SF1625 Calculus in one variable
Tentamen
Tuesday 24 October 2017

Time: 08:00-11:00
Available aid: None
Examinator: Roy Skjelnes

The exam consists of three parts; A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

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PART A

1. (a) Determine all primitive functions to $f(x) = \cos^3(x)\sin(x) + 2$. **(3 p)**
   (b) Determine the Taylor polynom of degree 2, around $x = 1$, of $\ln(1 + \frac{1}{2}x^2)$. **(3 p)**
   (c) Give the domain and the image of the function $f(x) = \arcsin(x)$, and determine the value of $\arcsin(\sin(\frac{3\pi}{4}))$. **(3 p)**
   (d) Draw the graph of the function $f(x) = \sqrt{\sin^2(x-1)}$. **(3 p)**

PART B

2. (a) Give an example of a function defined on a closed and bounded interval that does not have a maximum value. **(2 p)**
   (b) Show that the function

$$f(x) = \begin{cases} \frac{\sin(x-\frac{1}{2})}{x-\frac{1}{2}}e^x & \text{om } x \neq \frac{1}{2}, \\ \sqrt{e} & \text{om } x = \frac{1}{2} \end{cases}$$

   is continuous at $x = \frac{1}{2}$. **(2 p)**
   (c) Determine if the function $f(x)$ obtains a maximum and minimum on the closed interval $[0, 1]$. **(2 p)**

3. (a) Determine all primitive functions to $f(x) = e^{3x}\sin(2x)$. **(4 p)**
   (b) A curve is parametrised as $x(t) = \cos^3(t)$ and $y(t) = \sin^3(t)$, where t runs through the interval $[0, \pi/2]$. Determine the length of the curve. **(2 p)**

PART C

4. Show that $I(p) = \displaystyle\int_0^\infty \frac{x}{(1+x^2)^p}\,dx$ diverges when $0 < p \leq 1$. **(6 p)**

5. Show that the inequality

$$\sum_{k=1}^{n} \frac{1}{\sqrt{k}(\sqrt{k}+1)} \geq \ln(\frac{n}{4})$$

   holds for all $n \geq 1$. (Hint: The sum is an upper bound for an integral). **(6 p)**