KTH Teknikvetenskap

SF1624 Algebra and Geometry
Exam
Friday, October 18, 2019

Time: 08:00-11:00
No books/notes/calculators etc. allowed
Examiner: Danijela Damjanović

This exam consists of six problems, each worth 6 points.

Part A comprises the first two problems. The bonus points from the seminars will be automatically added to the total score of this part, which however cannot exceed 12 points.

The next two problems constitute part B, and the last two problems part C. The latter is mostly for achieving a high grade.

The thresholds for the respective grades are as follows:

Grade	A	B	C	D	E	Fx
Total sum	27	24	21	18	16	15
of which in part C	6	3	–	–	–	–

To get full score on a problem, your solution must be well-presented and easy to follow. In particular, you should define your notation; clearly explain the logical structure of your argument in words or symbols; and motivate and explain your argument. Solutions severely lacking in these respects will achieve at most 3 points.

PART A

1. (a) Find an equation for the plane Π that passes through the points $(2, 0, -1)$, $(4, 1, 2)$ and $(3, 1, 0)$. **(3 p)**

(b) The line

$$(2, 0, -1) + t(1, 1, 2), \quad -\infty < t < \infty,$$

and the normal vector to Π determine two angles, an acute angle θ and its supplement $\pi - \theta$. Find cosine of the acute angle θ. **(3 p)**

2. For which values of the constant x are the vectors $(1, 2, 3)$, $(3, 4, x)$ and $(4, x, 6)$ coplanar? (Vectors in $\mathbb{R}^3$ are *coplanar* if they are in the same plane.) **(6 p)**

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p.t.o.

Part B

3. The subspace V of $\mathbb{R}^4$ consists of all solution vectors of the system

$$\begin{cases} x+y+z+2w &= 0 \\ x-y-2z+w &= 0 \\ x+3y+4z+3w &= 0 \end{cases}$$

(a) Find a basis $\mathcal{B}$ for the vector space V. **(3 p)**

(b) Find the coordinate vector of $\begin{bmatrix} 2 & 4 & -2 & -2 \end{bmatrix}^T$ with respect to the basis $\mathcal{B}$. **(3 p)**

4. During an experiment, data were measured that should theoretically follow the model $x_2 = ax_1 + b$ for some constants a and b. Use the method of least squares to determine the constants a and b, if the measured data were given by the following table:

x_1	-2	-1	0	1	2
x_2	3	4	6	7	9

(6 p)

Part C

5. Consider a subspace V of $\mathbb{R}^3$ with orthonormal basis $(\vec{v}_1, \vec{v}_2)$.

(a) Show that the matrix of the orthogonal projection onto V is AA^T, where A is the matrix with columns $\vec{v}_1$ and $\vec{v}_2$. **(3 p)**

(b) Use (a) to find the orthogonal projection in $\mathbb{R}^3$ of the vector $\vec{u} = (2, -1, 3)$ onto the subspace $V = \operatorname{span}\{(-1, 1, 0), (-1, 0, 1)\}$. **(3 p)**

6. Consider the matrix

$$A = \begin{bmatrix} 1 & 4 & a \\ 4 & 7 & a(a+1) \\ 0 & 0 & -1 \end{bmatrix}.$$

(a) For which values of a is the matrix A diagonalizable? **(4 p)**

(b) Is there any value of a such that A is orthogonally diagonalizable? **(2 p)**

Justify your answer!