SF1685 (SF1625) Calculus in one variable
Tentamen
Friday, 8 March 2019

Time: 08:00-11:00
Available aid: None
Examinator: Roy Skjelnes

The exam consists of three parts; A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

Please turn page!

PART A

1. This problem is about the differential equation $y'' - y' - 2y = 12e^{-t}$.
   (a) Determine the coefficient A such that $y_P(t) = Ate^{-t}$ is a solution. **(2 p)**
   (b) Determine all solutions to the differential equation. **(2 p)**
   (c) Determine all solutions $y(t)$ to the differential equation such that $y(0) = 2$ and such that $\lim_{t\to\infty} y(t) = 0$. **(2 p)**

2. Determine all primitive functions to the rational function **(6 p)**

$$f(x) = \frac{x^3 + 2x^2 - 4x - 1}{x^2 - 1}.$$

PART B

3. The function $g(x) = \ln(1 + \sin x)$ is defined on the open interval $I = (-\pi/2, 3\pi/2)$.
   (a) Draw the graph of the function $g(x)$, and mark all of its extreme values. **(3 p)**
   (b) Give an example of a continuous function $f(x)$ defined on I that does not have extreme values. **(2 p)**

4. The function $g(x) = \ln(1 + \sin x)$ is defined on the open interval $I = (-\pi/2, 3\pi/2)$.
   (a) Determine the polynomial $P(x)$ of degree two that is the best approximation of the function $g(x)$ around $x = 0$. **(2 p)**
   (b) Let t be a number such that $0 \le t \le \pi$. Show that $|\, g(t) - P(t)\,| \le \frac{1}{6}t^3$. **(2 p)**
   (c) Determine an approximative value of $\int_0^{1/2} g(x)\,dx$ that does not differ more than $1/300$ from the exact value of the integral. **(3 p)**

PART C

5. (a) Show that for every integer $n \ge 1$ we have that **(6 p)**

$$\int_{\cos(1/n)}^{1} \frac{1}{t}dt \le \frac{\sin^2(1/n)}{\cos^2(1)}.$$

   (b) Show that the series

$$\sum_{n=1}^{\infty} -\ln(\cos(1/n))$$

   converges. **(6 p)**