SF1625 Calculus in one variable
Exam
Tuesday, 7 January 2020

Time: 08:00-11:00
Available aid: None
Examinator: Kristian Bjerklöv

The exam consists of three parts: A, B and C, each worth 12 points. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The grading will be performed according to the table

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

A necessity for full score on a problem is that your solution is well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

Please turn page!

PART A

1. (a) Let $g(x) = \arcsin(\sqrt{x})$. Find the domain of g and calculate $g'(x)$. **(2 p)**

   (b) Let $f(x) = x + \arctan(x)$. Find the domain and the range of f. Use the derivative to determine whether f is invertible or not. **(4 p)**

2. (a) Evaluate $\displaystyle\int_{\pi^2}^{4\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$. **(3 p)**

   (b) The function $f(x)$ satisfies $f'(x) = x\ln(x)$ and $f(1) = 1$. Find the function $f(x)$. **(3 p)**

PART B

3. Let $f(x) = xe^{-x},\ x \geq 1$.

   (a) Find the point (x_0, y_0) on the graph $y = f(x)$ which makes the area of the triangle with corners in $(0,0), (x_0, 0)$ and (x_0, y_0) maximal. **(4 p)**

   (b) Does there exist a point (x_1, y_1) on the graph $y = f(x)$ which makes the area of the triangle with corners in $(0,0), (x_1, 0)$ and (x_1, y_1) minimal? **(2 p)**

4. Let $F(x) = \displaystyle\int_0^x e^{-t^2} dt$.

   (a) Find the Taylor polynominal of order 1 for $F(x)$ about $x = 0$. **(2 p)**

   (b) Find an approximate value of $F(1/2)$ which differs by at most $1/8$ from the exact value. **(4 p)**

PART C

5. (a) Show that the inequality $\displaystyle\sum_{k=1}^{n} \frac{1}{k} > \ln(n)$ holds for all integers $n \geq 1$. **(3 p)**

   (b) Evaluate the limit **(3 p)**

$$\lim_{n\to\infty} \frac{1}{\ln(n)} \sum_{k=1}^{n} \frac{1}{k}.$$

6. Assume that the function $f(x)$ is defined on the whole real line and that $(f(x))^2 \leq x^4 + x^6$ for all x.

   (a) Determine whether f has to be continuous at $x = 0$ or not. **(3 p)**

   (b) Determine whether f has to be differentiable at $x = 0$ or not. **(3 p)**

   Justify your answers by providing proofs or counterexamples.