SF1685 (SF1625) Calculus in one variable
Exam
Thursday, 11 March 2021

Time: 08:00-11:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of six problems, each worth 6 points. To the score on Problem 1 your bonus points are added, up to a maximum of 6 points. The score on Problem 1 is at most 6 points, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

Please turn page!

PART A

1. Evaluate the following integrals: **(3+3 p)**

$$\int_e^{e^2} \frac{dx}{x \ln x} \quad \text{and} \quad \int \frac{dx}{x^2-4}.$$

2. Let $f(x) = \sqrt{1-x}$, $0 \leq x \leq 1$. Find the point (x_0, y_0) on the graph $y = f(x)$ which makes the rectangle with corners at the points $(0,0), (x_0, 0), (x_0, y_0)$ and $(0, y_0)$ as large as possible. Do not forget to explain why the area becomes maximal at this point. **(6 p)**

PART B

3. Determine whether there exists a solution $y(t)$ to the differential equation $y''(t) + 2y'(t) + 5y(t) = 0$ which satisfies

$$\lim_{t \to 0} \frac{y(t)}{t} = 1.$$

Find such a solution if such a solution exists, otherwise explain why there is no such solution. **(6 p)**

4. Find the points on the curve $y = e^{x^2+2x}$ at which the tangent line to the curve passes through the point $(1,0)$. (Note that the point $(1,0)$ does not lie on the curve.) **(6 p)**

PART C

5. Consider the integral $\displaystyle\int_1^{\infty} \frac{2}{2x^2 + \sin x + 1}\, dx$
   (a) Show that the integral converges. **(2 p)**
   (b) Find an approximate value of the integral for which the error is not larger than $\frac{1}{8}$. **(4 p)**

6. For each integer $n \geq 1$, let

$$A_n = \left(\frac{1}{n}\right)^{\left(\frac{1}{n^2}\right)} \cdot \left(\frac{2}{n}\right)^{\left(\frac{2}{n^2}\right)} \cdot \left(\frac{3}{n}\right)^{\left(\frac{3}{n^2}\right)} \cdots \left(\frac{n}{n}\right)^{\left(\frac{n}{n^2}\right)}.$$

Evaluate $\lim_{n \to \infty} A_n$. **(6 p)**