



**SF1685 (SF1625) Calculus in one variable**  
**Tentamen**  
**Friday 8th of June, 2018**

Time: 08:00-11:00

Available aid: None

Examinator: Roy Skjelnes

The exam consists of three parts; A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

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**PART A**

1. Compute the limit **(3 p)**

$$\lim_{x \rightarrow 0} \frac{e^{\sin(x)} - 1 - x}{\ln(1 + x^2)}.$$

2. Define what is meant with a function  $\varphi$  being continuous at a given point  $x$ . **(4 p)**
3. The curve  $y = \sqrt{1 + |x|^3}$  together with the lines  $x = -3$ ,  $x = 3$  and  $y = 0$  determine a bounded region in the plane. Determine the volume of the solid generated by rotating the region around the  $x$ -axis. **(5 p)**
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**PART B**

4. We have the function  $F(x) = 4 \arctan(x) - 4x + x^2$  defined for all real numbers  $x$ .
- (a) Solve the equation  $F'(x) = 0$ . **(2 p)**
  - (b) Show that  $F'(x) > 0$  for all  $x > 1$ . **(2 p)**
  - (c) Show that  $4 \arctan(x) > \pi - 3 + 4x - x^2$  for all  $x > 1$ . **(2 p)**
5. Let  $f(x) = \ln(1 + x)$ .
- (a) Determine the Taylor polynomial of degree  $n$  of  $f(x)$  around  $x = 0$ . **(2 p)**
  - (b) Estimate  $\ln(6/5)$  with a value having an error that is less than  $1/300$ . **(4 p)**
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**PART C**

6. Use the substitution  $x = 2 \sin(t)$  to find a primitiv function of **(6 p)**

$$f(x) = \sqrt{1 - \frac{x^2}{4}}.$$

7. Determine convergence or divergence of the series **(6 p)**

$$\sum_{n=1}^{\infty} \frac{2n\sqrt{n} - 1}{n^3 - n + 1}.$$

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