SF1625 (SF1685) Calculus in one variable
Exam
Thursday, 7 January 2021

Time: 08:00-11:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of six problems, each worth 6 points. To the score on Problem 1 your bonus points are added, up to a maximum of 6 points. The score on Problem 1 is at most 6 points, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

Please turn page!

PART A

1. (a) Evaluate the integral $\displaystyle\int_0^{\pi/4} \frac{\sin x}{\cos^2 x}\, dx$. **(3 p)**

   (b) Find a primitive function of $f(x) = \arctan x$. **(3 p)**

2. Let $f(x) = \ln(1 + x^2)$.

   (a) Find the domain of definition of f and calculate $f'(x)$. **(2 p)**

   (b) Find the Taylor polynomial of order 2 for f about $x = 0$. **(2 p)**

   (c) Evaluate the limit $\displaystyle\lim_{x\to 0} \frac{f(x)}{x^2}$. **(2 p)**

PART B

3. (a) Parametrize the curve $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$. Also sketch the curve. **(2 p)**

   (b) What is the maximal area of a rectangle if its corners are on the curve in (a) and its sides are parallel to the coordinate axes? **(4 p)**

4. Let $f(x) = \dfrac{\ln x}{x^2}$.

   (a) Make a sign chart for the derivative and find all local extreme points. Use the sign chart, together with appropriate limits, to sketch the curve $y = f(x)$. **(3 p)**

   (b) Determine which of the following improper integrals converge:

$$\int_0^1 f(x)\, dx, \qquad \int_1^\infty f(x)\, dx.$$

   Evaluate the integrals in case they converge. **(3 p)**

PART C

5. Let the function f be defined for all real numbers x by

$$f(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

   (a) At which points is f continuous? **(2 p)**

   (b) At which points is f differentiable? **(2 p)**

   (c) Find the range of f. **(2 p)**

6. Show that for every constant $c > 0$ the following holds: **(6 p)**

$$\frac{\pi}{2\sqrt{c}} \leq \sum_{n=0}^{\infty} \frac{1}{n^2 + c} \leq \frac{\pi}{2\sqrt{c}} + \frac{1}{c}$$