KTH Teknikvetenskap

SF1624 Algebra och geometri
Exam
January 9, 2020

Time: 08:00-11:00
No books/notes/calculators etc. allowed
Examiner: Danijela Damjanović

This exam consists of six problems, each worth 6 points.

Part A comprises the first two problems. The bonus points from the seminars will be automatically added to the total score of this part, which however cannot exceed 12 points.

The next two problems constitute part B, and the last two problems part C. The latter is mostly for achieving a high grade.

The thresholds for the respective grades are as follows:

Grade	A	B	C	D	E	Fx
Total sum	27	24	21	18	16	15
of which in part C	6	3	–	–	–	–

To get full score on a problem, your solution must be well-presented and easy to follow. In particular, you should define your notation; clearly explain the logical structure of your argument in words or symbols; and motivate and explain your argument. Solutions severely lacking in these respects will achieve at most 3 points.

Part A

1. Determine a second degree polynomial whose graph passes through the points

$$(1,3), \quad (2,-2), \quad (3,-5).$$

(6 p)

Tip: start with a second degree polynomial and solve a linear system where the coefficients of the polynomial are the unknowns.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which maps a point in the plane $\mathbb{R}^2$ to its mirror image with respect to the line $x + y = 0$.

(a) Find the matrix A for T in the standard basis of $\mathbb{R}^2$. **(2 p)**

(b) Find two linearly independent eigenvectors of $\vec{v}_1, \vec{v}_2$ of A. **(2 p)**

(c) Find the matrix B for T in the basis $\{\vec{v}_1, \vec{v}_2\}$. **(2 p)**

Please turn over!

PART B

3. Consider the vectors $\vec{v_1} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, and $\vec{v_3} = \begin{bmatrix} 2 \\ 0 \\ -7 \end{bmatrix}$.

(a) Show that $\vec{v_1}, \vec{v_2}, \vec{v_3}$ is a basis for $\mathbf{R}^3$. **(2 p)**

(b) Write $\vec{w} = \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix}$ as a linear combination of $\vec{v_1}, \vec{v_2}, \vec{v_3}$. **(2 p)**

(c) Determine the orthogonal projection of $\vec{w}$ on the plane spanned by $\vec{v_1}$ and $\vec{v_2}$. **(2 p)**

4. Let $A = \begin{pmatrix} -5 & 0 & 6 \\ -3 & 1 & 3 \\ -4 & 0 & 5 \end{pmatrix}$. Compute A^{1000}. **(6 p)**

PART C

5. (a) Assuming that we know that $\vec{u} \times \vec{v} = \vec{u} \times \vec{w}$ for some vector $\vec{u}$. Can we conclude that $\vec{v}$ and $\vec{w}$ are the same? If yes, explain why is it so. If not, then precisely what can we conclude about $\vec{v}$ och $\vec{w}$, and why? **(3 p)**

(b) Assume that we know that $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$ for all vectors $\vec{u}$. Can we conclude that $\vec{v}$ och $\vec{w}$ are the same? If yes, explain why is it so. If not, then precisely what can we conclude about $\vec{v}$ och $\vec{w}$, and why? **(3 p)**

6. Let A be a symmetric $(n \times n)$-matrix. Show that the equation

$$X^3 = A$$

has a solution. **(6 p)**