KTH Teknikvetenskap

SF1624 Algebra och geometri
Exam
Friday, October 19, 2018

Time: 08:00-11:00
No books/notes/calculators etc. allowed
Examiner: Danijela Damjanović, Tilman Bauer

This exam consists of six problems, each worth 6 points.

Part A comprises the first two problems. The bonus points from the seminars will be automatically added to the total score of this part, which however cannot exceed 12 points.

The next two problems constitute part B, and the last two problems part C. The latter is mostly for achieving a high grade.

The thresholds for the respective grades are as follows:

Grade	A	B	C	D	E	Fx
Total sum	27	24	21	18	16	15
of which in part C	6	3	–	–	–	–

To get full score on a problem, your solution must be well-presented and easy to follow. In particular, you should define your notation; clearly explain the logical structure of your argument in words or symbols; and motivate and explain your argument. Solutions severely lacking in these respects will achieve at most 3 points.

Part A

1. Let Π be the plane in $\mathbb{R}^3$ which contains the points $A = (1, 3, 1), B = (2, 0, 0), C = (0, 1, 1)$.

(a) Find an equation of the form $ax + by + cz = d$ for Π. **(4 p)**

(b) Determine whether $(0, 2, 0)$ lies in the plane Π. **(2 p)**

2. Consider the following matrix:

$$A = \frac{1}{13}\begin{bmatrix} -5 & 12 \\ 12 & 5 \end{bmatrix}.$$

(a) Determine all eigenvalues and corresponding eigenvectors of A. **(3 p)**

(b) Is the linear mapping given by the matrix representing reflection, projection, rotation, or something else? Motivate your answer. **(3 p)**

Part B

3. Consider the equation

$$\begin{bmatrix} 12t+6 & -7t-4 \\ -7t-3 & 4t+2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

(a) Find some value of t for which the equation has no solutions. **(3 p)**

(b) Find some value for t for which the equation has infinitely many solutions, and find those solutions. **(3 p)**

4. Let L be the linear map from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by

$$L(\vec{x}) = \vec{v} \times \vec{x}, \quad \text{where } \vec{v} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

(a) Find the standard matrix for L. **(3 p)**

(b) Find the matrix for L in the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ -4 \end{bmatrix} \right\}$. **(3 p)**

Part C

5. Consider the quadratic form $Q(x, y) = 5x^2 - 4xy + 8y^2$.

(a) Find the symmetric matrix A associated to Q. **(1 p)**

(b) Determine the type of Q. **(2 p)**

(c) Find a matrix P which orthogonally diagonalizes the above matrix A. **(3 p)**

6. Find the functions $f(n)$, $g(n)$ och $h(n)$, where n is a natural number, satisfying the system of equations

$$\begin{aligned} f(n+1) &= 3f(n) + g(n) \\ g(n+1) &= -g(n) + h(n) \\ h(n+1) &= -6g(n) + 4h(n) \end{aligned}$$

and the conditions $f(0) = 0$, $g(0) = 1$ och $h(0) = 0$. **(6 p)**

Hint: Write the system in matrix form.