Chapter 2

Linear Modulation: Amplitude Modulation

In the previous chapter, we have covered, what is modulation, its different types along with its need. Now here in this chapter we specifically deal with amplitude modulation in detail. After completing this chapter, students will come to know that amplitude-modulated wave is made up of a number of sinusoidal components having a specific relation to one another. They will be able to calculate the frequencies, current and voltage parameters associated with AM signal. We have also covered different methods of AM generation.

2.1 MODULATION

Modulation is the systematic alteration of one waveform called the carrier, according to the characteristics of another waveform, the modulating signal or message signal or intelligence signal or baseband signal. The term baseband (basic or original) is used to designate the band of frequencies representing the original signal as defined by the source of information.

We have two types of modulation i.e., linear and non-linear modulation. Amplitude modulation is of linear type. It is a modulation technique for which superposition theorem stands correct. The modulations for which the above theorem does not hold good are called non-linear modulations. Linear modulation is the direct frequency translation of the message spectrum. DSB modulation (Double sideband) is precisely that. Minor modification of the translated spectrum yields the conventional amplitude modulation, single sideband modulation (SSB) or vestigial sideband modulation. Each of these types have got their own merits and demerits from the applications point of view.

2.1.1 Amplitude Modulation

The process of amplitude modulation consists of varying the peak amplitude of a high frequency carrier wave in proportion with the instantaneous amplitude of

where, f_c = frequency of carrier signal
f_m = frequency of modulating signal
n = 1 for first pair of sidebands.

When the carrier is amplitude modulated, the proportionality constant is made equal to unity, and the instantaneous modulating voltage variations are superimposed onto the carrier amplitude. When there is no modulation the amplitude of the carrier is equal to its unmodulated value. Fig. 2.2 shows the amplitude of AM wave. If V_m is greater than V_c some distortion will occur. Also V_m/V_c ratio is the definition of modulation index, m.

$$m = \frac{V_m}{V_c} \tag{2.4}$$

Its value lies between 0 & 1 and often expressed as percentage and is called percentage modulation.

As AM voltage is addition of v_m & V_c we can write it as

$$A = v_m + V_c \text{ at any instant}$$
$$A = V_m \sin w_m t + V_c.$$
$$A = mV_c \sin w_m t + V_c \qquad \left(\because m = \frac{V_m}{V_c}\right)$$
$$A = V_c \{1 + m \sin w_m t\} \tag{2.5}$$

Now the instantaneous value of resulting AM wave will be

$$v = A \sin \theta.$$
$$v = A \sin w_c t$$
$$v = V_c (1 + m \sin w_m t) \sin w_c t \tag{2.6}$$

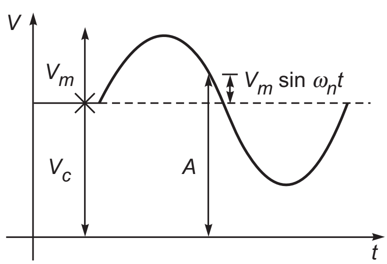

Fig. 2.2 Amplitude of AM wave.

Further Eqn. 2.6 can be splitted using trigonometric identity

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)].$$

we get

$$v = V_c \sin w_c t + \frac{mV_c}{2}\cos(w_c - w_m)t - \frac{mV_c}{2}\cos(w_c + w_m)t \tag{2.7}$$

Here we have three terms

(a) First is the same as unmodulated carrier i.e., $V_c \sin w_c\, t$.
(b) The second is frequency of lower sideband $(f_c - f_m)$.
(c) Third one is the frequency of upper sideband $(f_c + f_m)$.

Here from the above observations we can conclude that the B.W required for AM signal is twice that of the modulating signal or we can say that in AM broadcasting service the B.W required is double the highest modulating frequency.

The biggest merit of AM carrier signal is the simplicity with which the baseband signal can be recovered. The recovery can the done with the help of a simple circuit shown in Figs. 2.3 & 2.4.

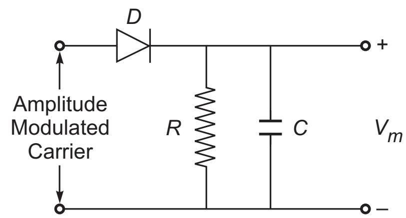

Fig. 2.3 A demodulator for an AM signal.

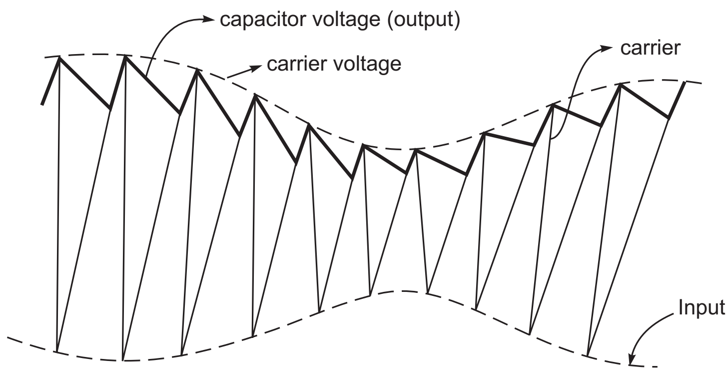

Fig. 2.4 Input and output voltage waveform across capacitor *C*.

Now let us see the working of the circuit. Also assume diode as ideal one i.e. zero resistance.

(a) Let the input be of fixed amplitude and say resistor '*R*' is not present in the circuit.
(b) Then the capacitor charges to peak voltage and does not allow the diode to conduct further.
(c) Now suppose the input amplitude is increased to a new level, then the capacitor again charges to new level because diode again becomes forward biased.

(d) In order to allow the capacitor voltage to follow the carrier's peak when the carrier amplitude is decreasing, it is necessary to include a resistance '*R*' in the circuitary, so that the capacitor may discharge through it.

(e) In case '*d*' we achieve capacitor waveform as shown in Fig 2.4. The time constant *RC* is selected so that the change in v_c between cycles is at least equal to the decrease in carrier amplitude between cycles.

In practice, the time interval between carrier cycles is extremely small in comparison with the time required for the envelope to make a sizeable change. Since, the carrier frequency is ordinarily quiet higher than the highest frequency of modulating signal, the sawtooth distortion of the envelop waveform is very easily removed by a filter.

2.1.3 Modulation Index

Modulation index defines the extent upto which amplitude of the carrier will be varied about an unmodulated maximum carrier.

Mathematically,
$$m = \frac{V_m}{V_c} \tag{2.8}$$

It is also known as depth of modulation, degree of modulation or modulation factor. It is also explained in terms of percentage modulation. Its value lies in between 0 and 1.

We know that baseband signal is preserved in the envelop of AM signal if and only if.

$$|V_m|_{\max} \leq V_c. \tag{2.9}$$

On the other hand we can say that, if the above condition does not prevail then the original baseband signal recovered from the AM modulated signal at the receiver end will be a distored one. Such a case where value of *m* exceeds 1 is called over modulation.

2.1.4 Power Content in AM Wave

It is observed from the expression of AM wave that the amplitude of the carrier component of AM wave is same as that of unmodulated carrier. Along with carrier in AM wave, we have two sidebands. It means that modulated wave bears more power as compared to unmodulated carrier. However, modulation index is responsible for the amplitudes of two sidebands.

$$P_{\text{total}} = \frac{V_{rc}^2}{R} + \frac{V_{r\,LSB}^2}{R} + \frac{V_{rUSB}^2}{R} \qquad \left\{ \because P = \frac{V^2}{R} \right\} \tag{2.10}$$

Here we have taken all the voltages in their r.m.s values, and R is the resistance in which power is dissipated. This first term of above equation is unmodulated carrier power and is given by

$$P_c = \frac{V_{rc}^2}{R} = \frac{(V_c/\sqrt{2})^2}{R}$$

$$P_c = \frac{V_c^2}{2R}. \tag{2.11}$$

Similarly,

$$P_{LSB} = \frac{V_{r\,LSB}^2}{R} = \frac{(V_{LSB}/\sqrt{2})^2}{R}$$

$$P_{LSB} = \frac{\left(\dfrac{mV_c}{2}\Big/\sqrt{2}\right)^2}{R} \qquad \left(\because V_{LSB} = \frac{mV_c}{2}\right)$$

$$P_{LSB} = \frac{m^2 V_c^2}{8R} \tag{2.12}$$

$$P_{USB} = \frac{m^2 V_c^2}{8R} \tag{2.13}$$

Substitute the above obtained values in 2.10.

$$P_{\text{total}} = \frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R}$$

$$P_{\text{total}} = P_c + \frac{m^2}{4}P_c + \frac{m^2}{4}P_c$$

$$P_{\text{total}} = P_c\left\{1 + \frac{m^2}{2}\right\}$$

$$\frac{P_{\text{total}}}{P_c} = 1 + \frac{m^2}{2} \tag{2.14}$$

It is interesting to note from equation (2.14) that the maximum power in AM wave $P_{\text{total}} = 1.5\ P_c$, where $m = 1$. This is an important fact, because it is the maximum power that relevant amplifier must be capable of handling without distortion.

2.1.5 Current Calculation in AM

It is often comfortable to calculate current than power. In such cases modulation index can be calculated from the modulated and unmodulated currents in the transmitter.

Assume I_c be the r.m.s value of the unmodulated current and I_t, the total r.m.s. current after modulation, of an AM transmitter. If 'R' is the resistance through which current flows then with the help of equation (2.14), we have

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \left\{1 + \frac{m^2}{2}\right\}$$

$$I_t = I_c \left\{1+\frac{m^2}{2}\right\}^{\frac{1}{2}}. \tag{2.15}$$

2.1.6 Modulation by Many Sine Waves

Generally, we modulate a carrier by several sine waves simultaneously. Now here we have to find out the resulting power conditions. In the above process we have to find out the total value of modulation index and then substitute the value of m into eqn. 2.14 to calculate total power. Modulation index in such a case can be found out in two different ways.

(a) If say V_1, V_2… etc. be the simultaneous modulating voltages, then the total modulating voltage V_t will be the square root of the sum of the squares of the individual voltage.

$$V_t = \sqrt{V_1^2 + V_2^2 + \ldots} \tag{2.16}$$

Divide both sides by V_c

$$\frac{V_t}{V_c} = \frac{\sqrt{V_1^2 + V_2^2 + \ldots}}{V_c}$$

$$m_t = \sqrt{\frac{V_1^2}{V_c^2} + \frac{V_2^2}{V_c^2} + \ldots}$$

$$m_t = \sqrt{m_1^2 + m_2^2 + \ldots} \tag{2.17}$$

(b) Another way to represent the same is squaring eqn. (2.17) on both sides. We get

$$m_t^2 = m_1^2 + m_2^2 + \ldots \tag{2.18}$$

Here we have to note that this total modulation should not exceed the value ‘1’ or distortion will occur as over modulation by a single sine wave.

NUMERICALS

EXAMPLE 2.1 We have two signals in case of $e_1(t)$ and $e_2(t)$, which are multiplied.

$$e_1(t) = 2\cos(2\pi f_1 t) + \cos(2\pi\cdot 2f_1 t)$$
$$e_2(t) = \cos(2\pi f_2 t) + 2\cos(2\pi\cdot 2f_2 t).$$

Plot amplitude-frequency characteristics of the resultant signal assuming

(a) $f_2 > 2f_1$ but not a harmonic of f_1

(b) $f_2 = 2f_1$.

Solution: $S(t) = e_1(t)\cdot e_2(t)$

$$= \{2\cos(2\pi f_1 t) + \cos(2\pi\cdot 2f_1 t)\}\ \{\cos(2\pi f_2 t) + 2\cos(2\pi\cdot 2f_2 t)\}.$$

$$= 2\cos(2\pi f_1 t)\cdot\cos(2\pi f_2 t) + 4\cos(2\pi f_1 t)\cdot\cos(2\pi\cdot 2f_2 t)$$

$$+ \cos(2\pi\cdot 2f_1t)\ \cos(2\pi f_2t) + 2\cos(2\pi\cdot 2f_1t)\cdot\cos(2\pi\cdot 2f_2t)$$
$$= \cos(2\pi(f_2 + f_1)t) + \cos(2\pi(f_2 - f_1)t) + 2\cos(2\pi\cdot(2f_2 + f_1)t)$$
$$+ 2\cos(2\pi(2f_2 - f_1)t) + \frac{1}{2}\cos(2\pi(f_2 + 2f_1)t) + \frac{1}{2}\cos(2\pi(f_2 - 2f_1)t)$$
$$+ \cos(2\pi(2f_2 + 2f_1)t) + \cos(2\pi(2f_2 - 2f_1)t).$$

Taking fourier transform on both sides.

$$S(f) = \frac{1}{2}[S(f - f_2 - f_1) + S(f + f_2 + f_1)] + \frac{1}{2}[S(f - f_2 + f_1) + S(f + f_2 - f_1)]$$
$$+ [S(f - 2f_2 - f_1) + S(f + 2f_2 + f_1)] + [S(f - 2f_2 + f_1) + S(f + 2f_2 - f_1)]$$
$$+ \frac{1}{4}[S(f - f_2 - 2f_1) + S(f + f_2 + 2f_1) + S(f - f_2 + 2f_1) + S(f + f_2 - 2f_1)]$$
$$+ \frac{1}{2}[S(f - 2f_2 - 2f_1) + S(f + 2f_2 + 2f_1) + S(f - 2f_2 + 2f_1) + S(f - 2f_2 - 2f_1)].$$

Now for case I $f_2 > 2f_1$. This response will occur at various frequencies mentioned above in the form of delta functions

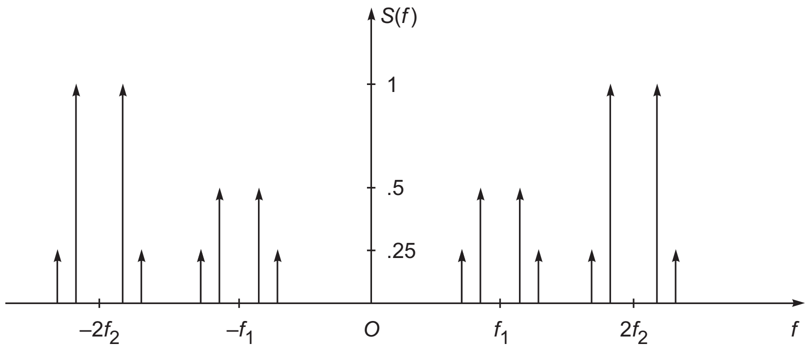

Fig. 2.5 Amplitude-frequency characteristics

- Now for case II

$f_2 = 2f_1$. Put this value in above final result of $S(t)$, we get

$$S(t) = \frac{1}{2} + \cos(2\pi f_1t) + \cos(2\pi\cdot 2f_1t) + 3\cos(2\pi\cdot 3f_1t)$$
$$+ \frac{1}{2}\cos(2\pi\cdot 4f_1t) + 2\cos(2\pi\cdot 5f_1t) + \cos(2\pi\cdot 6f_1t)$$

Taking fourier transform on both the sides, we get

$$S(t) = \frac{1}{4}S + \frac{1}{2}[S(f - f_1) + S(f + f_1)] + \frac{1}{2}[S(f - 2f_1) + S(f + 2f_1)]$$
$$+ \frac{3}{2}[S(f - 3f_1) + S(f + 3f_1)] + \frac{1}{4}[S(f - 4f_1) + S(f + 4f_1)]$$
$$+ [S(f - 5f_1) + S(f + 5f_1)] + \frac{1}{2}[S(f - 6f_1) + S(f + 6f_1)]$$

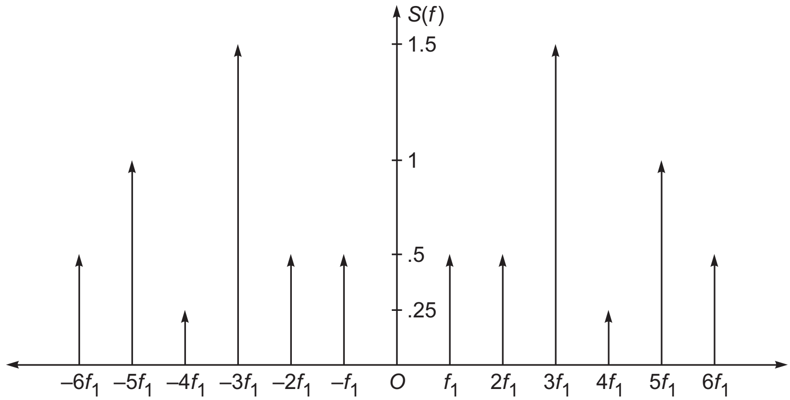

Fig. 2.6 Amplitude-frequency characteristics.

EXAMPLE 2.2 A 900 watt carrier is modulated to the depth of 80 per cent. Calculate the total power in the modulated wave. Assume modulating signal to be sinusoidal.

Solution: We know that

$$P_t = P_c\left\{1+\frac{m^2}{2}\right\}$$

$$P_t = 900\left\{1+\frac{(.80)^2}{2}\right\}$$

$$P_t = 1188 \text{ watts.}$$

EXAMPLE 2.3 A broadcast radio transmitter radiates 5 kW power when the modulation percentage is 60%. How much is the carrier power?

Solution: Again we know that

$$P_c = \frac{P_t}{\left\{1+\dfrac{m^2}{2}\right\}}$$

$$= 4.235 \text{ kW.}$$

EXAMPLE 2.4 The antenna current of an AM transmitter is 10A when only carrier is sent but it increases to 12A when a carrier is modulated by a single tone sinusoid. Find the percentage of modulation. Find antenna current when depth of modulation changes to 0.7.

Solution: From the current calculations of AM spectrum.

We have

$$m = \left\{2\left(\left(\frac{I_t}{I_c}\right)^2 - 1\right)\right\}^{\frac{1}{2}}$$

$$USB \text{ frequency} = 10^7 + 5000 = 10005 \text{ kHz}$$
$$LSB \text{ frequency} = 10^7 - 5000 = 9995 \text{ kHz.}$$
$$USB \text{ Amplitude} = 2.5 \text{ V.}$$
$$LSB \text{ Amplitude} = 2.5 \text{ V.}$$
$$\text{Carrier Amplitude} = 10 \text{ V.}$$

EXAMPLE 2.6 A certain transmitter radiates 9 kW with the carrier unmodulated, and 10.125 kW when carrier is sinusoidally modulated. Calculate the modulation index. If say two more sine waves, corresponding to 40% and 60% modulations are transmitted simultaneously, determine the total radiated power.

Solution: $m = \sqrt{2\left(\frac{P_t}{P_c} - 1\right)}$

$$m = \sqrt{2(.125)}$$
$$m = \sqrt{.250} = .50$$

For second part, the total MI will be

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2}$$
$$m_t = \sqrt{.25 + .16 + .36}$$
$$m_t = \sqrt{.77} = .8774964$$

Now we know that

$$P_t = P_c\left(1 + \frac{m_t^2}{2}\right)$$
$$P_t = 9\left(1 + \frac{(.8775)^2}{2}\right)$$
$$P_t = 12.465 \text{ kW.}$$

EXAMPLE 2.7 Represent the AM vectorially.

Solution: Since, AM is a vector quantity, it may be represented by rotating phasor diagrams as shown by diagram in Fig. 2.8. Here the carrier is regarded as stationary phasor, so that the upper sideband will have a relative angular velocity w_m. Similarly, the lower sideband will have angular velocity $-w_m$. The magnitude of the two sidebands is equal but they rotate in opposite direction. When the two vectors are in position 'C' and 'D', they neutralise each other and the resultant voltage is V_c. When both the phasors are in position 'A', we have minimum magnitude of $(V_c - V_m)$.

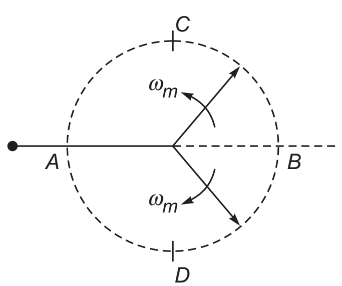

Fig. 2.8

Similarly, when both the phasors are in position '*B*', we have the maximum magnitude of $(V_c + V_m)$.

2.2 GENERATION OF AMPLITUDE MODULATED WAVE

There is a variety of modulator circuits employing tubes or solid-state devices to produce amplitude modulated waves. All these circuits may, however, be grouped as:

(a) Square law or non-linear modulator circuits
(b) Linear modulator circuits.

Another way of grouping is also there. This is based upon the power level at which modulation is carried out and may be termed:

(a) Low level modulation (LLM)
(b) High level modulation (HLM)

The power level at which modulation is carried out is low in LLM and high in HLM. Also it is worthwhile to note that, in general, square law modulators are low level modulators while linear modulators are high level modulators.

Now we will discuss the above AM generation techniques in a bit detail.

2.2.1 Low Level Modulation

Fig. 2.9 shows the block diagram for LLM. Here we are doing modulation at low level. Because of this the output power is also very low. Therefore, the power amplifiers are required to boost the amplitude modulated signals upto the desired output level. After this the AM signal (with carrier and two sidebands) is applied to a wideband power amplifier. A wideband power amplifier is used to preserve the sidebands of modulated signal. AM systems doing modulation at low power levels are called low level amplitude modulation transmitters. Square-Law diode and switching modulations are examples of low level modulation.

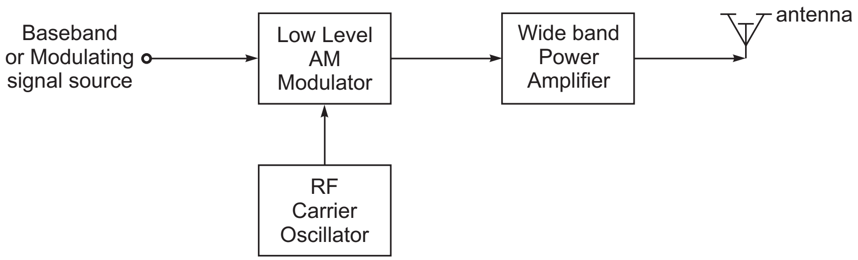

Fig. 2.9 Low level AM modulation.

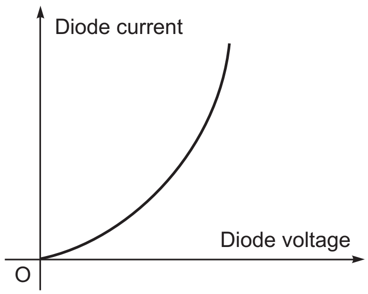

Fig. 2.11 V.I characteristics.

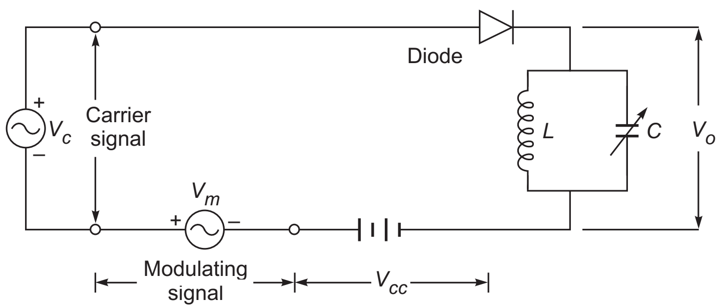

Fig. 2.12 Square law diode modulator.

We can treat the theoretical concept of AM in a much better way using mathematical expressions as explained below.

Let us consider the modulating and carrier signal be v_m and v_c.

$$v_c = V_c \cos w_c t \tag{2.19}$$
$$v_m = V_m \cos w_m t \tag{2.20}$$

Then total a.c voltage across the diode will be

$$v_t = v_c + v_m \text{ (at any instant)}$$
$$v_t = V_c \cos w_c t + V_m \cos w_m t \tag{2.21}$$

Now non-linear relationship between voltage and current across a diode may be expressed as

$$i = a + bv_t + c\,v_t^2 \tag{2.22}$$

where, a, b and c are constants, i = current, v_t = voltage (a.c).

Putting the value of v_t from (2.21) to (2.22), we get

$$i = a + b(V_c \cos w_c t + V_m \cos w_m t) + c(V_c \cos w_c t + V_m \cos w_m t)^2.$$
$$i = a + bV_c \cos w_c t + bV_m \cos w_m t + cV_c^2 \cos^2 w_c t + cV_m^2 \cos^2 w_m t$$
$$+\ 2cV_c V_m \cos w_c t \cos w_m t.$$
$$i = a + bV_c \cos w_c t + bV_m \cos w_m t + \frac{1}{2} cV_c^2 (2\cos^2 w_c t)$$
$$+ \frac{1}{2} cV_m^2 (2\cos^2 w_m t) + cV_c V_m (2\cos w_c t \cos w_m t).$$

$$i = a + bV_c \cos w_c t + bV_m \cos w_m t + \frac{1}{2}cV_c^2 (1 + \cos 2\, w_c t)$$
$$+ \frac{1}{2}cV_m^2 (1 + \cos 2\, w_m t) + cV_c V_m [(\cos(w_c + w_m)\, t + \cos (w_c - w_m) t]$$
$$\left[\begin{array}{l} \because 2\cos^2 Q - 1 = \cos 2\theta \\ \text{and } \cos(A+B) + \cos(A-B) = 2\cos A \cos B \end{array}\right] \begin{array}{l}\text{identities used in} \\ \text{above expression}\end{array}.$$
$$i = a + bV_c \cos w_c t + bV_m \cos w_m t + \frac{1}{2}cV_c^2 + \frac{1}{2}cV_c^2 \cos 2\, w_c t$$
$$+ \frac{1}{2}cV_m^2 + \frac{1}{2}cV_m^2 \cos 2\, w_m t + cV_c V_m \cos(w_c + w_m) t$$
$$+ cV_c V_m \cos(w_c - w_m) t.$$
$$i = \left[a + \frac{1}{2}cV_c^2 + \frac{1}{2}cV_m^2\right] \text{ is the d.c. term}$$
$$+ [bv_c \cos w_c t] \text{ is the carrier signal}$$
$$+ [bv_m \cos w_m t] \text{ is the modulating signal.}$$
$$+ \left[\frac{1}{2}cV_c^2 \cos 2\, w_c t + \frac{1}{2}cV_m^2 \cos 2\, w_m t\right]$$
is group of harmonics in modulating and carrier signals.
$$+ [cV_c V_m \cos(w_c + w_m)] \text{ is the upper sideband (USB)}$$
$$+ [cV_c V_m \cos(w_c - w_m)] \text{ is the lower sideband (LSB).} \qquad (2.23)$$

Now this signal is fed to tuned circuit which is tuned near the carrier frequency and only the frequency components which are actually developed in the output are w_c, $(w_c + w_m)$, $(w_c - w_m)$. The rest of the frequency components are rejected by the tuned circuit.

Therefore, the required AM output wave (current) will be as given below:

$$i_0 = bV_c \cos w_c t + cV_c V_m \cos(w_c + w_m) t + cV_c V_m \cos(w_c - w_m) t$$
$$i_0 = bV_c \cos w_c t + cV_c V_m [\cos(w_c + w_m) t + \cos(w_c - w_m) t]$$
$$i_0 = bV_c \cos w_c t + 2cV_c V_m \cos w_c t \cos w_m t.$$
$$i_0 = bV_c \left[1 + \frac{2cV_m}{b} \cos w_m t\right] \cos w_c t.$$
$$i_0 = bV_c [1 + m \,.\, \cos w_m t] \cos w_c t \qquad (2.24)$$

where, m = modulation index = $\dfrac{2cV_m}{b}$

Equation 2.24 is the required expression for the current output in an AM.

2.2.4 Switching Modulator

Efficient high-level modulators are arranged so that undesired modulation products never fully develop and need not be filtered out. This can be accomplished with the help of switching device. In Fig. 2.13, we have shown a modulator of this kind. Here we are considering the piece-wise linear model of a diode. The carrier wave v_c applied to diode has been assumed to be of large amplitude so as to swing right across the characteristic curve of the diode. The diode is again

(a) T_1 as radio frequency class C amplifier.
(b) T_2 as class B amplifier.

Working: At the base of T_1 the carrier signal is applied. V_{cc} makes the collector supply used for biasing purpose. T_2 amplifies the modulating signal. The baseband or modulating signal appears across the modulation transformer after amplified from T_2. Now this amplified modulating signal appears to be in series with the collector supply V_{cc}. The function of the capacitor is to provide low impedance path for high frequency carrier signal and hence we can avoid it to flow through modulating transformer.

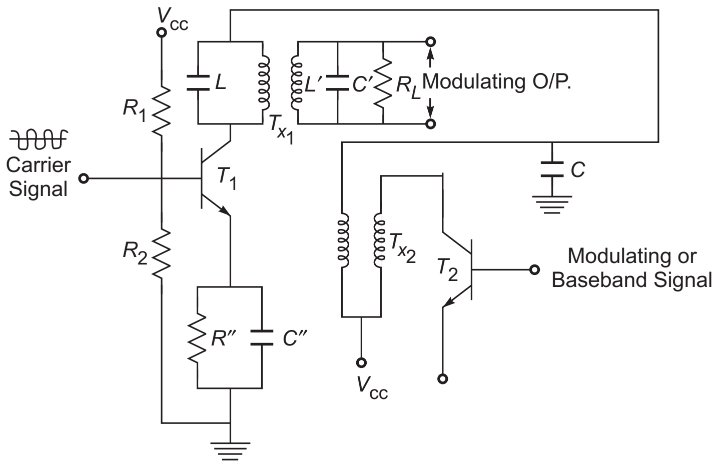

Fig. 2.14 Collector Modulation Method for AM generation.

Theory: It is a well-known fact that in a class C amplifier, the magnitude of *o*/*p* voltage is approximately equal to the supply voltage V_{cc}. In addition to this, a linear relationship exist between the o/p tank current I_t and variable supply voltage V_c (assume V_{cc} as varying and its value is denoted by V_c). This means that output is replica of input voltage and its magnitude is approximately equal to the carrier supply voltage V_{cc}. If R_L is resistance of output tank circuit, then

$$V_{cc} = R_L \cdot I_t$$

Therefore, the unmodulated carrier is amplified by a class C modulated amplifier using T_1 and its amplitude will remain constant at V_{cc}, since there appears no voltage across the modulating transformer in the absence of baseband or modulating signal.

Now when baseband modulating signal appears across T_{x_2}, this signal will be added to carrier supply voltage V_{cc}. This will show very slow variations in carrier supply voltage V_{cc}. This will change the magnitude of the carrier signal voltage at the output of the modulated class-C amplifier as shown in Fig. 2.15.

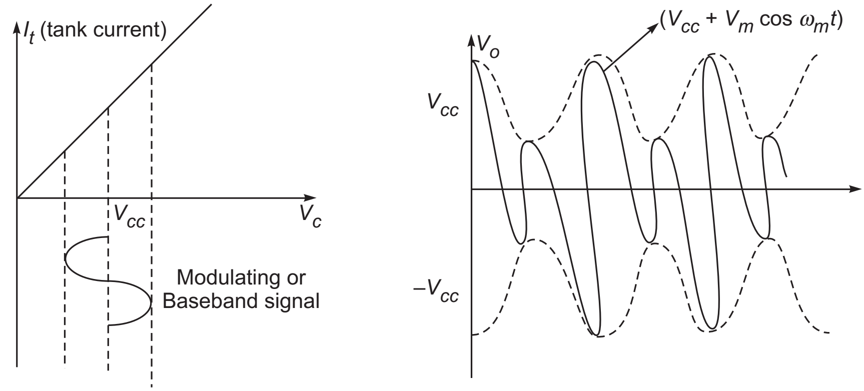

Fig. 2.15 Generation of AM.

Mathematical treatment:

We know that

$$V_c = V_{cc} + v_m$$

where, v_c = slowly changing carrier supply voltage

$$v_m = V_m \cos w_m t = \text{modulating signal}$$

now $$V_c = V_{cc} + V_m \cos w_m t \qquad (2.28)$$

but $$m = \frac{V_m}{V_{cc}}$$

$\therefore$ Eqn. 2.28 can be rewritten as

$$V_c = V_{cc} + mV_{cc} \cos w_m t \qquad (2.29)$$

Also modulated output voltage is

$$v_0 = V_c \cos w_c t$$
$$v_0 = V_{cc}(1 + m_a \cos w_m t) \cos w_c t \qquad (2.30)$$

which is the sufficient equation for A.M.

EXAMPLE 2.8 The output voltage of a transmitter is given by 500 (1 + 0.5 sin 2000 t) sin $6.28 \times 10^7 t$. This voltage is fed to a load of 1000 Ω resistance. Determine

(a) Carrier frequency
(b) Modulating frequency
(c) Mean power output
(d) Carrier power.

Solution: $v = 500\ (1 + 0.5 \sin 2000\ t) \sin 6.28 \times 10^7\ t$

(a) $w_c = 6.28 \times 10^7$

$$f_c = \frac{6.28 \times 10^7}{2\pi}$$

Solution: Under 100% modulation, *DC* power input to power amplifier, P_{dc} = 1500 W

Plate dissipation = 500 W

Out of this power, *DC* plate dissipation = $\frac{500 \times 2}{3}$ = 333.33 W and plate dissipation from modulating signal = $\frac{500 \times 1}{3}$ = 166.667 W

(a) Carrier Power (P_c) = 1500 – 333.33 = 1166.67 W.

Sideband Power (P_{sb}) = $\frac{P_c}{2} = \frac{1166.67}{2}$ = 583.33 W.

(b) $\therefore$ Modulator output power

$$P_0 = 583.33 + 166.67 = 750.0035 \text{ W}$$

Since efficiency is .6, d.c. power input to modulator is

$$P_{dc} = \frac{750.0035}{.6} = 1250.0058 \text{ W}$$

Now plate dissipation in modulator is given by

$$= P_{dc} - P_0 = 1250.0058 - 750.0035 = 500.0023.$$

EXAMPLE 2.11 A transmitter is adjusted to deliver 50 kW of carrier power to an antenna whose base impedance is 36 + *j* 40 ohms.

(a) What will be the base current in antenna when the transmitter is held at a sustained tone modulation of 40%?

(b) What will be the peak value of voltage appearing across the base insulator at the crest of modulation. Consider modulation of 100% for this part.

Solution: Since

$$P_c = 50 \text{ kW}$$

$$m = 0.4$$

$$\therefore \quad P_t = P_c \left\{1 + \frac{m^2}{2}\right\}$$

$$P_t = 50000 \left\{1 + \frac{(.4)^2}{2}\right\}$$

$$P_t = 54 \text{ kW.}$$

(a) This is the power dissipated across antenna resistance, therefore current flowing through antenna will be

$$I_{rms} = \sqrt{\frac{P_t}{R}} = \sqrt{\frac{54000}{36}} = 38.72 \text{ amperes.}$$

(b) R.m.s current for carrier

$$I_c = \sqrt{\frac{P_c}{R}} = \sqrt{\frac{50000}{3}} = 37.27 \text{ amperes.}$$

Peak carrier current = I_{cm} = 37.27 × $\sqrt{2}$ = 51.5 amperes. With 100% modulation, the current I_{cm} will be doubled.

∴ I_0 = 51.5 × 2 = 103 amperes.

Peak voltage across the base insulator during crest of modulation

$$V_{bp} = I_0 \cdot Z_{\text{out}} = 103 \cdot (36 + j\,40)$$
$$= 5541.4 \angle 48° \text{ volts.}$$

2.3 DEMODULATION OF AM WAVES

We are sending a weak signal along with the high frequency carrier signal over a channel and the process is called modulation. At the receiver end we are again required to retrieve the baseband signal in its original form from the modulated signal and the process is now known as demodulation. The systems we use for such processes are called demodulators or detectors. There are various types of demodulators which are explained here.

2.3.1 Square Law Demodulator

This kind of demodulator is used to detect the signals which are of small amplitude, i.e., nearly 1V. This is because, for demodulation we have to use non-linear portion of diode characteristics. Such demodulation is illustrated in Fig. 2.16. It may be observed that circuit is almost similar to square law modulator. The only difference lies in filter circuit used, which is a band pass in case of modulator and low pass filter in case of demodulator.

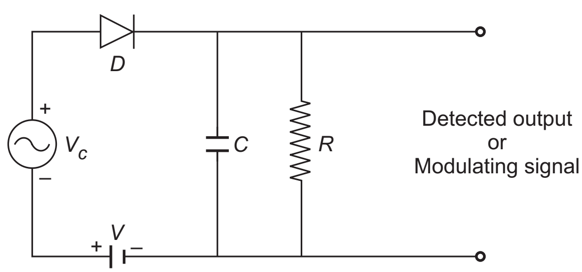

Fig. 2.16 Square law diode demodulator.

Here V is used as fixed power supply to obtain the fixed operating point in the non-linear region of diode characteristics. Here we are assuming that the device has square-law relationship between input and output signal with the help of mathematical identity

$$y = kx^2 \quad \text{where,} \quad y = \text{output},\ x = \text{input},\ K = \text{constant.}$$

Since, here we are only considering non-linear portion, the lower half portion of the modulated waveform is compressed. This will produce envelope applied and not so expensive, also at the same time it gives satisfactory reception for the broadcast programmes. In 2.18 we have shown the circuit diagram of envelope detector. In the input portion the tuned circuit is providing perfect tuning at desired carrier frequency. *RC* network is used to control the time constant. Whenever the signal is 1 volt or more we can use the linear portion of diode characteristics.

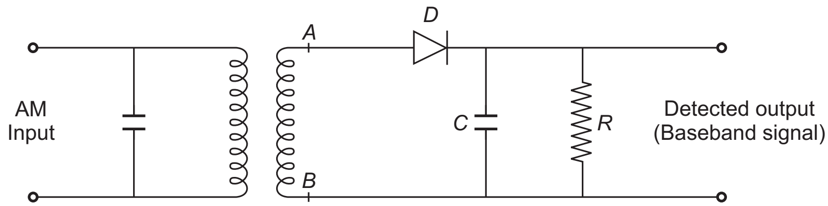

Fig. 2.18 Linear diode detector/demodulator.

In the circuit if we assume that '*C*' is absent, then it is a circuit of half wave rectifier, then the output will be half wave rectified modulated signal as shown in Fig 2.19. Now let us assume that '*C*' is being introduced.

(a) Now when there will be positive half cycle then *A* becomes positive w.r.t *B* and *D* conducts through *C*. Due to which *C* charges to the maximum amplitude of V_c.

(b) During negative half of cycle, diode becomes reverse biased and will not conduct. Therefore, now the capacitor will discharge through *R* with a time constant $C = RC$. This time period is such selected that capacitor voltage will not fall down completely in negative half cycle, and successively positive half starts to charge the capacitor again to peak value of V_c.

Hence, the output voltage across capacitor is spiky modulating signal. However these spikes are introduced due to the regular charging and discharging of capacitor.

We can remove these spikes by keeping $\tau = RC$ large so that that capacitor discharges negligibly small. However, large value of τ will produce another problem of clipping. Therefore, τ cannot be increased beyond a certain limit.

Time constant (*RC*) is an important parameter for envelope detector. We cannot keep it too high or too low.

(a) Let the time constant be quite low, then the discharge curve during off period of diode is almost vertical which results in large fluctuations in a output voltage.

(b) Let the time constant be quiet large, the discharge curve is then almost horizontal and then it misses several peaks of the rectified output during negative peak of AM wave as shown in graph of Fig. 2.20.

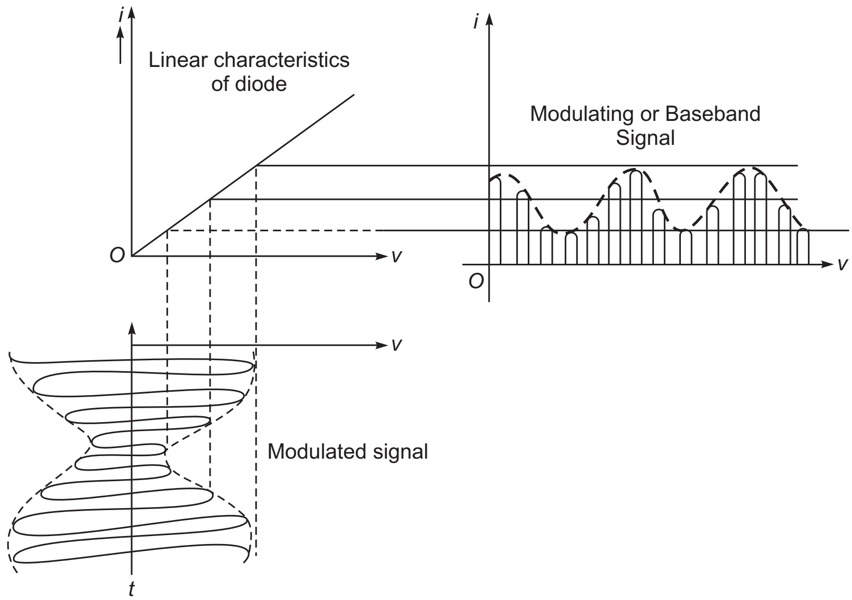

Fig. 2.19 Operation showing demodulation of baseband signal from AM.

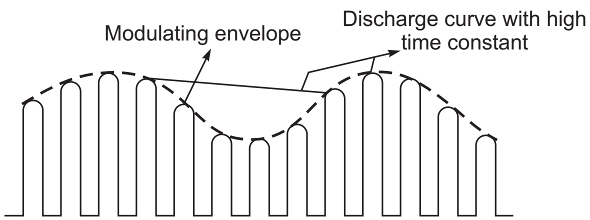

Fig. 2.20 When time constant is quite large.

Hence, we can conclude with the statement that increase the time constant upto the extent where clipping does not start during the negative half of the modulated wave. This maximum value of the time constant can also be detected mathematically as we know that

$$v = V_c(1 + m \cos w_m t) \tag{2.34}$$

where w_m = maximum modulating frequency permissible.

The slope of the envelope is given by

$$\frac{dv}{dt} = -V_c\, m\, w_m \sin w_m t \tag{2.35}$$

At time $t = t_0$ the value of envelope is

$$v_0 = V_c\{1 + m \cos w_m t_0\} \tag{2.36}$$

Thus, $$\left(\frac{dv}{dt}\right)_{t=t_0} = -w_m\, m\, V_c \sin w_m t_0 \tag{2.37}$$

(c) Equation (2.48) will give the maximum permissible value of time constant *RC* for no clipping.

We come across two distortions with the above ckt implementation.

(a) Proper setting of *RC* time constant. It should not be too small or too large as explained earlier.

(b) Second is the curvature of diode characteristics. Due to this curvature, the efficiency of rectification varies according to the amplitude of the envelope. This can be reduced by either selecting load resistance large as compared to anode resistance or by applying the carrier voltage of large amplitude.

2.4 SUPPRESSED CARRIER MODULATION

When a signal is amplitude modulated the outcome is three-frequency components i.e., original carrier frequency f_c, the upper sideband frequency ($f_c + f_m$) and the lower sideband frequency ($f_c - f_m$).

Here we know that the carrier does not convey any information. It is obvious that carrier component remains constant in amplitude, irrespective of what modulating signal does. We also know that

$$P_t : P_c = \left(1 + \frac{m^2}{2}\right) : 1 \text{ where } m \text{ is modulation index.}$$

Rather we can say that if there is 100% modulation then 67% of the total power is required for transmitting the carrier which does not contain any information. Thus, if we will be able to suppress the carrier and allow only two sidebands to be transmitted, then 67% ($2/3^{rd}$) of the power can be saved. In such a way our message signal will not be affected at all. Such a modulation is called DOUBLE SIDEBAND-SUPPRESSED CARRIER modulation and is the right alternative for the power wastage during transmission. In short we also call it DSB-SC.

Again we are aware of the fact that both the sidebands are images of each other, and both of them are affected by the changes in the modulating signal by a factor of $\frac{mV_c}{2}$. Here we can further conclude that whole information can be conveyed by single sideband or any one of the two. Here suppressing one of the two carriers will help us to further reduce the required power by an amount $\frac{P_c m^2}{4}$ from the total sideband power $\frac{P_c m^2}{2}$. This new scheme can now be nominated as single sideband-suppressed carrier (SSB-SC) modulation. Further this technique will reduce the bandwith requirement by half what we require in case of conventional AM or DSB-SC.

$$S_1(t) = A_c(1 + K_a\, e_m(t)) \cos(2\pi f_c t) \tag{2.50}$$
$$S_2(t) = A_c(1 - K_a\, e_m(t)) \cos(2\pi f_c t) \tag{2.51}$$

Subtracting $S_2(t)$ from $S_1(t)$, we get

$$S(t) = S_1(t) - S_2(t) \tag{2.52}$$
$$S(t) = 2A_c\, K_a\, e_m(t)\ \cos(2\pi f_c t) \tag{2.53}$$

Balanced modulator circuit can be realised with the help of transistorised push-pull balanced modulator or with the help of a block diagram as shown in Fig 2.22.

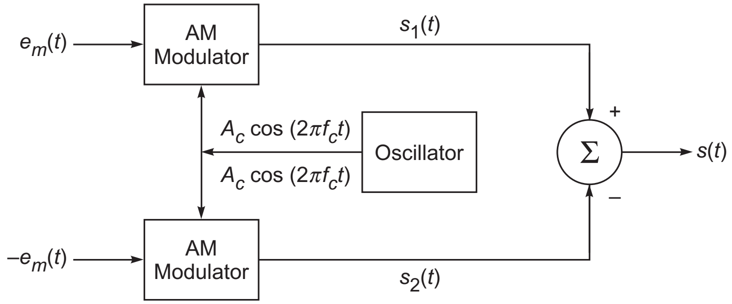

Fig. 2.22 Block diagram of balanced modulator.

Thus, except for a scaling factor $2K_a$, the balanced modulator output is equal to the product of the modulating wave and the carrier, which is nothing but the DSB-SC signal.

2.4.2.2 Ring Modulator

Another way of generating DSB-SC signal is with the help of a ring modulator. Here we are connecting four diodes in a manner of ring such that every diode point in the same way. The diodes are controlled by square wave carrier $e_c(t)$ of frequency f_c applied through a centre tapped transformer. For ideal diodes and perfectly balanced transformers, we have two cases.

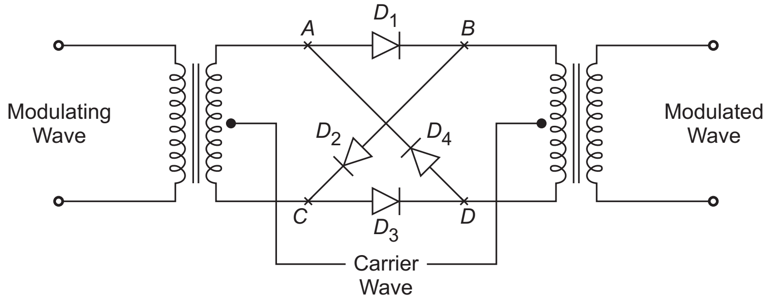

Fig. 2.23 Circuit diagram for ring modulator.

Case I: When the carrier signal is +*ve*, then diodes D_1 and D_3 conduct and D_2 and D_4 are switched off and thus offering very high impedance. In this case modulator multiplies the modulating signal by +1. Hence, fig. 2.23(a) shows the conducting paths.

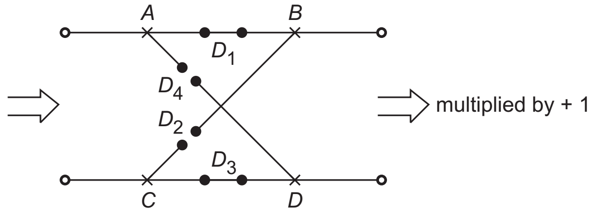

Fig. 2.23(a)

Case II: When the carrier signal is –*ve* then diodes D_2 and D_4 conduct but D_1 and D_3 are reverse biased and offer very high impedence. Here modulator multiplies the modulating signal by –1 as shown in fig. 2.23(b).

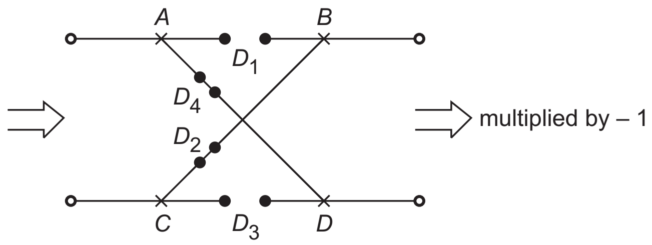

Fig. 2.23(b)

After considering both the cases we can ultimately realise a product modulator for square wave carrier and modulating signal.

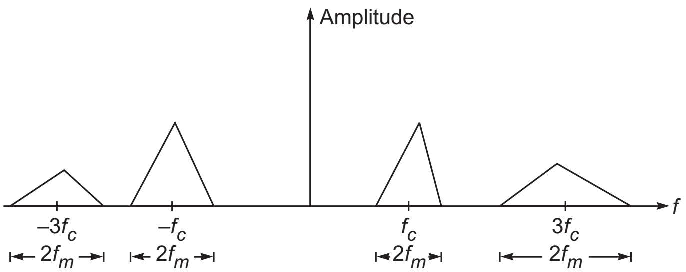

Fig. 2.24 Spectrum of DSB-SC output of ring modulator.

In order to recover $e_m(t)$ from the DSB-SC wave, we have to retranslate the spectrum back to original position ($f = 0$). The baseband signal can be easily recovered from a DSB-SC wave $S(t)$ by first multiplying $S(t)$ with a locally generated carrier and then low pass filtering the product as shown in the block diagram above. We may simply note down that the output of the product modulator.

$$S_c(t) = A_c\, e_m(t)\, \cos(2\pi f_c t)\, \cos(2\pi f_c t)$$
$$S_c(t) = A_c\, e_m(t) \cos^2 (2\pi f_c t).$$
$$S_c(t) = A_c \left\{ e_m(t)\, \frac{1}{2}(1 + \cos(2\pi 2 f_c t)) \right\}$$
$$S_c(t) = A_c \left\{ \frac{1}{2} e_m(t) + \frac{1}{2} e_m(t) \cos(2\pi 2 f_c t) \right\} \tag{2.54}$$

Thus, the baseband signal reappears after filtering out the high pass signal corresponding to the second term. The second term represents a DSB-SC wave with a carrier frequency $2f_c$.

Here in this case, frequency and phase of the locally generated carrier signal and the carrier signal at the transmitter must be same, otherwise the regenerated original baseband signal would be distorted one.

2.4.3.2 Envelope Detection Method

The other method with which we can demodulate baseband signal from DSB-SC modulated signal is accomplished by inserting a carrier generated at the receiver end with the help of a local oscillator.

We know that if we insert a sufficient carrier of same frequency and phase to DSB-SC signal, it converts DSB-SC signal into a conventional AM wave. Now this AM wave is demodulated by an envelope detector. However, two types of error are again noticed in the same manner as in case of synchronous detection.

The two errors are:

(i) Phase error: When local oscillator has identical frequency but arbitrary phase difference ϕ measured w.r.t to the carrier $e_c(t)$.

(ii) Frequency error: The local oscillator has identical phase but difference in frequency with respect to carrier frequency.

Let us analyse the process mathematically. We have DSB-SC signal as

$$S(t) = c(t)\cdot x(t) \tag{2.55}$$
$$S(t) = \mathrm{A}\, \cos(2\pi f_c t)\, x(t) \tag{2.56}$$
$$\text{Let} \quad A = 1 \text{ in above eqn.}$$
$$\therefore\ S(t) = \cos\, (2\pi f_c t)\, x(t) \tag{2.57}$$

We have inserted a carrier at the receiver with a phase difference of ϕ.

$$c'(t) = A\cos(2\pi f_c t + \phi) \tag{2.58}$$

Then the resulting signal will be

$$r(t) = S(t) + \text{re-inserted carrier at the receiver}$$
$$r(t) = S(t) + c'(t) \tag{2.59}$$

Substituting eqn. (2.55) and (2.58) in (2.59), we get

$$r(t) = \cos(2\pi f_c t)\, x(t) + A\cos(2\pi f_c t + \phi) \tag{2.60}$$
$$r(t) = \cos(2\pi f_c t)\, x(t) + A\cos(2\pi f_c t)\cos\phi - A\sin(2\pi f_c t)\sin\phi$$
$$r(t) = \cos(2\pi f_c t)\,\{x(t) + A\cos\phi)\} - A\sin\phi\sin(2\pi f_c t).$$
$$r(t) = e(t)\cos[(2\pi f_c t) + \theta(t)] \tag{2.61}$$

where, $e(t) = \sqrt{(A + x(t))^2 - 2A\, x(t)(1 - \cos\phi)}$

$$\theta(t) = \tan^{-1}\left[\frac{A\sin\phi}{x(t) + A\cos\phi}\right]$$

From eqn. (2.61) it is clear that $e(t)$ is the envelope of $r(t)$. Also if we consider $\phi = 0$. We are left with

$$e(t) = A + x(t) \tag{2.62}$$

From (2.62) we can make out that modulating signal $x(t)$ can be recovered from $r(t)$ using an envelope detector, since the $r(t)$ is basically a conventional AM wave given by

$$r(t) = [A + x(t)]\cos 2\pi f_c t \tag{2.63}$$

This is possible only when $[A + x(t)] > 0$.

Hence, modulating signal can be recovered from $r(t)$ using envelope detector.

Above case is possible by keeping $m < 1$. If $\phi \neq 0$ then phase error exists between the two carriers is given by:

$$e(t) = A\left\{1 + \frac{2x(t)}{A}\cos\phi + \left(\frac{x(t)}{A}\right)^2\right\}^{1/2} \tag{2.64}$$

If $A >> |x(t)|$, then we have

$$e(t) = A + x(t)\cos\phi \tag{2.65}$$

The desired signal in the above expression will now be $x(t)\cos\phi$. If $\phi = 0$ and there is a difference in frequency Δf between the two oscillators, then the envelope of resulting $r(t)$ will be given by

$$e(t) = A + x(t)\cos(2\pi\Delta f t) \quad \text{for} \quad A >> |x(t)| \tag{2.66}$$

2.5 QUADRATURE AMPLITUDE MODULATION (QAM)

QAM is also termed as quadrature carrier multiplexing. However, this scheme allows two DSB-SC signals to occupy the same bandwidth and they will be separated at the receiver end. It is, therefore, also known as bandwidth conversion scheme.

$$\frac{1}{2}Ax_1(t) \text{ and } \frac{1}{2}Ax_2(t)$$

Such kind of assembly finds wider applications in colour television.

2.6 SINGLE SIDEBAND SUPPRESSED CARRIER (SSB-SC) MODULATION

As it has been pointed out that single sideband is sufficient to convey information to the distant receiver. This reduces the power and bandwidth requirements considerably. Moreover the suppressed sidebands may be used as a second independent communication channel. Therefore, carrier and one of the two sidebands will be suppressed at the transmitter itself. Such a technique is known as single sideband suppressed carrier (SSB-SC) system. Figure 2.28 shows the concept of SSB-SC wtih the help of different frequency spectrums.

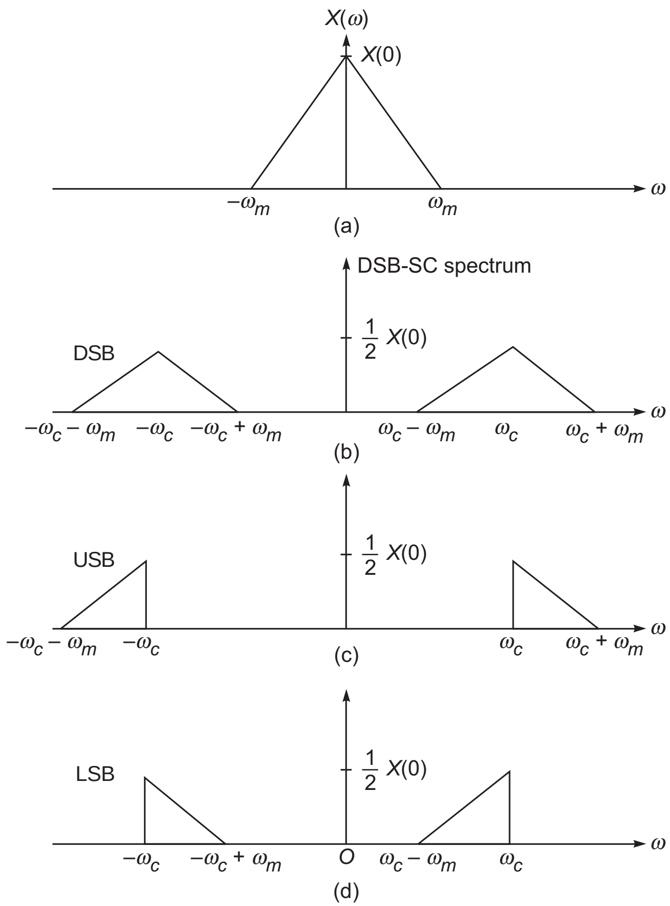

Fig. 2.28 (a) Spectrum of baseband signal (b) Spectrum of DSB-SC wave (c) Spectrum of SSB-SC wave with USB transmitted (d) Spectrum of SSB-SC wave with LSB transmitted.

2.6.1 Time Domain Description of SSB-SC Wave

To describe the SSB-SC signal in time domain we shall use the concept of Hilbert transform. The SSB-SC signal being a bandpass signal can be represented in the time domain in the canonical form as

$$S(t) = S_c(t)\cos(2\pi f_c t) - S_s(t)\sin 2\pi f_c(t) \tag{2.68}$$

where, $S_c(t)$ = in phase component of SSB-SC wave.

$S_s(t)$ = is its quadrature component.

The in-phase component can be derived from $S(t)$ by first multiplying it by $\cos(2\pi f_c t)$ and then passing it through a low pass filter. Similarly quadrature component can also be derived from $S(t)$ by first multiplying it with $\sin(2\pi f_c t)$ and passing the product through ideal low pass filter. We thus find that the fourier transform of $S_c(t)$ and $S_s(t)$ are related to that of the SSB-SC wave as follows.

$$\begin{aligned} S_c(f) &= S(f - f_c) + S(f + f_c) \quad \begin{cases} -W \le f \le W \\ \end{cases} \\ &= 0 \qquad\qquad\qquad\qquad\quad \text{elsewhere} \end{aligned} \tag{2.69}$$

and

$$\begin{aligned} S_s(f) &= j\{S(f - f_c) - S(f + f_c)\} \quad \begin{cases} -W \le f \le W \\ \end{cases} \\ &= 0 \qquad\qquad\qquad\qquad\qquad\quad \text{elsewhere} \end{aligned} \tag{2.70}$$

Where $-W \le f \le W$ defines the frequency band occupied by the message signal. Consider the case of SSB-SC wave that is obtained by transmitting the upper sideband. The spectrum of such a wave is shown in Fig 2.28 which is reproduced in Fig. 2.29 (a), (b) and (c) representing two frequency shifted spectra pertaining to $S(f - f_c)$ and $S(f + f_c)$, respectively. Spectra of in-phase component $S_c(t)$ and quadrature component $S_s(t)$ are shown in Fig. 2.29(d) and (e). On the basis of Fig 2.29(d), we can write

$$S_c(f) = \frac{1}{2} A_c M(f) \tag{2.71}$$

Where, $M(f)$ is the fourier transform of the message signal $e_m(t)$. Accordingly in-phase-component $S_c(t)$ is defined by

$$S_c(t) = \frac{1}{2} A_c e_m(t) \tag{2.72}$$

On the basis of Fig. 2.29 (e), we can write the equation as

$$S_s(f) = \frac{-j}{2} A_c M(f), \qquad f > 0 \tag{2.73}$$

$$S_s(f) = \frac{j}{2} A_c M(f), \qquad f < 0 \tag{2.74}$$

That is,

$$S_s(f) = -j\,\frac{A_c}{2}\; Sgn(f)\, M(f) \tag{2.75}$$

Where

$$\begin{aligned} Sgn(f) &= 1 \quad \text{for} \quad f > 0 \\ &= -1 \quad \text{for} \quad f < 0. \end{aligned} \tag{2.76}$$

where, the term $-j\, sgn(f)\, M(f)$ is the fourier transform of the Hilbert transform of $e_m(t)$.

Thus, we have

$$S_s(f) = \frac{A_c}{2}\,\hat{M}(f) \tag{2.77}$$

where, $\hat{M}(f)$ is the Fourier transform of the Hiltbert transform of $e_m(t)$. Taking inverse fourier transform on both sides of eqn. 2.77.

$$S_s(t) = \frac{A_c}{2}\;\hat{e}_m(t) \tag{2.78}$$

Where, $\hat{e}_m(t)$ is the Hilbert transform of $e_m(t)$. Substituting eqn. (2.72) and (2.78) in (2.68), we get the SSB-SC wave for upper sideband transmitted as

$$S(t) = \frac{A_c}{2}\,[e_m(t)\cos(2\pi f_c t) - \hat{e}_m(t)\;\sin(2\pi f_c t)] \tag{2.79}$$

In the same way, it is possible to show that the SSB-SC wave for lower-sideband transmitted takes a form.

$$S(t) = \frac{A_c}{2}\,[e_m(t)\cos(2\pi f_c t) + \hat{e}_m(t)\;\sin(2\pi f_c t)] \tag{2.80}$$

2.6.2 Hilbert Transform

It may be observed that the function $x_h(t)$ obtained by providing $(-\pi/2)$ phase shift to every frequency component present in $x(t)$, actually represents the Hilbert transform of $x(t)$. This makes us understand that $x_h(t)$ is the Hilbert transform of $x(t)$ defined as

$$x_h(t) = \frac{1}{\pi}\,x(t)\,\frac{1}{t} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\,d\tau \tag{2.81}$$

Also the inverse Hilbert transform can be stated as

$$x(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x_h(\tau)}{t-\tau}\,d\tau \tag{2.82}$$

Applications

(i) In the generation of SSB signal.
(ii) For the representation of band pass signal.
(iii) For the design of minimum phase shift filters.

Properties

(i) Hilbert transform has same energy density spectrum as that of signal.
(ii) Hilbert transform has same auto correlation function as that of signal.
(iii) Hilbert transform and the signal itself are orthogonal mutually.

$$\text{i.e.,}\quad \int_{-\infty}^{\infty} x(t)\,x_h(t)\,dt = 0 \tag{2.83}$$

(ii) The transition band of the filter should not exceed twice the minimum frequency component present in the baseband signal.

Here we are considering separation between passband and stop band as transition band of the filter.

2.6.3.2 Phase Discrimination Method

The phase discrimination method is based on the time domain description of the SSB-SC signal. It can be seen from eqns. 2.79 and 2.80 that SSB-SC signal can be generated by using two separate DSB modulations and combining them suitably depending on the desired sideband.

The method makes use of two balanced modulators and two phase shifting networks as shown in Fig 2.31. Here M_1 receives the carrier voltage shifted by 90° and the modulating voltage, whereas M_2 receives modulating voltage shifted by 90° and the carrier voltage.

Both the modulators produce an output consisting only of sidebands. Both the upper sidebands lead the input carrier voltage by 90°. One of the two lower sidebands leads the reference voltage by 90° and other lags it by 90°. Therefore, the two lower sidebands are out of phase, and when combined together in the adder, they get cancelled by each other.

The upper sidebands are in phase to each other and hence they add producing SSB in which the lower sideband has been cancelled.

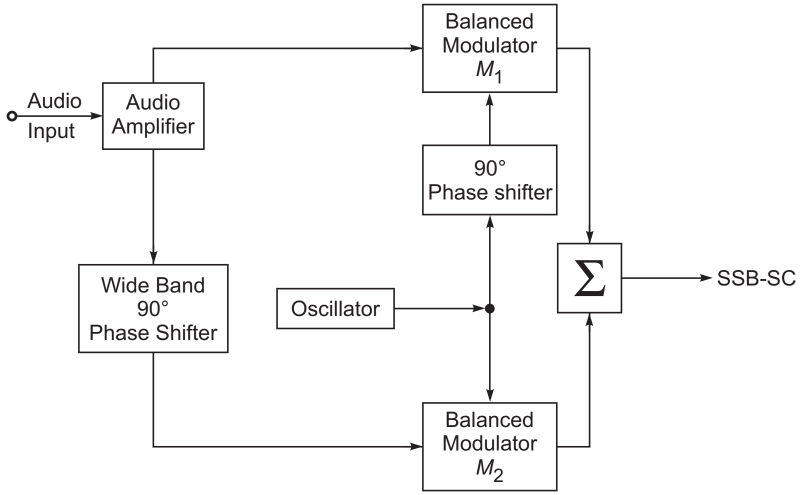

Fig. 2.31 Block diagram of phase discriminator method for SSB-SC generation.

Mathematical treatment to above method will be as follows:

Let $e_c(t)$ and $e_m(t)$ be the carrier and modulating signal.

Say M_1 receives $e_m(t)$ and $(e_c(t) + 90°)$ (2.85)

M_2 receives $e_c(t)$ and $(e_m(t) + 90°)$ (2.86)

At the output of M_1 we receive sum and difference frequencies.

Hence, $$V_1 = \cos[(w_c t + 90°) - w_m t] - \cos[(w_c t + 90°) + w_m t] \quad (2.87)$$

$$V_1 = \underset{\text{LSB}}{\cos[w_c t - w_m t + 90°]} - \underset{\text{USB}}{\cos[w_c t + w_m t + 90°]} \quad (2.88)$$

Similarly at the output of M_2, we have

$$V_2 = \cos[w_c t - w_m t - 90°] - \cos[w_c t + w_m t + 90°] \quad (2.89)$$

Therefore, the output of the adder will be

$$V_0 = V_1 + V_2 = 2\cos(w_c t + w_m + 90°] \quad (2.90)$$

which is nothing but SSB-SC.

2.6.4 Demodulation of SSB-SC

Here also we use synchronous detection technique for SSB-SC. The incoming signal is multiplied with locally generated sinusoid and then low filtered. The filter is chosen to have the same bandwidth as the message bandwidth W or somewhat larger. Here we are required to synchronise the local oscillator (LO) in phase and frequency with the carrier. Another method is also there which involves re-insertion of locally generated synchronous carrier and envelope detecting the resulting signal.

2.6.4.1 Synchronous Detection

The arrangement is similar to the one used in the detection of DSB-SC signal and is shown in Fig. 2.32. Here we are considering the input to be SSB-SC with upper sideband for the purpose of analysis, which is given by

$$S(t) = \frac{A_c}{2}\{e_m(t)\cos(2\pi f_c t) - \hat{e}_m(t)\sin(2\pi f_c t)\} \quad (2.91)$$

Assume carrier generated from local oscillator be $A'_c \cos(2\pi f_c t)$. Then, the output of the product modulator is given by

$$S_c(t) = \frac{1}{2} A_c A'_c \cos(2\pi f_c t)\ [e_m(t)\cos(2\pi f_c t) - \hat{e}_m(t)\sin(2\pi f_c t)] \quad (2.92)$$

$$= \frac{1}{2} A_c A'_c \left[\frac{e_m(t)}{2}(1+\cos(2\pi f_c t)) - \frac{\hat{e}_m(t)}{2}\sin(2\pi f_c t)\right] \quad (2.93)$$

$$= \frac{1}{4} A_c A'_c\, e_m(t) + \frac{1}{4} A_c A'_c\, [e_m(t)\cos(2\pi f_c t) - \hat{e}_m(t)\sin(2\pi f_c t)] \quad (2.94)$$

First term in above equation ⇒ demodulated signal

Second term ⇒ Another SSB-SC wave corresponding to a carrier frequency $2f_c$.

sated for the transmission of corresponding part of suppressed sideband. The technique is explained in Fig. 2.34. The bandwidth of VSB signal is

$$BW = f_c + f_v - f_c + W = W + f_v \tag{2.95}$$

where, W = Bandwidth of message signal.

f_v = Width of vestigial sideband.

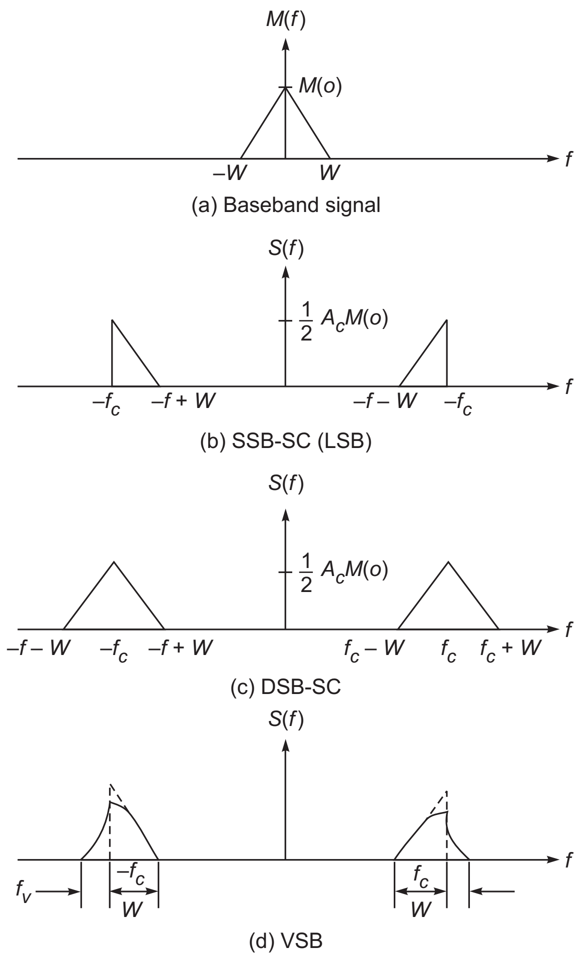

Fig. 2.34 Illustration of VSB Spectrum.

2.7.1 Generation of VSB

VSB can simply be generated by passing a DSB-SC signal through an appropriate filter having transfer function $H(f)$. It has been explained in the block diagram shown in Fig. 2.35. The frequency spectrum of VSB signal, $S(f)$ is therefore given by

$$S(f) = FT \text{ of } \{S(t)\} \text{ and } M(f) = FT \text{ of } e_m(t).$$

$$S(f) = \frac{A_c}{2}\,[M(f - f_c) + M(f + f_c)]\,H(f) \qquad (2.96)$$

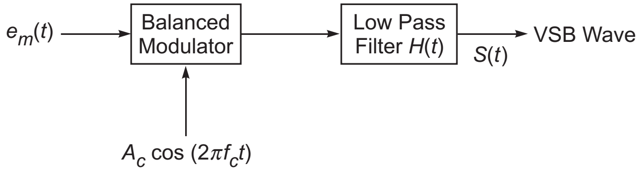

Fig. 2.35 VSB Generator.

Here we are supposed to determine the specifications of transfer function, $H(f)$ so that the spectrum given by eqn. (2.96) should correspond to the spectrum $S(f)$ of the VSB signal, $S(t)$.

Here is a very important fact that in VSB transmission, signal frequency from DC to .75 MHz are present equally in both sidebands whereas the remaining signal frequencies are present only in upper sideband. If these frequencies are amplified equally by receiver stages, then frequencies from 0 to 0.75 MHz will have a magnitude twice that of the remaining frequencies of the signal. In order to avoid this, the response of the TV receiver is adjusted in such a manner that amplification at these frequencies is reduced to half of the amplification at the other frequencies, so that the output conforms to the original shape.

2.7.2 Demodulation of VSB

VSB wave is fed to a balanced modulator along with a carrier from local oscillator i.e., $A_c' \cos(2\pi f_c t)$. The output of product modulator is $S_c(t)$ as shown in Fig. 2.36.

$$S_c(t) = A_c'\,S(t)\cos(2\pi f_c(t) \qquad (2.97)$$

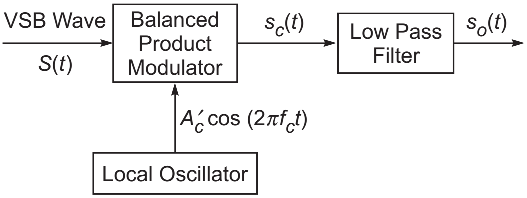

Fig. 2.36 Demodulation of VSB.

Taking Fourier transform on both sides of eqn. (2.97). we get

$$S_c(f) = \frac{A_c'}{2}\,[S(f - f_c) + S(f + f_c)] \qquad (2.98)$$

Put the value of $S(f)$ from (2.96) into (2.98), we get

$$S_c(f) = \frac{A_c A_c'}{4}\,[M(f - 2f_c) + M(f)]\,H(f - f_c)$$

$$+ \frac{A_c A_c'}{4} [M(f) + M(f + 2f_c)] H(f + f_c) \tag{2.99}$$

$$S_c(f) = \frac{A_c A_c'}{4} M(f) [H(f - f_c) + H(f + f_c)]$$
$$+ \frac{A_c A_c'}{4} [M(f - 2f_c) H(f - f_c) + M(f + 2f_c) H(f + f_c) \tag{2.100}$$

The first term of the eqn. (2.100) corresponds to the spectrum of the baseband signal while the second term corresponds to the spectrum of the VSB signal having carrier frequency $2f_c$. The second term can be removed by using a low pass filter as shown in Fig. 2.36. The spectrum of the signal $S_0(t)$ at the output of the LPF is given by

$$S_0(f) = \frac{A_c A_c'}{4} M(f) [H(f - f_c) + H(f + f_c)] \text{ as} \tag{2.101}$$

shown in Fig. 2.37.

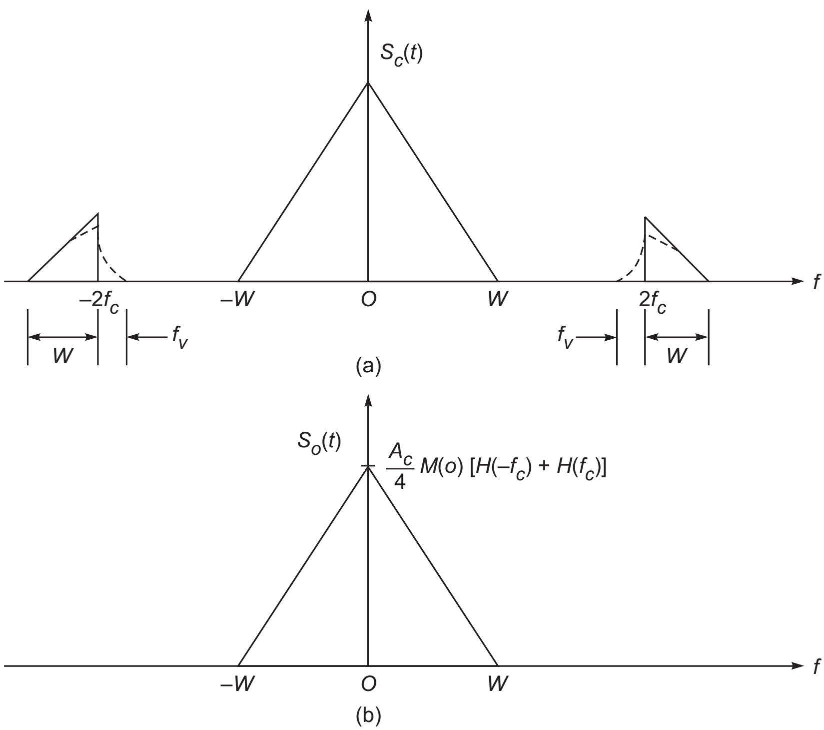

Fig. 2.37 (a) Spectrum of product modulator output. (b) Spectrum of Demodulated signal.

2.8 COMPARISON OF VARIOUS AM SYSTEMS

In this chapter, we have studied different techniques for AM modulation. As in conventional AM there are two side bands as well as the carrier. In suppressed

$\therefore$ No. of stations $= \dfrac{30 \times 10^6}{8 \times 10^3}$

$$N = 3333 \text{ stations}$$

EXAMPLE 2.12 Which of the following demodulators can be used for demodulating the signal

$$x(t) = f(1 + 2 \cos 1000\,\pi t) \sin 1000\,\pi t.$$

(a) Envelope demodulator
(b) Square law demodulator
(c) Synchronous demodulator
(d) All of the above
(e) None of the above

Solution: Since, above given signal is case of AM modulation (conventional) with a carrier and two sidebands. Therefore, all the three methods can be used to detect the said signal. Whereas envelope detection with any square law device can be used for the purpose.

$\therefore$ '*d*' option is correct.

EXAMPLE 2.13 Determine the power content of each of the sidebands and of the carrier of an AM signal that has a per cent modulation of 80% and contains 1000 W of total power.

Solution:

Given $M = 80\%, \quad m = .85 \quad P_t = 1000 \text{ W}.$

To be calculated:

$$P_c, \quad P_{USB}, \quad P_{LSB}$$

We know that

$$P_t = P_c \left\{1 + \frac{m^2}{2}\right\}$$

We get $P_c = 757.57$ watts.

We know further that,

$$P_c + P_{LSB} + P_{USB} = P_t$$

$$P_{SB} = (P_{LSB} + P_{USB}) = (1000 - 757.57) \text{ W} = 242.42 \text{ watts.}$$

Now $P_{LSB} = P_{USB} = \dfrac{P_{SB}}{2} = \dfrac{242.42}{2}$

$$= 121.21 \text{ watts.}$$

EXAMPLE 2.14 A *SSB* signal contains 4 kW. How much power is contained in the sidebands and in carrier?

Solution: Given $P_{SSB} = 4$ kW.

To be calculated: P_{SB} , P_C.

Here whatever power is transmitted is sent through single sideband only regardless of % age modulation.

$$\therefore \quad P_{SB} = 4 \text{ kW}$$
$$P_C = 0 \text{ kW} \quad (\because \text{ No carrier is transmitted in SSB})$$

EXAMPLE 2.15 Find the modulation index and percentage modulation of the signal shown in Fig. 2.38.

Solution:
$$m = \frac{\text{max peak to peak} - \text{min peak to peak}}{\text{max peak to peak} + \text{min peak to peak}}$$
$$\% \ m = \frac{100-50}{100+50} \times 100 = \frac{50}{150} = \frac{1}{3} \times 100$$
$$\% \ m = 33.33\%$$

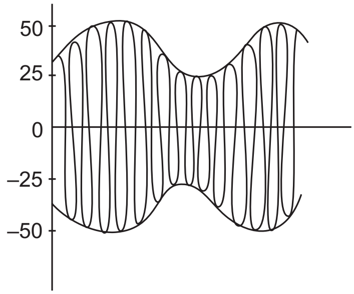

Fig. 2.38

EXAMPLE 2.16 Evaluate the effect of a small frequency error in the local oscillator on synchronous DSB modulation.

Solution: Let the frequency error be Δw. The local carrier is then be expressed as $\cos (w_c + \Delta w)t$.

$\therefore$ We can have

$$d(t) = m(t) \cos w_c t \cos(w_c + \Delta w)t$$
$$d(t) = \frac{1}{2} m(t) \cos(\Delta w)t + \frac{1}{2} m(t) \cos 2w_c t.$$
$$y(t) = \frac{1}{2} m(t) \cos (\Delta w)t.$$

Here $m(t)$ is multiplied by a low frequency sinusoid. This is called a "beating" effect and is a very undesirable distortion.

EXAMPLE 2.17 Show that a synchronous demodulator shown in Fig. 2.39 can demodulate an AM signal

$$x(t) = [A + m(t)] \cos w_c t \text{ regardless the value of } A.$$

$$-\frac{1}{2}\cos\{(2\pi(f_c - f_i)t) - \theta_i\} + \frac{1}{2}\cos\{(2\pi(f_c + f_i)t) + \theta_i\}$$

$$x(t) = \sum_{i=1}^{N} \cos\{(2\pi(f_c + f_i)t) + \theta_i\} = S_u(t)$$

Here $S_u(t)$ contains frequency components higher than the carrier frequency f_c and therefore SSB-SC signal with upper sideband.

(b) The *LSB* will be

$$S(t) = \sum_{i=1}^{N} [\cos(2\pi f_c t)\cos(2\pi f_i t + \theta_i) + \sin(2\pi f_c t)\sin(2\pi f_i t + \theta_i)]$$

$$= \sum_{i=1}^{N} \cos(2\pi(f_c - f_i)t - \theta_i) = S_L(t)$$

(c) The total DSB-SC signal will be

$$S_{DSB}(t) = S_u(\mathrm{t}) + S_L(t)$$

$$= 2\sum_{i=1}^{N} \cos(2\pi f_c t)\cos(2\pi f_i t + \theta_i)$$

$$= \left[2\sum_{i=1}^{N} \cos(2\pi f_i t + \theta_i)\right]\cos 2\pi f_c t.$$

EXAMPLE 2.19 The power of a SSB signal $\cos(2\pi(f_c + f_m)t)$ is 0.5 volt^2. The signal is to be detected by carrier reinsertion technique. Find the amplitude of the carrier to be reinserted, so that the power is recovered signal at the demodulator output is 90% of the normal power. Neglect the d.c component.

Solution: The waveform after carrier insertion will become

$$S(t) = A_c\cos(2\pi f_c t) + \cos(2\pi(f_c + f_m)t)$$
$$S(t) = (A_c + \cos(2\pi f_m t))\cos(2\pi f_c t) - \sin(2\pi f_m t)\sin(2\pi f_c t)$$

Here at demodulators output

$$e(t) = [A_c + \cos(2\pi f_m t))^2 + \sin^2(2\pi f_m t)]^{1/2}$$
$$= [A_c^2 + 1 + 2A_c\cos(2\pi f_m t)]^{1/2}$$

$$= \sqrt{A_c^2+1}\left[1+\frac{2A_c}{A_c^2+1}\cos(2\pi f_m t)\right]^{1/2}$$

$$= \sqrt{A_c^2+1}+\left[1+\frac{A_c\cos(2\pi f_m t)}{A_c^2+1}\right]$$

$$= \sqrt{A_c^2+1}+\frac{A_c\cos(2\pi f_m t)}{\sqrt{A_c^2+1}}$$

neglect the d.c. component, normalised power of the demodulated signal is

14. An AM signal contains a total 10 kW of power. Calculate the power being transmitted at the carrier frequency and at each of the sidebands when the percentage modulation is 100%.
15. List the advantages of SSB transmission over conventional double sideband system.
16. What is VSB transmission? How it is used in TV broadcast?
17. With the simple phasor diagram and block diagram explain the principle of generation of AM.
18. Enumerate the advantages and disadvantages of AM.
19. Compare and contrast DSB/AM and SSB-SC AM systems.
20. A certain AM transmitter is coupled to an antenna. The input power to the antenna is measured through monitoring the antenna current. With no modulation, the current is 10 A. With modulation the current rises to 13 A. Determine the depth of modulation explaining the significance of the formula used.
21. Derive an expression for the transmission efficiency of AM wave.
22. Explain the low level and high level AM modulation methods with the help of figures.
23. Derive the current and power relations for AM wave.