SF1685 (SF1625) Calculus in one variable
Exam
Wednesday, 3 June 2020

Time: 14:00-17:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of three parts: A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

Please turn page!

PART A

1. Evaluate the limits **(3+3 p)**

$$\lim_{x\to\infty} \frac{e^{x/2} + \ln(x) + 2e^{2x}}{3e^{2x} + x^{100} - 7} \quad \text{and} \quad \lim_{x\to 0} \frac{x - \sin(x)}{x - x\cos(x)}.$$

2. (a) Find a primitive function of $f(x) = 1/(x^2 + 5x + 6)$. **(3 p)**

   (b) Find the area of the plane region bounded by the curve $y = xe^{-x}$ and the lines $y = 0, x = 0$ and $x = 1$. **(3 p)**

PART B

3. Let $f(x) = \dfrac{x}{\sqrt{x^4 + 1}}$. **(6 p)**
   - Find all the intervals on which f is increasing and all the intervals on which f is decreasing. Find all local extreme points.
   - Find all asymptotes to the curve $y = f(x)$.

   Use the information above to sketch the curve $y = f(x)$. Does the function f have absolute maximum or absolute minimum values? Find these values if they exist.

4. Determine whether the following improper integrals are convergent or divergent. If they are convergent, evaluate them.

   (a) $\displaystyle\int_0^1 \frac{e^{-x}}{x} dx.$ **(3 p)**

   (b) $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^2} dx.$ **(3 p)**

PART C

5. Show that **(6 p)**

$$\ln\left(\left(\frac{1+x}{1-x}\right)^{\frac{1}{2x}}\right) > 1 \qquad \text{for all } 0 < x < 1.$$

6. Assume that the function f is defined on the whole real line and that f, f' and f'' are continuous everywhere. Assume also that $f(0) = f'(0) = 0$ and that we have $f''(x) \geq 0$ for all x. Show that the series $\displaystyle\sum_{k=1}^{\infty} f(1/k)$ is convergent. **(6 p)**