SF1625 Calculus in one variable
Tentamen
Monday 8, January 2018

Time: 08:00-11:00
Available aid: None
Examinator: Roy Skjelnes

The exam consists of three parts; A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

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PART A

1. Draw the graph of the function $f(x) = |x-3| + |x| - 4$. **(6 p)**

2. Determine all primitive functions to $f(x) = \dfrac{x-1}{x^2-5x+6}$. **(6 p)**

PART B

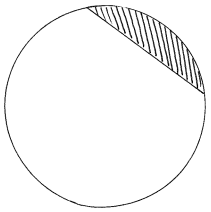
3. We have the function $f(x) = \dfrac{\sin x}{\cos^3 x}\sqrt{1+\cos^{-2}(x)}$. Use the substitution $u = 1/\cos x$ to determine $\displaystyle\int_0^{\pi/4} f(x)\,dx$. **(5 p)**

4. The polynomial $P(x) = \frac{2}{9} + \frac{8}{9}x - \frac{1}{9}x^2$ is the Taylor polynomial of degree 2, around $x = 1$, of the function $f(x) = x^{2/3}$.
   (a) Use Taylors Theorem to describe the function $E(x) = f(x) - P(x)$ around $x = 1$. **(2 p)**
   (b) Show that $|4^{1/3} - \frac{14}{9}| \leq \frac{4}{81}$. **(5 p)**

PART C

5. Determine whether the integral $\displaystyle\int_2^\infty \frac{1}{\sqrt{x^3-1}}\,dx$ converges or diverges. **(6 p)**

6. A circle segment S is the part of a disc determined by the arc and a straight line cutting the circle in two points. (The shadowed area in the figure.) Let the circle have radius r, and let the arc have angle t, seen from the center of the circle ($t < \pi$). This determines a circle segment S_t. Let $A(S_t)$ denote the area of the segment S_t. Compute the limit



$$\lim_{t\to 0}\frac{A(S_t)}{t^3}.$$

(6 p)