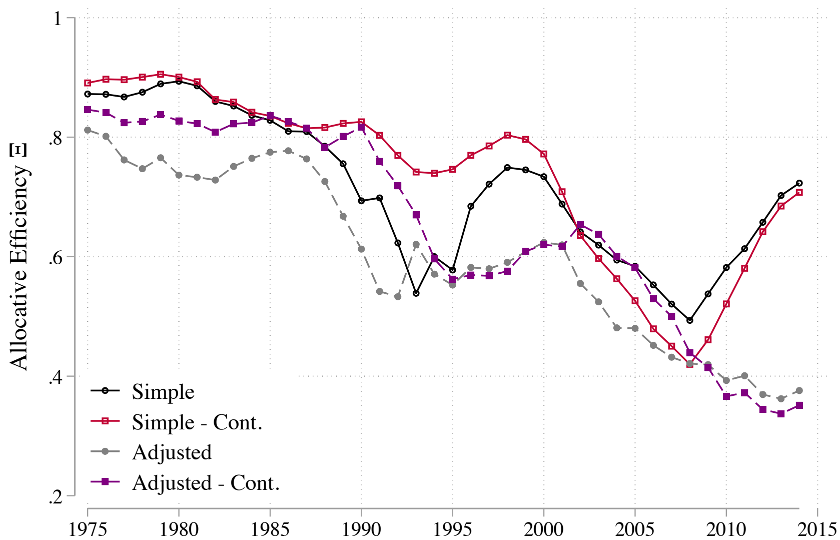
(b) Growth Impact

Notes: This figure plots the evolution of estimated allocative efficiency under alternative measurement assumptions. In panel (a), $\Delta Sales$ labeled lined use changes in sales instead of patent valuations to measure R&D output. In panel (b), the line labeled "Text-Impact" uses the text-based patent impact measure in Kelly et al. (2021) instead of patent citations to measure the growth impact of an innovation.

Sample composition. A natural question in light of these findings is whether declining allocative efficiency is driven by entry and exit or by continuing firms. In Figure 4, I compare my baseline results with a sample restricted to firms active for at least 75% of the sample.

The respective measures track each other tightly, implying that entry-and-exit do not drive the decline in allocative efficiency.

Figure 4: Allocative efficiency for all and continuing firms



Notes: This figure plots annual estimates of allocative efficiency using the formulas developed in Corollary 1 and Proposition 1 for the main sample and firms with 30 or more active years. The simple measure assumes a constant Impact-Value wedge across firms, in contrast to the adjusted measure. Lines marked "Cont." only include firms with 30 or more active years in the sample. The total sample length is 40 years.

Countervailing forces. Given the stark decline in measured allocative efficiency over time, one might ask whether the model also has something to say about potential countervailing forces that softened the blow. Indeed, the model highlight firm heterogeneity in R&D productivity as one such force due to gains from specialization. An economy with more heterogeneity in R&D productivity will have a larger economic growth rate at the same average R&D productivity as R&D activity is concentrated among high productivity firms. The appropriately weighted average R&D productivity then exceeds the simple average.

As detailed in Appendix C.5, I indeed find that rising heterogeneity in R&D productivity has more than offset the declining allocative efficiency such that the joint impact of the two sources of firm heterogeneity has become an increasingly positive force for economic growth. While this explains why economic growth has not completely collapsed, it also highlights the rising opportunity of improving efficiency in the market for R&D resources.

Market power in the input market. We can extend the framework directly to allow for some market power in the input market by assuming that wages are a function of firm's inventor demand. The primary effect of this change is to make the firm's profit function more concave and, thus, input demand less responsive to frictions. I estimate the impact of market power in the inventor market in companion paper Lehr (2022b).

Multiple production factors and abundant resources. It is straight-forward to show that the results derived in Proposition 1 directly apply in an economy with multiple production factors as long as their supply is perfectly inelastic. A positive supply elasticity, on the other hand, lowers the growth cost of factor misallocation as some of it translates into rising input prices and thus supply.

Semi-endogenous growth. Under semi-endogenous growth, the growth-rate impact of misallocation is only temporary as the long-run growth rate is determined by population growth only (Jones, 1995). Nonetheless, reducing misallocation will increase the growth-rate temporarily and thereby increase the long-run level of productivity. In the most simple semi-endogenous growth model, the long-run level shifts 1-for-1 with Ξ. In Appendix C.3 I compare my main results to the implications in a semi-endogenous growth model. My results suggests that comparable welfare effects across models, primarily driven by slow transition dynamics implied by semi-endogenous growth models. Even after 50 years, the growth-rate still maintains around 50% of the initial gains from solving R&D misallocation.

Multi-research line firms. A long tradition in endogenous growth theory has modeled the innovation sector with multi-research line firms (Klette and Kortum, 2004). Importantly, this literature often assumes that the distribution of research lines across firms in the economy is an endogenous object, driven by firm's R&D efforts and successes. In Appendix A.3 I show that my measurement directly apply in this alternative framework, however, the counterfactual treats the allocation of research lines across firms as constant. In this sense, allocative efficiency becomes a measure of immediate gains from solving misallocation, which might be larger or smaller in the long-run as the allocation of R&D lines adjusts.

5 Conclusion

There is a rising consensus that economic growth has declined significantly over the past decades, while R&D investments have remained stable. It thus appears that aggregate R&D productivity has declined significantly. I argue that this decline is partly driven by rising misallocation of R&D resources due to frictions.

Building on a workhorse endogenous growth model, I derive a close-form solution for the growth-rate impact of first-order condition wedges in firms' R&D problem. I take the model to the data for a sample of listed US firms with significant R&D activity. Measuring the relevant model inputs using recent advances in the measurement of R&D output, I provide estimates for allocative efficiency for the 1975-2014 period.

Two findings emerge. First, the estimates consistently suggest that the economy is significantly below its frontier growth rate and that there are significant gains from improving allocative efficiency. Second, allocative efficiency declined dramatically over the sample period, with estimates ranging from 25%-40%. The observed decline is a consistent finding under a range of alternative measurement strategies and suggests that declining allocative efficiency made a large contribution to the observed decline in aggregate R&D productivity.

These findings suggest important avenues for future research. Firstly, more research is necessary to understand the underlying forces driving rising dispersion. Lehr (2022b) suggests frictions in the market for inventors as one source, however, this mechanism doesn't explain the full dispersion in measured R&D wedges. Second, the degree to which policy could tap into the growth potential suggested by the allocative efficiency estimates it remains an open question. Improving allocative efficiency is naturally more imperative if we can rule out that it is driven by technology.

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Appendix

A Model Appendix

A.1 Proofs

A.2 Welfare

In this section I describe how we can translate changes in allocative efficiency into welfare terms. For this purpose, consider a simple household with log-preferences consumption sequence $\{c_t\}$ and discount factor β s.t. its welfare is given by

$$\mathcal{W}(\{c_t\}) = \sum_{t=0}^{\infty} \beta^t \ln c_t. \tag{A.1}$$

If consumption grows at a constant rate g, then we can simplify this expression to

$$\mathcal{W}(c_0, g) = \frac{1}{1-\beta}\left(\ln c_0 + \frac{\beta}{1-\beta}\ln(1+g)\right). \tag{A.2}$$

For any given change in g, we can thus solve for a permanent shift in the consumption stream that yields an equivalent change in welfare holding the growth rate constant as the solution to $\mathcal{W}(c_0 \cdot (1+\Delta_C), g) = \mathcal{W}(c_0, g+\Delta_g)$. Solving the system of equations, we have

$$\Delta_C \approx \exp\left(\frac{\beta}{1-\beta}\cdot \Delta_g\right) - 1. \tag{A.3}$$

The welfare cost of a given long-run level of misallocation Ξ are thus given by

$$\Delta_C \approx \exp\left(\frac{\beta}{1-\beta}\cdot \tilde{g} \cdot (1-\Xi)\right) - 1. \tag{A.4}$$

A.3 Multiple R&D lines

Consider an alternative version of the model with multiple R&D lines per firm. I will index a firm by $i \in \mathcal{I}$ and a R&D line by $j \in \mathcal{J}_i$.

Production function

$$z_{ij} = \varphi_{ij} \ell_{ij}^{\frac{1}{1+\phi}} \tag{A.5}$$

First order conditions at the R&D line level:

$$\frac{1}{1+\phi} \ell_{ij}^{-\frac{\phi}{1+\phi}} \gamma_{ij} = \frac{1}{\xi_{ij}} W \tag{A.6}$$

Return

$$\frac{\gamma_{ij} \ell_{ij}^{\frac{1}{1+\phi}}}{W \ell_{ij}} = \frac{1}{\xi_{ij}} \times (1+\phi) \tag{A.7}$$

Wages

$$\tilde{W} \equiv (1+\phi) W = L^{-\frac{\phi}{1+\phi}} \left(\int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_\rangle} (\gamma_{ij} \xi_{ij})^{\frac{1+\phi}{\phi}} \Big) di \right)^{\frac{\phi}{1+\phi}} \tag{A.8}$$

Growth rate.

$$\begin{aligned} g = & \int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} z_{ij} (\lambda_{ij} - 1) \Big) di = \int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} \zeta_{ij} \gamma_{ij} \ell_{ij}^{\frac{1}{1+\phi}} \Big) di & \text{(A.9)} \\ = & \int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} \frac{\zeta_{ij}}{\xi_{ij}} \tilde{W} \ell_{ij} \Big) di & \text{(A.10)} \\ = & L^{\frac{1}{1+\phi}} \frac{\int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} \tilde{\gamma}_{ij}^{\frac{1+\phi}{\phi}} (\xi_{ij}/\zeta_{ij})^{\frac{1}{\phi}} \Big) di}{\left(\int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} \tilde{\gamma}_{ij}^{\frac{1+\phi}{\phi}} (\xi_{ij}/\zeta_{ij})^{\frac{1+\phi}{\phi}} \Big) di \right)^{\frac{1}{1+\phi}}} & \text{(A.11)} \end{aligned}$$

Now consider the case of $\zeta_{ij} = \zeta$ first.

At the firm level, we have

$$\frac{1}{\xi_i} = \frac{\sum_{j \in \mathcal{J}_i} \gamma_{ij} \ell_{ij}^{\frac{1}{1+\phi}}}{\sum_{j \in \mathcal{J}_i} W \ell_{ij}} = \sum_{j \in \mathcal{J}_i} \frac{\ell_{ij}}{\ell_i} \frac{1}{\xi_{ij}} \tag{A.12}$$

$$\gamma_i = \frac{1}{\xi_i}\tilde{W}^{\frac{1}{1+\phi}}(\tilde{W}\ell_i)^{\frac{\phi}{1+\phi}} \tag{A.13}$$

$$\zeta_i = \frac{\sum_{j\in\mathcal{J}_i}\gamma_{ij}\ell_{ij}^{\frac{1}{1+\phi}}\zeta_{ij}}{\sum_{j\in\mathcal{J}_i}\gamma_{ij}\ell_{ij}^{\frac{1}{1+\phi}}} \tag{A.14}$$

Some algebra confirms that

$$g = L^{\frac{1}{1+\phi}} \frac{\int_{\mathcal{I}}\tilde{\gamma}_i^{\frac{1+\phi}{\phi}}(\xi_i/\zeta_i)^{\frac{1}{\phi}}di}{\left(\int_{\mathcal{I}}\tilde{\gamma}_i^{\frac{1+\phi}{\phi}}(\xi_i/\zeta_i)^{\frac{1+\phi}{\phi}}\right)^{\frac{1}{1+\phi}}}. \tag{A.15}$$

A.4 The Planner's Problem

A.5 Approximations of Allocative Efficiency

Assuming no productivity differences and homogeneous φ, we have

$$\begin{aligned}\ln\Xi = &\ln\left[\int_0^1((1+\Delta)(1+\varphi))^{-\frac{1}{\phi+\varphi(1+\phi)}}\,di\right]\\ &-\frac{1}{1+\phi}\ln\left[\int_0^1((1+\Delta)(1+\varphi))^{-\frac{1+\phi}{\phi+\varphi(1+\phi)}}\,di\right]\end{aligned} \tag{A.16}$$

I will define $X_i = ((1+\Delta_i)(1+\varphi_i))^{-\frac{1}{\phi+\varphi_i(1+\phi)}}$ and $Y_i = X_i^{1+\phi}$ together with their respective aggregate X and Y s.t. for $\varphi=\varphi_i$ and $\Delta_i=\Delta$ we have

$$\ln\Xi = \ln X - \frac{1}{1+\phi}\ln Y.$$

Note that $\frac{\partial\ln Y_i}{\partial\ln 1+\varphi_i} = (1+\phi)\frac{\partial\ln X_i}{\partial\ln 1+\varphi_i}$ and $\frac{\partial\ln Y_i}{\partial\ln 1+\varphi_i} = (1+\phi)\frac{\partial\ln X_i}{\partial\ln 1+\varphi_i}$

Lemma 1. *Consider a small, idiosyncratic change in Δ_i or φ_i around $\Delta_i=\Delta$ and $\varphi_i=\varphi$. Up to a first-order approximation the change leaves $\ln\Xi$ unaffected.*

Proof of Lemma 1. Firstly, note that $\Xi=0$ for $\Delta_i=\Delta$ and $\varphi_i=\varphi$. The derivative of $\ln Xi$ with respect to any change in z is given by

$$\frac{\partial \ln \Xi}{\partial \ln z_i} = \frac{X_i}{X}\frac{\partial \ln X_i}{\partial \ln z_i} - \frac{1}{1+\phi}\frac{Y_i}{Y}\frac{\partial \ln Y_i}{\partial \ln z_i}$$

The first-order Taylor approximation is then given by

$$\ln \Xi \approx \int_0^1 \frac{\partial \ln \Xi}{\partial \ln z_i}\Big|_{z_i=z} d\ln z_i.$$

It is straight-forward to show that in the symmetric equilibrium $Y_i = Y$ and $X_i = X$ s.t. that $\frac{\partial \Xi}{\partial \ln z_i}\Big|_{z_i=z} = 0$ and as a result $\ln \Xi \approx 0$ or $\Xi \approx 1$. □

Lemma 2. *Consider a small, idiosyncratic change in Δ_i around $\Delta_i = \Delta$. Up to a second-order approximation, we have*

$$\widehat{\ln \Xi} = -\frac{1}{2}\frac{\phi}{(\phi + \varphi(1+\phi))^2} Var(\ln(1+\Delta_i)). \tag{A.17}$$

B Data and Measurement Appendix

B.1 Constructing Adjusted Returns

I construct adjusted returns on R&D in three steps. Firstly, I construct a baseline using winsorized patent valuations and knowledge capital as my measures of R&D output and input respectively. Winsorizing patent valuations reduces the impact of outliers and thus dispersion in the measured returns. I winsorize the top 5% of valuations in each application year. Knowledge capital is the discounted aggregate of R&D expenditure and, thus, is a more long-term input measure. I use the knowledge capital measure constructed in Ewens et al. (2020).

Secondly, I adjust for the potential impact of acquiring innovative firms with patents in the pipeline. This could lead to measurement error as R&D expenditure is occurred by the firm before acquisition, while patents are realized post acquisition. Let C_{it} be the R&D expenditure, V_{it} the value created from the firm's R&D, A_{it} be the value of acquisitions and s the share of acquisition that effectively is R&D expenditure. Then measured returns are given by

$$\frac{V_{it}}{C_{it}} = \frac{\tilde{V}_{it}}{C_{it} + sA_{it}} \times \left(1 + s\frac{A_{it}}{C_{it}}\right) \tag{B.1}$$

If firms optimize appropriately, then the first component is a constant such that

$$\frac{\partial \ln(V_{it}/C_{it})}{\partial(A_{it}/C_{it})} = \frac{s}{1 + s\frac{A_{it}}{C_{it}}}, \tag{B.2}$$

which we can estimate and rearrange to solve for s. Using the estimated s we can add the R&D part of acquisitions to the knowledge capital input measure.

Finally, I adjust for industry heterogeneous and amenities by residualizing the log return with respect to NAICS3$\times$ industry effects as well as average temperatures, house prices, and income levels in the location of inventors working for a firm. The latter can account for cost to the firm that are not directly counted as R&D, but factor into their R&D decision.

C Empirical Appendix

C.1 Measurement Error

In the companion paper Lehr (2022b), I discuss this issue in detail and provide robustness across the measurement choices. Here I will discuss the explicit estimation of the measurement error contribution to the overall dispersion in the measured return on R&D in a parametric framework.

Suppose that the return on R&D follows an AR(1) process in logs with potentially heterogeneous intercepts across firms. I will define $y_{it} \equiv \ln(1 + \Delta_{it})$ for convenience.

$$y_{it} = (1-\rho)\mu_i + \rho y_{it-1} + \varepsilon_{it}, \quad \text{with} \quad \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2) \quad \text{and} \quad \mu_i \sim N(0, \sigma_\mu^2) \tag{C.1}$$

The researcher, however, only observes this process with i.i.d. measurement error

$$\tilde{y}_{it} = y_{it} + \nu_{it} \quad \text{with} \quad \nu_{it} \sim N(0, \sigma_\nu^2). \tag{C.2}$$

It follows that the variance of observed returns is increasing in the measurement error:

$$Var(\tilde{y}_{it}) = \underbrace{\sigma_\mu^2 + (1-\rho^2)^{-1}\sigma_\varepsilon^2}_{=Var(y_{it})} + \sigma_\nu^2 \tag{C.3}$$

Furthermore, as per Corollary 2 and 3, the increased variance is likely going to reduce the measured allocative efficiency. In other words, measurement error leads to a downwards bias on allocative efficiency.

To understand the extend of this bias, I estimate the measurement error variance. For this purpose, I will exploit the auto-covariance structure of R&D returns. In particular, one can show that the parameters of the auto-regressive process are identified by the covariance of returns with themselves as well as their first three lags.

Lemma 3. *The parameters* $\{\rho, \sigma_\varepsilon^2, \sigma_\mu^2, \sigma_\nu^2\}$ *are exactly identified by the covariance of* $\tilde{y}_{it}$ *with* $\tilde{y}_{it-s}$ *for* $s = 0, 1, 2, 3$.

We can implement this insight using sample moments. Furthermore, standard GMM arguments together with the Delta method allows us to quantify the uncertainty around the parameter estimates. I provide the associated results in Appendix C.1.

One of the challenges is that we need to specify the time-horizon at which the process operates. I will 1 year, which is the smallest possible horizon that I can exploit with my data. To understand the extend of measurement error at the 5-year horizon, I will then need to aggregate appropriately. I will discuss this issue after presenting the parameter estimates.

The estimates reported in column (1) in Table C.1 suggest that measurement error is surprisingly small. I find that returns are highly auto-correlated, which suggests that there is a systematic component. The estimated variances suggest that around 96% of the overall variance is driven by idiosyncratic shocks 4% for measurement error. Interestingly, the estimates suggest no role for permanent firm differences.

Column (2) reports the estimates for R&D returns adjusted for the impact-value wedge. The results are similar, however, the measurement error contribution is somewhat larger at slightly below 10%.

Table C.1: GMM results for AR(1) with Noise

Parameter	R&D Return	
	Baseline	Adjusted
ρ	0.896***	0.942***
	(0.055)	(0.096)
σ^2_{ε}	0.174***	0.130***
	(0.020)	(0.016)
σ^2_{μ}	-0.053	-0.049
	(1.002)	(2.816)
σ^2_{ν}	0.043**	0.123***
	(0.018)	(0.015)
Observations	7,428	7,428

Note: Parameter estimates from structural measurement error estimation. All returns at the annual level and restricted to observations with at least 10 patents. Baseline returns are adjusted for NAICS× year effects. Adjusted returns are R&D returns times the impact-value wedge, where both variables are independently residualized for NAICS× year effects. Standard errors clustered at the NAICS6 level and reported in brackets.

Aggregating the returns to the 5-year level requires a weighted average across years:

$$\frac{\sum_{s=0}^{4} \texttt{Pat. Val.}_{it+s}}{\sum_{s=0}^{4} \texttt{R\&D Exp.}_{it-1+s}} = \sum_{s=0}^{4} \left(\frac{\texttt{R\&D Exp.}_{it-1+s}}{\sum_{s=0}^{4} \texttt{R\&D Exp.}_{it-1+s}} \right) \times \frac{\texttt{Pat. Val.}_{it+s}}{\texttt{R\&D Exp.}_{it-1+s}}. \tag{C.4}$$

The model allows us to construct the appropriate weights across periods depending on R&D productivity and the return on R&D. I implement this by specifying a productivity process and calibrating it via moment matching. I then simulate data and aggregate using the model implied formulae.

Columns (1) and (2) of Table C.2 reports the associated results for the unadjusted R&D wedge. At the 5-year level, measurement error contributes 1.3% of the variance if returns compared to 4.6% at the 1-year level. The aggregation thus further reduces the measurement error contribution as some of it averages out. Similarly, the results for the adjusted wedge in columns (3) and (4) show that measurement error is small once we aggregate to the 5-year level, explaining less than 3% of the variation.

Table C.2: Variance Estimates for R&D wedges

	Baseline		Adjusted	
	1-Year	5-Year	1-Year	5-Year
Variance	0.93	0.78	1.28	1.09
Variance with $\sigma_\nu = 0$	0.88	0.77	1.15	1.06
$\Delta\%$	4.6%	1.3%	9.6%	2.6%

Note: Estimates based on simulation with parameters from Table C.1. Adjusted values take into account the impact-value wedge. Aggregation to 5-year level based on formulas developed in Appendix C.1. Final row reports the gap in percent between the first and second row.

Overall, these results suggest that classical measurement error, perhaps surprisingly, contributes a negligible share of the variance of returns on R&D and, thus, will not significantly bias measured allocative efficiency.

Consider a stationary, AR(1) process $\{y_{it}\}$:

$$y_{it} = (1-\rho)\mu_i + \rho y_{it-1} + \varepsilon_{it} \quad \text{with} \quad \varepsilon_{it} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \quad \text{and} \quad \mu_i \sim N(0, \sigma_\mu^2). \tag{C.5}$$

The econometrician observes the process with i.i.d. normal measurement error:

$$\tilde{y}_{it} \equiv y_{it} + \nu_{it} \quad \nu_{it} \overset{iid}{\sim} N(0, \sigma_\nu^2). \tag{C.6}$$

Lemma 4. *Define $\Delta\tilde{y}_{it} \equiv \tilde{y}_{it} - \tilde{y}_{it-1}$, then under $\rho \in (0,1)$, we have*

$$\begin{aligned}
m_1 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it}) = \frac{1}{1+\rho}\sigma_\varepsilon^2 + \sigma_\nu^2 \\
m_2 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it-1}) = \frac{\rho}{1+\rho}\sigma_\varepsilon^2 \\
m_3 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it-2}) = \frac{\rho^2}{1+\rho}\sigma_\varepsilon^2 \\
m_4 &\equiv Cov(\tilde{y}_{i,t}, \tilde{y}_{it-1}) = \sigma_\mu^2 + \frac{\rho}{1-\rho^2}\sigma_\varepsilon^2.
\end{aligned}$$

Proposition 2. *If $\rho \in (0,1)$, we can solve for $\{\rho, \sigma_\mu, \sigma_\varepsilon, \sigma_\nu\}$ using the population auto-covariance structure of $\tilde{y}_{it}$ and $\Delta\tilde{y}_{it} \equiv y_{it} - y_{it-1}$:*

$$\beta \equiv \begin{bmatrix} \rho \\ \sigma_\varepsilon^2 \\ \sigma_\mu^2 \\ \sigma_\nu^2 \end{bmatrix} = \begin{bmatrix} \frac{m_3}{m_2} \\ \frac{(m_2)^2}{m_3} + m_2 \\ m_4 - \frac{(m_2)^2}{m_2 - m_3} \\ m_1 - \frac{(m_2)^2}{m_3} \end{bmatrix}$$

Let Ω be the covariance matrix of m and denote the sample moments by $\hat{m}$, then

$$\hat{\beta} \sim N(\beta, \Sigma) \quad \textit{and a feasible estimator is} \quad \hat{\Sigma} = \left(\frac{\partial \hat{\beta}}{\partial m}\right)' \hat{\Omega} \left(\frac{\partial \hat{\beta}}{\partial m}\right),$$

where $\partial\beta/\partial m$ is evaluated at $\hat{m}$ and given by

$$\frac{\partial \beta}{\partial m} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -\frac{m_3}{(m_2)^2} & 2\frac{m_2}{m_3} + 1 & m_2\left(\frac{m_2 - 2m_3}{(m_2 - m_3)^2}\right) & -2\frac{m_2}{m_3} \\ \frac{1}{m_2} & -\left(\frac{m_2}{m_3}\right)^2 & -\left(\frac{m_2}{m_2 - m_3}\right)^2 & -\left(\frac{m_2}{m_3}\right)^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Proof. The first part follows by rearranging the moments expressions. The second part follows from the Law of Large Numbers for the moment vector and the Delta method. □

Note that this methodology does not aggregate. In particular, if we assume that Expected Return on R&D follows an AR(1) in logs at the annual level, we cannot implement the above methodology at the 5-year horizon directly as the 5-year Expected Return on R&D is a weighted-average of the annual Return in levels, which does not translate into logs:

$$\frac{\sum_{s=0}^{4} \text{Pat. Val.}_{it+s}}{\sum_{s=0} \text{R\&D Exp.}_{it-1+s}} = \sum_{s=0}^{4} \frac{\text{R\&D Exp.}_{it-1+s}}{\sum_{w=0} \text{R\&D Exp.}_{it-1+w}} \times \frac{\text{Pat. Val.}_{it+s}}{\text{R\&D Exp.}_{it-1+s}}.$$

To address this concern, I will estimate the system at the 1-year level and propose a methodology to estimate the importance of measurement error at the 5-year level. I restrict my sample to 1-year returns with at least 5 patents and provide additional estimates for an adjusted measure with at least 10 patents per return, winsorized patent valuations, and only within industry-year variation.

The GMM estimates presented in Table C.1 suggest that measurement error constitutes little of the overall variation in the 1-year Return on R&D. The estimated measurement error variation is around 0.04 for both return measures, but only significantly different from 0 at the 5% level for the adjusted returns. In addition, I consistently find that the Return on R&D is highly auto-correlated with significant variation due to idiosyncratic shocks. The results for permanent differences across firms are mixed. While the baseline estimates suggest some role for them, the estimated coefficient for the adjusted returns is very close to 0. One interpretation is that there are permanent differences across industries, which do not show up for the adjusted returns as they are residualized. Note, however, that the standard errors around the estimates for σ_μ^2 are very large.

As discussed before, we cannot immediately translate these estimates into measurement error contributions at the 5-year level due to aggregation. I address this challenge by adding some structure on the firm R&D process. In particular, I will assume that each firm in my data solves the simple maximization problem

$$\max_{\ell_{it}} \left\{ \varphi \ell_{it}^{\frac{1}{1+\phi}} - \Delta_{it} \times W \ell_{it} \right\}. \tag{C.7}$$

The source of R&D returns in this framework is Δ_{it} and I will consequently assume that it follows an AR(1) process, which the researcher observed with i.i.d. measurement error.

Lemma 5. *Under above assumptions, the 5-year Return on R&D is given by*

$$\textit{Expected Return on R\&D}_{it} = (1+\phi) \times \frac{\sum_{s=0}^{4} \Delta_{it}^{-\frac{1+\phi}{\phi}} \times \tilde{\Delta}_{it}}{\sum_{s=0}^{4} \Delta_{it}^{-\frac{1+\phi}{\phi}}}.$$

Proof. The solution to the firm optimization problem is given by

$$\ell_{it} = (\Delta_{it}(1+\phi)W)^{-\frac{1+\phi}{\phi}} \times (\varphi)^{\frac{1+\phi}{\phi}}$$

The annual return on R&D is proportional to Δ_{it}:

$$\frac{\varphi \ell_{it}^{\frac{1}{1+\phi}}}{W \ell_{it}} = (1+\phi) \times \Delta_{it}.$$

By definition, we can then express the overall return as measured in the data as

$$\frac{\sum_{s=0}^{4} W\ell_{it+s} \times \tilde{\Delta}_{it+s}}{\sum_{s=0}^{4} W\ell_{it+s}} = (1+\phi) \times \frac{\sum_{s=0}^{4} \Delta_{it}^{-\frac{1+\phi}{\phi}} \times \tilde{\Delta}_{it}}{\sum_{s=0}^{4} \Delta_{it}^{-\frac{1+\phi}{\phi}}}.$$

□

Using this framework, we can simulate data based on the estimates in C.1 and aggregate to the 5-year level as suggested above. To estimate the importance of measurement error, we can then compare baseline estimates against a counterfactual with $\sigma_\nu^2 = 0$. I follow the literature and set $\phi = 1$ for the purpose of this exercise Acemoglu et al. (2018).

Table C.2 reports the results, which suggest that measurement error makes a minor contribution to the dispersion in the Expected Return on R&D. For baseline and adjusted returns I find that measurement error contributes less than 1% to the overall dispersion in the 5-year Expected Return on R&D. The importance of measurement error is decreasing in the time-horizon considered as the individual shocks average out.

C.2 Alternative Measurements

In this section, I discuss the impact of using alternative input measures. I will split the discussion into two sections, one focusing on the simple measure and one one the adjusted measure.

C.3 Welfare Cost under Semi-Endogenous Growth

In this section I show that using a semi-endogenous growth framework yields quantitatively similar conclusions of the welfare cost of misallocation as the ones estimated in the endogenous growth model in the main text (Jones, 1995; Bloom et al., 2020). Consider a simple semi-endogenous growth model, where the growth-rate of the technology depends on aggregate innovator input L_t, the current level of technology A_t, and misallocation term Ξ_t according to:

$$g_t \equiv \frac{\dot{A}_t}{A_t} = L_t \cdot \Xi_t \cdot A_t^{-\phi}. \tag{C.8}$$

The parameter $\phi > 0$ captures the fishing-out effect that ensures existence of a balanced growth path with constant growth rate. It is straight-forward to show that a constant growth rate along the balanced growth path has to satisfy

$$g = \frac{n}{\phi}. \tag{C.9}$$

Thus, along the balanced growth path, the level of technology is given by

$$A_t = \left(\frac{\phi}{n} \times L_t \times \Xi\right)^{\frac{1}{\phi}}. \tag{C.10}$$

In the long-run, misallocation thus has a level effect in the semi-endogenous growth model instead of the growth-rate effect implied by an endogenous growth model.

I calibrate the model using the parameter reported in Table C.3. I set the population growth rate at 1% p.a., which is the approximately the long-run average for the US in the post-war period. I then set ϕ to ensure a long-run growth rate of 1.5% p.a. at initial levels of R&D misallocation, which I also impose in the endogenous growth model. Finally, I set the discount rate to a standard macro value of $\rho = 0.03$.

Table C.3: Parameter Values For Semi-Endogenous Growth

Parameter	Value	Source
ϕ	0.67	Implied by growth rate of 1.5%
n	0.01	Long-run average
ρ	0.03	Standard macro value

We can quantify the welfare gains from setting $\Xi_t = 1$ numerically, using the growth-rate formula together with parameteric assumptions. I will assume that the economy was on the balanced growth path prior to solving misallocation. I then set $\Xi_t = 1$ and use the growth rate formula to characterize the evolution of A_t. I then translate these values into welfare and welfare equivalent by assuming that the planner values a per-capita consumption stream c_t, which is directly linked to the productivity level as

$$\mathcal{W}(\{c_t\}) = \int_0^\infty e^{-(\rho-n)t} \ln(c_t) dt. \tag{C.11}$$

Note that this formulation discounts the future at rate $\rho - n$ instead of ρ, which is the discount rate in a model without population growth. I will use the same discount rate when comparing the welfare implications in the semi-endogenous and endogenous growth model.

We can then calculate the consumption-equivalent welfare cost of R&D misallocation as

$$\Delta_c = \exp\left(\rho \cdot (\mathcal{W}(\{c_t^*\}) - \mathcal{W}(\{c_t\}))\right) - 1, \tag{C.12}$$

where $\{c_t\}$ is the consumption sequence with permanent misallocation and $\{c_t^*\}$ is the consumption sequence induced by solving misallocation. As the growth-rate is not constant over time in this model, we cannot solve this equation directly and I will instead calculate the necessary inputs numerically.

Table C.4 reports the results for the average level of misallocation according to the simple and adjusted allocative efficiency measure. The welfare cost of R&D misallocation are slightly smaller in the semi-endogenous growth model, but of comparable magnitudes across models. For example, the simple estimate of allocative efficiency is associated with potential consumption-equivalent welfare gains of 36% in the endogenous growth model compared to 32% for the semi-endogenous growth model.

Table C.4: Welfare Cost of R&D Misallocation Across Models

Measure	Allocative Efficiency	Welfare Cost Endogenous	Welfare Cost Semi-endogenous
Simple	0.704	36%	32%
Adjusted	0.583	69%	56%

Notes: This table reports the welfare gains from getting rid of misallocation implied by endogenous and semi-endogenous growth models. Both models discount the future at rate $\rho - n$, but have alternative paths for technology due to the nature of growth.

C.4 Decomposing the Evolution in the Adjusted Measure

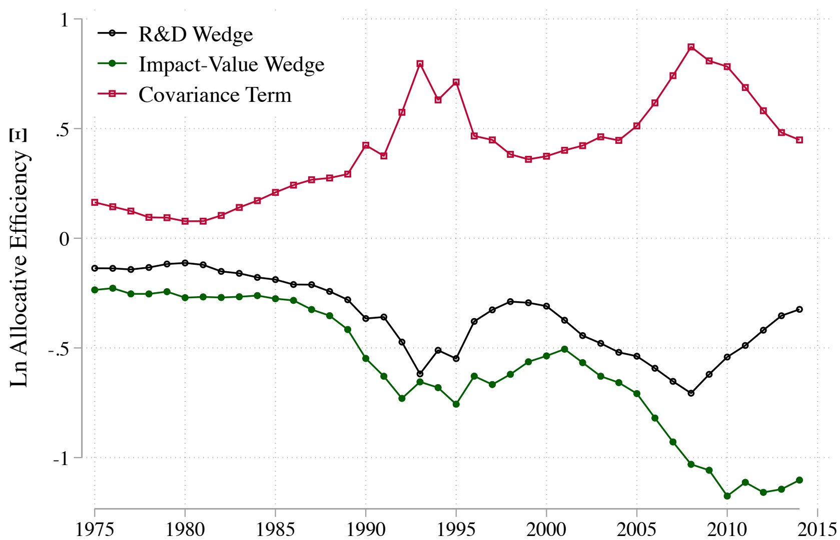
We can decompose the adjusted Allocative Efficiency Measure into three parts:

$$\Xi(\{\zeta_{it} \cdot (1 + \Delta_{it})\}) = \Xi(\{(1 + \Delta_{it})\}) \times \Xi(\{\zeta_{it}\}) \times \frac{\Xi(\{\zeta_{it} \cdot (1 + \Delta_{it})\})}{\Xi(\{(1 + \Delta_{it})\}) \times \Xi(\{\zeta_{it}\})}$$

The first and second component measure the independent impact of the R&D and Impact-Value wedge on allocative efficiency. The final term is effectively a correlation term that considers the joint impact of both wedges relative to their individual contribution. This term is going to be approximately 1 if both terms are independent, larger than 1 if they partly offset each other, and vice versa.

Figure C.1 plots the decomposition terms in logs. Both individual wedge terms decline steadily throughout the sample, however, there is a divergence post 2005 as the impact-value wedge term continues to decline, while the R&D wedge term slightly rebounds. The correlation term continually offsets the impact of the individual components with increasingly large impact. Negative correlation between the wedges thus offsets some of their direct effect.

Figure C.1: Decomposing Allocative Efficiency



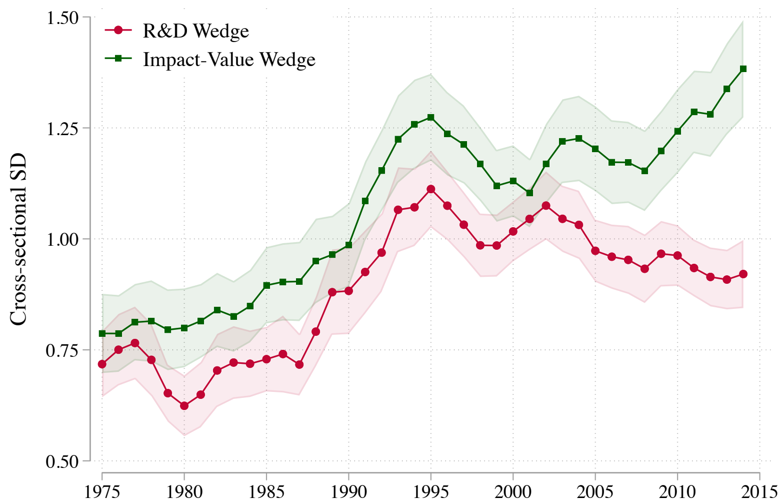
Another way to decompose the evolution is to use the approximation result that links the adjusted allocative efficiency term to the variance of the adjusted R&D wedge. Following the decomposition, we have

$$\ln \Xi_{adjusted} \approx -\frac{1}{2\phi}\left(\sigma_{\Delta}^2 + \sigma_{\zeta}^2 + 2 \cdot \sigma_{\Delta} \cdot \sigma_{\zeta} \cdot \rho_{\Delta,\zeta}\right), \tag{C.13}$$

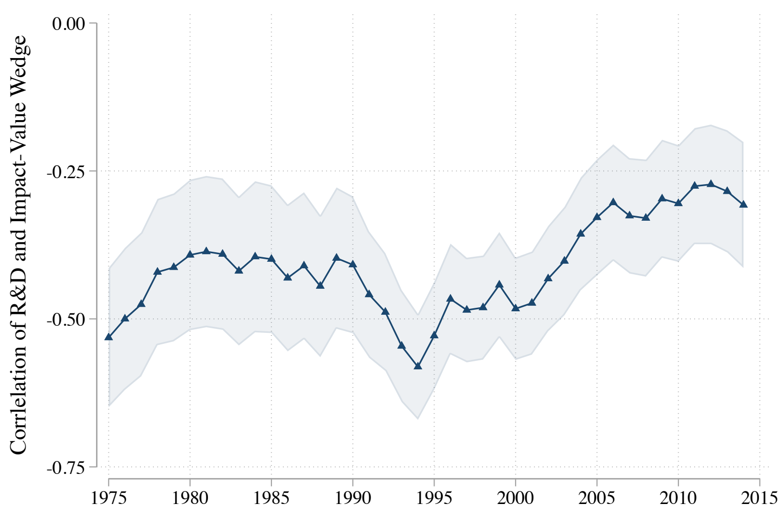
where $\sigma_{\Delta}^2 \equiv \sigma_{\omega}^2(\{\ln(1+\Delta_{it})\})$, $\sigma_{\zeta}^2 \equiv \sigma_{\omega}^2(\{\ln \zeta_{it}\})$, and $\rho_{\Delta,\zeta} = \rho_{\omega}(\{\ln(1+\Delta_{it})\}, \{\ln \zeta_{it}\})$. Up to a second order approximation, these three factors thus shape the evolution of allocative efficiency.

Figure C.2 reports the evolution of the components. Three observations emerge. First, the standard deviation of R&D wedge and Impact-Value wedge rises monotonically from 1975 to 1995, while their evolution diverges afterwards. The dispersion in the impact value wedge continues to rise post 1995, while dispersion in the R&D wedge stabilizes and slightly declines. Second, both wedges are consistently negatively correlated as shown in panel (b), however, the magnitude of the correlation decline post 1995. Rising misallocation is thus driven primarily by rising dispersion in wedges in the first half of the sample and by rising correlation of both in the second half.

Figure C.2: The Three Components of Approximated Allocative Efficiency



(a) Standard Deviation of Wedges



(b) The Correlation of R&D and Impact-Value Wedge

Notes: Shaded areas cover 95% confidence interval. Standard errors calculated using influence functions and the Delta method.

C.5 Countervailing Forces

The frontier growth rate can be further decomposed into a baseline component and a firm specialization component as detailed in the following Lemma.

Lemma 6. *The frontier growth-rate can be decomposed into two components:*

$$\tilde{g}_t = \bar{g}_t \times \Phi_t, \tag{C.14}$$

where $\bar{g}_t \equiv \int)^1 \gamma_{it} \cdot \zeta_{it} di$ *is the frontier growth in absence of firm heterogeneity and* Φ_t *measures the specialization gains from heterogeneous R&D productivity:*

$$\Phi_t \equiv \frac{\left[\int_0^1 (\gamma_{it} \cdot \zeta_{it})^{\frac{1+\phi}{\phi}} di\right]^{\frac{\phi}{1+\phi}}}{\int_0^1 (\gamma_{it} \cdot \zeta_{it}) \, di}. \tag{C.15}$$

It is straight-forward to show that Φ_t is independent of the common level of R&D productivity and Impact-Value wedge, however, as per Jensen's inequality, it is increasing in dispersion in their product. The economic intuition for this rests on specialization. In particular, heterogeneous R&D productivity implies good firms can do more R&D and vice versa. This reallocation of R&D resources increases the appropriately aggregated R&D productivity beyond the simple mean of R&D productivities across firms by weighting high R&D productivity firms more. Thus, even with constant average R&D productivity, aggregate R&D productivity improves if firm-level R&D productivities are more dispersed. This effect is weaker with more concave R&D production functions, which limit the extend to which R&D productivity differences translate into R&D efforts.

I estimate the firm specialization term in the data using the parameter estimation approach introduced in Section 3. I then implement the formulas as

$$\hat{\Phi}_t = \frac{\left[\frac{1}{N_t} \sum \left(\hat{\gamma}_{it} \cdot \hat{\zeta}_{it}\right)^{\frac{1+\phi}{\phi}} di\right]^{\frac{\phi}{1+\phi}}}{\frac{1}{N_t} \sum \left(\hat{\gamma}_{it} \cdot \hat{\zeta}_{it}\right) di}, \tag{C.16}$$

where N_t is the number of active firms in the sample. This approach preserve's the formula's the expectation character and is thus independent of the number of active firms. The latter can have an independent effect in the model due to the love of variety effect inherent in decreasing returns to scale.[18]

[18] In particular, one can show that $\int_0^M (X/M)^\alpha di$ is increasing in M for $\alpha < 1$. The growth-rate in the model shares this structure such that more active firms always imply a larger growth rate as long as the number of scientists in constant.

Panel (a) of Figure C.3 reports the results. I find that the gains from specialization have rising significantly over the sample across a range of alternative specifications with similar magnitudes. While specialization hovered around 1.25 in the beginning of the sample, it increase to around 2 in 2005, a 60% increase. This finding holds true whether we focus on the simple measure or take into account differences in the impact-value wedge as in the adjusted measure.

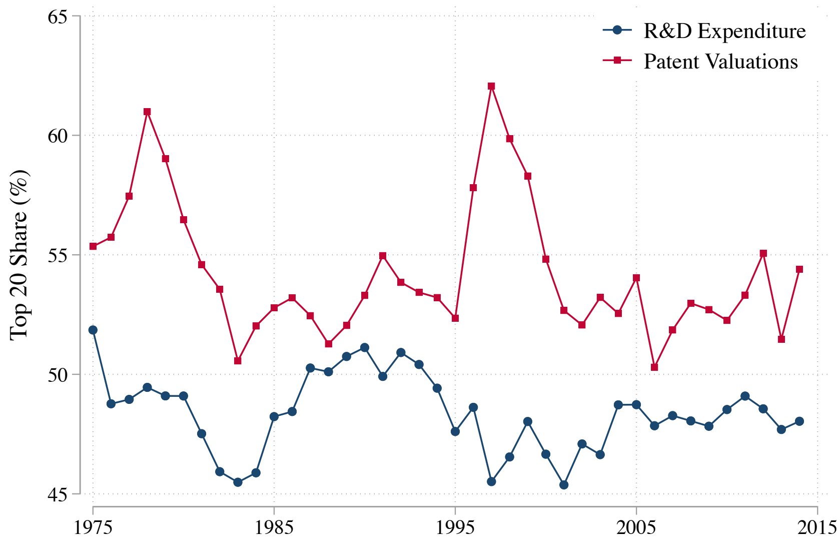
What is then the joint impact of firm heterogeneity? On the one hand, I documented in the text that allocative efficiency has declined significantly over time, driven by rising heterogeneity in the R&D wedge. On the other hand, gains from specialization in R&D have risen significantly. Panel (b) of Figure C.3 plots their joint impact, i.e. their product. Two findings emerge. First, the estimate from the simple measure is consistently above 1, which suggest that the economy gains from the observed heterogeneity. This finding is overturned if we either use alternative measures of the private value created from R&D or take into account heterogeneity in the impact value wedge. For these alternative measures, I find values close to, but below one until about 2005, which suggest that the economy might lose from firm heterogeneity.

Second, the joint impact rises significantly over the sample. For example, for the adjusted measure, the full impact is approximately constant at 0.9 from 1975 to 2005 follows by a strong increase to above 1.1 in the 2010s. This rise is more gradual and monotonic for the alternative measure, but the overall conclusion holds. Thus, the observed rise in gains from firm specialization dominates the decline in allocative efficiency making firm heterogeneity an ever more positive force for economic growth.

Jointly, it appears that rising firm heterogeneity had a positive impact on economic growth, however, it could have been much larger if allocative efficiency had remained constant. We might expect that some of this decline was inevitable in light of rising firm heterogeneity, however, the latter also increases the stakes for policy makers in creating a more efficient R&D environment with fewer frictions.

C.6 Concentration in R&D

Figure C.4: Concentration of R&D Has Remained Stable

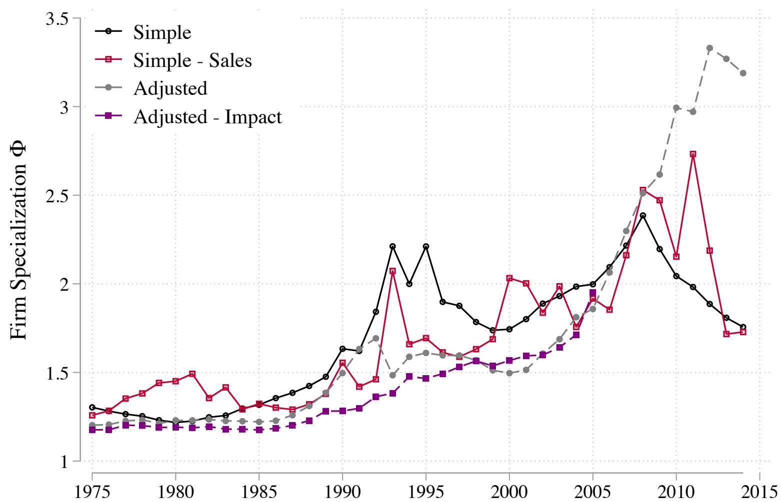


Notes: Author's calculations based on sample.

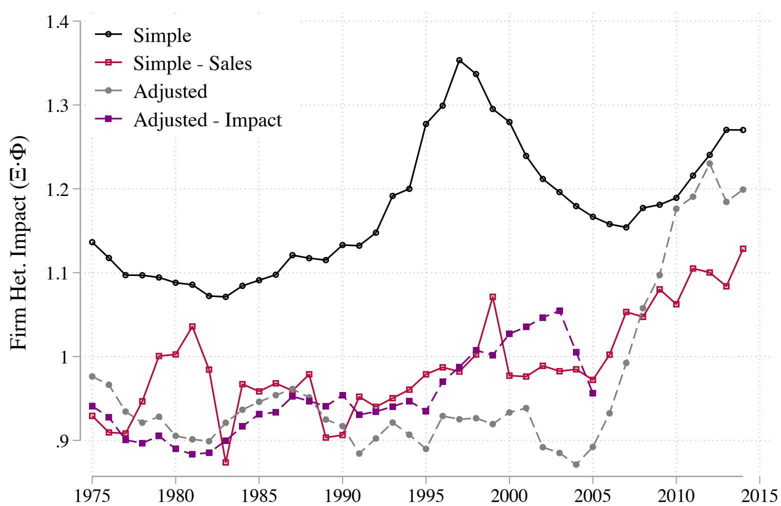
D The Value of Innovation

TBD.

Figure C.3: Firm Heterogeneity and Growth



(a) Firm Specialization



(b) Firm Heterogeneity

Notes: This figure plots the firm R&D specialization Φ_t and the full impact of firm heterogeneity on growth. The latter is calculated as the product of firm R&D specialization and R&D allocative efficiency Ξ_t. The simple measure assumes a constant Impact-Value wedge. The version labelled "Sales" uses sales changes instead of patent valuations to measure R&D output. The adjusted measures take into account heterogeneity in the Impact-Value wedge. The version labelled "Impact" uses the text-based patent impact measure from Kelly et al. (2021) instead of forward-citations to measure growth impact.