**SF1625 Calculus in one variable**
**Exam**
**Tuesday 23th, October 2018**

Time: 08:00-11:00
Available aid: None
Examinator: Roy Skjelnes

The exam consists of three parts; A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

| Grade | A | B | C | D | E | Fx |
|---|---|---|---|---|---|---|
| Total score | 27 | 24 | 21 | 18 | 16 | 15 |
| score on part C | 6 | 3 | – | – | – | – |

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

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## Part A

1. Let $f(x) = \dfrac{x^2-1}{|x-1|}$, with $x \neq 1$.
   (a) Compute the limit $\lim_{x\to 1+}(f(x))$. **(2 p)**
   (b) Determine if the $\lim_{x\to 1}(f(x))$ exists. **(2 p)**
   (c) For which $x$ do we have that $f(x) < 1$. **(2 p)**

2. Compute the integrals.
   (a) $\displaystyle\int_0^{\pi/2} \frac{\cos(x)}{1+\sin^2(x)}dx.$ **(3 p)**
   (b) $\displaystyle\int_0^{1} \frac{x^2}{1+x^2}\,dx.$ **(3 p)**

## Part B

3. A model for a population $P(t)$ is given by the integral equation
$$P'(t) = 2P(t) - 2\int_0^t P(s)\,ds - e^{2t},$$
   where $t$ represents time.
   (a) If you differentiate the integral equation you obtain an ordinary differential equation of second order (ODE). Determine this ODE. **(2 p)**
   (b) Determine $P(t)$ if the start population $P(0) = 10$. **(4 p)**

4. The function $f(x) = \sqrt{x}$ is defined for all real numbers.
   (a) Determine the Taylor polynomial $P(x)$ of degree 2 to $f$ around $x = 4$. **(2 p)**
   (b) Determine an approximation of $\sqrt{5}$ not being more than $1/200$ from the actual value. **(4 p)**

## Part C

5. Show that $\ln(n!) > 1 + n(\ln(n) - 1)$ for all $n \geq 2$. **(6 p)**

6. Assume that the function $\phi\colon \mathbb{R} \to \mathbb{R}$ is twice differentiable and that $\phi''(x)$ is everywhere continuous. Show that if $\phi''(x) > x^2$ forr all $x$, and if $\phi(0) = -1$, then there exist a number $c > 0$ such that $\phi(c) = 0$. **(6 p)**