SF1685 (SF1625) Calculus in one variable
Tentamen
Friday 7, June 2019

Time: 14:00-17:00
Available aid: None
Examinator: Roy Skjelnes

The exam consists of three parts; A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

Please turn page!

Part A

1. Determine the Taylor polynomial of $f(x) = \arctan(2x)$ of degree three, around $x = 0$. **(4 p)**

2. Determine a primitive function to $g(x) = x\cos^3(2x^2)$. **(3 p)**

3. The curve $y = \sin x$, with $0 \le x \le \pi/2$ is rotated around the y-axis, and form a vase V. Determine the volume of the vase V. **(5 p)**

Part B

4. We have the function $f(x) = \begin{cases} x^2\cos(1/x) & \text{när} \quad x \neq 0 \\ 2 & \text{när} \quad x = 0. \end{cases}$
   (a) What does it mean for a function to be continuous in a given point? **(2 p)**
   (b) Show that $\lim_{x\to 0}(x^2\cos(1/x)) = 0$. **(3 p)**
   (c) Does there exist a continuous function F defined for all real numbers that coincides with f when $x \neq 0$? **(2 p)**

5. We have the function $g(x) = \displaystyle\int_0^x \frac{1-t}{1+t^{7/2}}\,dt$, defined for all positive $x \ge 0$.
   (a) Determine the number x where the function g achieves its maximum. **(2 p)**
   (b) Determine whether the limit $\lim\limits_{x\to\infty} g(x)$ exists. **(3 p)**

Part C

6. There exists an integer n such that $\displaystyle\sum_{k=1}^{n} k^3 = 90000$.
   (a) Show that $n > 23$. **(4 p)**
   (b) Determine the number n. **(4 p)**

7. A function $f\colon \mathbb{R} \to \mathbb{R}$ is *uniform continuous* if for every $\epsilon > 0$ exists $\delta > 0$ such that for all x and y we have that
$$|x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon.$$
   Show that the function $f(x) = x^2$ is not uniform continuous. **(4 p)**