SF1624, -66, -67, -75, -84 Algebra och geometri
Exam
Friday, 6 April 2018

KTH Teknikvetenskap

Time: 08:00-11:00
No books/notes/calculators etc. allowed
Examiner: Tilman Bauer

This exam consists of six problems, each worth 6 points.

Part A comprises the first two problems. The bonus points from the seminars will be automatically added to the total score of this part, which however cannot exceed 12 points.

The next two problems constitute part B, and the last two problems part C. The latter is mostly for achieving a high grade.

The thresholds for the respective grades are as follows:

Grade	A	B	C	D	E	Fx
Total sum	27	24	21	18	16	15
of which in part C	6	3	–	–	–	–

To get full score on a problem, your solution must be well-presented and easy to follow. In particular, you should define your notation; clearly explain the logical structure of your argument in words or symbols; and motivate and explain your argument. Solutions severely lacking in these respects will achieve at most 3 points.

PART A

1. Consider the following matrix:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & -3 \\ 3 & -1 & 5 \end{bmatrix}.$$

(a) Find all solutions $\vec{x}$ for $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. **(3 p)**

(b) Determine whether the line $(2, 3, 11) + t(1, 2, 1)$ is mapped by A to a line or to a point. **(3 p)**

2. The line L in $\mathbb{R}^2$ is given by the scalar equation $3x + 4y = 2$.

(a) Give the line L in parameter form. **(2 p)**

(b) Find a scalar equation of the form $ax + by = c$ for a line that goes through $Q = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and is orthogonal to L. **(4 p)**

PART B

3. For a linear map $F\colon \mathbb{R}^3 \to \mathbb{R}^3$ we have that

$$F(1,0,0) = (1,0,1) \quad \text{and} \quad F(1,0,1) = (2,1,0).$$

Furthermore, the vector $\vec{u} = (1,1,-1)$ is an eigenvector of F with eigenvalue 2. Determine the matrix for F in

(a) the standard basis, **(3 p)**

(b) the basis $\mathcal{B} = \{(1,0,0),(1,0,1),(1,1,-1)\}$. **(3 p)**

4. The subspace V of $\mathbb{R}^4$ is spanned by the vectors

$$\begin{bmatrix}1 & -1 & 1 & 1\end{bmatrix}^T \quad \text{and} \quad \begin{bmatrix}2 & 1 & -2 & -1\end{bmatrix}^T.$$

(a) Find on orthonormal basis for V. **(3 p)**

(b) Compute the projection of the vector $\begin{bmatrix}0 & 1 & 1 & 1\end{bmatrix}^T$ onto V. **(3 p)**

PART C

5. Let A be a 2×2-matrix such that $A^{10} = 0$.

(a) Show that $\det A = 0$. **(2 p)**

(b) Show that $\operatorname{tr} A = 0$, where $\operatorname{tr} \begin{bmatrix}a & b \\ c & d\end{bmatrix} = a + d$ is the trace of$A$.

Hint: change of basis does not affect the trace. **(4 p)**

6. The radioactive element saturnium decays in such a way that $1/2$ of all atoms turn into the element xenium each Christmas. Xenium is also radioactive: $1/3$ of a xenium atoms turn into lead each Christmas. Lead is stable.

(a) Express the radioactive decay of the three elements by a 3×3-matrix M. **(2 p)**

(b) Find all eigenvalues and corresponding eigenvectors for M and write $M = SDS^{-1}$, where D is a diagonal matrix.

(c) In how many mole saturnium/xenium/lead does 1 mole saturnium decay after 100 Christmasses? **(2 p)**