SF1685 Calculus in one variable
Tentamen
Monday 12th of March, 2018

Time: 08:00-11:00
Available aid: None
Examinator: Roy Skjelnes

The exam consists of three parts; A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained. Solutions that are clearly inadequate in these respects will be awarded no more than 2 points.

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Part A

1. Determine all primitive functions to $f(x) = \dfrac{(\ln(x))^3}{x}$. **(4 p)**

2. Determine the Taylor polynomial of degree 2, around $x = 1$, of $g(x) = e^{-x^2}$. **(4 p)**

3. Give the definition of the derivative of a function φ at a point x. **(4 p)**

Part B

4. Show that the equation $x^7 + 3x^5 - \frac{3}{2x} + 2 = 0$ has a unique solution in the open interval $(0, 1)$. **(4 p)**

5. Compute the integral $\displaystyle\int_0^1 x\sqrt{1+x}\,dx$. **(4 p)**

6. The function $f(x) = \dfrac{x^3}{x^2 - 4}$ is defined on the open interval $(-2, 2)$. Show that $f(x)$ has an inverse. **(4 p)**

Part C

7. To every integer $N > 0$ we let $S_N = \sum_{n=1}^{N} \frac{1}{n\sqrt{n}}$. The series converges to a number S. We want to approximate the number S with S_N. Determine an integer N that guarantees that the error in the approximation is less than $1/100$. **(6 p)**

8. For every integer $n > 0$ we define the interval $I_n = [-\frac{1}{n}, \frac{1}{n}]$. Let φ be a continuous function, defined for every real number x. Let φ_n denote the maximum value the function φ obtains for x in the interval I_n. Show that the limit
$$\lim_{n\to\infty} \varphi_n$$
exists, and compute its value. **(6 p)**