SF1685 Calculus in one variable
Exam
Thursday, 9 June 2022

Time: 14:00-17:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of six problems, each worth 6 points. To the score on Problem 1 your bonus points are added, up to a maximum of 6 points. The score on Problem 1 is at most 6 points, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

Please turn page!

Part A

1. Let $f(x) = \tan x$ and $g(x) = \sin(\ln x)$.
   (a) Is the line $y = x$ tangent to the curve $y = f(x)$ at the point $(0, f(0))$? **(3 p)**
   (b) Find the Taylor polynomial of degree 2 for g about $x = 1$. **(3 p)**

2. Find the area of the bounded region in the plane bounded by the curve $y = \dfrac{1}{1+4x^2}$ and the line $y = 1/2$. Simplify your answer as much as possible. **(6 p)**

Part B

3. Find the range of the function $f(x) = 2xe^{x-x^2}$. **(6 p)**

4. Which of the following inequalities are true? (Do not forget to properly justify your answer.)
   (a) $\displaystyle\int_0^1 \frac{dx}{1+\arctan x} \leq 1.$ **(2 p)**
   (b) $\displaystyle\int_1^\infty \frac{x}{2x^2 - \sin^2 x}\, dx \leq 1.$ **(2 p)**
   (c) $\displaystyle\int_1^\infty \frac{x}{x^3 + \ln x}\, dx \leq 1.$ **(2 p)**

Part C

5. Is there a shortest line segment having one endpoint on the x-axis, the other endpoint on the y-axis, and passing through the point $(9, \sqrt{3})$? Find the length of such a shortest line segment if it exists, otherwise explain why there is no such shortest line segment. **(6 p)**

6. Show that the following holds for every integer $n \geq 1$: **(6 p)**

$$2\ln(2) - 1 \leq \frac{1}{n}\sum_{k=1}^{n} \ln\left(1 + \frac{k}{n}\right) \leq 2\ln(2) - 1 + \frac{\ln 2}{n}.$$