

Classical and Bayesian Statistics - Exercises

Table of contents

Problems 1	6
a	6
b	6
c	6
I	6
II	6
III	6
IV	6
d	7
Problem 1.2	7
a	7
b	7
c	7
d	8
e	8
f	8
g	8
h	9
Problem 1.3	9
Problems 2	10
Probmel 2.1	10
a	10
b	11
c	11
d	12
Problem 2.2	12
a	12
b	12

Problem 2.3	13
Problem 2.4	15
a	15
b	15
c	15
Problem 2.5	15
a	15
b	15
c	15
d	16
e	16
f	16
Problem 2.6	16
a	16
b	17
c	17
d	17
e	17
Problem 2.7	17

Problems 3	18
Problem 3.1	18
Problem 3.2	18
a	18
b	18
c	18
d	18
e	19
Problem 3.3	19
a	19
b	19
c	19
d	19
Problem 3.4	19
a	19
b	20
c	20
d	20
Problem 3.5	20
Problem 3.6	20
a	20
b	21

Problems 4	21
Problem 4.2	21
a	21
b	22
c	22
d	22
Problem 4.3	22
Problem 4.4	22
a	23
b	23
c	23
d	23
e	23
Problem 4.5	24
a	24
b	24
Problems 5	24
Problem 5.1	24
Problem 5.2	25
a	25
b	26
Problems 6	26
Problem 6.1	26
a	26
b	26
c	27
Problem 6.2	27
a	27
b	27
c	27
Problem 6.3	28
a	28
b	28
c	28
d	29
e	29
Problem 6.4	29
a	29
b	29
c	29
d	30

e	30
f	30
Problem 6.5	31
a	31
b	31
c	31
d	31
e	32
f	32
Problem 6.6	32
a	33
b	34
III	35
Problem 6.7	35
a	35
b	35
c	35
d	35
e	36
f	37
Problem 6.8	37
a	37
b	37
c	37
d	37
e	37
Problems 7	37
Problem 7.1	37
a	37
b	38
c	38
d	39
Problem 7.2	39
a	39
b	39
c	40
d	41
Problem 7.3	42
a	42
b/c	43
d	45
Problem 7.4	45

a	45
b	45
c	46
d	46
e	46
f	47
g	47
h	47
i	48
Problems 8	48
Problem 8.1	48
a	49
b	50
c	51
d	52
Problem 8.2	52
a	53
b	54
c	55
d	56
Problem 8.3	57
a	57
b	57
c	58
d	58
e	58
f	58
g	59
Problems 10	59
Problem 10.1	59
Problem 10.2	60
a	60
b	60
Problem 10.4	60
Problem 10.5	60

Problems 1

a

It seems that, in total, the Allied and Axis countries had about the same number of civilian deaths. However, if we compare the numbers per party member, we see that the Axis countries had a higher average amount of deaths. On the other hand, the data from the Allies is inconsistent (see Denmark).

b

The percentages in the chart do not sum up to 100%.

c

I

We can assume that Linda is a bank teller and is active in the feminist movement. Her experiences from the past could have influenced her behavior and thinking on certain topics. She may have a sense for justice and equality, regardless of the topic.

II

We can assume that Steve is a librarian because of his helpful personality and his need for order and structure, much like what's found in a library. His passion for details may also be connected to a desire for knowledge.

III

A Ball costs \$0.05.

IV

There are more death by heart diseases than accidents.

d

Since the engine is the only thing that keeps a plane in the air, it makes sense for it to be more armoured than the rest of the plane. Even if other parts have more bullet holes on average (e.g. the fuselage), the plane could still fly.

Problem 1.2

```
# Define vectors
winner <- c(193, 183, 191, 185, 185, 182, 182, 188, 188, 188, 185, 185, 177,
182, 182, 193, 183, 179, 179, 175)

opponent <- c(163, 191, 165, 187, 175, 193, 185, 187, 188, 173, 180, 177, 183,
185, 180, 180, 182, 178, 178, 173)
```

a

```
# Determine length
cat("Length of vector winners =", length(winner), "\n") # Add line break
```

Length of vector winners = 20

```
cat("Length of vector opponent =", length(opponent))
```

Length of vector opponent = 20

b

```
cat("Entries 6 to 10 =", winner[6:10]) # Index starts at 1
```

Entries 6 to 10 = 182 182 188 188 188

c

```
cat("Some values from winner:", winner[c(3, 5, 10, 12)]) # Passing a vector for selection
```

Some values from winner: 191 185 188 185

d

```
cat("Current values:",winner[c(8, 9)], "\n") # Check values
```

Current values: 188 188

```
winner[c(8, 9)] <- 189 # Reassign  
cat("New values:", winner[c(8, 9)]) # Check values
```

New values: 189 189

e

```
mu_winner <- mean(winner)  
mu_opponent <- mean(opponent)  
  
cat("Mean higth of winner vs. opponent:", mu_winner, "vs.", mu_opponent)
```

Mean higth of winner vs. opponent: 184.35 vs. 180.15

f

```
mu_diff <- mu_winner - mu_opponent  
  
cat("Differences between means =", mu_diff)
```

Differences between means = 4.2

g

```

var_winner <- var(winner)
sd_winner <- sd(winner)

cat("Variance / Std. deviation of winner:", var_winner, "/", sd_winner)

```

Variance / Std. deviation of winner: 25.08158 / 5.008151

h

```

my_variance <- function(data){
  mu <- mean(data)
  sum_of_squares <- sum((data - mu)^2)
  variance <- sum_of_squares / (length(data) - 1)

  return(variance)
}

my_stdDeviation <- function(variance){
  stdDevition <- sqrt(variance)

  return(stdDevition)
}

my_var_winner <- my_variance(winner)
my_sd_winner <- my_stdDeviation(my_var_winner)

cat("Variance of Winner =", my_var_winner, "\n") # Add line break

```

Variance of Winner = 25.08158

```
cat("Variance of Winner =", my_sd_winner)
```

Variance of Winner = 5.008151

Problem 1.3

```

grades <- c(4.2, 2.3, 5.6, 4.5, 4.8, 3.9, 5.9, 2.4, 5.9, 6, 4, 3.7, 5, 5.2, 4.5, 3.6, 5, 6, 2)

grades <- sort(grades) # Sort values

original_mu <- mean(grades)
originla_median <- median(grades)

grades[9:11] <- 1 # Reassign values
new_mu <- mean(grades)
new_median <- median(grades)

cat("Original vs. New mean:", original_mu, "vs.", new_mu, "\n") # Add line break

```

Original vs. New mean: 4.5125 vs. 4.1

```

cat("Median stays the same:", originla_median, "=", new_median)

```

Median stays the same: 4.65 = 4.65

Problems 2

Problem 2.1

a

```

# Read data
data <- read.csv('/home/nils/dev/mscids-notes/hs25/sa/data/husband_wife.csv')
head(data)

```

	age.husband	height.husband	age.wife	height.wife
1	49	180	43	159
2	25	184	28	156
3	40	165	30	162
4	52	177	57	154
5	58	161	52	142
6	32	169	27	166

b

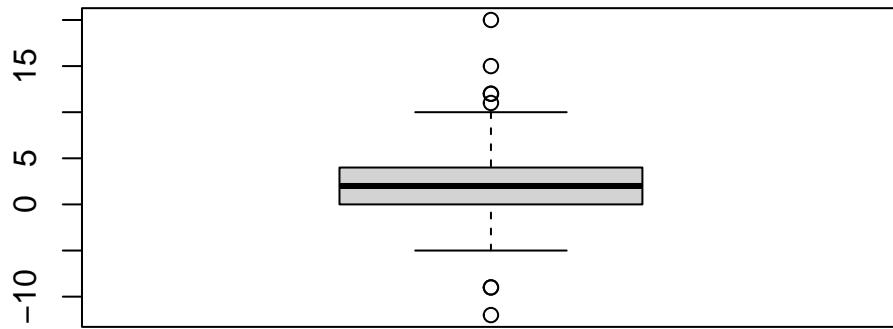
```
summary(data)
```

	age.husband	height.husband	age.wife	height.wife
Min.	:20.00	Min. :155.0	Min. :18.00	Min. :141.0
1st Qu.	:33.00	1st Qu.:169.0	1st Qu.:32.00	1st Qu.:156.0
Median	:43.50	Median :172.0	Median :41.00	Median :160.0
Mean	:42.92	Mean :172.8	Mean :40.68	Mean :160.3
3rd Qu.	:53.00	3rd Qu.:177.0	3rd Qu.:50.00	3rd Qu.:165.0
Max.	:64.00	Max. :190.0	Max. :64.00	Max. :176.0

For each column, we see a brief summary with quantitative and qualitative information about the data.

c

```
age_diff <- data$age.husband - data$age.wife # Calc age difference  
boxplot(age_diff)
```



d

- The median of age_diff is about 2.5. On average, the age difference between husbands and wives is around 2.5 years.
- 50% of the differences lie between approximately 0 and 5 years.
- There are more upper than lower outliers, meaning that extreme cases where the husband is much older than the wife occur more frequently.
- In addition, the values of the upper outliers are larger than those of the lower ones.

Problem 2.2

```
head(InsectSprays) # Preview data from head
```

	count	spray
1	10	A
2	7	A
3	20	A
4	14	A
5	14	A
6	12	A

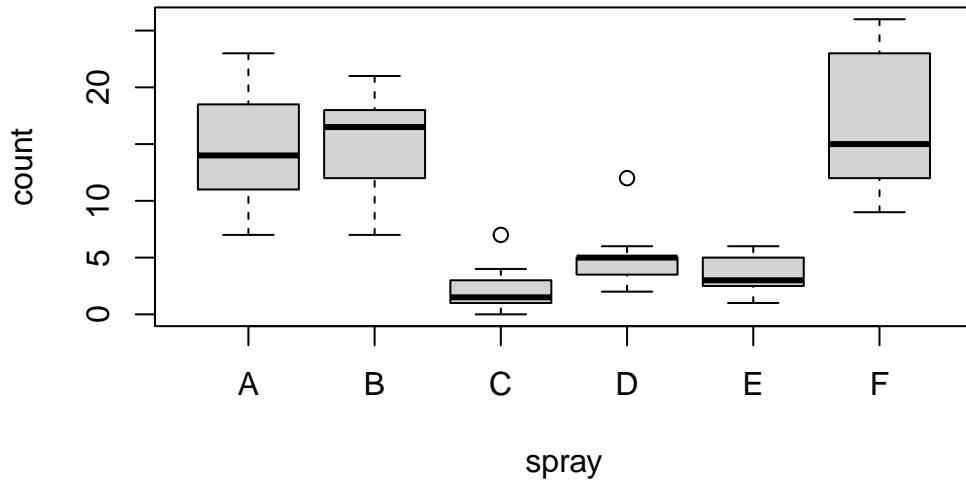
a

```
tapply(InsectSprays$count, InsectSprays$spray, mean)
```

	A	B	C	D	E	F
1	14.500000	15.333333	2.083333	4.916667	3.500000	16.666667

b

```
boxplot(count ~ spray,  
        data = InsectSprays)
```



Problem 2.3

```
data <- read.csv('/home/nils/dev/mscids-notes/hs25/sa/data/Diet.csv')
head(data)
```

	Person	gender	Age	Height	pre.weight	Diet	weight6weeks
1	25	NA	41	171	60	2	60.0
2	26	NA	32	174	103	2	103.0
3	1	0	22	159	58	1	54.2
4	2	0	46	192	60	1	54.0
5	3	0	55	170	64	1	63.3
6	4	0	33	171	64	1	61.1

```
# Add column weight.loss
data$weight.loss <- data$weight6weeks - data$pre.weight
data$weight.loss
```

```
[1]  0.0  0.0 -3.8 -6.0 -0.7 -2.9 -2.8 -2.0 -2.0 -8.5 -1.9 -3.1 -1.5 -3.0 -3.6
[16] -0.9  2.1 -2.0 -1.7 -4.3 -7.0 -0.6 -2.7 -3.6 -3.0 -2.0 -4.2 -4.7 -3.3  0.5
[31] -7.0 -5.6 -3.4 -6.8 -7.8 -5.4 -6.8 -7.2 -7.0 -7.3 -0.9 -7.6 -4.1 -6.3 -5.0
```

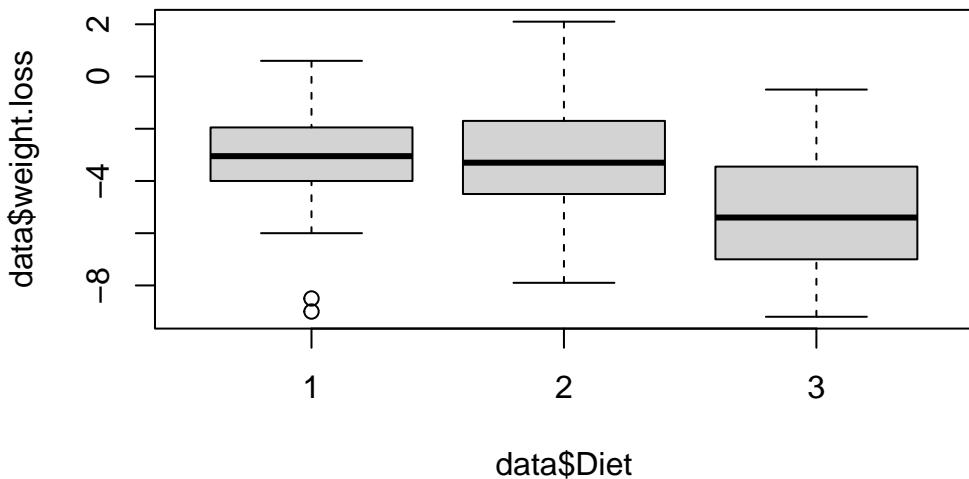
```
[46]  0.6 -1.1 -4.5 -4.1 -9.0 -2.4 -3.9 -3.5 -5.1 -3.5 -4.2 -2.4 -5.8 -3.5 -5.3  
[61] -1.7 -5.4 -6.1 -7.9  1.4 -4.3 -2.5 -0.9 -3.5 -0.5 -2.8 -8.6 -4.5 -2.8 -4.1  
[76] -5.3 -9.2 -6.1
```

```
tapply(data$weight.loss, data$Diet, mean)
```

```
1           2           3  
-3.300000 -3.025926 -5.148148
```

According to the data, diet 3 appears to have the greatest effect on weight loss over the 6-week therapy period. Diets 1 and 2 show more or less the same effect, although patients following diet 2 lost slightly less weight on average.

```
boxplot(data$weight.loss ~ data$Diet)
```



- Even though diet 3 appears to have the greatest effect according to the median, it also has the largest interquartile range (IQR) among the three diets.
- Diet 2 shows the greatest overall spread across the entire boxplot
- Diet 1 is influenced by several lower outliers.

Problem 2.4**a**

The probabilities of ‘heads’ and ‘tails’ do not add up to 1.

b

The calculated probability is negative. That’s not possible by definition.

c

The union of the quantities S and M cannot be 0.7, because men cannot be pregnant.

Problem 2.5**a**

Sample space of the experiment:

$$\Omega = \{(i, j) \mid i, j \in \{1, 2, 3, 4, 5, 6\}\}$$

b

$$p(\omega_n) = \frac{1}{36} = 0.02\bar{7}$$

c

Events, where the sum is 7:

$$E_1 = (1, 6), (2, 5), (3, 4)$$

Note: Since there are two dices, we can multiply the number of favourable results by 2.

Now, we can calculate the probability:

$$p(E_1) = \frac{6}{36} = 0.1\bar{6}$$

d

$$E_2 = (1, 1), (1, 2), (2, 1)$$

$$p(E_2) = \frac{3}{36} = 0.08\bar{3}$$

e

$$E_3 = \{(i, j) \mid i, j \in \{1, 3, 5\}\}$$

$$p(E_3) = \frac{9}{36} = 0.25$$

f

```
p_e2 <- 3/36
p_e3 <- 9/36
p_intersection <- 1/36

p_annual <- p_e2 + p_e3 - p_intersection

print(p_annual)
```

[1] 0.3055556

Problem 2.6

```
p_A <- 3/4
p_B <- 2/3
```

a

```
p_bothEvents <- p_A * p_B
print(p_bothEvents)
```

[1] 0.5

b

```
p_atLeastOne <- p_A + p_B - p_A * p_B  
print(p_atLeastOne)
```

[1] 0.9166667

c

```
p_atMostOne <- 1 - p_A * p_B  
cat(p_atMostOne)
```

0.5

d

```
p_noEvent <- 1 - (p_A + p_B - p_A * p_B)  
print(p_noEvent)
```

[1] 0.08333333

e

```
p_exactlyOneEvent <- p_A + p_B - 2 * p_A * p_B  
print(p_exactlyOneEvent)
```

[1] 0.4166667

Problem 2.7

```

p_earthquake <- 0.04
p_typhoon <- 0.08

p_annual <- p_earthquake + p_typhoon - p_earthquake * p_typhoon

print(p_annual)

```

[1] 0.1168

Problems 3

Problem 3.1

$$p_2 = 1 - 0.3 - 0.1 - 0.2 - 0.3 = 0.1$$

Problem 3.2

a

The probabilities in the table sum to one, so it is a probability distribution.

$$\sum P(X = k) = 1$$

b

$$p(2 \leq k \leq 4) = 0.2 + 0.2 + 0.1 = 0.5$$

c

$$p(k > 2) = 0.2 + 0.1 + 0.1 = 0.4$$

d

$$p(k \leq 4) = 1 - 0.1 = 0.9$$

e

$$p(k > 1) = 1 - 0.4 = 0.6$$

Problem 3.3

a

$$p(k \leq 13) = 0.992$$

b

$$p(k \geq 10) = 1 - 0.939 = 0.061$$

c

$$p(k = 15) = 1 - 0.999 = 0.001$$

d

$$p(9 \leq k \leq 12) = 0.989 - 0.711 = 0.282$$

Problem 3.4

a

$$\Omega = \{\text{TTT}, \text{TTH}, \text{THT}, \text{HTT}, \text{THH}, \text{HTH}, \text{HHT}, \text{HHH}\}$$

$$P(X = 0) = \frac{1}{8}$$

$$P(X = 1) = \frac{3}{8}$$

$$P(X = 2) = \frac{3}{8}$$

$$P(X = 3) = \frac{1}{8}$$

b

$$p(x = 2) = \frac{3}{8}$$

c

$$p(X \geq 2) = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$$

d

$$p(X \leq 1) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$$

Problem 3.5

```
x_k <- c(-5, -4, 1, 3, 6)
p <- c(0.3, 0.1, 0.2, 0.3)
p_k <- 1 - sum(p) # Calc p_k
cat("Probability of -4 =", p_k, "\n")
```

Probability of -4 = 0.1

```
p <- c(0.3, p_k, 0.1, 0.2, 0.3) # Reassign p
mu <- sum(x_k * p) # Calc expected value
mu
```

[1] 0.6

Problem 3.6

a

$$p(x) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

b

```
x <- 2:12 # Sum of eyes
p <- c(1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1) / 36 # Probability of summed eyes

mu = sum(x * p)
mu
```

[1] 7

```
var = sum((x - mu)**2 * p)
var
```

[1] 5.833333

```
sd = sqrt(var)
sd
```

[1] 2.415229

Problems 4

Problem 4.2

```
# Define distr. paras
mu <- 4
sd <- 1.25
```

a

```
# Assume normal distr.
pnorm(q = 2.5, mean = mu, sd = sd)
```

[1] 0.1150697

b

```
1 - pnorm(q = 5.0, mean = mu, sd = sd)
```

```
[1] 0.2118554
```

c

```
pnorm(q = 4.5, mean = mu, sd = sd) - pnorm(q = 3.0, mean = mu, sd = sd)
```

```
[1] 0.4435663
```

d

```
qnorm(0.98, mean=mu, sd=sd)
```

```
[1] 6.567186
```

Problem 4.3

```
# Define distr. paras
mu <- 2.2
sd <- 0.3

# Assume normal distr.
1 - pnorm(q=3.1, mean=mu, sd=sd/sqrt(100))
```

```
[1] 0
```

Problem 4.4

```
# Define distr. paras
mu <- 8.2
sd <- 6.0
```

a

```
# Assume normal distr.
pnorm(q=10.0, mean=mu, sd=sd/sqrt(36))
```

```
[1] 0.9640697
```

b

```
pnorm(q=10.0, mean=mu, sd=sd/sqrt(36)) - pnorm(q=5.0, mean=mu, sd=sd/sqrt(36))
```

```
[1] 0.9633825
```

c

```
1- pnorm(q=20.0, mean=mu, sd=sd/sqrt(36))
```

```
[1] 0
```

d

It's small, but not impossible. We also assume a normal distribution. The real distribution probability may differ from the normal distribution. We also use a very small sample size of 36.

e

Yes, the i.i.d. assumption holds here because each of the 36 passengers is an individual who is independent of the others.

Problem 4.5

```
# Define distr. paras
mu <- 77
sd <- 15

course_1 <- 25
course_2 <- 64
```

a

```
# Assume normal distr.
pnorm(q=82, mean=mu, sd=sd/sqrt(course_1)) - pnorm(q=72, mean=mu, sd=sd/sqrt(course_1))
```

```
[1] 0.9044193
```

b

```
pnorm(q=82, mean=mu, sd=sd/sqrt(course_2)) - pnorm(q=72, mean=mu, sd=sd/sqrt(course_2))
```

```
[1] 0.9923392
```

For a larger group, the probability is more likely to be at the mean compared to a smaller group (CLT).

Problems 5

Problem 5.1

- $H_0: \mu = \mu_0 = 70$
- $H_A: \mu < 70$

Rejection range:

```

data <- c(71, 69, 67, 68, 73, 72, 71, 71, 68, 72, 69, 72)
sd <- 1.5

mu_hat <- mean(data)

qnorm(p = 0.05, mean = 70, sd = 1.5/sqrt(12))

```

[1] 69.28776

Test:

```
pnorm(q = mu_hat, mean = 70, sd = 1.5 / sqrt(12))
```

[1] 0.7181486

We do not reject the null hypothesis.

- p-value: 0.718

The mean of the sample does not statistically deviate from the producers claimed mean.

Problem 5.2

a

- $H_0: \mu = \mu_0 = 50$
- $H_A: \mu < 50$

```

data <- c(46, 48, 52, 49, 46, 51, 52, 47, 49, 44, 48, 51, 49, 50, 53, 47)
sd <- 3.0

mu_hat <- mean(data)

pnorm(q = mu_hat, mean = 50, sd = 3.0/sqrt(16))

```

[1] 0.0668072

We do not reject the null hypothesis.

- p-value: 0.0668072

b

```
data <- c(46, 48, 52, 49, 46, 51, 52, 47, 49, 44, 48, 51, 49, 50, 53, 47)
sd <- 3.0

mu_hat <- mean(data)

pnorm(q = mu_hat, mean = 50, sd = 3.0/sqrt(100))
```

[1] 8.841729e-05

We reject the null hypothesis.

Problems 6

Problem 6.1

a

- Paired samples: We use the same people for the before and after smoking measurements.
- One-sided: We are only interested in increasing platelet accumulation.
- Null hypothesis: The amount of platelets is the same before and after smoking.
- Alternative hypothesis: The number of platelets is higher after smoking than before.

b

- Paired: The height of each self-pollinated seedling corresponds to the height of the cross-pollinated ‘partner’.
- One-sided: We are only interested if the plants grow bigger.
- Null hypothesis: There is no difference between cross-pollinated and self-pollinated plants. Alternative hypothesis: There is a significant difference between the two groups.

c

- Unpaired: We have two distinct groups.
- Two-sided: We are interested in any effect on blood pressure.
- Null hypothesis: There is no difference in blood pressure between the two groups.
- Alternative hypothesis: There is a difference between the two groups. #### d
- Unpaired: We have two distinct groups.
- Two-sided: We are interested in the number of iron forms.
- Null hypothesis: There is no difference in the amount of iron between the groups/forms.
- Alternative hypothesis: There is a difference between the groups.

Problem 6.2

a

These are paired samples. Measurements are taken at the same location with both gauges.

b

It's a one-sided test because we are assuming that the values from gauge B are larger.

c

```
gauge_a <- c(120, 265, 157, 187, 219, 288, 156, 205, 163)
gauge_b <- c(127, 281, 160, 185, 220, 298, 167, 203, 171)

t.test(x=gauge_a, y=gauge_b, alternative="less", paired=TRUE, conf.level=0.95)
```

```
Paired t-test

data: gauge_a and gauge_b
t = -2.7955, df = 8, p-value = 0.01168
alternative hypothesis: true mean difference is less than 0
95 percent confidence interval:
```

```
-Inf -1.93449
sample estimates:
mean difference
-5.777778
```

There is a statistically significant difference.

Problem 6.3

a

The samples are unpaired because we are comparing two different groups: males and females.

b

- Null hypotheses: There is no difference in length between the two groups.
- Alternative hypothesis: There is a difference in length between the two groups.

c

```
male <- c(120, 107, 110, 116, 114, 111, 113, 117, 114, 112)
female <- c(110, 111, 107, 108, 110, 105, 107, 106, 111, 111)

t.test(x=males, y=females, alternative="two.sided", paired=FALSE, conf.level=0.95)
```

Welch Two Sample t-test

```
data: male and female
t = 3.4843, df = 14.894, p-value = 0.00336
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 1.861895 7.738105
sample estimates:
mean of x mean of y
 113.4      108.6
```

There is a statistically significant difference.

d

```
wilcox.test(x=male, y=female, alternative="two.sided", paired=FALSE, conf.level=0.95)
```

```
Warning in wilcox.test.default(x = male, y = female, alternative = "two.sided",
: cannot compute exact p-value with ties
```

```
Wilcoxon rank sum test with continuity correction
```

```
data: male and female
W = 87.5, p-value = 0.004845
alternative hypothesis: true location shift is not equal to 0
```

There is a statistically significant difference.

e

The result of the Wilcoxon-test is more trustworthy because, unlike the t-test, it does not assume that the data are normally distributed and we cannot verify this condition in any way.

Problem 6.4

a

Unpaired test: We investigated the calorie content of two different groups.

b

Two-sided: We are interested in any difference.

c

- Null hypotheses: There is no difference between the two groups.
- Alternative hypothesis: There is a difference between the two groups.

d

```
beef <- c(186, 181, 176, 149, 184, 190, 158, 139, 175, 148, 152, 111, 141, 153, 190, 157, 13  
poultry <- c(129, 132, 102, 106, 94, 102, 87, 99, 170, 113, 135, 142, 86, 143, 152, 146, 144  
  
mean_beef <- mean(beef)  
mean_poultry <- mean(poultry)  
  
cat(mean_beef, "vs", mean_poultry)
```

156.85 vs 122.4706

The calorie content of beef hot dogs seems to be much higher than that of poultry hot dogs.
The null hypothesis may be rejected

e

Since there is no indication whether the data are normally distributed, we choose a Wilcoxon test as a precautionary measure.

f

```
wilcox.test(x=beef, y=poultry, alternative="two.sided", paired=FALSE, conf.level=0.95)
```

```
Warning in wilcox.test.default(x = beef, y = poultry, alternative =  
"two.sided", : cannot compute exact p-value with ties
```

```
Wilcoxon rank sum test with continuity correction  
  
data: beef and poultry  
W = 285.5, p-value = 0.0004549  
alternative hypothesis: true location shift is not equal to 0
```

There is a statistically significant difference.

Problem 6.5

a

```
zh <- c(16.3, 12.7, 14.0, 53.3, 117, 62.6, 27.6)
bl <- c(10.4, 8.91, 11.7, 29.9, 46.3, 25.0, 29.4)

mean_zh <- mean(zh, na.rm=FALSE)
mean_bl <- mean(bl, na.rm=FALSE)

sd_zh <- sd(zh, na.rm=FALSE)
sd_bl <- sd(bl, na.rm=FALSE)

cat("ZH: Mean =", mean_zh, "and SD =", sd_zh, "\n")
```

ZH: Mean = 43.35714 and SD = 38.02301

```
cat("BL: Mean =", mean_bl, "and SD =", sd_bl)
```

BL: Mean = 23.08714 and SD = 13.66495

b

The samples are unpaired if we argue that the cities constitute the experimental units.

c

- Null hypotheses: There is no difference between the two groups.
- Alternative hypothesis: There is a difference between the two groups.

d

```
t.test(x=zh, y=bl, alternative="two.sided", paired=FALSE, conf.level=0.95)
```

```
Welch Two Sample t-test
```

```
data: zh and bl
t = 1.3273, df = 7.5245, p-value = 0.2233
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
-15.33677 55.87677
sample estimates:
mean of x mean of y
43.35714 23.08714
```

There is not a statistically significant difference.

e

```
[−15.33677, 55.87677]
```

f

```
wilcox.test(x=zh, y=bl, alternative="two.sided", paired=FALSE, conf.level=0.95)
```

```
Wilcoxon rank sum exact test
```

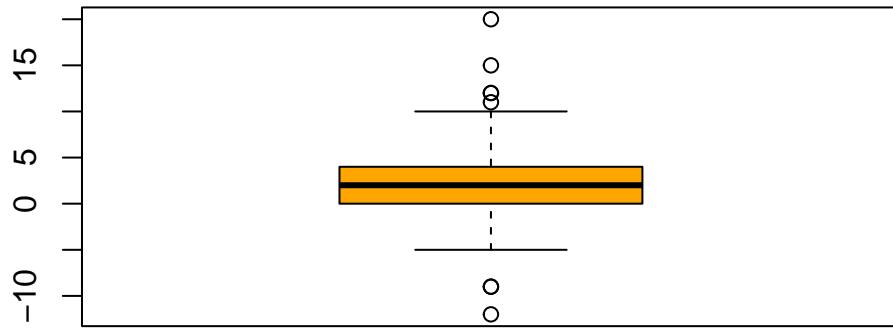
```
data: zh and bl
W = 34, p-value = 0.2593
alternative hypothesis: true location shift is not equal to 0
```

There is not a statistically significant difference.

Problem 6.6

```
mf <- read.csv("/home/nils/dev/mscids-notes/hs25/sa/data/husband_wife.csv")
diff <- mf$age.husband - mf$age.wife

boxplot(diff, col = "orange")
```



a

I

It is a paired test. For each test unit (married couple) there are two associated measurements (age husband, age wife).

II

We are not sure whether the husbands are really older than their wives. It is simply our impression and not a fact. So perform do a two-sided test.

III

- Null hypotheses: There is no difference between the two groups.
- Alternative hypothesis: There is a difference between the two groups.

```
t.test(x=mf$age.husband, y=mf$age.wife, alternative="two.sided", paired=TRUE, conf.level=0.99)
```

Paired t-test

```
data: mf$age.husband and mf$age.wife
t = 7.1518, df = 169, p-value = 2.474e-11
alternative hypothesis: true mean difference is not equal to 0
95 percent confidence interval:
 1.618286 2.852302
sample estimates:
mean difference
 2.235294
```

There is a statistically significant difference.

IV

```
wilcox.test(x=mf$age.husband, y=mf$age.wife, alternative="two.sided", paired=TRUE, conf.level=
```

Wilcoxon signed rank test with continuity correction

```
data: mf$age.husband and mf$age.wife
V = 9460, p-value = 3.977e-12
alternative hypothesis: true location shift is not equal to 0
```

There is a statistically significant difference.

b

I

It is an unpaired and a two-sided test.

II

- Null hypotheses: There is no difference between the two groups.
- Alternative hypothesis: There is a difference between the two groups.

III

```
t.test(x=mf$height.husband, y=mf$height.wife, , mu=13, alternative="two.sided", paired=FALSE)
```

Welch Two Sample t-test

```
data: mf$height.husband and mf$height.wife
t = -0.63293, df = 336.53, p-value = 0.5272
alternative hypothesis: true difference in means is not equal to 13
95 percent confidence interval:
 11.18772 13.92993
sample estimates:
mean of x mean of y
 172.8471 160.2882
```

There is not a statistically significant difference.

Problem 6.7

a

The test is paired because we measure the temperature of the same patients both before and after treatment.

b

One-sided: We are interested in its fever-lowering effect.

c

- Null hypotheses: There is no difference between the two groups.
- Alternative hypothesis: There is a significant difference between the two groups.

d

```
t1 <- c(39.1, 39.3, 38.9, 40.6, 39.5, 38.4, 38.6, 39.0, 38.6, 39.2)
t2 <- c(38.1, 38.3, 38.8, 37.8, 38.2, 37.3, 37.6, 37.8, 37.4, 38.1)

t.test(x=t1, y=t2, alternative="greater", paired=TRUE, conf.level=0.95)
```

Paired t-test

```
data: t1 and t2
t = 5.6569, df = 9, p-value = 0.0001554
alternative hypothesis: true mean difference is greater than 0
95 percent confidence interval:
 0.7976252      Inf
sample estimates:
mean difference
                 1.18
```

There is a statistically significant difference.

e

```
wilcox.test(x=t1, y=t2, alternative="greater", paired=TRUE, conf.level=0.95)
```

```
Warning in wilcox.test.default(x = t1, y = t2, alternative = "greater", :
  cannot compute exact p-value with ties
```

Wilcoxon signed rank test with continuity correction

```
data: t1 and t2
V = 55, p-value = 0.002865
alternative hypothesis: true location shift is greater than 0
```

There is a statistically significant difference.

f

The p-value of the Wilcoxon-test is greater than the p-value of the t-test. Since the Wilcoxon-test assumes less (no normal distribution) than the t-test, there is an additional uncertainty. The null hypothesis is less strongly rejected.

Problem 6.8

a

True

b

True

c

True

d

True

e

True

Problems 7

Problem 7.1

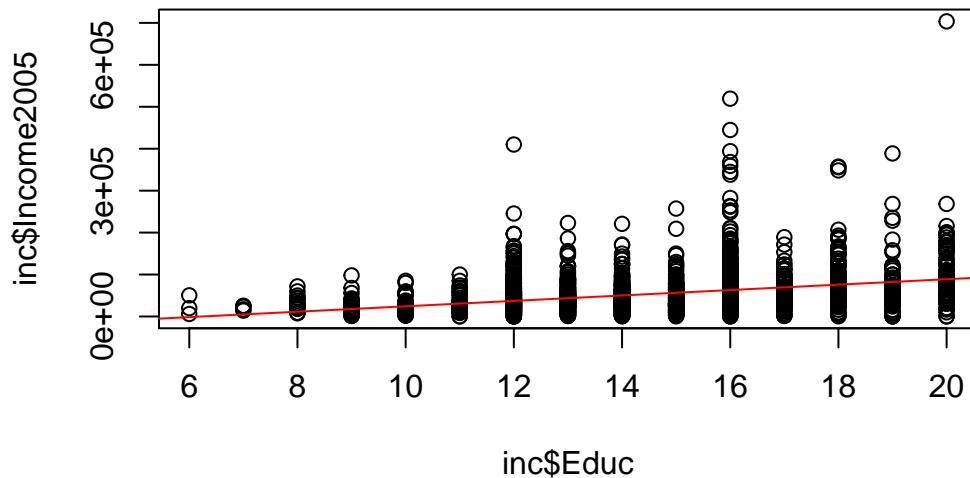
a

```
inc <- read.table("/home/nils/dev/mscids-notes/hs25/sa/data/income.dat", header=TRUE)
```

b

```
# Plot data
plot(inc$Educ, inc$Income2005)

# Add linear reg to plot
abline(lm(inc$Income2005 ~inc$Educ), col="red")
```



c

```
m <- lm(inc$Income2005 ~inc$Educ)
m$coefficients
```

(Intercept)	inc\$Educ
-40199.575	6451.475

- a: The regression line crosses the y axis at the point $x = -40199.575$.
- b: For one step at the direction x (one year of education), we increase the salary by 6451.475.

d

```
cat("The correlaiton between the education and income is:", cor(inc$Educ, inc$Income2005))
```

The correlaiton between the education and income is: 0.3456474

The corralation value ist near to O. The data points correlate loosly.

Problem 7.2

a

```
head(anscombe)
```

	x1	x2	x3	x4	y1	y2	y3	y4
1	10	10	10	8	8.04	9.14	7.46	6.58
2	8	8	8	8	6.95	8.14	6.77	5.76
3	13	13	13	8	7.58	8.74	12.74	7.71
4	9	9	9	8	8.81	8.77	7.11	8.84
5	11	11	11	8	8.33	9.26	7.81	8.47
6	14	14	14	8	9.96	8.10	8.84	7.04

b

```
par(mfrow=c(2,2))
plot(anscombe$x1, anscombe$y1)
reg <- lm(anscombe$y1 ~ anscombe$x1)
abline(reg)
title("x1, y1")

plot(anscombe$x2, anscombe$y2)
reg <- lm(anscombe$y2 ~ anscombe$x2)
abline(reg)
title("x2, y2")

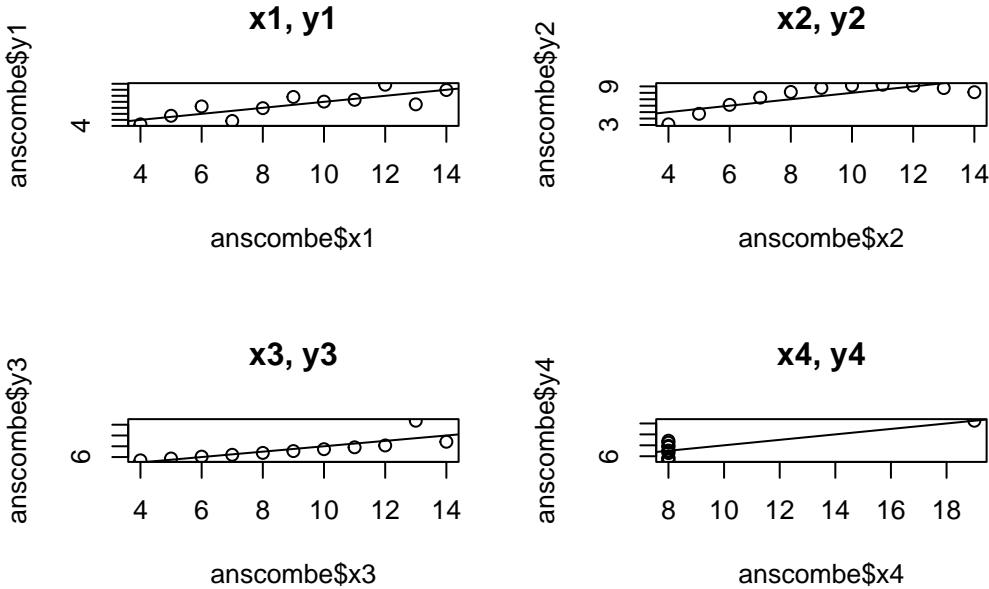
plot(anscombe$x3, anscombe$y3)
```

```

reg <- lm(anscombe$y3 ~ anscombe$x3)
abline(reg)
title("x3, y3")

plot(anscombe$x4, anscombe$y4)
reg <- lm(anscombe$y4 ~ anscombe$x4)
abline(reg)
title("x4, y4")

```



c

```

lm(y1 ~ x1, data = anscombe)

```

```

Call:
lm(formula = y1 ~ x1, data = anscombe)

Coefficients:
(Intercept)          x1
3.0001            0.5001

```

```
lm(y2 ~ x2, data = anscombe)
```

Call:

```
lm(formula = y2 ~ x2, data = anscombe)
```

Coefficients:

(Intercept)	x2
3.001	0.500

```
lm(y3 ~ x3, data = anscombe)
```

Call:

```
lm(formula = y3 ~ x3, data = anscombe)
```

Coefficients:

(Intercept)	x3
3.0025	0.4997

```
lm(y4 ~ x4, data = anscombe)
```

Call:

```
lm(formula = y4 ~ x4, data = anscombe)
```

Coefficients:

(Intercept)	x4
3.0017	0.4999

The model coefficients are almost identical.

d

```
cat("The correlaiton between the x1 and y1 is:", cor(anscombe$x1, anscombe$y1), "\n")
```

The correlaiton between the x1 and y1 is: 0.8164205

```
cat("The correlaiton between the x2 and y2 is:", cor(anscombe$x2, anscombe$y2), "\n")
```

The correlaiton between the x2 and y2 is: 0.8162365

```
cat("The correlaiton between the x3 and y3 is:", cor(anscombe$x3, anscombe$y3), "\n")
```

The correlaiton between the x3 and y3 is: 0.8162867

```
cat("The correlaiton between the x4 and y4 is:", cor(anscombe$x4, anscombe$y4))
```

The correlaiton between the x4 and y4 is: 0.8165214

The correlation value is almost identical.

Problem 7.3

```
# install.packages("ISLR")
library(ISLR)
```

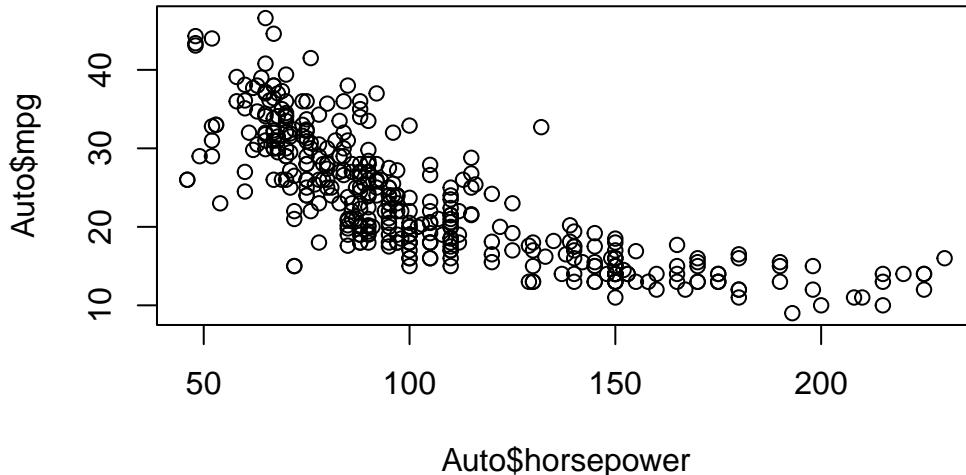
a

```
head(Auto)
```

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
1	18	8	307	130	3504	12.0	70	1
2	15	8	350	165	3693	11.5	70	1
3	18	8	318	150	3436	11.0	70	1
4	16	8	304	150	3433	12.0	70	1
5	17	8	302	140	3449	10.5	70	1
6	15	8	429	198	4341	10.0	70	1
			name					
1	chevrolet	chevelle	malibu					
2		buick	skylark	320				
3		plymouth	satellite					
4			amc	rebel	sst			
5				ford	torino			
6				ford	galaxie	500		

b/c

```
plot(Auto$horsepower, Auto$mpg)
```



```
model <- lm(mpg ~ horsepower, data=Auto)
summary(model)
```

Call:
lm(formula = mpg ~ horsepower, data = Auto)

Residuals:

Min	1Q	Median	3Q	Max
-13.5710	-3.2592	-0.3435	2.7630	16.9240

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	39.935861	0.717499	55.66	<2e-16 ***
horsepower	-0.157845	0.006446	-24.49	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
Residual standard error: 4.906 on 390 degrees of freedom
Multiple R-squared:  0.6059,    Adjusted R-squared:  0.6049
F-statistic: 599.7 on 1 and 390 DF,  p-value: < 2.2e-16
```

I

The fuel consumption depends on the horsepower.

II

The y value at position $x = 0$ value has no practical meaning here.

III

```
confint(model)
```

	2.5 %	97.5 %
(Intercept)	38.525212	41.3465103
horsepower	-0.170517	-0.1451725

The confidence interval indicates the most likely range of values.

IV

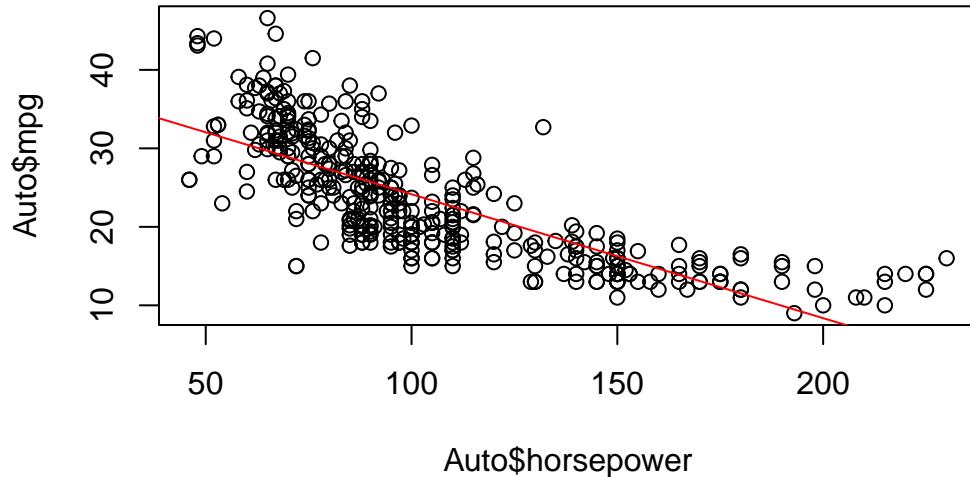
```
summary(model)$r.squared
```

```
[1] 0.6059483
```

The R^2 value is 0.606. This indicates that the variability to 60 % is through the model.

d

```
plot(Auto$horsepower, Auto$mpg)
model <- lm(mpg ~ horsepower, data=Auto)
abline(model, col="red")
```



Problem 7.4

a

```
# ?MASS::Boston
```

b

```
library(MASS)
colnames(Boston)
```

```
[1] "crim"      "zn"        "indus"      "chas"       "nox"        "rm"        "age"
[8] "dis"        "rad"        "tax"        "ptratio"    "black"      "lstat"      "medv"
```

c

```
attach(Boston)
```

d

I/II

```
model <- lm(medv ~ lstat, data=Boston)
summary(model)
```

Call:
lm(formula = medv ~ lstat, data = Boston)

Residuals:

Min	1Q	Median	3Q	Max
-15.168	-3.990	-1.318	2.034	24.500

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)							
(Intercept)	34.55384	0.56263	61.41	<2e-16 ***							
lstat	-0.95005	0.03873	-24.53	<2e-16 ***							

Signif. codes:	0	'***'	0.001	'**'	0.01	'*'	0.05	'..'	0.1	' '	1

Residual standard error: 6.216 on 504 degrees of freedom
Multiple R-squared: 0.5441, Adjusted R-squared: 0.5432
F-statistic: 601.6 on 1 and 504 DF, p-value: < 2.2e-16

e

```
names(model)
```

```
[1] "coefficients"   "residuals"      "effects"        "rank"
[5] "fitted.values"  "assign"         "qr"             "df.residual"
[9] "xlevels"         "call"          "terms"          "model"
```

f

```
coef(model)
```

```
(Intercept)      lstat  
34.5538409 -0.9500494
```

At the data point $x = 0$, we start at a value of 34.6. For every step in x direction, we lose -0.95. The p-value for `lstat` is close to 0 and therefore highly significant.

g

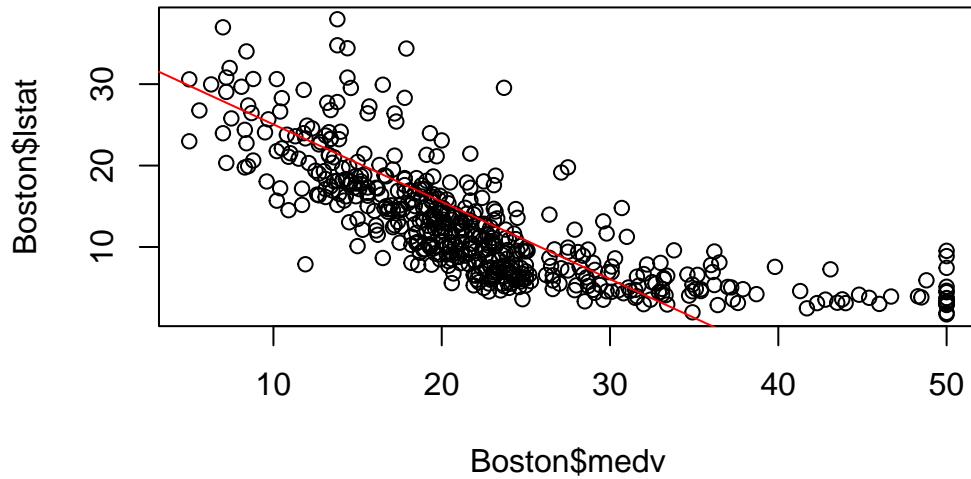
```
confint(model)
```

```
2.5 %      97.5 %  
(Intercept) 33.448457 35.6592247  
lstat       -1.026148 -0.8739505
```

The model shows that the true x and y values lies between this ranges.

h

```
par(mfrow=c(1,1))  
plot(Boston$medv, Boston$lstat)  
abline(model, col="red")
```



i

```
summary(model)$r.squared
```

```
[1] 0.5441463
```

The R^2 -value is 0.5441, so about 54 % of the variability is explained by the model.

Problems 8

Problem 8.1

```
Auto <- read.csv("/home/nils/dev/mscids-notes/hs25/sa/data/auto.csv")

# Read data
head(Auto)
```

	X	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
1	1	18	8	307	130	3504	12.0	70	1
2	2	15	8	350	165	3693	11.5	70	1
3	3	18	8	318	150	3436	11.0	70	1
4	4	16	8	304	150	3433	12.0	70	1
5	5	17	8	302	140	3449	10.5	70	1
6	6	15	8	429	198	4341	10.0	70	1

name

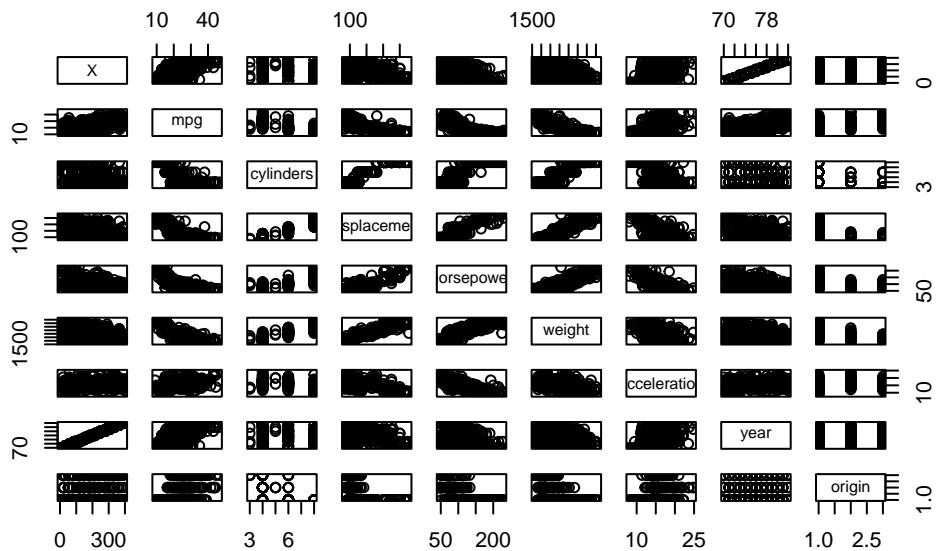
1	chevrolet	chevelle	malibu
2	buick	skylark	320
3	plymouth	satellite	
4	amc	rebel	sst
5	ford	torino	
6	ford	galaxie	500

```
# Remove var "name"
Auto_1 <- within(Auto, rm(name))
head(Auto_1)
```

	X	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
1	1	18	8	307	130	3504	12.0	70	1
2	2	15	8	350	165	3693	11.5	70	1
3	3	18	8	318	150	3436	11.0	70	1
4	4	16	8	304	150	3433	12.0	70	1
5	5	17	8	302	140	3449	10.5	70	1
6	6	15	8	429	198	4341	10.0	70	1

a

```
pairs(Auto_1)
```



b

```
cor(Auto_1)
```

	X	mpg	cylinders	displacement	horsepower
X	1.0000000	0.5863298	-0.3602752	-0.3871458	-0.4229250
mpg	0.5863298	1.0000000	-0.7776175	-0.8051269	-0.7784268
cylinders	-0.3602752	-0.7776175	1.0000000	0.9508233	0.8429834
displacement	-0.3871458	-0.8051269	0.9508233	1.0000000	0.8972570
horsepower	-0.4229250	-0.7784268	0.8429834	0.8972570	1.0000000
weight	-0.3217474	-0.8322442	0.8975273	0.9329944	0.8645377
acceleration	0.2909849	0.4233285	-0.5046834	-0.5438005	-0.6891955
year	0.9967805	0.5805410	-0.3456474	-0.3698552	-0.4163615
origin	0.2005760	0.5652088	-0.5689316	-0.6145351	-0.4551715
	weight	acceleration	year	origin	
X	-0.3217474	0.2909849	0.9967805	0.2005760	
mpg	-0.8322442	0.4233285	0.5805410	0.5652088	
cylinders	0.8975273	-0.5046834	-0.3456474	-0.5689316	
displacement	0.9329944	-0.5438005	-0.3698552	-0.6145351	
horsepower	0.8645377	-0.6891955	-0.4163615	-0.4551715	
weight	1.0000000	-0.4168392	-0.3091199	-0.5850054	

```

acceleration -0.4168392      1.0000000  0.2903161  0.2127458
year          -0.3091199      0.2903161  1.0000000  0.1815277
origin         -0.5850054      0.2127458  0.1815277  1.0000000

```

The scatter plot and correlation value show a high positive correlation between the `horsepower` and `displacement` variables.

c

```

model <- lm(mpg ~ ., data=Auto_1)
summary(model)

```

```

Call:
lm(formula = mpg ~ ., data = Auto_1)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.6234 -2.1948 -0.1499  1.8294 12.9947 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7.800e+01  4.259e+01 -1.831  0.06785 .
X             -2.829e-02  1.971e-02 -1.436  0.15196  
cylinders    -4.564e-01  3.239e-01 -1.409  0.15956  
displacement  1.715e-02  7.744e-03  2.215  0.02735 *  
horsepower   -1.431e-02  1.389e-02 -1.030  0.30348  
weight        -6.378e-03  6.546e-04 -9.745  < 2e-16 *** 
acceleration  7.505e-02  9.878e-02  0.760  0.44790  
year          1.622e+00  6.092e-01  2.663  0.00807 ** 
origin        1.455e+00  2.785e-01  5.224  2.88e-07 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.323 on 383 degrees of freedom
Multiple R-squared:  0.8224,    Adjusted R-squared:  0.8187 
F-statistic: 221.7 on 8 and 383 DF,  p-value: < 2.2e-16

```

|

The variables predict the response variable, `mpg`, statistically.

II

It seems that the variables `horsepower` and `year` have the greatest impact on the response variable `mpg`.

III

The variable `year` indicates a high positive correlation with the response variable `mpg`.

d

```
model <- lm(mpg ~ weight * year, data=Auto_1)
summary(model)
```

Call:

```
lm(formula = mpg ~ weight * year, data = Auto_1)
```

Residuals:

Min	1Q	Median	3Q	Max
-8.0397	-1.9956	-0.0983	1.6525	12.9896

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-1.105e+02	1.295e+01	-8.531	3.30e-16 ***
weight	2.755e-02	4.413e-03	6.242	1.14e-09 ***
year	2.040e+00	1.718e-01	11.876	< 2e-16 ***
weight:year	-4.579e-04	5.907e-05	-7.752	8.02e-14 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.193 on 388 degrees of freedom

Multiple R-squared: 0.8339, Adjusted R-squared: 0.8326

F-statistic: 649.3 on 3 and 388 DF, p-value: < 2.2e-16

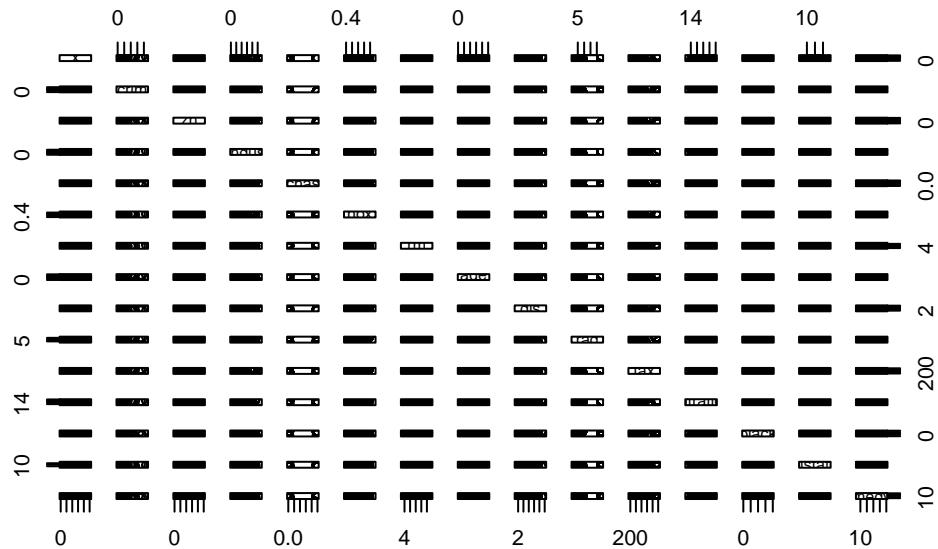
Problem 8.2

```
boston <- read.csv("/home/nils/dev/mscids-notes/hs25/sa/data/boston.csv")
head(boston)
```

	X	crim	zn	indus	chas	nox	rm	age	dis	rad	tax	ptratio	black	lstat
1	1	0.00632	18	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3	396.90	4.98
2	2	0.02731	0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.90	9.14
3	3	0.02729	0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	392.83	4.03
4	4	0.03237	0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94
5	5	0.06905	0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.90	5.33
6	6	0.02985	0	2.18	0	0.458	6.430	58.7	6.0622	3	222	18.7	394.12	5.21
		medv												
1		24.0												
2		21.6												
3		34.7												
4		33.4												
5		36.2												
6		28.7												

a

```
pairs(boston)
```



```
model <- lm(medv ~ lstat + age, data=boston)
summary(model)
```

Call:
`lm(formula = medv ~ lstat + age, data = boston)`

Residuals:

Min	1Q	Median	3Q	Max
-15.981	-3.978	-1.283	1.968	23.158

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)							
(Intercept)	33.22276	0.73085	45.458	< 2e-16 ***							
lstat	-1.03207	0.04819	-21.416	< 2e-16 ***							
age	0.03454	0.01223	2.826	0.00491 **							

Signif. codes:	0	'***'	0.001	'**'	0.01	'*'	0.05	'..'	0.1	' '	1

Residual standard error: 6.173 on 503 degrees of freedom

Multiple R-squared: 0.5513, Adjusted R-squared: 0.5495

F-statistic: 309 on 2 and 503 DF, p-value: < 2.2e-16

- $\hat{\beta}_0 = 33.22$: In neighborhoods where there is no population of lower status and no units build before 1940, the medium value of houses is \$ 33 220.
- $\hat{\beta}_1 = -1.03$: For each additional percent of population of lower status, the medium value decreases by \$ 1030.
- $\hat{\beta}_2 = 0.03$: For each additional percent of units build before 1949, the medium value increases by \$ 30.
- All p-values are significant (below the significance level of 5 %), so all estimates individually contribute significantly to the model.
- The R^2 value is 0.5513, therefore about 55 % of the variation is explained by the model.
- The p-value of the F value is below the significance level and therefore significant. The null hypothesis is rejected.

b

```
model <- lm(medv ~ ., data=boston)
summary(model)
```

```

Call:
lm(formula = medv ~ ., data = boston)

Residuals:
    Min      1Q  Median      3Q     Max 
-15.8948 -2.7585 -0.4663  1.7963 26.0911 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 36.461352   5.100994   7.148 3.21e-12 ***
X             -0.002526   0.002080  -1.215 0.225046  
crim          -0.108762   0.032855  -3.310 0.001000 **  
zn              0.048031   0.013785   3.484 0.000538 *** 
indus          0.019932   0.061468   0.324 0.745871  
chas            2.705245   0.861298   3.141 0.001786 **  
nox             -17.541602  3.822390  -4.589 5.66e-06 *** 
rm              3.839225   0.418422   9.175 < 2e-16 *** 
age             -0.001938   0.013380  -0.145 0.884866  
dis             -1.493304   0.199892  -7.471 3.68e-13 *** 
rad              0.324925   0.068111   4.771 2.43e-06 *** 
tax             -0.011598   0.003807  -3.046 0.002443 **  
ptratio         -0.947985   0.130822  -7.246 1.67e-12 *** 
black            0.009357   0.002685   3.485 0.000536 *** 
lstat            -0.526184   0.050704 -10.377 < 2e-16 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.743 on 491 degrees of freedom
Multiple R-squared:  0.7414,    Adjusted R-squared:  0.734 
F-statistic: 100.6 on 14 and 491 DF,  p-value: < 2.2e-16

```

The p-value is almost 1, so not significant at all. But in the first model, the p-value is 0.005, which is significant. That means that the variable age must correlate strongly with other variables.

c

The more variables you have the bigger the R^2 value. That means that the R^2 is not a good indicator to compare different models.

d

```
model <- lm(medv ~ lstat * age, data=boston)
summary(model)
```

Call:

```
lm(formula = medv ~ lstat * age, data = boston)
```

Residuals:

Min	1Q	Median	3Q	Max
-15.806	-4.045	-1.333	2.085	27.552

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)							
(Intercept)	36.0885359	1.4698355	24.553	< 2e-16 ***							
lstat	-1.3921168	0.1674555	-8.313	8.78e-16 ***							
age	-0.0007209	0.0198792	-0.036	0.9711							
lstat:age	0.0041560	0.0018518	2.244	0.0252 *							

Signif. codes:	0	'***'	0.001	'**'	0.01	'*'	0.05	'..'	0.1	' '	1

Residual standard error: 6.149 on 502 degrees of freedom

Multiple R-squared: 0.5557, Adjusted R-squared: 0.5531

F-statistic: 209.3 on 3 and 502 DF, p-value: < 2.2e-16

- $\hat{\beta}_0 = 36.10$: In neighborhoods where there is no population of lower status and no units build before 1940, the medium value of houses is \$ 36 100.
- $\hat{\beta}_1 = -1.39$: For each additional percent of population of lower status, the medium value decreases by \$ 1930.
- $\hat{\beta}_2 = -0.00072$: For each additional percent of units build before 1949, the medium value decreases by \$ 0.27. As you can imagine, this value is not significant, as you can see from the output.
- $\hat{\beta}_{12} = 0.004$: This coefficient is somewhat difficult to interpret and we didn't do it in class.
- Not all p-values are significant (below the significance level of 5 %) any- more.
- The R^2 value is 0.56, therefore about 56 % of the variation is explained by the model.
- The p-value of the F value is below the significance level and therefore significant. The null hypothesis H_0 is rejected.

Problem 8.3

a

```
cs <- read.csv("/home/nils/dev/mscids-notes/hs25/sa/data/carseats.csv")
head(cs)
```

	X	Sales	CompPrice	Income	Advertising	Population	Price	ShelveLoc	Age	Education
1	1	9.50	138	73	11	276	120	Bad	42	17
2	2	11.22	111	48	16	260	83	Good	65	10
3	3	10.06	113	35	10	269	80	Medium	59	12
4	4	7.40	117	100	4	466	97	Medium	55	14
5	5	4.15	141	64	3	340	128	Bad	38	13
6	6	10.81	124	113	13	501	72	Bad	78	16
		Urban	US							
1		Yes	Yes							
2		Yes	Yes							
3		Yes	Yes							
4		Yes	Yes							
5		Yes	No							
6		No	Yes							

b

```
model <- lm(Sales ~ Price + Urban + US, data=cs)
summary(model)
```

Call:

```
lm(formula = Sales ~ Price + Urban + US, data = cs)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.9206	-1.6220	-0.0564	1.5786	7.0581

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	13.043469	0.651012	20.036	< 2e-16 ***
Price	-0.054459	0.005242	-10.389	< 2e-16 ***

```

UrbanYes    -0.021916   0.271650  -0.081     0.936
USYes       1.200573   0.259042   4.635 4.86e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.472 on 396 degrees of freedom
Multiple R-squared:  0.2393,    Adjusted R-squared:  0.2335
F-statistic: 41.52 on 3 and 396 DF,  p-value: < 2.2e-16

```

c

- According to the model, 13.04 this is the average sales figures in shops reached in rural areas outside the USA, with the price of child seats still being \$0 (not very realistic).
- The coefficient -0.05 indicates that for an increase of one dollar, an average of 0.05 units of child seats are sold less.
- The coefficient -0.021 means that on average 0.021 less units are sold in urban areas compared to rural areas. However, the p value is very high, so this is more of a random variation.
- The 1.2 coefficient means that 1.2 more units are sold within the US compared to shops outside the USA. Perhaps child seats are compulsory in the USA.

d

$$\text{Sales} = \beta_0 + \beta_1 \cdot \text{Price} + \beta_2 \cdot \text{Urban} + \beta_3 \cdot \text{US}$$

Note: General model!

e

For all except Urban.

f

```

model <- lm(Sales ~ Price + Urban, data=cs)
summary(model)

```

```

Call:
lm(formula = Sales ~ Price + Urban, data = cs)

Residuals:
    Min      1Q  Median      3Q     Max 
-6.5324 -1.8441 -0.1443  1.6662  7.5000 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 13.621458   0.655230 20.789 <2e-16 ***
Price       -0.053104   0.005367 -9.895 <2e-16 ***
UrbanYes    0.034095   0.278293  0.123    0.903    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.535 on 397 degrees of freedom
Multiple R-squared:  0.198, Adjusted R-squared:  0.194 
F-statistic: 49.01 on 2 and 397 DF,  p-value: < 2.2e-16

```

g

The model `lm(Sales ~ Price + Urban + US, data=cs)` is a mutible regression while the smaller model `model <- lm(Sales ~ Price + Urban, data=cs)` is a simple liniear regression.

Problems 10

Problem 10.1

	Pop	Lib	NotLib
A	40%	50%	50%
B	25%	60%	40%
C	35%	35%	65%

$$P(B|L) = \frac{P(L|B)P(B)}{P(L)}$$

- $P(L|B) = 60\%$

- $P(B) = 25\%$
- $P(L) = [P(L | A) \cdot P(A)] + [P(L | B) \cdot P(B)] + [P(L | C) \cdot P(C)] = 0.20 + 0.15 + 0.1225 = 0.4725$

$$P(B|L) = \frac{0.6 \cdot 0.25}{0.4725} \approx 0.3175 = 31.75\%$$

Problem 10.2

a

	1	2	3	4
Model A	1/4	1/4	1/4	1/4
Model B	1/10	2/10	3/10	4/10
Model C	12/25	12/50	12/75	12/100

- Model A: Fairest model since all values have the same likelihood.
- Model B: Unfair model. Value 4 has the highest chance of appearing.
- Model C: Unfair model. Value 1 has the highest chance of appearing.

b

First try: After 100 throws, the results seem evenly distributed and reflect the distribution from model A. Second try: We can see that value 1 appears much more frequently than value 4. It resembles model C.

Problem 10.4

Since we don't know whether it's heads or tails, we need to assume a priori a 50% probability for each outcome.

Problem 10.5

	Ice Cream	Fruits	French Fries	Pop
1st graders	0.3	0.6	0.1	0.2
6th graders	0.6	0.3	0.1	0.2
11th graders	0.3	0.1	0.6	0.6
Overall	0.36	0.24	0.4	1