Nowcasting Modeli

$$(1) \quad ADS_t = \lambda^{ADS} f_t^{ADS} + \varepsilon_t^{ADS}, \quad \varepsilon_t^{ADS} \sim N(0, \Sigma) \tag{1}$$

Here, $\mathbf{\Sigma}$ is of diagonal structure with the diagonal elements as $\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2$.

$$(2) \quad f_t^{ADS} = \phi f_{t-1}^{ADS} + u_t, \quad u_t \sim N(0, \sigma_u^2) \tag{2}$$

$$\begin{aligned}(3) \quad WEI_t &= 1 \cdot ADS_t + 2 \cdot ADS_{t-1} + 3 \cdot ADS_{t-2} + 4 \cdot ADS_{t-3} \\ &\quad + 3 \cdot ADS_{t-4} + 2 \cdot ADS_{t-5} + 1 \cdot ADS_{t-6} \\ &= \sum_{s=0}^{6} w_s \cdot ADS_{t-s}, \quad w_s = (1,2,3,4,3,2,1) \\ &= \sum_{s=0}^{6} w_s (\lambda^{ADS} f_{t-s}^{ADS} + \varepsilon_{t-s}^{ADS}) \\ &= \lambda^{WEI} f_t^{WEI} + \varepsilon_t^{WEI}, \quad \varepsilon_t^{WEI} \sim N(0, \sigma_q^2) \\ &\text{where } f_t^{WEI} = \sum_{s=0}^{6} w_s f_{t-s}^{ADS}, \quad t = 7k,\ k = 1, \ldots, K\end{aligned}$$

1. Measurement Equation

a) Basit hali

$$\begin{pmatrix} \mathrm{ADS}_t \\ \mathrm{WEI}_t \end{pmatrix} = \begin{pmatrix} \lambda^{\mathrm{ADS}} & 0 \\ 0 & \lambda^{\mathrm{WEI}} \end{pmatrix} \begin{pmatrix} f_t^{\mathrm{ADS}} \\ f_t^{\mathrm{WEI}} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^{\mathrm{ADS}} \\ \varepsilon_t^{\mathrm{WEI}} \end{pmatrix}.$$

b)Geniş hali

$$\begin{pmatrix} \mathrm{ADS}_t \\ \mathrm{WEI}_t \end{pmatrix} = \underbrace{\begin{pmatrix} \lambda^{\mathrm{ADS}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda^{\mathrm{ADS}} & 2\lambda^{\mathrm{ADS}} & 3\lambda^{\mathrm{ADS}} & 4\lambda^{\mathrm{ADS}} & 3\lambda^{\mathrm{ADS}} & 2\lambda^{\mathrm{ADS}} & \lambda^{\mathrm{ADS}} \end{pmatrix}}_{H_f} \begin{pmatrix} f_t^{\mathrm{ADS}} \\ f_{t-1}^{\mathrm{ADS}} \\ f_{t-2}^{\mathrm{ADS}} \\ f_{t-3}^{\mathrm{ADS}} \\ f_{t-4}^{\mathrm{ADS}} \\ f_{t-5}^{\mathrm{ADS}} \\ f_{t-6}^{\mathrm{ADS}} \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{pmatrix}}_{H_\varepsilon} \begin{pmatrix} \varepsilon_t^{\mathrm{ADS}} \\ \varepsilon_{t-1}^{\mathrm{ADS}} \\ \varepsilon_{t-2}^{\mathrm{ADS}} \\ \varepsilon_{t-3}^{\mathrm{ADS}} \\ \varepsilon_{t-4}^{\mathrm{ADS}} \\ \varepsilon_{t-5}^{\mathrm{ADS}} \\ \varepsilon_{t-6}^{\mathrm{ADS}} \end{pmatrix}$$

2. State Equation

$$\begin{pmatrix} f_t^{\mathrm{ADS}} \\ f_{t-1}^{\mathrm{ADS}} \\ f_{t-2}^{\mathrm{ADS}} \\ f_{t-3}^{\mathrm{ADS}} \\ f_{t-4}^{\mathrm{ADS}} \\ f_{t-5}^{\mathrm{ADS}} \\ f_{t-6}^{\mathrm{ADS}} \end{pmatrix} = \begin{pmatrix} \phi & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} f_{t-1}^{\mathrm{ADS}} \\ f_{t-2}^{\mathrm{ADS}} \\ f_{t-3}^{\mathrm{ADS}} \\ f_{t-4}^{\mathrm{ADS}} \\ f_{t-5}^{\mathrm{ADS}} \\ f_{t-6}^{\mathrm{ADS}} \\ f_{t-7}^{\mathrm{ADS}} \end{pmatrix} + \begin{pmatrix} u_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$