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<u>Prop 7.11:</u> • $(L,\partial)/(k,\partial)$ PV-extension

• k alg. closed

• $(L,\partial) \supseteq (M,\partial) \supseteq (k,\partial)$ intermediate differential field

Then: $\mathrm{Aut}_\partial(L/M) \subseteq \mathrm{Aut}_\partial(L/k)$ algebraic subgroup

<u>Proof:</u> Prop 7.10

$\Rightarrow (L,\partial)/(M,\partial)$ PV-extension

$\Rightarrow \mathrm{Aut}_\partial(L/M)$ algebraic group

• $\mathcal{L} \in k[\partial]$ monic diff. operator s.t. $(L,\partial)/(k,\partial)$ PV-ext. for $\mathcal{L}$

• $S = k[Y_{ij}][W^{-1}]$ full univ. sol. algebra

• $S_M := M \otimes_k S$ —— $\|$ ——

over M (identify S with [illegible]) over M (identify S with image in S_M)

- $V \subseteq S_M \cap S$, $V := \mathrm{span}_k \{ y_{0i} \mid 1 \leq i \leq n \}$

- If $\mathfrak{a} \subset S$ proper differential ideal

$\Rightarrow \mathfrak{a}_M := M \otimes_k \mathfrak{a} \subset S_M$

proper differential ideal

- If $\mathfrak{m} \subset S$ max. diff. ideal

$\mathfrak{m}' \subset S_M$ —— || ——

$\Rightarrow \mathfrak{m}' \cap S = \mathfrak{m}$

- $L = Q(S/\mathfrak{m}) \cong Q(S_M/\mathfrak{m}')$

Thm 7.1 $\Rightarrow \mathrm{Aut}_\partial(L/k) \cong Gl(V)_{\mathfrak{m}}$

$\mathrm{Aut}_\partial(L/M) \cong Gl(V)_{\mathfrak{m}'}$

- $S \subset S_M$ $Gl(V)$-submodule and

$\mathfrak{m} = S \cap \mathfrak{m}'$

$\Rightarrow Gl(V)_{\mathfrak{m}'} \subseteq Gl(V)_{\mathfrak{m}} \subseteq Gl(V)$

algebraic subgroups $\square$

<u>Ex. 13</u>: $(L,\partial)/(k,\partial)$ PV-extension

for $\mathcal{L} = \partial^n + \sum_{i=1}^{n} a_i \partial^{n-i} \in k[\partial]$

· Assume k alg. closed

· $V = \mathrm{span}_k\{y_1, \dots, y_n\}$, $y_1, \dots, y_n$ full set of solutions of $\mathcal{L} = 0$

· $w = w(y_1, \dots, y_n)$ Wronskian

· Then: $M = K(w)$ is a PV-extension for $\partial + a_1\partial^0$

Hint: show that $\partial(w) = -a_1 w$

<u>Prop 7.12.</u> Notation as in Ex. 13 we have $\mathrm{Aut}_\partial(L/M) = \mathrm{Aut}_\partial(L/K) \cap Sl(V)$ where we view $\mathrm{Aut}_\partial(L/K) \subseteq Gl(V)$

<u>Proof:</u> Let $\sigma \in \mathrm{Aut}_\partial(L/K)$

Ex. 12 $\Rightarrow \sigma(w) = \det(\sigma) \cdot w$

$\Rightarrow \mathrm{Aut}_\partial(L/K)_w = \{\sigma \in \mathrm{Aut}_\partial(L/K) \mid \det(\sigma) = 1\}$

$\Rightarrow \mathrm{Aut}_\partial(L/M) = \mathrm{Aut}_\partial(L/K) \cap Sl(w)$ $\square$