SF1625 Calculus in one variable
Exam
Tuesday, 22 October 2019

Time: 08:00-11:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of three parts; A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

Please turn page!

PART A

1. Evaluate the integrals **(3+3 p)**

$$\int_{-1}^{1} x^2 \ln(2+x^3)dx \quad \text{and} \quad \int \frac{3}{2+8x^2}dx.$$

2. Let $f(x) = xe^{-x^2}$ be defined for $x > 0$.
   (a) Find all local extreme values of f and classify them (max/min). **(2 p)**
   (b) Find the intervals on which f is increasing and on which f is decreasing. **(2 p)**
   (c) Does f have absolute maximum or absolute minimum values? Find these values if they exist. **(2 p)**

PART B

3. Let $f(x) = \sqrt{x}$ and let $P(x)$ be the Taylor polynomial of order 2 for f about $x = 1$.
   (a) Find the polynomial $P(x)$. **(2 p)**
   (b) According to Taylor's formula the reminder $E(x) = f(x) - P(x)$ can be written as an expression involving the third derivative of f. Which is the expression? **(1 p)**
   (c) Is it true that $|\sqrt{3/2} - P(3/2)| < 1/100$? **(3 p)**

4. (a) Evaluate the improper integral $\displaystyle\int_1^\infty \frac{3}{x^2+3x}dx$. **(4 p)**
   (b) Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{3}{k^2+3k}$ converges or diverges. **(2 p)**

PART C

5. (a) Evaluate the limit $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{2}{n} + \frac{2}{n}\sqrt{\frac{2k}{n}}\right)$. **(3 p)**
   (b) Evaluate the limit **(3 p)**

$$\lim_{n\to\infty} \int_n^{n+1} t \sin\left(\frac{1}{t}\right) dt.$$

6. (a) Assume that the function $f(x)$ is continuous on the closed interval $[0, 1]$ and that $0 \le f(0) \le 1$ and $0 \le f(1) \le 1$. Show that there exists (at least) one point p in the interval $[0, 1]$ for which $f(p) = p$. **(3 p)**
   (b) Assume that the function $g(x)$ is differentiable on the whole real line and that we have $|g'(x)| \le 1/2$ for every x. Show that there exists (at least) one point p for which $f(p) = p$. **(3 p)**