SF1624 Algebra och geometri
Exam
Wednesday, 9 January 2019

KTH Teknikvetenskap

Time: 08:00-11:00
No books/notes/calculators etc. allowed
Examiner: Danijela Damjanović, Tilman Bauer

This exam consists of six problems, each worth 6 points.

Part A comprises the first two problems. The bonus points from the seminars will be automatically added to the total score of this part, which however cannot exceed 12 points.

The next two problems constitute part B, and the last two problems part C. The latter is mostly for achieving a high grade.

The thresholds for the respective grades are as follows:

Grade	A	B	C	D	E	Fx
Total sum	27	24	21	18	16	15
of which in part C	6	3	–	–	–	–

To get full score on a problem, your solution must be well-presented and easy to follow. In particular, you should define your notation; clearly explain the logical structure of your argument in words or symbols; and motivate and explain your argument. Solutions severely lacking in these respects will achieve at most 3 points.

DEL A

1. (a) Find the volume of the parallellepiped P spanned by the vectors

$$\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

(3 p)

(b) Let T be the linear transformation that is given by the standard matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

Find the volume of $T(P)$, where P is the parallellepiped from (a). **(3 p)**

2. Use the least squares method to determine the straight line $y = a + bx$ which best fits the five points

$$(x, y) = (1, 5),\ (2, 4),\ (3, 1),\ (4, -1),\ (5, -3).$$

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DEL B

3. Let L_1 be the straight line in $\mathbb{R}^3$ given by $(x, y, z) = (2, 2, 0) + t(3, 0, 2)$.

(a) Determine the plane containing the line L_1 and the point $A = (8, 2, 3)$. **(2 p)**

(b) The line L_2 is given by $(x, y, z) = (5, 1, 0) + t(2, 1, 1)$. Determine an equation for the line that passes through the point $A = (8, 2, 3)$ and meets both L_1 and L_2. **(4 p)**

4. Let V be the subspace of $\mathbb{R}^4$ consisting of all solutions $\begin{bmatrix} x & y & z & w \end{bmatrix}^T$ of the system of equations

$$\begin{cases} x & +2y & -3z & +3w & = 0 \\ x & -y & & -3w & = 0 \end{cases}.$$

(a) Find an orthonormal basis for V. **(3 p)**

(b) Compute the projection of the vector $\begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^T$ onto V. **(3 p)**

DEL C

5. (a) Show that eigenvectors corresponding to different eigenvalues of a symmetric matrix are orthogonal. **(3 p)**

(b) Find a symmetric matrix having eigenvalues $\lambda_1 = 1$, $\lambda_2 = -1$, $\lambda_3 = 3$, where an eigenvector corresponding to λ_1 is

$$\vec{v}_1 = [1, 2, 2]^T$$

and an eigenvector corresponding to λ_2 is

$$\vec{v}_2 = [2, 1, -2]^T.$$

(3 p)

6. Let A be a 3×2-matrix and B a 2×3-matrix. Suppose that $\operatorname{rank}(A) = \operatorname{rank}(B) = 2$.

(a) Show that the (quadratic) matrix AB is never invertible. **(2 p)**

(b) Show that BA is invertible if and only if the only vector which lies in both $\operatorname{col}(A)$ and $\operatorname{null}(B)$ is the zero vector. **(4 p)**