SF1685 Calculus in one variable
Exam
Friday, 11 March 2022

Time: 8:00-11:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of six problems, each worth 6 points. To the score on Problem 1 your bonus points are added, up to a maximum of 6 points. The score on Problem 1 is at most 6 points, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

PART A

1. Find primitive functions of $f(x) = x^{2022} \ln x$ and $g(x) = \dfrac{(\ln x)^{2022}}{x}$. **(3+3 p)**

2. Let $f(x) = \arctan(x^2)$.
   - (a) Find the Taylor polynomial of order 2 for f about $x = 0$. **(3 p)**
   - (b) Evaluate the limit $\displaystyle\lim_{x \to 0} \dfrac{\arctan(x^2)}{x^2 + x^3}$. **(3 p)**

PART B

3. We consider the function $f(x) = \dfrac{1}{x \ln(x)}$. **(6 p)**
   - Solve the inequality $f(x) < 0$.
   - Find all intervals on which f is increasing and all intervals on which f is decreasing, and find all local extreme points.
   - Find all asymptotes of the curve $y = f(x)$.

   Use the information above to sketch the curve $y = f(x)$.

4. (a) Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \dfrac{k}{e^k}$ is convergent or divergent. **(3 p)**

   (b) Determine whether the improper integral $\displaystyle\int_0^1 \dfrac{dx}{x + \sqrt{x}}$ is convergent or divergent.

   (3 p)

PART C

5. (a) Assume that the function f is differentiable at the point x_0. Show that f is continuous at the point x_0. **(3 p)**

   (b) Assume that the function f is differentiable at the point x_0 and that $f(x_0) \neq 0$. Show that the function $1/f$ is differentiable at the point x_0. **(3 p)**

6. Find a tangent to the curve $y = e^{2x} - 2e^{-x} + x$ which is not parallel to any other tangent to the curve. **(6 p)**