SF1625 (SF1685) Calculus in one variable
Exam
Thursday, 15 October 2020

Time: 08:00-11:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of six problems, each worth 6 points. To the score on Problem 1 your bonus points are added, up to a maximum of 6 points. The score on Problem 1 is at most 6 points, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

Please turn page!

PART A

1. (a) Find all functions $y(t)$ that satisfy $y'' + y' - 2y = 0$. **(2 p)**
   (b) Fins all functions $y(t)$ that satisfy **(4 p)**

$$\begin{cases} y'' + y' - 2y = 4e^{-t} \\ \lim_{t\to\infty} y(t) = 0 \\ y(0) = 5. \end{cases}$$

2. Evaluate the following integrals: **(3+3 p)**

$$\int_0^{\ln 2} \frac{e^x}{\sqrt{1+e^x}}\,dx \quad \text{och} \quad \int \frac{x+4}{x^2+2x}\,dx$$

PART B

3. Let $f(x) = x \ln x$.
   (a) Find the Taylor polynomial of degree 2 for f about $x = 1$ and use it to find an approximate value of $3 \ln 3$. **(2 p)**
   (b) Is it true that the approximate value found in (a) differs by at most $1/10$ from the true value of $3 \ln 3$? **(3 p)**
   (c) Is the approximate value larger or smaller than $3 \ln 3$? **(1 p)**

4. How many solutions does the equation $\dfrac{1}{x} + 2 \arctan x = 3$ have?

PART C

5. Show that the following holds for all positive integers m och n:

$$\sum_{k=n+1}^{2n} \frac{1}{k} \leq \ln 2 \leq \sum_{k=m}^{2m-1} \frac{1}{k}.$$

6. (a) Assume that $f(0) = 0$ and that $|f(x)| > \sqrt{|x|}$ for all $x \neq 0$. Show that the function f is not differentiable at $x = 0$. **(3 p)**
   (b) Assume that the function g is defined on the whole real line and satisfies the following conditions: $g'(0) = k,\ g(0) \neq 0$ and $g(x+y) = g(x)g(y)$ for all x and y. Show that $g(0) = 1$ and that $g'(x) = kg(x)$ for all x. **(3 p)**