SF1685 (SF1625) Calculus in one variable
Exam
Friday, 11 June 2021

Time: 14:00-17:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of six problems, each worth 6 points. To the score on Problem 1 your bonus points are added, up to a maximum of 6 points. The score on Problem 1 is at most 6 points, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

Please turn page!

PART A

1. (a) Find a solution of the differential equation $y''(x) + 2y'(x) = 12\cos(2x) + 4\sin(2x)$ of the form $y(x) = A\cos(2x) + B\sin(2x)$ where A and B are constants. **(2 p)**
   (b) Find the solution of the differential equation $y''(x)+2y'(x) = 12\cos(2x)+4\sin(2x)$ which satisfies $y(0) = 0$ and $y'(0) = 4$. **(4 p)**

2. Find the area of the region bounded by the curve $y = \dfrac{1}{4+x^2} + \dfrac{x}{x^2+3x+2}$ and the lines $y = 0, x = 0$ och $x = 2$. Simplify your answer. **(6 p)**

PART B

3. Let $f(x) = e^{x-\frac{x^2}{2}}$.
   (a) Find the points on the curve $y = f(x)$ where the tangent is horisontal. **(2 p)**
   (b) Determine whether there is some point on the curve $y = f(x)$ where the slope is maximal. Find such a point if such a point exists; otherwise explain why there is no such point. **(4 p)**

4. (a) Let $f(x) = \dfrac{1}{x(\ln x)^2}$. Use derivative to show that f is decreasing on the interval $(1,\infty)$. **(2 p)**
   (b) Use suitable integral estimates to show that **(4 p)**

$$\sum_{k=3}^{\infty} \frac{1}{k(\ln k)^2} \leq \frac{1}{\ln 2}.$$

PART C

5. The function f is an infinitely differentiable function, defined in some open neighborhood I of $x = e$, and determined by the conditions

$$\begin{cases} e^{f(x)} - x\,(f(x))^2 = 0 \\ f(e) = 1 \end{cases}$$

   Find the second-order Taylor polynomial for f about the point $x = e$. **(6 p)**

6. (a) Assume that the function g is continuous everywhere (but we do not assume that g is differentiable). Let $f(x) = xg(x)$. Show that the function f is differentiable at $x = 0$ and find $f'(0)$. **(3 p)**
   (b) Assume that the function $h : \mathbb{R} \to \mathbb{R}$ satisfies $|h(x) - h(y)| \leq |x - y|^2$ for all real numbers x, y. Show that h is a constant function. **(3 p)**