**SF1685 (SF1625) Calculus in one variable**
**Exam**
**Monday, 9 March 2020**

Time: 08:00-11:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of three parts: A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

| Grade | A | B | C | D | E | Fx |
|---|---|---|---|---|---|---|
| Total score | 27 | 24 | 21 | 18 | 16 | 15 |
| score on part C | 6 | 3 | – | – | – | – |

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

*Please turn page!*

## Part A

1. (a) Evaluate $\displaystyle\int_1^e \frac{(1+\ln(x))^2}{x}dx$. **(3 p)**
   (b) Find all primitive functions to $f(x) = x\sin(x)$. **(3 p)**

2. (a) Let $L$ be the tangent line to the curve $y = \arctan(x^2)$ at the point on the curve that has $x$-coordinate $1$. Find an equation of the line $L$. **(3 p)**
   (b) Let $f(x) = e^{-x}$ and let $P(x)$ be the Taylor polynomial of degree $1$ for $f$ about $x = 0$. Show that $0 < f(1/3) - P(1/3) < 1/10$. **(3 p)**

## Part B

3. Let $f(x) = (x^2 - 2x - 2)e^x$.
   (a) Does the function $f$ have absolute maximum or absolute minimum values? Find these values if they exist. **(4 p)**
   (b) How many solutions does the equation $f(x) = -1$ have? **(2 p)**

4. (a) Determine whether the integral $\displaystyle\int_0^\infty \frac{2+\sin(x)}{1+x^2}dx$ converges or diverges. **(4 p)**
   (b) Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \cos(1/k^2)$ converges or diverges. **(2 p)**

## Part C

5. Let $n \geq 1$ be an integer, and let $P_n$ be the partition of the interval $[2,3]$ into $n$ subintervals, each of length $1/n$. Let $L(P_n)$ denote the lower Riemann sum for the function $f(x) = 3x$ on the interval $[2,3]$ with respect to the given partition $P_n$.
   (a) Find a formula, only depending on $n$, for $L(P_n)$. **(4 p)**
   (b) Find the limit $\displaystyle\lim_{n\to\infty} L(P_n)$. **(2 p)**

6. Assume that the function $f$ is differentiable with the derivative $f'(x) = 0$ for all real numbers $x$.
   (a) Show that $f$ is continuous on the whole real line (that is, show that differentiability implies continuity). **(3 p)**
   (b) Use the mean value theorem (from differential calculus) to show that $f$ is constant. **(3 p)**