

# Flows - secondary slides

## Proseminar Algorithmen auf Graphen

Nils Wagner

RWTH Aachen University

May 21, 2024

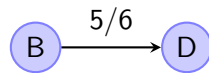


# Ford-Fulkerson Method

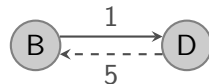
## Ford-Fulkerson Method

- 1: initialize flow  $f$  with 0
- 2: **while** there exists an augmenting path  $p$  in the residual network  $G_f$  **do**
- 3:   augment flow  $f$  along  $p$
- 4: **end while**
- 5: **return**  $f$

edge in flow  
network  $G$



edge in residual  
network  $G_f$



## Ford-Fulkerson Method (with DFS)

```
1: for each edge  $(u, v) \in E$  do
2:    $f(u, v) \leftarrow 0$ 
3: end for
4: while there exists an augmenting path  $p$  in the residual network  $G_f$  do
5:    $c_f(p) \leftarrow \min\{c_f(u, v) \mid (u, v) \in p\}$ 
6:   for each edge  $(u, v) \in p$  do
7:     if  $(u, v) \in E$  then
8:        $f(u, v) \leftarrow f(u, v) + c_f(p)$ 
9:     else
10:       $f(v, u) \leftarrow f(v, u) - c_f(p)$ 
11:    end if
12:  end for
13: end while
14: return  $f$ 
```

$O(E)$

DFS:  $O(E)$

$O(|f|)$  times

$O(E)$

$O(|f|E)$



## Ford-Fulkerson Method (with BFS) / Edmonds-Karp Algorithm

```
1: for each edge  $(u, v) \in E$  do
2:    $f(u, v) \leftarrow 0$ 
3: end for
4: while there exists an augmenting path  $p$  in the residual network  $G_f$  do
5:    $c_f(p) \leftarrow \min\{c_f(u, v) \mid (u, v) \in p\}$ 
6:   for each edge  $(u, v) \in p$  do
7:     if  $(u, v) \in E$  then
8:        $f(u, v) \leftarrow f(u, v) + c_f(p)$ 
9:     else
10:       $f(v, u) \leftarrow f(v, u) - c_f(p)$ 
11:    end if
12:  end for
13: end while
14: return  $f$ 
```

 $O(E)$ BFS:  $O(E)$  $O(VE)$   
times $O(E)$  $O(VE^2)$