Did R&D Misallocation Contribute to Slower Growth?*

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Abstract

This paper identifies worsening R&D allocative efficiency as a potential driver of declining US economic growth. Within a simple endogenous growth framework, I develop a closed-form solution of the growth rate that can be decomposed into a frontier growth-rate, only achievable with the growth-maximizing resource allocation, and an allocative efficiency measure, measuring the gap between realized and frontier growth. Combining model with data on the innovation activity of US firms I estimate that allocative efficiency declined significantly from 1975 to 2014. Comparing the 1975-94 period to the 2005-14 period, I find that declining allocative efficiency predicts 40% slower economic growth in the latter period, which can explain the entire concurrent decline in economic growth as documented in the literature. I discuss potential drivers of declining allocative efficiency including waning federal support of R&D, institutional and technological change, and increasing labor market power over inventors.

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1 Introduction

Economic growth has declined in the last 20 years due to a slowdown in productivity growth. While productivity grew at a pace of 0.7 p.p. per year in 1975-1994, this rate declined to 0.5 p.p. in 2005-2020.[1] Through the lens of workhorse growth models, this decline could be driven by two factors: a decline in research and development (R&D), i.e. fewer resources allocated to generating growth, or a decline in the economy's productivity at translating resources into productivity growth. Empirically, the US R&D to output ratio has remained stable, implying that declining growth is driven by declining aggregate R&D productivity.

$$\text{Growth } (\downarrow) = \text{R\&D Expenditure } (-/\uparrow) \times \text{R\&D Productivity } (\downarrow)$$

In this paper, I provide evidence suggesting that lower aggregate R&D productivity is partly due to declining resource allocation efficiency in the R&D sector. My evidence suggests that some firms conduct too much, while others conduct too little R&D, and increasingly so. Lower growth is, thus, not only driven by declining firm-level R&D productivity, as argued in Bloom et al. (2020), but also by declining allocative efficiency among innovative firms.

At the core of this paper is a simple growth accounting framework that allows me to directly estimate the contribution of the R&D allocation across firms to aggregate R&D productivity. I derive this framework in a simple endogenous growth model nesting workhorse models in the literature.[2] In the model, firms with potentially different *R&D productivity* hire inventors to maximize the private value created from innovation. I introduce private frictions flexibly by allowing for exogenous *R&D wedges* in firms' first-order conditions. Finally, growth occurs as a by-product of innovation, however, I allow for a potential gap between the private value created from innovation and its growth impact, which I will refer to as *impact-value wedge*. This term captures the concerns about externalities emphasized in the growth literature.

Within this framework, I derive a closed form solution of the economic growth rate depending on the joint distribution of R&D productivities, R&D wedges, and impact-value wedges. Growth can be decomposed into a frontier growth rate, reflecting the growth-maximizing R&D allocation, and an allocative efficiency adjustment term measuring the fraction of maximal growth that is achieved. In absence of heterogeneity in impact-value wedges, variation in R&D wedges reduces allocative efficiency by pushing the distribution of relative R&D

[1] I calculate these number using the TFP growth measures from Comin et al. (2022) . I annualize the quarterly growth rate as $g_{ann} = (1 + g_{qrt})^4 - 1$.

[2] See e.g. Romer (1990); Aghion and Howitt (1992); Acemoglu and Cao (2015)

efforts away from their growth-maximizing optimum — an R&D sector equivalent finding of the results in Hsieh and Klenow (2009). Similarly, in absence of R&D wedges, variation in impact-value wedges reduces allocative efficiency by creating a divergence between privately optimal and growth maximizing R&D allocation, as in de Ridder (2021) and Aghion et al. (2022a). With joint heterogeneity, the same insights apply to adjusted R&D wedges, which synthesize both effects. These formulae thus give us a tool to quantify the unified impact of frictions and incentive misalignment on aggregate R&D productivity and economic growth.

Next, I measure the relevant parameters using data on patents and R&D expenditure of US listed firms from 1975 to 2014. I measure the private value created from R&D using patent valuations and construct a proxy for the growth-impact of an innovation using citation measures. Together with information on R&D expenditures, these data allow me to measure the model parameters and, thus, estimate the level and evolution of allocative efficiency.

Before applying the model formulae, I investigate drivers of measured wedges. R&D wedges, which capture frictions through the lens of the model, are surprisingly hard to predict. For example, they are uncorrelated with the return on capital and measures of subsidies. Variation in investment Q and inventor employment can account for a significant share of variation in R&D wedges. Impact-value wedges, which measure incentive misalignment, are easier to predict, with a significant share of the variation accounted for by proxies for profitability or firm size and age.[3] The strongest predictors of combined wedges are firm employment and R&D expenditure. Both exhibiting a strong negative correlation, which suggests that large firms conduct excessive R&D as argued e.g. in Acemoglu et al. (2018).

Combining measured wedges and model, I estimate that allocative efficiency has declined remarkably over time. Comparing 1975-94 to 2005-14, I find that declining allocative efficiency predicts a decline in the economic growth rate around 40%, which accounts for the entire documented decline in economic growth. Annual estimates of allocative efficiency suggest a stable decline. I estimate an average level of allocative efficiency around 60%, suggesting a large gap between the realized and frontier growth rate. For comparison, for the US production sector Hsieh and Klenow (2009) estimate an allocative efficiency around 70%, while Berger et al. (2022) estimate a value of 83% based on monopsony power only.[4] Reassuringly, my findings remain robust through an extensive set of robustness checks.

[3] Acemoglu et al. (2018); de Ridder (2021); Aghion et al. (2022a) have models in line with these findings.

[4] Hsieh and Klenow (2009) estimate that US productivity improved by 40% under first-best allocation, while Berger et al. (2022) estimate a 21% output improvement in absence of monopsony. I translate these values in allocative efficiency as $1/(1 + 40\%) \approx 70\%$ and $1/(1 + 21\%) \approx 83\%$.

Literature. This paper contributes to three strands of the literature. First, a growing literature documents the recent decline in the economic growth rate and investigates its origins.[5] Similar to Bloom et al. (2020), I argue for declining aggregate R&D productivity as a core driver, however, I attribute this change to declining allocative efficiency instead of declining micro-level R&D productivity, which is similar to the perspective advanced in Aghion et al. (2022b) and de Ridder (2021). My framework offers a unified treatment of externalities and frictions, the latter of which have not been the focus of this literature.

Second, I provide new findings potentially linked to the drivers of aggregate R&D resources allocation and productivity.[6] My framework allows for a closed-form decomposition of the growth rate, and clarifies how private frictions and externalities shape economic growth through their impact on the R&D resource allocation. My estimates suggest that both matter quantitatively. Closely connected, de Ridder (2021) and Aghion et al. (2022a) argue that heterogeneity in growth externalities give rise to R&D misallocation.[7] My framework speaks to these findings, but offers a joint treatment of the impact of externalities and frictions on R&D resource allocation and economic growth.

Third, the documented R&D return dispersion speaks to the literature on factor misallocation.[8] Focusing on R&D instead of static production factors such as capital and labor introduces a dynamic component linking factor return heterogeneity to the growth rate instead of static production efficiency. Nonetheless, my framework allows for a closed-form solution and direct link to the data as developed in Hsieh and Klenow (2009) for the production sector. My estimates suggest that allocation efficiency is not only low in the production sector, but also in the R&D sector.

Organization. The remainder of this paper is structured as follows. Section 2 introduces the theory and develops the main formulas. Section 3 introduces the data and discusses measurement of core model parameters. Section 4 combines data and theory to investigate the role of resource allocation for economic growth. I conclude in Section 5.

[5]See e.g. Gordon (2016); Syverson (2017); Bloom et al. (2020); Akcigit and Ates (2021); de Ridder (2021); Aghion et al. (2022b,a); Olmstead-Rumsey (2022); Liu et al. (2022)

[6]For related literature, see e.g. Romer (1990); Aghion and Howitt (1992); Acemoglu and Cao (2015); Acemoglu et al. (2018); Akcigit and Kerr (2018a); Peters (2020); de Ridder (2021); Aghion et al. (2022b); Terry (2022).

[7]Similarly, Akcigit et al. (2022) investigate optimal policy in a model with heterogeneous externalities and private frictions using a mechanism design approach.

[8]See Restuccia and Rogerson (2008); Hsieh and Klenow (2009); Midrigan and Xu (2014); David et al. (2016); David and Venkateswaran (2019); David et al. (2021) for more recent advances.

2 Theory

This section introduces an endogenous growth model and derives the economic growth rate therein, which can be decomposed into a growth frontier and an allocation efficiency term.

2.1 Model Setup

Time is infinite, discrete, and indexed by t. There is a unit mass of workers in the economy.

Static production. Aggregate output per capita in the economy is the product of aggregate productivity A_t and production labor input $1 - L$:

$$Y_t = A_t(1 - L), \tag{1}$$

Aggregate productivity encompasses technological efficiency and static production frictions such as markups.[9] I focus on its evolution based on changes in technological efficiency only.

Firms. There is a unit mass of innovative firms indexed by i. Firms assign value V_{it} to successful innovations and hire R&D input ℓ_{it}, which could be a composite good, at input price W_t to achieve innovation arrival rate z_{it}:[10]

$$z_{it} = \varphi_{it} \ell_{it}^{\frac{1}{1+\phi}}. \tag{2}$$

Finally, firms are subject to first-order condition wedge Δ_{it} such that their equilibrium labor choice ℓ_{it}^* satisfies

$$\frac{\partial z_{it}}{\partial \ell_{it}}\Big|_{\ell_{it}=\ell_{it}^*} V_{it} = (1 + \Delta_{it}) \times W_t. \tag{3}$$

Note that the left-hand side is the marginal benefit of research input, while the right-hand side is the marginal cost adjusted for the R&D wedge. If $\Delta_{it} = 0$, we recover the frictionless benchmark in which firms equalize marginal benefit to marginal cost. Otherwise, firms' choices are distorted relative to the benchmark with larger level of Δ_{it} implying too little demand for R&D resources and vice versa. There are many potential interpretations of Δ_{it} including adjustment frictions, financial frictions, labor market power, and R&D subsidies.

[9] Peters (2020) derives a formulation in a similar setup that further decomposes production efficiency into a term depending on markup heterogeneity and a technological efficiency term. My results are compatible with this interpretation as long as markup heterogeneity is constant and independent of other frictions.

[10] In workhorse endogenous growth models, the value of innovation is linked to discounted future profits and innovation opportunities. Here, I take it as given.

Factor markets. I assume that the input factor is fixed at the aggregate level:

$$L = \int_0^1 \ell_{it} di. \tag{4}$$

Mechanically, this assumption focuses the attention on the allocation within the R&D sector as total R&D resources are fixed. Intuitively, this assumption captures that research talent is scarce and supplied relatively inelastically (Goolsbee, 2003; Wilson, 2009; Akcigit et al., 2017). Another way to think about this assumption is that policy already fixed the optimal amount of R&D resources in the economy and that the following results thus only concern the allocation across firms and not across the production and research sector.

Growth. I assume that the value created by a firm is linked to its productivity impact via impact-value wedge ζ_{it}, which acts as an exchange rate between value received by the firm and the growth impact of an innovation. Firms with a large impact-value wedge produce more growth per dollar of private value created and vice versa. The growth rate is given by

$$g_t \equiv \frac{\dot{A}_t}{A_t} = \int_0^1 \zeta_{it} \cdot z_{it} \cdot V_{it} \cdot di. \tag{5}$$

Note that ζ_{it} has a prominent role in the endogenous growth literature as it determines the degree to which firms' incentives are aligned with a growth-maximizing planner. Variation therein can arise e.g. because some firms are better at taking advantage of their inventions or due to heterogeneity in the innovation quality and knowledge externalities .[11]

Closing the model. I will focus exclusively on innovation sector in this economy and thus forgo an explicit household sector. Accordingly, I will use a simplified equilibrium definition. For welfare calculations, I will assume CRRA utility with discount factor β and intertemporal elasticity of substitution σ. The interest rate in the economy only affects the present discounted value of innovation, which I take as given.

Definition 1. *A Growth Equilibrium is a sequence* $\{\{V_{it}, \varphi_{it}, \Delta_{it}, \zeta_{it}, \ell_{it}\}_{i\in[0,1]}, W_t, g_t\}_{t=0,\ldots,\infty}$ *satisfying equations (2)-(5).*

[11] For the former, see e.g. de Ridder (2021); Mezzanotti (2021); Aghion et al. (2022b,a). For the latter, see e.g. Akcigit and Kerr (2018b); Akcigit and Ates (2021).

2.2 Results

With the model in place, we can derive the main result of this section in Proposition 1.

Proposition 1. *Under equations (2)-(5), we can express the economic growth rate as the product of two terms:*

$$g_t = \tilde{g}_t \times \Xi_t, \tag{6}$$

where $\tilde{g}_t$ measures the growth frontier, achievable by the growth-maximizing allocation, and $\Xi_t \in (0,1]$ measures the allocative efficiency, i.e. the fraction of maximal growth-rate that the economy is actually achieving. Let $\gamma_{it} \equiv \varphi_{it} V_{it}$, then allocative efficiency is given by

$$\Xi_t = \frac{\int_0^1 \tilde{\omega}_{it} \left((1+\Delta_{it}) \cdot \zeta_{it}\right)^{-\frac{1}{\phi}} di}{\left(\int_0^1 \tilde{\omega}_{it} \left((1+\Delta_{it}) \cdot \zeta_{it}\right)^{-\frac{1+\phi}{\phi}} di\right)^{\frac{1}{1+\phi}}} \quad \textit{with} \quad \tilde{\omega}_{it} = \frac{(\gamma_{it}\zeta_{it})^{\frac{1+\phi}{\phi}}}{\int_0^1 (\gamma_{jt}\zeta_{jt})^{\frac{1+\phi}{\phi}} dj}, \tag{7}$$

The economy achieves the growth frontier if $\zeta_{it}(1+\Delta_{it})$ is constant across firms.

According to the proposition, we can derive a summary statistic for the impact of frictions and incentive misalignment on the economic growth rate. R&D resources are exploited well in an economy with high allocative efficiency and vice versa. Importantly, Corollary 1 shows that the key to an efficient allocation is low dispersion in adjusted frictions $(1+\Delta_{it})\zeta_{it}$. Note, that their level does not matter in my context as we have a fixed R&D input supply, such that insufficient aggregate demand is adjusted for by the R&D input price and vice versa.

Corollary 1. *Up to a second order approximation, Ξ_t is given by*

$$\Xi_t \approx \exp\left(-\frac{1}{2\phi} \cdot \sigma^2_\omega(\ln(1+\Delta_{it}) + \ln \zeta_{it})\right), \tag{8}$$

where $\sigma^2_\omega(\cdot)$ is the weighted variance with observation weights $\{\omega_{it}\}$. If $\{1+\Delta_{it}, \zeta_{it}, \gamma_{it}\}$ are jointly log-normal, then the approximation is precise using the unweighted variance.

To develop some intuition for this result and connect it to the existing literature, I will discuss two simplified cases first, focusing on one wedge at a time. Consider the case of no R&D wedges, or equivalently constant R&D wedges, first. Corollary 2 shows that allocative efficiency is a function of $\{\zeta_{it}\}$ only in this case with rising dispersion leading to lower economic growth. The key intuition is that firms do not take into account the impact-value wedge and, thus, the allocation of R&D inputs is inefficient from the perspective of growth maximization. This case is at the heart of the endogenous growth literature and directly captures the mechanism at the hearth of de Ridder (2021) and Aghion et al. (2022a). In both models, firms have heterogeneous markups which are unrelated to the productivity impact of their invention. As a result, firms with high exogenous markups will earn higher profits on a given innovation and, thus, have lower impact-value wedges. Differences in these exogenous markups then leads to dispersion in impact-value wedges, which reduces allocative efficiency.

Corollary 2. *Let the R&D wedge be constant across firms, i.e.* $\Delta_{it} = \Delta_t$*, then*

$$\Xi_t = \frac{\int_0^1 \omega_{it} \left(\zeta_{it}\right)^{-\frac{1}{\phi}} di}{\left(\int_0^1 \omega_{it} \left(\zeta_{it}\right)^{-\frac{1+\phi}{\phi}} di\right)^{\frac{1}{1+\phi}}} \quad \text{with} \quad \omega_{it} = \frac{\left(\gamma_{it}\zeta_{it}\right)^{\frac{1+\phi}{\phi}}}{\int_0^1 \left(\gamma_{jt}\zeta_{jt}\right)^{\frac{1+\phi}{\phi}} dj}. \tag{9}$$

Consider the case of constant impact-value wedges next. Corollary 3 highlights that allocative efficiency is a function of the distribution of R&D wedges in this case with allocative efficiency declining in their dispersion. The result is connected to Hsieh and Klenow (2009), who show that misallocation in the production sector reduces the aggregate productivity level. Here, R&D wedges reduce allocative efficiency in the R&D sector leading to low aggregate R&D productivity with downstream consequences for productivity growth and not levels. This case has received little attention in the endogenous growth literature, however, it is closely connected to the literature on financial frictions in innovation and intangible capital (Brown et al., 2009; Peters and Taylor, 2017; Howell, 2017; Ewens et al., 2020).

Corollary 3. *Let the Impact-Value wedge be constant across firms, i.e.* $\zeta_{it} = \zeta_t$*, then*

$$\Xi_t = \frac{\int_0^1 \omega_{it} \left(1+\Delta_{it}\right)^{-\frac{1}{\phi}} di}{\left(\int_0^1 \omega_{it} \left(1+\Delta_{it}\right)^{-\frac{1+\phi}{\phi}} di\right)^{\frac{1}{1+\phi}}} \quad \text{with} \quad \omega_{it} = \frac{\gamma_{it}^{\frac{1+\phi}{\phi}}}{\int_0^1 \gamma_{jt}^{\frac{1+\phi}{\phi}} dj}. \tag{10}$$

Returning to the main formula in Proposition 1 we can see that the main result combines both insights, such that dispersion in the adjusted R&D wedge $(1 + \Delta_{it})\zeta_{it}$ reduces allocative efficiency and, thus, economic growth. From a theoretical perspective, combining both frictions can enhance or dampen their dispersion. For example, startups might be are both financially constrained and less able to take advantage of their ideas than established firms, which enhances the individual impact of both frictions. On the other hand, growth maximizing R&D subsidies, which would be directly reflected in Δ_{it}, perfectly offset dispersion in ζ_{it} such that $(1 + \Delta_{it})\zeta_{it}$ is a constant. I derive this result formally in Appendix A.5.

Combined, these results allow us to investigate the importance of R&D resource allocation for growth. Furthermore, since the results do not require a constant growth rate across time, they also allow us to directly investigate the evolution of allocative efficiency and its impact on economic growth. What remains is to measure $\{\phi, \{\gamma_{it}, 1 + \Delta_{it}, \zeta_{it}\}\}$.

2.3 Extensions

I consider a range of extension of the model in the Appendix, which I briefly highlight here.

Free entry. I show that free-entry can amplify the cost of private frictions in Appendix A.4. Low allocative efficiency reduces the expected profits of innovative firms leading to lower entry. Fewer innovative firm implies a larger mass of researchers per firm, which reduces their average productivity due to decreasing returns to scale and, thus, lowers growth.

Semi-endogenous growth. Under semi-endogenous growth, the growth-rate impact of allocative efficiency is temporary as long-run growth is driven by population growth (Jones, 1995). Improving allocative efficiency increases the growth-rate temporarily and thereby increase the long-run level of productivity. In a simple semi-endogenous growth model, the long-run level shifts 1-for-1 with Ξ. I derive the relevant formulae in Appendix C.3.

Multi-research line firms. A long tradition in endogenous growth has modeled the innovation sector with multi-research line firms (Klette and Kortum, 2004). This literature often assumes that the distribution of research lines is an endogenous object, driven by firms' innovation. In Appendix A.3, I show that my results extend to this alternative framework, however, the counterfactual holds constant the distribution of research lines across firms.[12]

Market power in the input market. We can extend the framework to allow for market power in the input market by assuming that firm-specific prices are a function of their demand. Resultingly, firms' profit function more concave and, thus, input demand less responsive to frictions. I focus on this case in companion paper Lehr (2022b).

Multiple production factors and abundant resources. It is straight-forward to show that the results derived in Proposition 1 directly apply in an economy with multiple production factors as long as their supply is perfectly inelastic. A positive supply elasticity lowers the cost of misallocation as some of it is offset by rising supply due rising prices.

[12]This extends also to the measurement approach developed in Section 3.

3 Data and Measurement

This section introduces the data and discusses measurement of the model parameters.

3.1 Data

I estimate the level and evolution of allocative efficiency using data on US listed firms. I choose this sample as there is sufficient data available to measure the underlying model primitives and directly apply the formulas developed above. My measurement approach requires three pieces of information on R&D: expenditure, value created, and growth impact.

I obtain annual firm-level R&D expenditure directly from WRDS Compustat, which collects the information from mandatory filings. In addition, this data reports information on industry classification of the firm, which will take advantage of as well.

I rely on patents to measure both the private value created from R&D as well as their growth impact. Patents are arguably the most direct measure of R&D output available to researchers. They capture an invention that the issuing patent office, in my case the US Patent and Trademark Office (USPTO), deemed both new and useful. A patent grants the owner exclusive rights to the use of the invention described therein, which gives firms strong incentives to patent their inventions.[13]

I use patent valuation estimates from Kogan et al. (2017) to measure the private value created from innovation. An advantage patent valuations is that they directly capture the private value of an invention, which is directly linked to firms' incentives to innovate. In contrast, other patent-based measures of innovation such as (quality-adjusted) patent counts only capture the quantity of innovation, but not its value to the firm (Lerner, 1995; Kogan et al., 2017; Kelly et al., 2021). These concepts can diverge e.g. due to externalities or because some firms are better equipped than other to take advantage of an invention.[14]

I use forward-citations as my primary measure of patent growth impact motivated by a large literature arguing in favor of this interpretation (Lerner, 1995; Bloom et al., 2013; Akcigit and Kerr, 2018b). At the patent-level, I construct forward-citations, i.e. citations received, by the patent within the first 5 years since the patent grant using the combined citations files of USPTO Patentsview, Kogan et al. (2017), and Berkes (2016). I then normalize this

[13] Note, not all inventions are patented and, thus, patents are an imperfect measure (Cohen et al., 2000).

[14] See e.g. Akcigit and Kerr (2018a); de Ridder (2021); Aghion et al. (2022b)

measure by the average forward-citations within an application year to make the measure comparable across years as in Kogan et al. (2017).

I aggregate citations and patent valuation up the firm-level using the exclusive mapping between firms and patent developed in Kogan et al. (2017). Patents are recorded in their application year to reflect the timing of innovation. The final dataset thus has annual observations of total R&D expenditure, patent valuations, and forward-citations.

I restrict the sample to 1975-2014 and drop firms with consistently low R&D expenditure (less than 2.5m 2012 USD per year), low patenting (less than 2.5 patents per year) or less than 5 sample years. The final sample covers more than 80% of R&D expenditure in Compustat and patent valuations in Kogan et al. (2017) for the 1975-2014 period as well as 40% of the R&D recorded in BEA accounts. See Appendix B for further data details.

3.2 Measurement

Measurement turns out to be relatively straight-forward. First, there is a strong consensus in the literature on setting $\phi = 1$ (Acemoglu et al., 2018; Akcigit and Kerr, 2018a). This value implies an elasticity of R&D expenditure to unit cost around 1.

Second, we can measure the first order condition wedge up to a constant factor directly from the R&D return, i.e. the ratio of value created from R&D divided by its cost:

$$\frac{z_{it} V_{it}}{W_t \ell_{it}} = (1+\phi) \times (1+\Delta_{it}), \tag{11}$$

which I implement using 5-year windows with a 1-year lag between R&D expenditure and patent valuations:

$$\widehat{1+\Delta_{it}} \equiv \frac{1}{1+\phi} \times \frac{\sum_{s=0}^{4} \texttt{Patent Valuations}_{it+s}}{\sum_{s=0}^{4} \texttt{R\&D Expenditure}_{it-1+s}}. \tag{12}$$

Note that the relevant formulas are HD(0) in $1+\Delta_{it}$ such that the factor $\frac{1}{1+\phi}$ has no bearing on the aggregate measures. I restrict the sample to observations with at least 50 patents to create a measure of expected returns as required by the model.

Third, we can measure R&D productivity from the firms' first order conditions, which can be rearranged to

$$\gamma_{it} = (1+\Delta_{it}) \times (W_t \ell_{it})^{\frac{\phi}{1+\phi}} \times W_t^{-\frac{1}{\phi}}. \tag{13}$$

Note, again, that the formula for allocative efficiency is independent of the scale of γ_{it} such that I can drop the final wage intercept without loss of generality as it is common across all firms. I thus measure R&D efficiency as

$$\hat{\gamma}_{it} \equiv \widehat{1+\Delta_{it}} \times \left(\sum_{s=0}^{4} \texttt{R\&D Expenditure}_{it-1+s} \right)^{\frac{1}{1+\phi}}. \tag{14}$$

Finally, I measure the impact-value wedge as the ratio of patent citations to valuations. This measurement is accurate if citations measure the growth impact of an invention up to a constant factor, which is broadly in line with the interpretation in the existing literature.

$$\hat{\zeta}_{it} \equiv \frac{\sum_{s=0}^{4} \texttt{Patent Citations}_{it+s}}{\sum_{s=0}^{4} \texttt{Patent Valuations}_{it-1+s}}. \tag{15}$$

One potentially concern with the measurement approach developed above is unrelated industry heterogeneity. For the R&D wedge, differences in the scale elasticity ϕ across industries are a potential source of variation in the return on R&D that is independent of the R&D wedge. For the impact-value wedge, a more imminent threat is heterogeneity in citation conventions that affect the relative frequency of citations across industries even if growth impacts are comparable.[15] I will address both concerns by residualizing the respective parameters with respect to industry×year fixed effects.

Finally, following Proposition 1 I estimate allocative efficiency as

$$\hat{\Xi}_{t,adjusted} = \frac{\int_0^1 \hat{\omega}_{it} \left((\widehat{1+\Delta_{it}}) \cdot \hat{\zeta}_{it} \right)^{-\frac{1}{\phi}} di}{\left(\int_0^1 \hat{\omega}_{it} \left((\widehat{1+\Delta_{it}}) \cdot \hat{\zeta}_{it} \right)^{-\frac{1+\phi}{\phi}} di \right)^{\frac{1}{1+\phi}}} \quad \text{with} \quad \hat{\omega}_{it} = \frac{(\hat{\zeta}_{it}\hat{\gamma}_{it})^{\frac{1+\phi}{\phi}}}{\int_0^1 (\hat{\zeta}_{it}\hat{\gamma}_{jt})^{\frac{1+\phi}{\phi}} dj}. \tag{16}$$

I collapse the annual estimates across periods using simple means. We can interpret them as the fractions of potential growth the economy actually achieved during the period.

[15] For example, in some technology classes such as computer processors invention might be more cumulative over time, implying a large citation counts for important inventions in absolute terms. On the other hand, in other technology classes such as drugs, invention might be more independent on average, implying lower absolute citation count regardless of the respective growth impact.

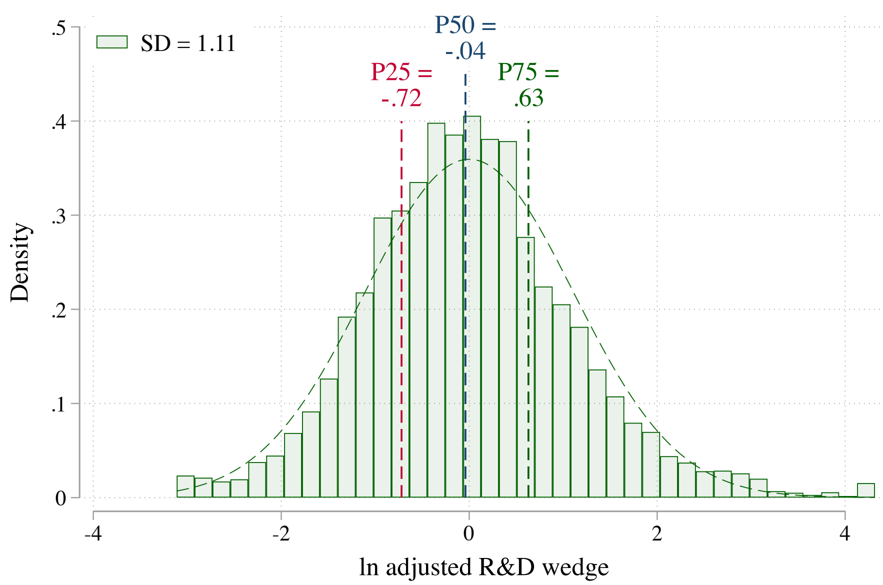
4 Allocative Efficiency and its Evolution

This section combines theory and data to investigate the level and evolution of allocative efficiency in the US. I also analyze potential microfoundations for measured wedges.

4.1 R&D and Impact-Value Wedges

Before discussing the results for estimated allocative efficiency, I want to first explore its underlying components. As shown in Figure 1, measured adjusted R&D wedges are highly dispersed. The gap between a firm at the 75th percentile and the median is approximately factor 2 with a similar gap between the 50th and 25th percentile. The decomposition in Appendix Figure C.2 highlight that R&D wedge and impact-value wedge are both highly dispersed, however, they negatively correlated such that they partly offsets each other.

Figure 1: Adjusted R&D Wedges are Highly Dispersed



Notes: Histogram of the log adjusted R&D wedges. SD refers to the standard deviation. See Section 3 and Appendix B for data and measurement details.

What drives this dispersion? The literature suggests potentially different drivers for the R&D wedge and impact value wedge and I, thus, consider each in turn. For the purpose of this exploration I run simple OLS regression with proxies for potential micro-foundations on the right hand side and wedges on the left hand side. I detail the construction of the respective proxies in Appendix B and report an extensive set of regression tables in Appendix C.1. Here, I will highlight the most informative or promising correlations.

R&D Wedges. The R&D wedge captures frictions or taxes affecting the choice of R&D expenditure. I provide evidence on three potential sources in Table 1. First, a natural starting point are proxies for financial or investment frictions such as the return on capital and investment Q, i.e. the ratio of market to book value (David et al., 2016; Philippon and Gutiérrez, 2017). Interestingly, I find that the R&D wedge is uncorrelated with the return on capital with an $R^2 < 1\%$, however, I find a strong correlation with investment Q with an $R^2 \approx 10\%$. I do not find strong correlations with other proxies for financial frictions. A second potential source for R&D wedges are subsidies, however, I do not find any evidence that they matter quantitatively across a range of proxies (Hsieh and Klenow, 2009). For example, state-level R&D tax credits are uncorrelated with R&D wedges. A final potential source is input market power, the focus of companion paper Lehr (2022a) wherein I provide evidence in favor of monopsony over inventors. Here, I provide suggestive evidence showing that R&D wedges are increasing with firms' inventor workforce. This finding is robust to focusing on firm dominance in inventor hiring within its technology fields.

Table 1: Correlations with R&D Wedges

	(1)	(2)	(3)	(4)	(5)
			ln **R&D Wedge**		
ln Return on Capital	0.069				
	(0.075)				
ln Investment Q		0.190***			
		(0.021)			
$\ln 1 - \tau$			-0.517		
			(0.700)		
ln Inventor employment				0.196***	
				(0.036)	
ln Dominance					0.108**
					(0.052)
R2	0.002	0.099	0.001	0.050	0.006
Observations	9,147	8,837	8,785	9,147	8,190

Note: This table reports OLS coefficient estimates. Return on capital is the ratio of revenue to capital stock. Investment Q is the ratio of market to book value. Subsidy rates based on state-level R&D tax credits. See Appendix B. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Impact-Value Wedges. Impact-value wedges measure a misalignment between public and private incentives, which can arise, e.g.,if firms have heterogeneous markups or profitability unrelated to the quality of their innovations (de Ridder, 2021; Aghion et al., 2022a). Indeed, Table 2 reports that impact-value wedges are smaller for firms with high mark-ups or profit rates. Another potential channel is that large and old firms conduct too much R&D in order to stay in business (Acemoglu et al., 2018). Indeed, I find that young firms have larger impact-value wedges, while larger firms, both in terms of employment and R&D expenditure,have lower R&D returns on average. The correlations with profit rates and firm size explain more than 10% of the variation each. Finally, optimal policy suggests that the government should offset the impact-value wedge with subsidies. Empirically, I do not find a significant relationship of the R&D wedge with state-level R&D subsidies nor with the share of patent valuation connected to state actors. Furthermore, it appears that firms engaged in R&D specific lobbying tend to have lower, not larger R&D wedges. Thus, I cannot find evidence that policy makers responds to the R&D wedge as suggested by the model.

Table 2: Correlations with Impact-Value Wedges

	(1)	(2)	(3)	(4)	(5)
		ln **Impact-Value Wedge**			
ln Markup	-0.556**				
	(0.216)				
Profit rate		-3.859***			
		(0.517)			
$\ln 1 - \tau$			-1.100		
			(1.195)		
ln Employment				-0.366***	
				(0.035)	
ln R&D Expenditure					-0.372***
					(0.050)
R2	0.033	0.124	0.002	0.262	0.239
Observations	8,446	9,147	8,785	9,147	9,147

Note: Markups from Loecker et al. (2020). Profitability is the ratio of earnings before taxes divided by revenue. Subsidy rates based on state-level R&D tax credits. See Appendix B. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Adjusted R&D Wedges. Ultimately, allocative efficiency rests on the behavior of adjusted R&D wedges $(1 + \Delta_{it})\zeta_{it}$. Empirically, I find that many predictors of its individual components become less predictive for the combined measure with only three valuables explaining more than 2% of the overall variation: firm dominance in its inventor markets, total employment, and total R&D budget. The correlation with lobbying suggests that firms focused on R&D lobbying tend to conduct too much R&D, such that they have low adjusted R&D wedges. On the other hand, the correlation with inventor market dominance is positive, suggesting too little R&D as predicted by e.g. inventor monopsony. Finally, the correlation with overall firm size and firms' R&D budget suggest that large firms and those conducting a lot of R&D tend to do too much R&D, i.e. they have low adjusted R&D wedges. Both of these correlations account for more than 20% of the variation. Furthermore, the evidence discussed above suggests that this is primarily due to the low impact-value wedges for these firms, which could be interpreted e.g. through the lens of Acemoglu et al. (2018).

Table 3: Correlations with Adjusted R&D Wedges

	(1)	(2)	(3)
	ln **Adjusted R&D Wedge**		
ln Employment	-0.320***		
	(0.034)		
ln R&D Expenditure		-0.424***	
		(0.038)	
ln Dominance			0.295***
			(0.087)
R2	0.182	0.282	0.032
Observations	9,147	9,147	8,190

Note: See Appendix B. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Overall, the evidence gives us an understanding of some of the underlying forces driving these wedges, however, a large share of variation remains unexplained. Here, I will take this variation as given and instead focus on estimating its impact on allocative efficiency in the R&D sector.

4.2 Average Allocative Efficiency and its Evolution

The level of allocative efficiency. The first column in Table 4 reports estimates of allocative efficiency for the full sample using the formula in Proposition 1. The estimated allocative efficiency is surprisingly low, suggesting that the US only achieved 60% of its growth potential over the period. Against a realized growth-rate of 1.5%, the estimate suggests a frontier growth-rate of 2.5%. The model thus estimates that US productivity would have been around 50% larger at the end of the sample if the US had achieved its frontier growth rate. Considering the individual components in the second and third row, we can see that both contribute to low overall allocative efficiency. Ignoring the variation in the impact value wedge yields an estimate of 70%, while we get an estimate of 55% if we ignore variation in R&D wedges. For comparison, Hsieh and Klenow (2009) estimate that US productivity would be 40% larger in absence of to capital misallocation, while Berger et al. (2022) estimate that US output would be 21% larger in absence of monopsony power in the production sector. My estimates thus suggests that misallocation is significantly worse in the R&D sector compared to the production sector.

Table 4: Estimating Allocative Efficiency

Measure	Period		
	1975-2014	1975-1994	2005-2014
Baseline	58.3%	70.4%	40.9%
	(0.017)	(0.012)	(0.058)
Only R&D Wedge	70.4%	78.5%	59.2%
	(0.013)	(0.019)	(0.018)
Only Impact-Value Wedge	55.1%	68.9%	35.9%
	(0.006)	(0.007)	(0.017)

Note: Estimates of allocative efficiency using Proposition 1. See text for variable definition. Allocative efficiency measures are first constructed at the annual measure and then averaged over the relevant sample using the geometric mean. assumes a constant Impact-Value wedge and vice versa. The adjusted measure uses citations to measure the growth impact of patents. Standard errors are reported in paranthesis and constructed via the Delta-method.

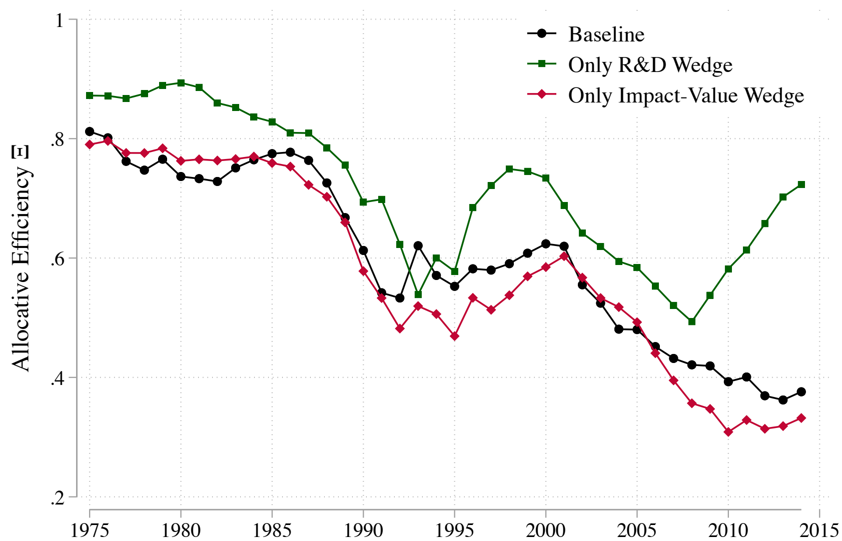
The estimated growth-rate cost have large aggregate implications. For example, the baseline estimate suggests that US productivity could be 50% larger at the end of my sample under the growth-maximizing resource allocation.[16] More systematically, when I translate growth-

[16]My sample spans 40 years. Assuming a realized growth rate of 1.5% per annum, the baseline estimate

rate cost into consumption-equivalent cost, I find a 30% welfare improvement from moving towards frontier growth assuming an intertemporal elasticity of substitution of 2 and a welfare improvement of 70% under log consumption preferences.[17]

It goes without saying that not all R&D misallocation might be preventable. For example, Asker et al. (2014) argue capital misallocation is partly driven by technological constraints and adjustment frictions in particular. Similar arguments might apply in the case of R&D misallocation. For example, in the companion paper Lehr (2022b) I argue that this misallocation is partly driven by friction in the market for inventors, which a policy maker might not be able to fully address.

Figure 2: Allocative efficiency has declined steadily since 1980



Notes: This figure plots annual estimates of allocative efficiency using the formulas developed in Corollary 3 and Proposition 1. The simple measure assumes a constant Impact-Value wedge across firms, in contrast to the adjusted measure.

The Evolution of Allocative Efficiency. Figure 2 plots the annual estimates of allocative efficiency and reveals a relatively steady decline. As with the overall level, the decline is driven by both R&D and Impact-Value wedges as shown by the individual lines. The most notable difference across lines that the R&D wedge only measure experiences a strong

for allocative efficiency implies a frontier growth rate of 2.57%. We thus have $(1.0257/1.015)^{40} \approx 1.5$ or a 50% larger productivity after 50 years.

[17] The general formula for an intertemporal elasticity of substitution $\sigma \neq 1$ can be derived as $\Delta_C = \left(\frac{\rho+(\sigma-1)\tilde{g}}{\rho+(\sigma-1)g}\right)^{\frac{1}{\sigma-1}} - 1$. With log preferences, i.e. $\sigma = 1$, we have $\Delta_C = \exp\left(\frac{g}{\rho}\frac{1-\Xi}{\Xi}\right) - 1$. My calculations assume $\rho = 0.02$ as in Acemoglu et al. (2018). See A.5 for details.

rebound after 2008, while the Impact-Value wedge only measure continues its decline. Furthermore, there is a temporary improvement during the dot-com boom in the R&D wedge only measure that is absent in the other measures.

The magnitudes of the decline are remarkable. Estimated allocative efficiency starts out at around 80% for the first decade from 1975 to 1984 followed by a strong decline toward c. 60% for the 1990-2000 period. From 2000 to 2014, allocative efficiency further decline at a constant pace reach around 40% in 2010. The broad decline across the sample is driven by both individual wedges in tandem, however, there is some divergence towards the end of the sample.[18] While the R&D wedge only measure rebounds to above 70% in 2014, the impact-value wedge only measure continues its decline to ultimately less than 40%. Taking simple averages, column (2) and (3) in Table 4 suggest that allocative efficiency declined 30 percentage points when comparing the 1975-94 and 2005-2014 period. All else equal, these estimates suggest that rising misallocation should have led a decline in the annual growth-rate by 40%, which is somewhat larger than the observed decline in economic growth (30%, see introduction).

Magnitudes. The level of estimated allocative efficiency is surprisingly low a first, especially for the adjusted measure. Comparison with the literature, however, puts this into context. Using a similar research design, Hsieh and Klenow (2009) estimate that US output could be 40% larger in absence of capital misallocation in the production sector, which is close to my unadjusted estimate. The gap to the full estimate is then attributable to measured misalignment of innovation incentives, which other papers have argued to be significant as well (de Ridder, 2021; Aghion et al., 2022a). Nonetheless, healthy skepticism might be useful when considering the level of measured misallocation. That being said, the evolution of measured misallocation is even more surprising as my estimate suggest that allocative efficiency has essentially halved during my sample. I discuss below how we can explain the less dramatic decline in economic growth through the lens of the model as well as how alternative measurement choices affect the picture. The estimated magnitudes suggest two alternative interpretations: either, misallocation is a much larger problem than previously recognized by the literature and requires significantly more attention by the literature, or, our models are a surprisingly bad description of the world and more research is required to bring model and data closer together.

[18] This divergence arises as the correlation between the two wedges becomes less negative in the final years. I documented this in greater detail in Appendix C.4.

4.3 Discussion

Specification. I have three main choices in the construction of my measures: the time-window in consideration, the lag between R&D expenditure and output, and the minimum number of patents required. For my baseline, I chose a window of 5 years, a lag of 1 year, and a minimum of 50 patents. Table 5 confirms that neither choice is driving my results. Extending the time-window leaves measured allocative efficiency unaffected, while extending the lag between R&D inputs and outputs decreases it. Finally, requiring at least 200 patents raises estimated allocative efficiency by 5 percentage points, however, it also excludes about 50% of the observations and focuses the attention only on the largest firms in the sample.

Table 5: Estimating Allocative Efficiency — Alternative Specifications

Measure	Period		
	1975-2014	1975-1994	2005-2014
A. Minimum Patents			
50 Patents (Baseline)	58.1%	70.3%	40.5%
100 Patents	60.3%	74.6%	39.8%
200 Patents	62.7%	77.6%	39.4%
B. Time Horizon			
5-Year (Baseline)	58.1%	70.3%	40.5%
10-Year	59.7%	72.3%	41.6%
20-Year	58.0%	73.9%	41.6%
C. Investment-Realization Gap			
1-Year (Baseline)	58.1%	70.3%	40.5%
2-Year	56.4%	68.8%	38.8%
5-Year	52.8%	64.6%	38.5%

Note: Estimates of allocative efficiency using Proposition 1 under alternative specifications. See text for variable definition. Allocative efficiency measures are first constructed at the annual measure and then averaged over the relevant sample using the geometric mean.

Measurement choices. I make two important measurement assumptions when estimating allocative efficiency. First, I measure the output created from R&D using patent valuations, which might not capture all the value created as some inventions are not patented (Cohen et al., 2000). Following Bloom et al. (2020) I construct an alternative measure of R&D output from cumulative, non-negative sales changes, which might be a more comprehensive measure, although likely less precise. Panel (a) in Figure 3 shows that estimated allocative efficiency is essential identical to my main specification. Even when focusing R&D wedges only, the estimates track each other closely.

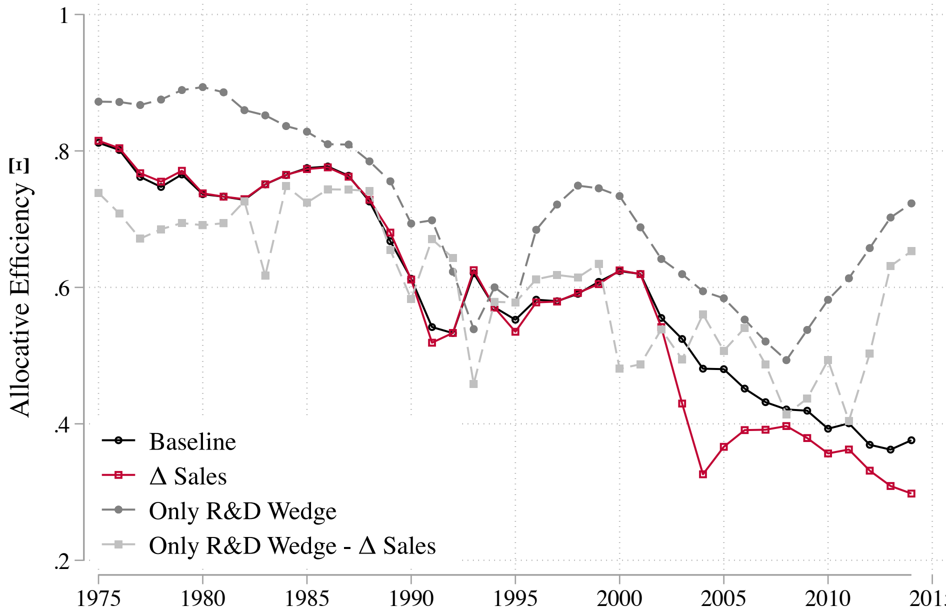
A second measurement assumption was using patent citations as a proxy for the growth impact. I consider the text-based patent impact measure developed in Kelly et al. (2021) as an alternative measure, which is available until 2007. Panel (b) in Figure 3 reports the results and confirms highly similar trends using both measure. The estimate are tightly aligned in the early and late sample with slight divergence around 1990. Thus, the main conclusion are robust to this alternative measure of a patent's growth impact.

Measurement error. An important challenge when measuring the underlying model parameter is measurement error, which would naturally inflate the dispersion in adjusted R&D wedges and thus decrease measured allocative efficiency. I propose and implement a two complementary approaches to estimating the importance of measurement error in Appendix C.2 using both a structural and bootstrapping approach. Perhaps surprisingly, I find that it contributes a small fraction of the overall variance in measured R&D wedges. I implement both approaches using a rolling window and construct measurement adjusted measures of allocative efficiency as shown in Figure 4. Measurement adjustment improves the level of estimated allocative efficiency, but does not change its evolution. Even with measurement error adjusted, allocative efficiency declines by about 40% comparing the 2005-2014 period to 1975-1995 as reported in Appendix Table C.11.

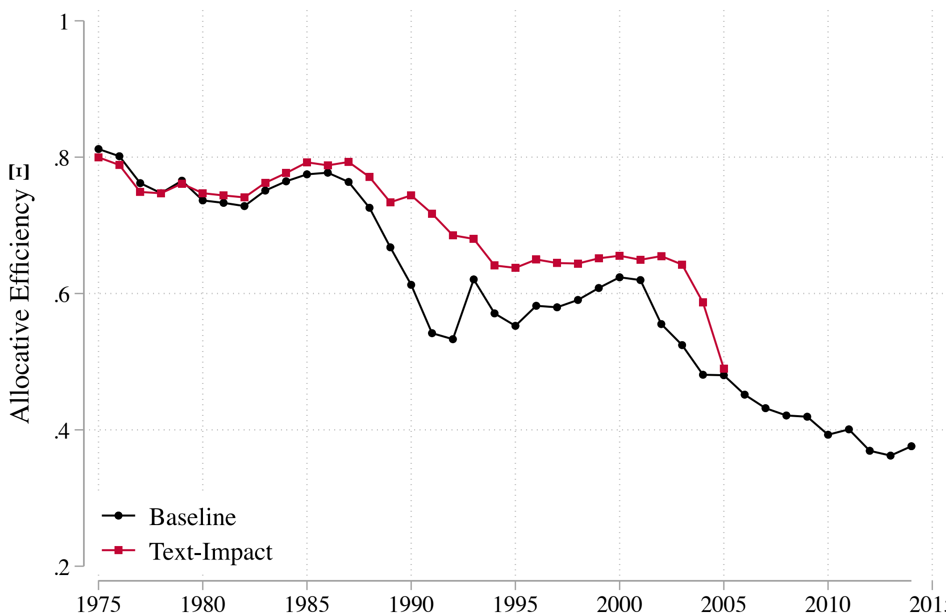
Parametric choices. Another important assumption was setting $\phi = 1$ in line with the broad consensus in the literature (Acemoglu and Restrepo, 2018; de Ridder, 2021), however, some papers have argued for potentially larger values, i.e. more concave R&D production functions.[19] The formulae in Corollary 1 suggest that larger values of ϕ will indeed dampen the effect of dispersion in adjusted R&D wedges. Figure 5 reports alternative estimates of allocative efficiency using larger values of ϕ, which, as expected, yield larger values of estimated allocative efficiency and a flatter evolution. Nonetheless, even when doubling ϕ, estimated allocative efficiency is around 27% larger in the early compared to the late period.

[19] Terry (2022) structurally estimate a value around 1.3, close to the upper bound estimated in Guceri (2018). Dechezleprêtre et al. (2019) find significantly larger values for small and medium size firms, but argue that this is partly due to financial frictions. Similar findings emerge in Guceri and Liu (2019).

Figure 3: Allocative Efficiency Under Alternative Measurement



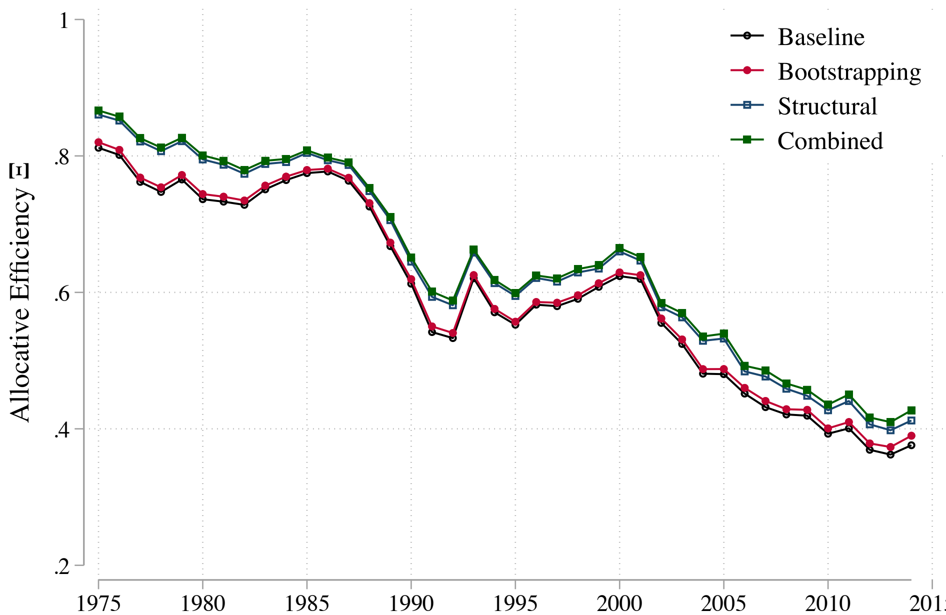
(a) Private Value Created



(b) Growth Impact

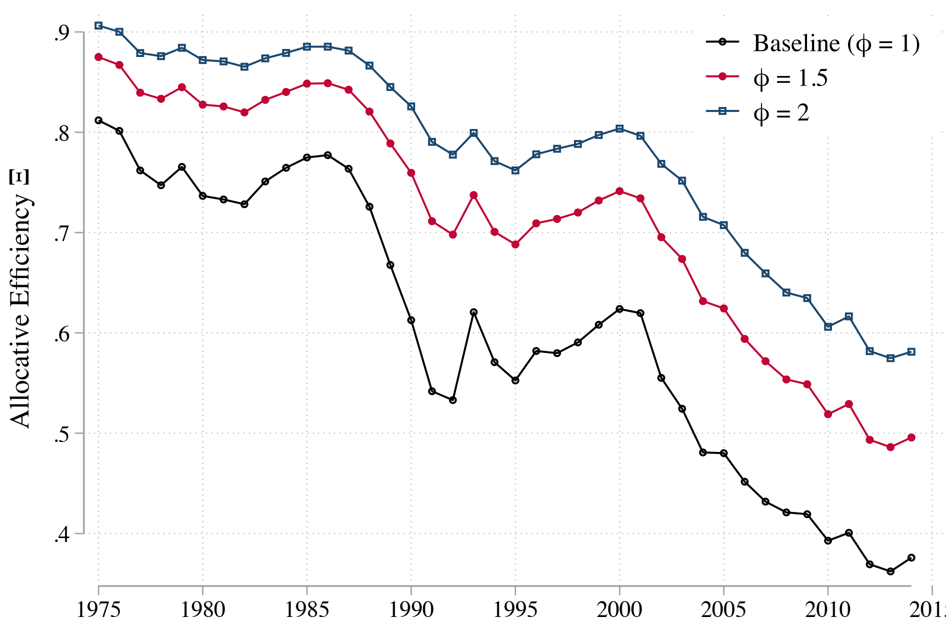
Notes: This figure plots the evolution of estimated allocative efficiency under alternative measurement assumptions. In panel (a), $\Delta Sales$ labeled lined use changes in sales instead of patent valuations to measure R&D output. In panel (b), the line labeled "Text-Impact" uses the text-based patent impact measure in Kelly et al. (2021) instead of patent citations to measure the growth impact of an innovation.

Figure 4: Allocative Efficiency with Measurement Error Adjustment



Notes: This figure plots annual estimates of allocative efficiency adjusted for measurement error. See Appendix C.2 for details on the adjustment.

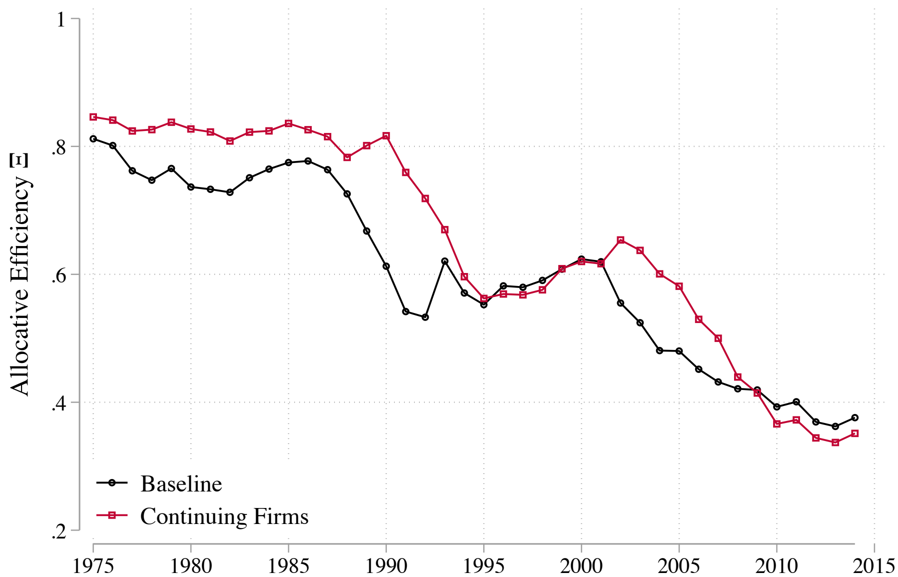
Figure 5: Allocative Efficiency Under Alternative R&D Scale Elasticities



Notes: This figure plots the evolution of estimated allocative efficiency under alternative assumptions for R&D scale elasticity ϕ.

Entry-and-Exit. I investigate entry-and-exit in Figure 6 by comparing my baseline estimates with a sample restricted to firms active for at least 75% of the sample. The tight fit of both measures suggests that entry-and-exit does not drive declining allocative efficiency.

Figure 6: Allocative efficiency for all and continuing firms



Notes: This figure plots annual estimates of allocative efficiency using the formulas developed in Corollary 3 and Proposition 1 for the main sample and firms with 30 or more active years. The simple measure assumes a constant Impact-Value wedge across firms, in contrast to the adjusted measure. Lines marked "Cont." only include firms with 30 or more active years in the sample. The total sample length is 40 years.

Semi-endogenous growth. I compare my main results to the implications in a semi-endogenous growth model in Appendix C.3. My results suggests that comparable welfare effects across models, primarily driven by slow transition dynamics. Even after 50 years, the growth-rate still maintains around 50% of the initial gains from solving R&D misallocation.

Countervailing forces. The model identifies a potential countervailing force to declining allocative efficiency: gains from specialization due to rising R&D productivity dispersion. In the model, heterogeneity in R&D productivity leads to a faster economic growth rate as R&D activity is skewed towards high productivity firms, raising the activity-weighted average R&D productivity. In Appendix C.5 I indeed find that rising heterogeneity in R&D productivity offsets declining allocative efficiency, especially post 2005. While this explains why economic growth has not completely collapsed, it also highlights the rising opportunity of improving efficiency in the allocation of R&D resources.

4.4 Potential Drivers of Declining Allocative Efficiency

The strong decline throughout the sample raises the question as to which forces are driving the observed pattern. A range of alternative explanations are possible. First, it is well documented that federal involvement in R&D decreased significantly during the sample period. As shown in Figure 7, the share of federal in total R&D expenditure declined from 45% in 1975-94 to 29% in 2005-14. Even more important, the share of business R&D funded by the federal government declined from 28% in the early period to 11% in the late period. It is possible that this decline reduced the governments ability to guide private R&D towards high impact innovation, especially those whose benefits are not as easily captured by innovating firms.[20] Alternatively, these research funds might have alleviated potential financial constraints or given firms access to a larger pool of R&D talent.

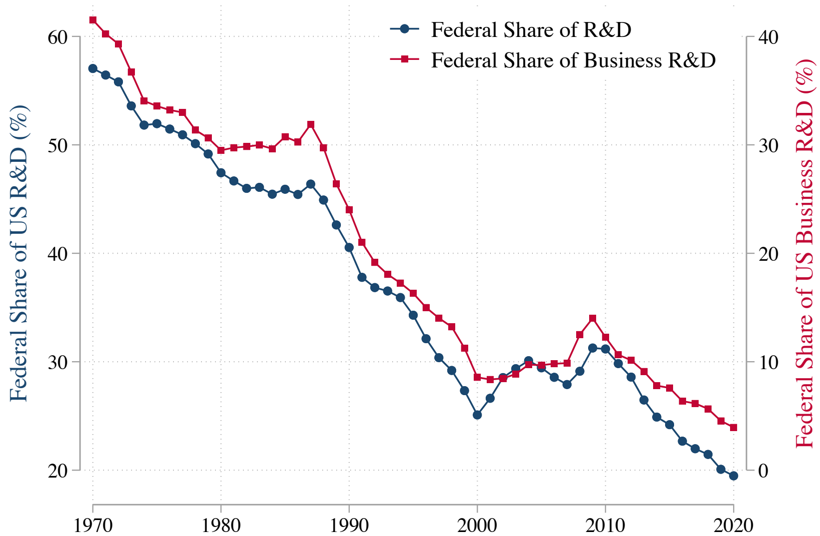
Second, the growing dispersion in the impact-value wedge could be due to changing institutional environment or technology. A growing literature is arguing the patent system itself and changes therein have deteriorated US innovation infrastructure due to litigation risk, blocking of follow on innovation, and giving firms incentives to patent even minor inventions (Jaffe and Lerner, 2007; Kim, 2017; Mezzanotti, 2021). These changes directly impact the gap between the private value and growth impact of an innovation and, thus, could have contributed to rising dispersion in the impact-value wedge. On the technology side, de Ridder (2021) and Aghion et al. (2022b) argue that the IT revolution has created a new class of firm that is able to profit more from its innovation to due static productivity advantages linked to IT investments and data. These differences naturally would also be reflected in the impact-value wedge.

Finally, another possibility is that the superstar firm phenomena and the associated rise in concentration has increased firm's market power in the market for inventors. Labor market power can increase misallocation as firms artificially lower inventor demand to keep wages low.[21] In Lehr (2022a) I provide evidence that firms indeed have market power and propose a model that links market power to firm size as in Card et al. (2018) with the implication that rising dispersion in R&D activity indeed leads to lower allocative efficiency.

[20]One example here could be so-called fundamental research, which lays the groundwork for future innovation, but does not always immediately lead to new products (Akcigit et al., 2020).

[21]See e.g. Autor et al. (2020); Seegmiller (2021); Berger et al. (2022); Yeh et al. (2022). Berger et al. (2022) find that local concentration, which is the relevant statistic for firms' market power in their model, has decreased for workers. Arguably the market for inventors is more structured by human capital specificity rather than geography. Note also that R&D expenditure and patent valuations have not experienced the same rising concentration as documented in Autor et al. (2020). See Appendix Figure C.4.

Figure 7: Federal Support of R&D Has Decline Since 1975



Notes: Author's calculations based on NSF National Pattern.

5 Conclusion

Economic growth has declined significantly over the past decades due to declining aggregate R&D productivity. I argue that this decline is partly driven by rising misallocation of R&D resources due to frictions. Building on a workhorse endogenous growth model, I derive a close-form solution for impact of frictions and incentive misalignment on allocative efficiency and economic growth. I take the model to the data for a sample of listed US firms with significant R&D activity and estimate allocative efficiency for the 1975-2014 period.

Two findings emerge. First, the estimates suggest that growth is significantly below potential due to low allocative efficiency. Second, allocative efficiency declined dramatically over the sample, with estimates ranging from 25%-40%. The observed decline is a consistent finding under a range of alternative measurement strategies and suggests that declining allocative efficiency made a large contribution to the observed decline in aggregate R&D productivity.

These findings suggest important avenues for future research. Most importantly, more research is needed to understand the underlying forces driving rising dispersion. Lehr (2022b) suggests frictions in the market for inventors as one source, however, this mechanism doesn't explain the full dispersion in measured R&D wedges. A thorough understanding of the variation in adjusted R&D wedges will allow for the development of potentially targeted policies and is thus essential for improving allocative efficiency in the R&D sector.

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Appendix

A Model Appendix

A.1 Proofs

A.2 Welfare

In this section I describe how we can translate changes in allocative efficiency into welfare terms. For this purpose, consider a simple household with log-preferences consumption sequence $\{c_t\}$ and discount factor β s.t. its welfare is given by

$$\mathcal{W}(\{c_t\}) = \sum_{t=0}^{\infty} \beta^t \ln c_t. \tag{A.1}$$

If consumption grows at a constant rate g, then we can simplify this expression to

$$\mathcal{W}(c_0, g) = \frac{1}{1-\beta}\left(\ln c_0 + \frac{\beta}{1-\beta}\ln(1+g)\right). \tag{A.2}$$

For any given change in g, we can thus solve for a permanent shift in the consumption stream that yields an equivalent change in welfare holding the growth rate constant as the solution to $\mathcal{W}(c_0 \cdot (1+\Delta_C), g) = \mathcal{W}(c_0, g+\Delta_g)$. Solving the system of equations, we have

$$\Delta_C \approx \exp\left(\frac{\beta}{1-\beta}\cdot \Delta_g\right) - 1. \tag{A.3}$$

The welfare cost of a given long-run level of misallocation Ξ are thus given by

$$\Delta_C \approx \exp\left(\frac{\beta}{1-\beta}\cdot \tilde{g} \cdot (1-\Xi)\right) - 1. \tag{A.4}$$

A.3 Multiple R&D lines

Consider an alternative version of the model with multiple R&D lines per firm. I will index a firm by $i \in \mathcal{I}$ and a R&D line by $j \in \mathcal{J}_i$.

Production function

$$z_{ij} = \varphi_{ij} \ell_{ij}^{\frac{1}{1+\phi}} \tag{A.5}$$

First order conditions at the R&D line level:

$$\frac{1}{1+\phi} \ell_{ij}^{-\frac{\phi}{1+\phi}} \gamma_{ij} = \frac{1}{\xi_{ij}} W \tag{A.6}$$

Return

$$\frac{\gamma_{ij} \ell_{ij}^{\frac{1}{1+\phi}}}{W \ell_{ij}} = \frac{1}{\xi_{ij}} \times (1+\phi) \tag{A.7}$$

Wages

$$\tilde{W} \equiv (1+\phi) W = L^{-\frac{\phi}{1+\phi}} \left(\int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_{\rangle}} (\gamma_{ij} \xi_{ij})^{\frac{1+\phi}{\phi}} \Big) di \right)^{\frac{\phi}{1+\phi}} \tag{A.8}$$

Growth rate.

$$\begin{aligned} g = & \int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} z_{ij} (\lambda_{ij} - 1) \Big) di = \int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} \zeta_{ij} \gamma_{ij} \ell_{ij}^{\frac{1}{1+\phi}} \Big) di & \text{(A.9)} \\ = & \int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} \frac{\zeta_{ij}}{\xi_{ij}} \tilde{W} \ell_{ij} \Big) di & \text{(A.10)} \\ = & L^{\frac{1}{1+\phi}} \frac{\int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} \tilde{\gamma}_{ij}^{\frac{1+\phi}{\phi}} (\xi_{ij}/\zeta_{ij})^{\frac{1}{\phi}} \Big) di}{\left(\int_{\mathcal{I}} \Big(\sum_{j \in \mathcal{J}_i} \tilde{\gamma}_{ij}^{\frac{1+\phi}{\phi}} (\xi_{ij}/\zeta_{ij})^{\frac{1+\phi}{\phi}} \Big) di \right)^{\frac{1}{1+\phi}}} & \text{(A.11)} \end{aligned}$$

Now consider the case of $\zeta_{ij} = \zeta$ first.

At the firm level, we have

$$\frac{1}{\xi_i} = \frac{\sum_{j \in \mathcal{J}_i} \gamma_{ij} \ell_{ij}^{\frac{1}{1+\phi}}}{\sum_{j \in \mathcal{J}_i} W \ell_{ij}} = \sum_{j \in \mathcal{J}_i} \frac{\ell_{ij}}{\ell_i} \frac{1}{\xi_{ij}} \tag{A.12}$$

$$\gamma_i = \frac{1}{\xi_i} \tilde{W}^{\frac{1}{1+\phi}} (\tilde{W} \ell_i)^{\frac{\phi}{1+\phi}} \tag{A.13}$$

$$\zeta_i = \frac{\sum_{j \in \mathcal{J}_i} \gamma_{ij} \ell_{ij}^{\frac{1}{1+\phi}} \zeta_{ij}}{\sum_{j \in \mathcal{J}_i} \gamma_{ij} \ell_{ij}^{\frac{1}{1+\phi}}} \tag{A.14}$$

Some algebra confirms that

$$g = L^{\frac{1}{1+\phi}} \frac{\int_{\mathcal{I}} \tilde{\gamma}_i^{\frac{1+\phi}{\phi}} (\xi_i/\zeta_i)^{\frac{1}{\phi}} di}{\left(\int_{\mathcal{I}} \tilde{\gamma}_i^{\frac{1+\phi}{\phi}} (\xi_i/\zeta_i)^{\frac{1+\phi}{\phi}} \right)^{\frac{1}{1+\phi}}}. \tag{A.15}$$

A.4 Misallocation with Free Entry

Assuming that frictions Δ_{it} show up directly in the firm's cost function, we have

$$\begin{aligned} \mathcal{V}_{it} &\equiv \max \left\{ \varphi_{it} \ell_{it}^{\frac{1}{1+\phi}} V_{it} - W_t \ell_{it} (1 + \Delta_{it}) \right\} \\ &= \frac{\phi}{1+\phi} \times \gamma_{it}^{\frac{1+\phi}{\phi}} \times (\tilde{W}_t (1 + \Delta_{it}))^{-\frac{1}{\phi}}. \end{aligned} \tag{A.16}$$

Let M be the mass of active firms, then the equilibrium wage satisfies

$$\tilde{W}_t = \left(\frac{M}{L} \right)^{\frac{\phi}{1+\phi}} \left(\int_0^1 \gamma_{it}^{\frac{1+\phi}{\phi}} (1 + \Delta_{it})^{-\frac{1+\phi}{\phi}} di \right)^{\frac{\phi}{1+\phi}}. \tag{A.17}$$

Imposing the free-entry condition $\mathbb{E}[\mathcal{V}_{it}] = \phi_E \frac{\phi}{1+\phi}$, we can solve for the equilibrium mass of firms as

$$\phi_E = \tilde{\Phi} \times \tilde{\Xi} \times \left(\frac{L}{M} \right)^{\frac{1}{1+\phi}}, \tag{A.18}$$

where $\tilde{\Phi} = \left(\int_0^1 \gamma_{it}^{\frac{1+\phi}{\phi}} di \right)^{\frac{1+\phi}{\phi}}$ is a measure of expected firm-level R&D productivity and $\tilde{\Xi}$ is the allocation efficiency term under constant impact-value wedge. We can then solve for the equilibrium growth rate for a given mass of firms M as

$$g = M^{\frac{\phi}{1+\phi}} (L)^{\frac{1}{1+\phi}} \times \Phi \times \Xi, \tag{A.19}$$

where $\Phi = \left(\int_0^1 (\zeta_{it}\gamma_{it})^{\frac{1+\phi}{\phi}} di\right)^{\frac{1+\phi}{\phi}}$ is a measure of growth-impact adjusted firm-level R&D productivity and Ξ is allocative efficiency.

Finally, combining growth-rate formula and free-entry condition, we have

$$g = L \times \phi_E^{-\frac{\phi}{1+\phi}} \times \left(\tilde{\Phi}^{\phi} \times \Phi\right) \times \left(\tilde{\Xi}^{\phi} \times \Xi\right). \tag{A.20}$$

The combined allocative efficiency term $\tilde{\Xi}^{\phi} \times \Xi$ has additional sensitivity to private frictions. In particular, under joint log-normality, we have

$$\ln\left(\tilde{\Xi}^{\phi} \times \Xi\right) = -\frac{1}{2}\left(\frac{1+\phi}{\phi}\sigma^2(\ln(1+\Delta_{it})) + \frac{1}{\phi}\sigma^2(\ln\zeta_{it}) + \frac{2}{\phi}\sigma(\ln(1+\Delta_{it}), \ln\zeta_{it})\right), \tag{A.21}$$

which now has a weight of $\frac{1+\phi}{\phi}$ on the variance of $\ln(1+\Delta_{it})$ instead of $\frac{1}{\phi}$ as in the baseline case. Thus, the cost of private frictions has become more costly to growth.

A.5 The Planner's Problem

Consider a growth-maximizing planner solving the following problem:

$$\max_{\{\ell_{it}\}} g = \int_0^1 \zeta_{it} z_{it} V_{it} di \quad \text{s.t.} \quad \int_0^1 \ell_{it} di = L \quad \text{and} \quad z_{it} = \varphi_{it}\ell_{it}^{\frac{1}{1+\phi}}. \tag{A.22}$$

It is straight-forward to show that the growth maximizing allocation is given by

$$\frac{\ell_{it}}{L} = \frac{(\gamma_{it}\zeta_{it})^{\frac{1+\phi}{\phi}}}{\int_0^1 (\gamma_{jt}\zeta_{jt})^{\frac{1+\phi}{\phi}} dj}. \tag{A.23}$$

The associated growth rate is equivalent to setting $\Xi_t = 1$ in Proposition 1. Furthermore, the planner can implement the allocation with subsidies τ_{it} set such that

$$1 - \tau_{it} = \frac{1}{(1+\Delta_{it})\zeta_{it}}, \tag{A.24}$$

where Δ_{it} measures any frictions other than the subsidies.

A.6 Approximations of Allocative Efficiency

Assuming no productivity differences and homogeneous φ, we have

$$\ln \Xi = \ln \left[\int_0^1 ((1+\Delta)(1+\varphi))^{-\frac{1}{\phi+\varphi(1+\phi)}} \, di \right] - \frac{1}{1+\phi} \ln \left[\int_0^1 ((1+\Delta)(1+\varphi))^{-\frac{1+\phi}{\phi+\varphi(1+\phi)}} \, di \right] \tag{A.25}$$

I will define $X_i = ((1+\Delta_i)(1+\varphi_i))^{-\frac{1}{\phi+\varphi_i(1+\phi)}}$ and $Y_i = X_i^{1+\phi}$ together with their respective aggregate X and Y s.t. for $\varphi = \varphi_i$ and $\Delta_i = \Delta$ we have

$$\ln \Xi = \ln X - \frac{1}{1+\phi} \ln Y.$$

Note that $\frac{\partial \ln Y_i}{\partial \ln 1+\varphi_i} = (1+\phi)\frac{\partial \ln X_i}{\partial \ln 1+\varphi_i}$ and $\frac{\partial \ln Y_i}{\partial \ln 1+\varphi_i} = (1+\phi)\frac{\partial \ln X_i}{\partial \ln 1+\varphi_i}$

Lemma 1. *Consider a small, idiosyncratic change in Δ_i or φ_i around $\Delta_i = \Delta$ and $\varphi_i = \varphi$. Up to a first-order approximation the change leaves $\ln \Xi$ unaffected.*

Proof of Lemma 1. Firstly, note that $\Xi = 0$ for $\Delta_i = \Delta$ and $\varphi_i = \varphi$. The derivative of $\ln Xi$ with respect to any change in z is given by

$$\frac{\partial \ln \Xi}{\partial \ln z_i} = \frac{X_i}{X}\frac{\partial \ln X_i}{\partial \ln z_i} - \frac{1}{1+\phi}\frac{Y_i}{Y}\frac{\partial \ln Y_i}{\partial \ln z_i}$$

The first-order Taylor approximation is then given by

$$\ln \Xi \approx \int_0^1 \frac{\partial \ln \Xi}{\partial \ln z_i}\Big|_{z_i=z} d \ln z_i.$$

It is straight-forward to show that in the symmetric equilibrium $Y_i = Y$ and $X_i = X$ s.t. that $\frac{\partial \Xi}{\partial \ln z_i}\Big|_{z_i=z} = 0$ and as a result $\ln \Xi \approx 0$ or $\Xi \approx 1$. □

Lemma 2. *Consider a small, idiosyncratic change in Δ_i around $\Delta_i = \Delta$. Up to a second-order approximation, we have*

$$\widehat{\ln \Xi} = -\frac{1}{2}\frac{\phi}{(\phi+\varphi(1+\phi))^2} Var(\ln(1+\Delta_i)). \tag{A.26}$$

B Data and Measurement Appendix

B.1 Additional Measures

Mapping patents to firms. I assign patents to firms based on the crosswalk between patents and `PERMNO`s in Kogan et al. (2017), which I extend to `GVKEY`s using the mapping provided by WRDS.

Measuring inventor employment. Let $\mathcal{P}_{it\to t+4}$ be the set of successful patent applications for firm i between t and $t+4$ and $\mathcal{I}_{it\to t+4}$ be the set of associated inventors. I will denote the number of patents assigned to firm i and listing j as inventor at time t as P_{ijt} and the total number of patents listing j as inventor as P_{jt}

$$\texttt{Inventors}_{it\to t+4} = \sum_{j\in\mathcal{I}_{it\to t+4}} \frac{\sum_{s=0}^{4} P_{ijt+s}}{\sum_{s=0}^{4} P_{jt+s}}. \tag{B.1}$$

I use two additional measure in robustness checks. Firstly, I use the raw size of $|\mathcal{I}_{it\to t+4}|$, which forgoes the full-time equivalent adjustment, and, secondly, I construct the measure first at the 1-year horizon and then aggregate over the 5-year window. Note that the former is identical to the main measure when all inventors are only listed on patents that are also assigned to the firm.

Dominance. I construct the dominance measure used in Appendix C.1 in two steps. Firstly, for each of the firm's new patent within a 5-year window, I calculate the share of inventors working for the firm among those that worked on patents of the exactly same technology class classification. For the latter, I use the complete CPC classification of the patent, which has more than 600 technology classes, which are non-exclusive at the patent level. Patents of the same technology class are thus those that have exactly the same classifications as the patent in consideration. As before, I distinguish between inventors using the USPTO disambiguation and link inventors to a firm if they are listed on a firm's new patent for the 5-year window in consideration. Secondly, I aggregate the patent-based measure to the firm-level by taking a simple average over the firm's new patents. Note that the resulting measure is between 0 and 1 by construction with 1 implying maximal dominance and vice versa.

Specialization. I construct the specialization measure used in Appendix C.1 in two steps. Firstly, I calculate inventor specialization for a given 5-year window as the average cosine similarity between patent classifications in an inventors portfolio of new patents. I rely on CPC classifications of patents, which has more than 600 non-exclusive patent categories. For each patent I then create an indicator vector over the set of available patent classification, where I weight individual categories by their inverse frequency. I then calculate the average cosine similarity across all patents in the portfolio and take the simple average across all patents. This measure is between 0 and 1 by construction with 0 implying completely different patents and 1 implying that all patents have the same technology classification. I aggregate this measure up to the firm-level by taking a patent-weighted average across inventor associated with a firm, where the weight reflect the number of new patents shared by the inventor and firm. I interpret a larger value in this measure as more specialized inventors and vice verse following the logic that specialized inventors work on similar patents.

Alternative R&D Output Measures. I follow Bloom et al. (2020) and construct alternative measure of R&D output based on positive changes in revenue, employment, or labor productivity defined as revenue per employee. The alternative measures of the Return on R&D are thus defined as

$$\begin{aligned} \text{R\&D Return}_{it}^{X} &\equiv \frac{\sum_{s=0}^{4} \max\{\mathrm{X}_{it+s} - X_{it-1+s}, 0\}}{\sum_{s=0}^{4} \text{R\&D Expenditure}_{it-1+s}} \\ \text{with} \quad & \mathrm{X} \in \{\text{Revenue, Employment, Labor Productivity}\}. \end{aligned} \tag{B.2}$$

Alternative R&D Input Measures. I use the knowledge capital series from Ewens et al. (2020), which reflects discounted R&D and overhead expenses, to construct an alternative measure of the Return on R&D as

$$\text{R\&D Return}_{it}^{K} \equiv \frac{\sum_{s=0}^{4} \text{Patent valuations}_{it+s}}{\sum_{s=0}^{4} \text{Knowledge capital}_{it-1+s}}. \tag{B.3}$$

Return on Capital. Following David et al. (2021), I measure the return on capital as the ratio of sales to beginning of period capital stock. As for the R&D return, I construct the measure at the 5-year level:

$$\texttt{Return on Capital}_{it} \equiv \frac{\sum_{s=0}^{4} \texttt{Sales}_{it+s}}{\sum_{s=0}^{4} \texttt{Capital}_{it+s}}. \tag{B.4}$$

Investment Q. I define the (physical) investment Q as the ratio of firm valuation to physical capital (ppeqgt). I calculate firm valuation as stock price times outstanding shares plus debt net of cash holdings ($\texttt{prcc\_f} \times \texttt{csho} + \texttt{dltt} + \texttt{dlc} - \texttt{act}$).

Liquidity. I define liquidity as cash holdings divided by assets ch/at.

Profit rate. I calculate the profit rate as earnings before extraordinary items (ib) divided by revenue (revt), both of which I first aggregate at the 5-year horizon.

Public value share. I identify patents as connected to the public actors either (a) if they are assigned to a government entity, research lab, or university or (b) if they have a government interest statement. I then calculate the public value share as the ratio of patent valuations connected to public actors divided by total patent valuations for the releveant 5-year window.

Lobbying. I calculate two measures of lobbying based on the firm-level data from Kim (2018). First, for each year I record whether a firm has lobbied at all and whether the lobbying was related to R&D topics. I then calculate the "R&D Lobbying Intensity" as the ratio of years with R&D specific lobbying to years with any lobbying at the 5-year window. Second, I create measure "R&D-lobbying expenditure" as the ratio of lobbying expenditure on R&D related topics divided by total R&D expenditure at the 5-year horizon. Both measures are normalized with the intention to make them ex-ante size neutral.

C Empirical Appendix

C.1 Regression Evidence

Table C.1: R&D Wedges and Measures of Investment Frictions

	(1)	(2)	(3)	(4)	(5)
			ln **R&D Wedge**		
ln Return on Capital	0.069				
	(0.075)				
ln Investment Q		0.190***			
		(0.021)			
Young Firm			-0.094		
			(0.141)		
ln Liquidity				0.010	
				(0.027)	
β_{CAPM}					0.020
					(0.074)
R2	0.002	0.099	0.000	0.000	0.000
Observations	9,147	8,837	9,147	8,230	5,662

Note: This table reports OLS coefficient estimates. "Young Firm" and "No Dividend Payout" are indicators variable for firm age in Compustat of less than 20 years and no dividend payments, respectively. Liquidity measures the firms cash holdings relative to its book assets. β_{CAPM} uses the stockmarket beta from the CRSP/Compustat dataset. See Appendix **??**. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Table C.2: R&D Wedges and Inventor Markets

	(1)	(2)	(3)
		ln **R&D Wedge**	
ln Inventor employment	0.196***		
	(0.036)		
ln Dominance		0.108**	
		(0.052)	
ln Inventor specialization			0.226*
			(0.119)
R2	0.050	0.006	0.003
Observations	9,147	8,190	9,138

Note: This table reports OLS coefficient estimates. Dominance measures the firms' inventor share in their technology classes. Specialization measures the average specialization of inventors employed by the firm. See Appendix **??**. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Table C.3: R&D Wedges and Measures of State Involvement

	(1)	(2)	(3)	(4)
	ln **R&D Wedge**			
$\ln 1-\tau$	-0.517			
	(0.700)			
Public value share		1.209*		
		(0.618)		
R&D Lobbying Intensity			0.291***	
			(0.097)	
R&D-lobbying expenditure				48.448***
				(12.380)
R2	0.001	0.004	0.010	0.013
Observations	8,785	9,147	4,354	4,354

Note: This table reports OLS coefficient estimates. Subsidy rates based on state-level R&D tax credits. Public value share is share of patent valuation that are also assigned to a public entity or have a government interest statement. Lobbying R&D Focus is the ratio of year in which a firm engaged in R&D specific lobbying normalized by overall lobbying activity. R&D Lobbying Intensity is the ratio of lobbying expenditure on R&D-related topics to overall R&D expenditure. See Appendix **??**. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Table C.4: Impact-Value Wedges, Profitability, and Firm Size

	(1)	(2)	(3)	(4)	(5)	(6)
			ln **Impact-Value Wedge**			
ln Markup	-0.556**					
	(0.216)					
Profit rate		-3.859***				
		(0.517)				
ln Investment Q			-0.118***			
			(0.020)			
Young Firm				0.631**		
				(0.264)		
ln Employment					-0.366***	
					(0.035)	
ln R&D Expenditure						-0.372***
						(0.050)
R2	0.033	0.124	0.027	0.013	0.262	0.239
Observations	8,446	9,147	8,837	9,147	9,147	9,147

Note: This table reports OLS coefficient estimates. Markups from Loecker et al. (2020). Profitability is the ratio of earnings before taxes divided by revenue. Investment Q is the ratio of market to book value. Young firm is an indicator for firm-age less than 20 years. See Appendix **??**. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Table C.5: Impact-Value Wedges and State Involvement

	(1)	(2)	(3)	(4)
	ln **Impact-Value Wedge**			
$\ln 1 - \tau$	-1.100			
	(1.195)			
Public value share		-0.318		
		(0.670)		
R&D Lobbying Intensity			-0.708***	
			(0.149)	
R&D-lobbying expenditure				-34.037**
				(17.075)
R2	0.002	0.000	0.041	0.004
Observations	8,785	9,147	4,354	4,354

Note: This table reports OLS coefficient estimates. Subsidy rates based on state-level R&D tax credits. Public value share is share of patent valuation that are also assigned to a public entity or have a government interest statement. Lobbying R&D Focus is the ratio of year in which a firm engaged in R&D specific lobbying normalized by overall lobbying activity. R&D Lobbying Intensity is the ratio of lobbying expenditure on R&D-related topics to overall R&D expenditure. See Appendix **??**. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Table C.6: Adjusted R&D Wedges, Firm Age, and Firm Scale

	(1)	(2)	(3)	(4)	(5)	(6)
			ln **Adjusted R&D Wedge**			
ln R&D Expenditure	-0.424***					
	(0.038)					
ln Employment		-0.320***				
		(0.034)				
Young Firm			0.538**			
			(0.223)			
ln Inventor employment				-0.110**		
				(0.050)		
ln Dominance					0.295***	
					(0.087)	
ln Inventor specialization						0.369*
						(0.203)
R2	0.282	0.182	0.009	0.010	0.032	0.005
Observations	9,147	9,147	9,147	9,147	8,190	9,138

Note: This table reports OLS coefficient estimates. Markups from Loecker et al. (2020). Profitability is the ratio of earnings before taxes divided by revenue. Investment Q is the ratio of market to book value. Young firm is an indicator for firm-age less than 20 years. Return on capital is the ratio of revenue to capital stock. β_{CAPM} uses the stockmarket beta from the CRSP/Compustat dataset.See Appendix **??**. All regressions control for NAICS3$\times$ Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Table C.7: Adjusted R&D Wedges, Profitability, and Investment Frictions

	(1)	(2)	(3)	(4)	(5)
			ln **Adjusted R&D Wedge**		
ln Markup	-0.343				
	(0.247)				
Profit rate		-1.370			
		(0.840)			
ln Investment Q			0.072***		
			(0.025)		
ln Return on Capital				0.069	
				(0.075)	
β_{CAPM}					0.020
					(0.074)
R2	0.011	0.014	0.009	0.002	0.000
Observations	8,446	9,147	8,837	9,147	5,662

Note: This table reports OLS coefficient estimates. Markups from Loecker et al. (2020). Profitability is the ratio of earnings before taxes divided by revenue. Investment Q is the ratio of market to book value. Young firm is an indicator for firm-age less than 20 years. Return on capital is the ratio of revenue to capital stock. β_{CAPM} uses the stockmarket beta from the CRSP/Compustat dataset.See Appendix **??**. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

Table C.8: Adjusted R&D Wedges and State Involvement

	(1)	(2)	(3)	(4)
	ln **Adjusted R&D Wedge**			
$\ln 1-\tau$	-1.607			
	(1.300)			
Public value share		0.895		
		(0.868)		
R&D Lobbying Intensity			-0.412***	
			(0.151)	
R&D-lobbying expenditure				14.391
				(25.612)
R2	0.004	0.001	0.012	0.001
Observations	8,785	9,147	4,354	4,354

Note: This table reports OLS coefficient estimates. Subsidy rates based on state-level R&D tax credits. Public value share is share of patent valuation that are also assigned to a public entity or have a government interest statement. Lobbying R&D Focus is the ratio of year in which a firm engaged in R&D specific lobbying normalized by overall lobbying activity. R&D Lobbying Intensity is the ratio of lobbying expenditure on R&D-related topics to overall R&D expenditure. See Appendix **??**. All regressions control for NAICS3× Year effects and standard errors are clustered at the NAICS6 level.

Standard errors in parentheses. Significance levels: * 10% , ** 5%, *** 1%.

C.2 Measurement Error

Dispersion in adjusted R&D wedges could be due to measurement error arising, e.g., from the expectation-realization gap, patent valuation estimation, or misreporting of R&D expenditure.[22] In this section I propose two complementary approaches to estimating the contribution of measurement error to measured adjusted R&D wedges. I begin by taking a structural approach using a GMM estimator to investigate the importance of classical measurement error. In addition, I use bootstrapping to estimating the potential measurement error due the uncertainty around patent valuations.

GMM Estimation of Measurement Error. Consider a stationary, AR(1) process $\{y_{it}\}$:

$$y_{it} = (1-\rho)\mu_i + \rho y_{it-1} + \varepsilon_{it} \quad \text{with} \quad \varepsilon_{it} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \quad \text{and} \quad \mu_i \sim N(0, \sigma_\mu^2). \tag{C.1}$$

The econometrician observes the process with i.i.d. normal measurement error:

$$\tilde{y}_{it} \equiv y_{it} + \nu_{it} \quad \nu_{it} \overset{iid}{\sim} N(0, \sigma_\nu^2). \tag{C.2}$$

Lemma 3. *Define $\Delta\tilde{y}_{it} \equiv \tilde{y}_{it} - \tilde{y}_{it-1}$, then under $\rho \in (0,1)$, we have*

$$\begin{aligned}
m_1 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it}) = \frac{1}{1+\rho}\sigma_\varepsilon^2 + \sigma_\nu^2 \\
m_2 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it-1}) = \frac{\rho}{1+\rho}\sigma_\varepsilon^2 \\
m_3 &\equiv Cov(\tilde{y}_{i,t}, \Delta\tilde{y}_{it-2}) = \frac{\rho^2}{1+\rho}\sigma_\varepsilon^2 \\
m_4 &\equiv Cov(\tilde{y}_{i,t}, \tilde{y}_{it-1}) = \sigma_\mu^2 + \frac{\rho}{1-\rho^2}\sigma_\varepsilon^2.
\end{aligned}$$

Proposition 2. *If $\rho \in (0,1)$, we can solve for $\{\rho, \sigma_\mu, \sigma_\varepsilon, \sigma_\nu\}$ using the population auto-*

[22] Note that R&D expenditure is expensed in US GAAP accounting, giving firms an incentive to fully report R&D expenditure to reduce their tax liability. Terry et al. (2022) argue that managers still might misreport when attempting to hit short-run earnings targets or smooth earnings. See also Dukes et al. (1980); Baber et al. (1991); Lev et al. (2005); Chen et al. (2021); Terry (2022).

covariance structure of $\tilde{y}_{it}$ and $\Delta\tilde{y}_{it} \equiv y_{it} - y_{it-1}$:

$$\beta \equiv \begin{bmatrix} \rho \\ \sigma_\varepsilon^2 \\ \sigma_\mu^2 \\ \sigma_\nu^2 \end{bmatrix} = \begin{bmatrix} \frac{m_3}{m_2} \\ \frac{(m_2)^2}{m_3} + m_2 \\ m_4 - \frac{(m_2)^2}{m_2 - m_3} \\ m_1 - \frac{(m_2)^2}{m_3} \end{bmatrix}$$

Let Ω be the covariance matrix of m and denote the sample moments by $\hat{m}$, then

$$\hat{\beta} \sim N(\beta, \Sigma) \quad \textit{and a feasible estimator is} \quad \hat{\Sigma} = \left(\frac{\partial \hat{\beta}}{\partial m}\right)' \hat{\Omega} \left(\frac{\partial \hat{\beta}}{\partial m}\right),$$

where $\partial\beta/\partial m$ is evaluated at $\hat{m}$ and given by

$$\frac{\partial \beta}{\partial m} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -\frac{m_3}{(m_2)^2} & 2\frac{m_2}{m_3} + 1 & m_2\left(\frac{m_2 - 2m_3}{(m_2 - m_3)^2}\right) & -2\frac{m_2}{m_3} \\ \frac{1}{m_2} & -\left(\frac{m_2}{m_3}\right)^2 & -\left(\frac{m_2}{m_2 - m_3}\right)^2 & -\left(\frac{m_2}{m_3}\right)^2 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Proof. The first part follows by rearranging the moments expressions. The second part follows from the Law of Large Numbers for the moment vector and the Delta method. □

Note that this methodology does not aggregate. In particular, if we assume that adjusted R&D wedges follows an AR(1) in logs at the annual level, we cannot implement the above methodology at the 5-year horizon directly as the 5-year expected adjusted R&D wedge is a weighted-average of the annual wedges in levels, which does not translate into logs:

$$\frac{\sum_{s=0}^{4} \text{Pat. Cit.}_{it+s}}{\sum_{s=0} \text{R\&D Exp.}_{it-1+s}} = \sum_{s=0}^{4} \frac{\text{R\&D Exp.}_{it-1+s}}{\sum_{w=0} \text{R\&D Exp.}_{it-1+w}} \times \frac{\text{Pat. Cit.}_{it+s}}{\text{R\&D Exp.}_{it-1+s}}.$$

I, thus, estimate the system at the 1-year level, which will likely overstate the extend of measurement error as it misses aggregation over time. I restrict my sample to 1-year adjusted R&D wedges with at least 10 patents in line with the requirement that they should have 50 patents or more over 5 years.

Table C.9: GMM results for AR(1) with Noise

Parameter	Estimate
ρ	0.942***
	(0.096)
σ_ε^2	0.130***
	(0.016)
σ_μ^2	-0.049
	(2.816)
σ_ν^2	0.123***
	(0.015)
Observations	7,428

Note: Estimates from structural measurement error estimation. All wedges at the annual level and restricted to observations with at least 10 patents. Adjusted R&D wedges residualized for NAICS$\times$ year effects. Standard errors clustered at the NAICS6 level and reported in brackets.

The GMM estimates presented in Table C.9 suggest that measurement error constitutes a small, but significant share of the overall variation at the 1-year level. The estimated measurement error variance is 0.12, or 10% of the overall variation. In addition, I find that the adjusted R&D wedge is highly auto-correlated with most variation due to idiosyncratic shocks. The estimates suggests that permanent difference constitute little of the overall variation, however, the standard errors around the estimate for σ_μ^2 are very large.

Bootstrap Estimation for Valuation Uncertainty. In addition to the investigation of classical measurement error, I consider the role of patent valuation uncertainty explicitly. In a bootstrap procedure I redraw patent valuations from the realized patent portfolio and construct Returns on R&D assuming that the first targets a return proportional the expected value of patent valuations ex-ante. Repeating this exercise for 1000 iteration I then calculate an estimated dispersion in the measured Expected Return on R&D based on uncertain valuation outcomes only.

Each iteration in my procedure proceeds as follows:

1. For each firm and 5-year window in which the firm has at least 50 patents:
   (a) From the portfolio of patent valuations for the firm-period, draw with replacement an alternative portfolio with as many valuations as the firm had patents in the period.
   (b) Calculate the return as the ratio the valuations in the alternative portfolio divided by the valuation of the true portfolio.
2. Calculate the standard deviation of Return on R&D for the simulated data.

I repeat this procedure until I have 1000 bootstrap estimates of the standard deviation of Returns on R&D. Note that the resulting dispersion in the Return on R&D is driven exclusively by the variability of patent valuations and would yield 0 variation if all patents had the same value.

One way to interpret this approach is that the realized patent portfolio is a good approximation for the true uncertainty faced by the firm around its innovation outcomes. The procedure ignores all variation coming from shifts in the level of expected patent valuation and instead considers the dispersion conditional on the average value only. As a result, the procedure will overstate the associated measurement error if firms are aware that certain project are low or high expected value within their research portfolio.

Table C.10 reports estimates suggesting that the measurement error due to patent valuation uncertainty could account for up to $(0.067/1.1)^2 = 6\%$ of the variance of the measured adjusted R&D returns. Unsurprisingly, the estimated measurement error declines with the size of the minimum patent portfolio and is precisely estimated with tight confidence intervals.

Table C.10: Measurement Error Estimates using Bootstrap Procedure

Minimum patents	Estimate	Standard error	95% Confidence Interval
30	0.172	(0.002)	[0.168,0.176]
50	0.143	(0.002)	[0.139,0.147]
100	0.111	(0.002)	[0.109,0.115]
200	0.086	(0.001)	[0.083,0.088]

Note: Measurement error estimates based on distribution of patent valuations for different cut-offs levels of minimum patent counts.

Adjustment Factors and Rolling Windows. I implement a measurement adjustment by calculating shrinkage factors according to the simple formula:

$$\beta = \sqrt{\frac{\sigma^2 - \sigma_\nu^2}{\sigma^2}}, \tag{C.3}$$

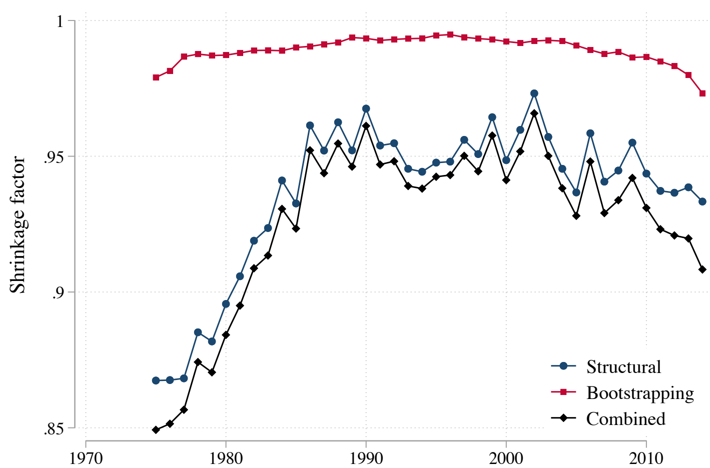
where σ^2 is the overall variation in adjusted R&D wedges and σ_ν^2 is the estimated measurement error variance. The adjustment factor has the simple property that

$$Var(\beta \times (1 + \Delta_{it})\zeta_{it}) = \beta^2 \times Var((1 + \Delta_{it})\zeta_{it}) = \frac{\sigma^2 - \sigma_\nu^2}{\sigma^2} \times \sigma^2 = \sigma^2 - \sigma_\nu^2. \tag{C.4}$$

Such that the adjusted R&D wedges have the estimated true underlying variance without measurement error component.

I calculate adjustment factors over time for the structural method using 5-year rolling windows and for the bootstrap method by using the year-specific data. Figure C.1 reports the estimated shrinkage factors. Clearly, the structural method estimates a significantly larger measurement error component on average as indicated by smaller shrinkage factors. Post 1985, shrinkage factors are very stable. I also calculate a combined factor, which is the product of both shrinkage factors.

Figure C.1: Measurement Error Shrinkage Factors



Notes: Figure plots shrinkage factors for measurement error adjustment using alternative approaches.

Table C.11 reports the aggregated allocative efficiency estimates using the shrinkage factors. Measurement error adjustment matter somewhat, but do not dramatically change the picture.

Table C.11: Estimating Allocative Efficiency

Measure	Period		
	1975-2014	1975-1994	2005-2014
Baseline	58.3%	70.4%	40.9%
Structural adjustment	62.3%	74.6%	44.7%
Bootstrapping adjustment	59.0%	71.0%	41.8%
Combined adjustment	63.0%	75.2%	45.7%

Note: Estimates of allocative efficiency using Proposition 1. See text for variable definition. Allocative efficiency measures are first constructed at the annual measure and then averaged over the relevant sample using the geometric mean. Rows 2-4 assume alternative measurement adjustment factors as detailed in the text.

C.3 Welfare Cost under Semi-Endogenous Growth

In this section I show that using a semi-endogenous growth framework yields quantitatively similar conclusions of the welfare cost of misallocation as the ones estimated in the endogenous growth model in the main text (Jones, 1995; Bloom et al., 2020). Consider a simple semi-endogenous growth model, where the growth-rate of the technology depends on aggregate innovator input L_t, the current level of technology A_t, and misallocation term Ξ_t according to:

$$g_t \equiv \frac{\dot{A}_t}{A_t} = L_t \cdot \Xi_t \cdot A_t^{-\phi}. \tag{C.5}$$

The parameter $\phi > 0$ captures the fishing-out effect that ensures existence of a balanced growth path with constant growth rate. It is straight-forward to show that a constant growth rate along the balanced growth path has to satisfy

$$g = \frac{n}{\phi}. \tag{C.6}$$

Thus, along the balanced growth path, the level of technology is given by

$$A_t = \left(\frac{\phi}{n} \times L_t \times \Xi\right)^{\frac{1}{\phi}}. \tag{C.7}$$

In the long-run, misallocation thus has a level effect in the semi-endogenous growth model instead of the growth-rate effect implied by an endogenous growth model.

I calibrate the model using the parameter reported in Table C.12. I set the population growth rate at 1% p.a., which is the approximately the long-run average for the US in the post-war period. I then set ϕ to ensure a long-run growth rate of 1.5% p.a. at initial levels of R&D misallocation, which I also impose in the endogenous growth model. Finally, I set the discount rate to a standard macro value of $\rho = 0.03$.

Table C.12: Parameter Values For Semi-Endogenous Growth

Parameter	Value	Source
ϕ	0.67	Implied by growth rate of 1.5%
n	0.01	Long-run average
ρ	0.03	Standard macro value

We can quantify the welfare gains from setting $\Xi_t = 1$ numerically, using the growth-rate formula together with parameteric assumptions. I will assume that the economy was on the balanced growth path prior to solving misallocation. I then set $\Xi_t = 1$ and use the growth rate formula to characterize the evolution of A_t. I then translate these values into welfare and welfare equivalent by assuming that the planner values a per-capita consumption stream c_t, which is directly linked to the productivity level as

$$\mathcal{W}(\{c_t\}) = \int_0^\infty e^{-(\rho-n)t} \frac{c_t^{1-\sigma}}{1-\sigma} dt. \tag{C.8}$$

Note that this formulation discounts the future at rate $\rho - n$ instead of ρ, which is the discount rate in a model without population growth. I will use the same discount rate when comparing the welfare implications in the semi-endogenous and endogenous growth model.

We can then calculate the consumption-equivalent welfare cost of R&D misallocation as

$$\Delta_c = \left(\frac{\mathcal{W}(\{c_t^*\})}{\mathcal{W}(\{c_t\})} \right)^{\frac{1}{1-\sigma}} - 1, \tag{C.9}$$

where $\{c_t\}$ is the consumption sequence with permanent misallocation and $\{c_t^*\}$ is the consumption sequence induced by solving misallocation. As the growth-rate is not constant over time in this model, we cannot solve this equation directly and I will instead calculate the necessary inputs numerically. Following Acemoglu et al. (2018) I set $\sigma = 2$.

Table C.13 reports the results for the average level of misallocation according to the simple and adjusted allocative efficiency measure. The welfare cost of R&D misallocation are slightly smaller in the semi-endogenous growth model, but of comparable magnitudes across models. For example, the simple estimate of allocative efficiency is associated with potential consumption-equivalent welfare gains of 36% in the endogenous growth model compared to 32% for the semi-endogenous growth model.

Table C.13: Welfare Cost of R&D Misallocation Across Models

Measure	Allocative Efficiency	Welfare Cost Endogenous	Welfare Cost Semi-endogenous
Baseline	58.3%	22%	24%
Only R&D Wedge	70.4%	14%	13%
Only Impact-Value Wedge	55.1%	27%	25%

Notes: This table reports the welfare gains from getting rid of misallocation implied by endogenous and semi-endogenous growth models. Both models discount the future at rate $\rho - n$, but have alternative paths for technology due to the nature of growth.

C.4 Decomposing the Evolution in the Adjusted Measure

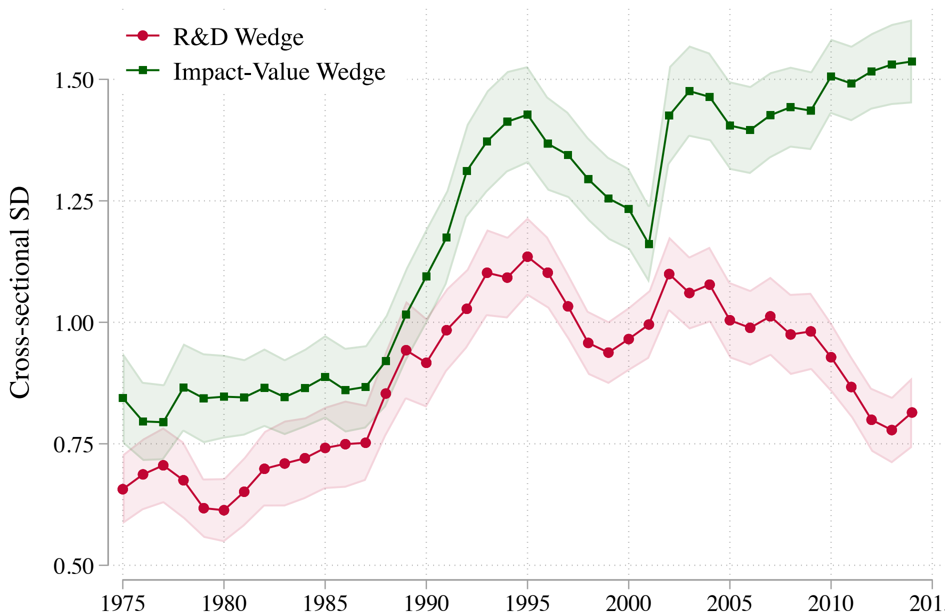
One way to decompose the evolution is to use the approximation result that links the adjusted allocative efficiency term to the variance of the adjusted R&D wedge. Following the decomposition, we have

$$\ln \Xi_{adjusted} \approx -\frac{1}{2\phi}\left(\sigma_\Delta^2 + \sigma_\zeta^2 + 2 \cdot \sigma_\Delta \cdot \sigma_\zeta \cdot \rho_{\Delta,\zeta}\right), \tag{C.10}$$

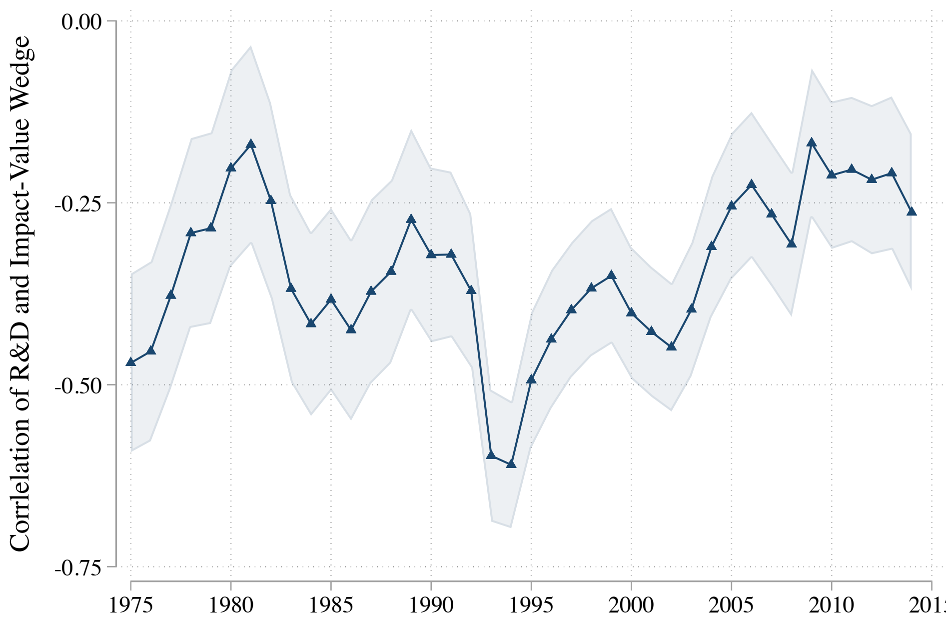
where $\sigma_\Delta^2 \equiv \sigma_\omega^2(\{\ln(1+\Delta_{it})\})$, $\sigma_\zeta^2 \equiv \sigma_\omega^2(\{\ln \zeta_{it}\})$, and $\rho_{\Delta,\zeta} = \rho_\omega(\{\ln(1+\Delta_{it})\}, \{\ln \zeta_{it}\})$. Up to a second order approximation, these three factors thus shape the evolution of allocative efficiency.

Figure C.2 reports the evolution of the components. Three observations emerge. First, the standard deviation of R&D wedge and Impact-Value wedge rises monotonically from 1975 to 1995, while their evolution diverges afterwards. The dispersion in the impact value wedge continues to rise post 1995, while dispersion in the R&D wedge stabilizes and slightly declines. Second, both wedges are consistently negatively correlated as shown in panel (b), however, the magnitude of the correlation decline post 1995. Rising misallocation is thus driven primarily by rising dispersion in wedges in the first half of the sample and by rising correlation of both in the second half.

Figure C.2: The Three Components of Approximated Allocative Efficiency



(a) Standard Deviation of Wedges



(b) The Correlation of R&D and Impact-Value Wedge

Notes: Shaded areas cover 95% confidence interval. Standard errors calculated using influence functions and the Delta method.

C.5 Countervailing Forces

The frontier growth rate can be further decomposed into a baseline component and a firm specialization component as detailed in the following Lemma.

Lemma 4. *The frontier growth-rate can be decomposed into two components:*

$$\tilde{g}_t = \bar{g}_t \times \Phi_t, \tag{C.11}$$

where $\bar{g}_t \equiv \int_0^1 \gamma_{it} \cdot \zeta_{it} di$ is the frontier growth in absence of firm heterogeneity and Φ_t measures the specialization gains from heterogeneous R&D productivity:

$$\Phi_t \equiv \frac{\left[\int_0^1 (\gamma_{it} \cdot \zeta_{it})^{\frac{1+\phi}{\phi}} di\right]^{\frac{\phi}{1+\phi}}}{\int_0^1 (\gamma_{it} \cdot \zeta_{it})\, di}. \tag{C.12}$$

It is straight-forward to show that Φ_t is independent of the common level of R&D productivity and Impact-Value wedge, however, as per Jensen's inequality, it is increasing in dispersion in their product. The economic intuition for this rests on specialization. In particular, heterogeneous R&D productivity implies good firms can do more R&D and vice versa. This reallocation of R&D resources increases the appropriately aggregated R&D productivity beyond the simple mean of R&D productivities across firms by weighting high R&D productivity firms more. Thus, even with constant average R&D productivity, aggregate R&D productivity improves if firm-level R&D productivities are more dispersed. This effect is weaker with more concave R&D production functions, which limit the extend to which R&D productivity differences translate into R&D efforts.

I estimate the firm specialization term in the data using the parameter estimation approach introduced in Section 3. I then implement the formulas as

$$\hat{\Phi}_t = \frac{\left[\frac{1}{N_t}\sum\left(\hat{\gamma}_{it} \cdot \hat{\zeta}_{it}\right)^{\frac{1+\phi}{\phi}} di\right]^{\frac{\phi}{1+\phi}}}{\frac{1}{N_t}\sum\left(\hat{\gamma}_{it} \cdot \hat{\zeta}_{it}\right) di}, \tag{C.13}$$

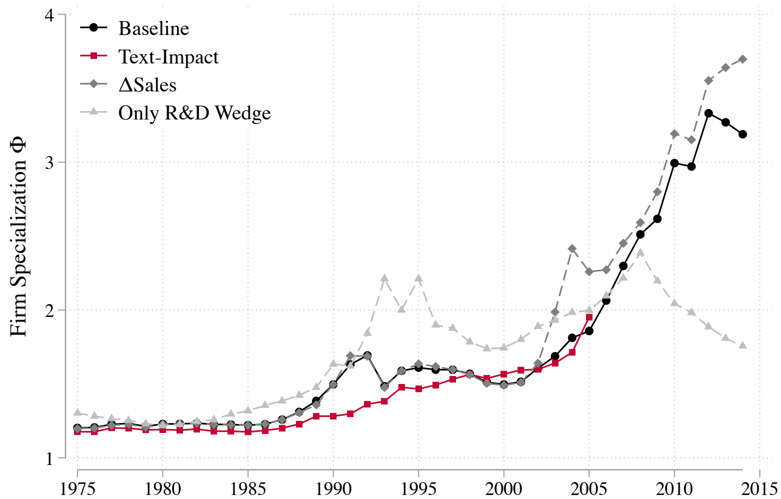
where N_t is the number of active firms in the sample. This approach preserves the formula's expectation character and is thus independent of the number of active firms. The latter can have an independent effect in the model due to the love of variety effect inherent in decreasing returns to scale.[23]

Panel (a) of Figure C.3 reports the results. I find that the gains from specialization have rising significantly over the sample across a range of alternative specifications with similar magnitudes. While specialization hovered around 1.25 in the beginning of the sample, it increase to around 2 in 2005, a 60% increase. This finding holds true regardless whether we focus on the baseline measure or ignore differences in the impact-value wedge.

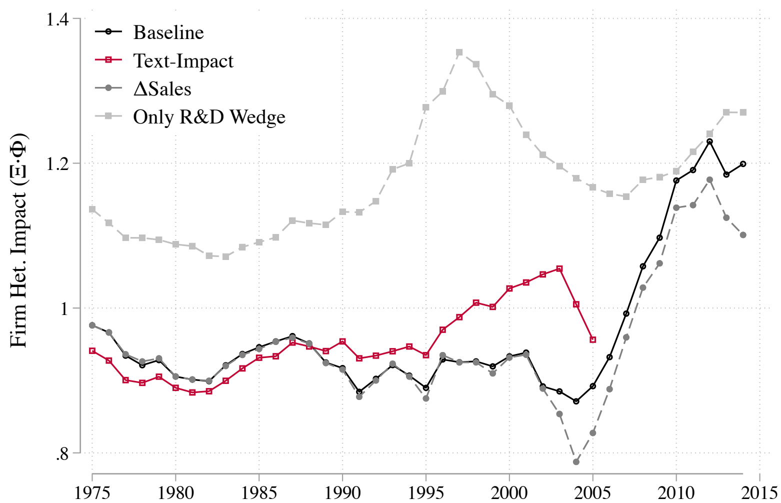
What is then the joint impact of firm heterogeneity? On the one hand, I documented in the text that allocative efficiency has declined significantly over time, driven by rising heterogeneity in the R&D wedge. On the other hand, gains from specialization in R&D have risen significantly. Panel (b) of Figure C.3 plots their joint impact, i.e. their product. Two findings emerge. First, the estimated joint impact is consistently below 1 for the baseline measure until 2005. Second, the joint impact rises significantly during the 2005-14 period going from around 0.9 in 2005 to 1.2 in 2010. Thus, the observed rise in gains from firm specialization dominates the decline in allocative efficiency making firm heterogeneity a positive force for economic growth post 2007. Jointly, it appears that rising firm heterogeneity had a positive impact on economic growth, however, it could have been much larger if allocative efficiency had remained constant. We might expect that some of this decline was inevitable in light of rising firm heterogeneity, however, the latter also increases the stakes for policy makers in creating a more efficient R&D environment with fewer frictions.

[23] In particular, one can show that $\int_0^M (X/M)^\alpha di$ is increasing in M for $\alpha < 1$. The growth-rate in the model shares this structure such that more active firms always imply a larger growth rate as long as the number of scientists in constant.

Figure C.3: Firm Heterogeneity and Growth



(a) Firm Specialization

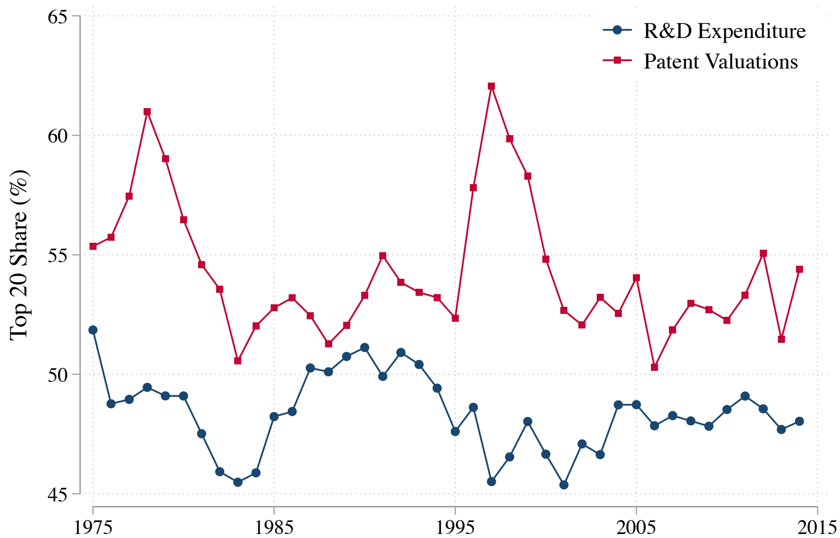


(b) Firm Heterogeneity

Notes: This figure plots the firm R&D specialization Φ_t and the full impact of firm heterogeneity on growth. The latter is calculated as the product of firm R&D specialization and R&D allocative efficiency Ξ_t. The simple measure assumes a constant Impact-Value wedge. The version labelled "Sales" uses sales changes instead of patent valuations to measure R&D output. The adjusted measures take into account heterogeneity in the Impact-Value wedge. The version labelled "Impact" uses the text-based patent impact measure from Kelly et al. (2021) instead of forward-citations to measure growth impact.

C.6 Concentration in R&D

Figure C.4: Concentration of R&D Has Remained Stable



Notes: Author's calculations based on sample.