Carlstrom and Fuerst (1997) to US data 1980Q1-2019Q4

Niloufar Eslah

Feb 17

Contents

1. Introduction

This report presents a replication and extension of ”Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis” by Carlstrom and Fuerst (1997) using updated U.S. data from 1980Q1 to 2019Q4. The original paper investigates the role of financial frictions in amplifying business cycle fluctuations, particularly through the interaction between firm borrowing constraints and economic shocks. Our objective is to estimate and validate their model using Bayesian methods in Dynare while incorporating a modified preference structure for households.

The replication follows a structured approach. First, we derive and present the system of equilibrium equations governing households, firms, and entrepreneurs, adjusting the preference specification to explore its implications for optimal decisions. We then calibrate the model based on empirical data sources, ensuring consistency with historical macroeconomic trends. The next step involves solving for the steady-state equilibrium and estimating the structural parameters using a Bayesian framework. Specifically, we estimate two key shocks—total factor productivity (TFP) and financial (monitoring cost) shocks—using observed GDP and corporate bond spreads. Our estimation strategy employs Markov Chain Monte Carlo (MCMC) techniques to infer the posterior distributions of key parameters. Convergence diagnostics, including Brooks-Gelman and Geweke tests, are used to assess estimation reliability. We further validate the model by analyzing impulse response functions, variance decomposition, and business cycle statistics. The results provide insights into the relative importance of financial frictions in explaining macroeconomic fluctuations, revealing that risk shocks (SE_eM) play a dominant role in investment and credit market dynamics.

2. Type out system of equilibrium equations:

Here we have a different form of household problem since we assume the preference has the form below:

$$u(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \chi \frac{L^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

2.1 Household’s problem

For calculating the first-order condition, households maximize utility subject to their budget constraint:

$$C + q(K_{t+1} - (1-\delta)K_t) = wL + rK_t$$

Since we are facing a dynamic problem:

$$v(C,L) = \max_{C,K'} \{U(C,L) + \beta E[v(C',L')]$$
$$\text{s.t.} \quad C + q(K' - (1-\delta)K) = wL + rK\}$$

2.1.1 Euler Equation

Now, taking the derivative with respect to C:

$$C^{-\delta} = \lambda$$

And since households make decisions between current consumption and investment to see how much they can consume for the next period, taking the derivative with respect to K:

$$\beta E[C'(r' + q'(1-\delta))] = \lambda q$$
$$\Rightarrow \beta E\left[\frac{C'(r' + q'(1-\delta))}{q}\right] = \lambda$$

Comparing these two conditions, we have:

$$C_t^{-\sigma} = \beta E_t\left[C_{t+1}^{-\sigma}\frac{r_{t+1} + (1-\delta)q_{t+1}}{q_t}\right]$$

Now, let us assume $\sigma = 2$:

$$\boxed{C_t^{-2} = \beta E_t\left[C_{t+1}^{-2}\frac{r_{t+1} + (1-\delta)q_{t+1}}{q_t}\right]}$$

2.1.2 Labor Supply Condition (Intra-temporal choice between consumption and labor):

Once again we have to take the derivation with respect to labor:

$$-\chi L^{\frac{1}{\nu}} = w\lambda$$

Using condition for cunsumption with substituting in λ we have:

$$\chi L_t^{\frac{1}{\nu}} = C_t^{-\sigma} w_t$$

Substitute $\nu = 2$:

$$\boxed{\chi L_t^{\frac{1}{2}} = C_t^{-2} w_t}$$

2.2 Firm's problem

Firms hire capital and labor to maximize profits:

$$Y_t = A_t K_t^{\alpha} H_t^{\zeta} H_{e,t}^{1-\alpha-\zeta}$$

Firm must solve its problem by the constraint of:

$$r_t L_t + w_t H_t + w_{e,t} H_{e,t}$$

Thus the problem that Firm is facing is:

$$\max_{K_t, H_t, H_{e,t}} A_t K_t^{\alpha} H_t^{\zeta} H_{e,t}^{1-\alpha-\zeta} - r_t K_t - w_t H_t - w_{e,t} H_{e,t}$$

Therefore for r_t we have:

$$\boxed{r_t = \alpha A_t K_t^{\alpha-1} H_t^{\zeta} H_{e,t}^{1-\alpha-\zeta}}$$

For w_t:

$$\boxed{w_t = \zeta A_t K_t^{\alpha} H_t^{\zeta-1} H_{e,t}^{1-\alpha-\zeta}}$$

For $w_{e,t}$:

$$\boxed{w_{e,t} = (1-\alpha-\zeta) A_t K_t^{\alpha} H_t^{\zeta} H_{e,t}^{-(\alpha+\zeta)}}$$

2.3 Entrepreneur's Problem

Entrepreneurs maximize their discounted expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t C_{e,t}$$

Their budget constraint:

$$C_{e,t} + q_t K_{e,t+1} = \omega_t I_t - (1 + R_{b,t})(I_t - N_t)$$

Evolution of net worth composed of three elements:

- Labor income, $w_{e,t}$
- Returns from capital,$(r_t + (1-\delta)q_t)\, K_{e,t}$
- Shocks to capture unexpected changes in wealth

$$\boxed{N_t = w_{e,t} + (r_t + (1-\delta)q_t)\, K_{e,t} + \text{shock term}}$$

The lender's break-even condition requires that the expected return equals the amount lent. This is because lenders are risk-neutral and only provide funding if the expected payoff covers the loan.

The expected return to the lender is:

$$q_t g(\bar{\omega}_t) I_t = I_t - N_t$$

rearranging this term, we get:

$$\boxed{I_t = \frac{N_t}{1 - q_t g(\bar{\omega}_t)}}$$

Also, we can think of the leverage formula for the investment with a rate of $q_t g(\bar{\omega}_t)$.
The net worth of the entrepreneur N_t determines the scale of investment, but external finance allows him to invest more than their own funds.
The optimal contract condition is given by:

$$q_t = \frac{1}{1 - \mu\Phi(\bar{\omega}_t) - \frac{\mu\phi(\bar{\omega}_t)f(\bar{\omega}_t)}{1-\Phi(\bar{\omega}_t)}}$$

The expected return condition is given by:

$$q_t = \beta\gamma E_t \left[(r_{t+1} + q_{t+1}(1-\delta)) \frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right]$$

2.4 Aggregate Equilibrium Conditions

Capital Market Clearing is:

$$K_t = (1-\eta)K_{c,t} + \eta K_{e,t}$$

Goods Market Clearing is:

$$(1-\eta)C_t + \eta C_{e,t} + I_t = Y_t$$

Law of Motion for Capital is:

$$K_{t+1} = (1-\delta)K_t + I_t(1 - \mu\Phi(\bar{\omega}_t))$$

Shocks Dynamics is:

$$A_t = (1-\rho_A) + \rho_A A_{t-1} + \sigma_A \epsilon_A$$

$$\mu_t = (1-\rho_\mu) + \rho_\mu \mu_{t-1} + \sigma_\mu \epsilon_\mu$$

3. Calibration

The calibration of model parameters follows the methodology used in Carlstrom and Fuerst (1997), using U.S. data from 1980Q1–2019Q4, obtained from the Federal Reserve Economic Data (FRED)[1]. The following parameters are calibrated based on empirical estimates:

- **Capital share in production** (α): The capital share is calibrated as $\alpha = 0.29868$, reflecting its role in the Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} H_t^{\zeta} \tag{3.1}$$

  where α represents the output elasticity of capital.

- **Labor share adjustment** (ζ): Given the capital share α, labor's share in production is determined as:

$$\zeta = 1 - \alpha - 0.0001 \tag{3.2}$$

  where the small adjustment ensures numerical consistency.

- **Discount factor** (β): The intertemporal discount factor is calibrated as $\beta = 0.91872$. It is derived from the Euler equation for consumption:

$$C_t^{-\sigma} = \beta E_t \left[C_{t+1}^{-\sigma} \frac{r_{t+1} + (1 - \delta)}{1} \right] \tag{3.3}$$

  ensuring consistency with observed interest rates.

- **Depreciation rate** (δ): The quarterly depreciation rate is calibrated as $\delta = 0.040853$, inferred from the steady-state capital accumulation condition:

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{3.4}$$

- **Labor disutility parameter** (χ): The labor disutility parameter $\chi = 0.0096672$ is determined from the labor supply equation:

$$\chi L_t^{1/\nu} = C_t^{-\sigma} w_t \tag{3.5}$$

  ensuring consistency with observed labor supply behavior.

[1] https://fred.stlouisfed.org

Parameter	Definition / Formula	Calibrated Value
α	Capital share in production: $Y_t = A_t K_t^{\alpha} H_t^{\zeta}$	0.29868
ζ	Labor share: $\zeta = 1 - \alpha - 0.0001$	0.70122
β	Discount factor from Euler equation: $C_t^{-\sigma} = \beta E_t \left[C_{t+1}^{-\sigma} \frac{r_{t+1}+(1-\delta)}{1} \right]$	0.91872
δ	Depreciation rate from capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t$	0.040853
χ	Labor disutility parameter from labor supply: $\chi L_t^{1/\nu} = C_t^{-\sigma} w_t$	0.0096672

Table 1: Summary of Calibrated Parameters

4. Steady-State Results

The steady-state values for key variables in the model are computed and summarized in Table 2. These values provide insights into the equilibrium relationships between capital, labor, consumption, and financial markets.

Variable	Steady-State	Variable	Steady-State
K	15.1423	q	1.0323
k_e	3.00184	n	3.37773
H	4.80584	i	6.19836
H_e	0.1	ω_b	0.442865
LA	1.40832	C	6.87293
h	5.33982	C_c	6.82628
Cc	7.58475	C_e	0.0466476
cc	7.58475	ce	0.466476
w	0.987541	w_e	0.00676815
Y	6.76815	$\log Y$	0
r	0.133501	I	0.619836
Bankruptcy	0.0019781	R_b	1.00463
rp_{BANK}	0.004633	rp_{ENT}	0.165477
lev	1.83506	r_{if}	1.05552
Φ	0.0019781	ϕ	0.052496
f	0.5572	g	0.440822
A	1	μ	1

Table 2: Steady-State Values of Model Variables

4.1 Interpretation of Steady-State Results

The steady-state values confirm that the model maintains a balanced equilibrium.

- Capital and Investment: The steady-state capital stock $K = 15.14$ and investment $I = 0.6198$ are consistent with a stable accumulation path.
- Labor Market: Hours worked in equilibrium are $H = 4.8058$, with entrepreneurial labor $H_e = 0.1$.
- Financial Markets: The risk premium for entrepreneurs $rp_{\text{ENT}} = 0.1655$ is significantly higher than the banking sector spread $rp_{\text{BANK}} = 0.0046$, reflecting higher credit risk.
- Consumption and Output: Aggregate consumption is $C = 6.8729$, with consumption split between entrepreneurs and workers.

4.2 Eigenvalue Stability Analysis

To verify model stability, we examine the eigenvalues of the system in Table 3. The model satisfies the Blanchard-Kahn conditions since the number of eigenvalues greater than one in modulus matches the number of forward-looking variables.

Modulus	**Real Part**	**Imaginary Part**
3.598×10^{-16}	3.598×10^{-16}	0
2.412×10^{-15}	2.412×10^{-15}	0
0.32	0.32	0
0.3923	0.3923	0
0.8164	0.8164	0
0.85	0.85	0
1.301	1.301	0
20.85	20.85	0
2.72×10^{18}	-2.72×10^{18}	0
1.162×10^{19}	-1.162×10^{19}	0
2.772×10^{19}	-2.772×10^{19}	0

Table 3: Eigenvalue Stability Analysis

There are five eigenvalues greater than one in modulus, which corresponds to the number of forward-looking variables. Thus, the model satisfies the rank condition, ensuring a unique and stable equilibrium.

5. Comparison of Theoretical and Bayesian Estimation Approaches

Bayesian estimation and stochastic simulation serve different purposes in DSGE modeling. Bayesian estimation is a statistical method that combines prior beliefs with observed data to estimate the probability distributions of model parameters, allowing for uncertainty quantification and better handling of identification issues. In contrast, stochastic simulation takes fixed parameter values—either estimated or calibrated—and examines how the model responds to different shocks by generating artificial data. While Bayesian estimation aligns the model with empirical data, stochastic simulation helps analyze business cycle dynamics, impulse responses, and variance decompositions.

6. Bayesian Estimation of Productivity and Financial Shocks

Bayesian estimation is used to infer the two structural shocks in the model: the productivity shock (e_A) and the financial (monitoring cost) shock (e_M). To estimate these shocks, we rely on two observed macroeconomic time series: 1. **Log TFP** ($\log A$) is used to estimate the productivity shock e_A. 2. **Log Corporate Spread** ($\log rpBANK$) is used to estimate the financial (monitoring cost) shock e_M.

The estimation process integrates prior distributions with empirical data to derive the posterior distributions of these shocks, allowing us to assess their persistence and volatility. The table below presents the key results.

	Prior Mean	Posterior Mode	Posterior Mean	Std. Dev.	90% HPD Interval
Persistence Parameters					
ρ_A (Productivity Shock)	0.850	0.9472	0.9468	0.0008	[0.9462, 0.9472]
ρ_μ (Financial Shock)	0.320	0.2766	0.2758	0.0035	[0.2612, 0.2883]
Standard Deviations of Shocks					
σ_A (Productivity Shock)	0.005	0.0121	0.0123	0.0007	[0.0112, 0.0135]
σ_M (Financial Shock)	0.005	10.8004	10.9376	1.0006	[9.4702, 12.2859]

Table 4: Bayesian Estimation Results for Productivity and Financial Shocks

The results indicate that the estimated persistence of the productivity shock is high, with $\rho_A \approx 0.95$, suggesting that changes in technology have long-lasting effects on the economy. The financial shock, on the other hand, is much less persistent, with $\rho_\mu \approx 0.28$, implying that fluctuations in financial frictions tend to dissipate more quickly. However, the standard deviation of the financial shock is significantly larger than that of the productivity shock, indicating that financial disruptions introduce substantial short-term volatility into the system.

The log data density values, which measure how well the estimated parameters fit the data, are:

- Laplace Approximation: 378.024575

- Modified Harmonic Mean: 377.160392

These values confirm that the Bayesian estimation successfully aligns the model with observed data.

6.1 Graphical Analysis of Estimation Results

Figure 1 displays the smoothed estimates of $\log A$ (TFP) and $\log rpBANK$ (corporate spread). The black lines represent the estimated series, while the red lines correspond to their smoothed versions. These results confirm that corporate spreads exhibit significantly larger volatility compared to TFP.

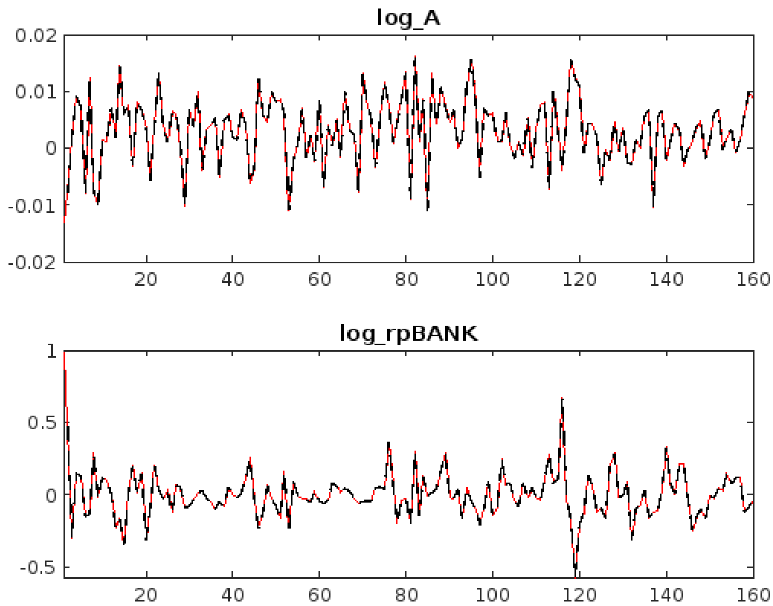

Figure 1: Smoothed Estimates for $\log A$ (TFP) and $\log rpBANK$ (Corporate Spread)

Figure 2 presents the estimated structural shocks: e_A (productivity shock) and e_M (financial shock). The financial shock shows much larger fluctuations, reinforcing the earlier result that financial frictions generate significant macroeconomic volatility.

Figure 3 illustrates the MCMC estimation of ρ_μ, the persistence of financial shocks. The blue and red lines represent different MCMC chains, demonstrating good convergence across iterations.

Figure 4 shows the inefficiency factors and MCMC convergence diagnostics for key parameters. The high inefficiency factor for financial shocks suggests that these shocks are more volatile and

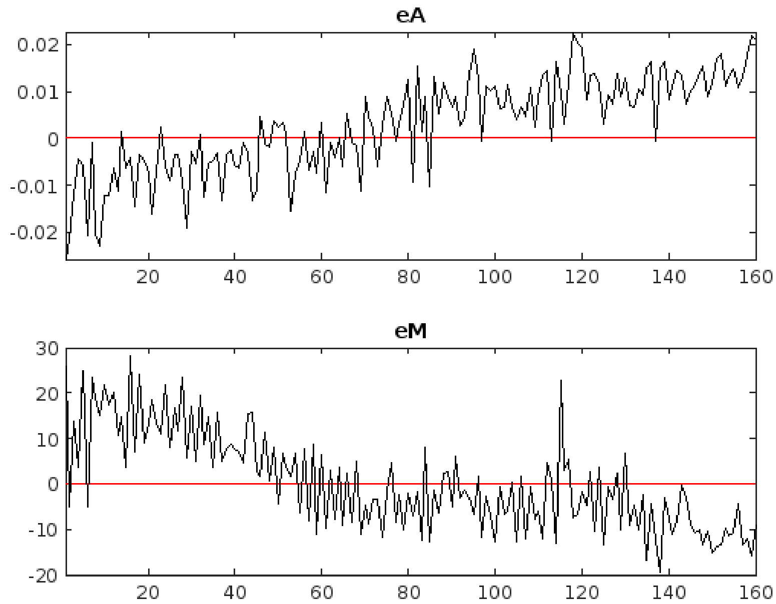

Figure 2: Estimated Structural Shocks: e_A (Productivity Shock) and e_M (Financial Shock)

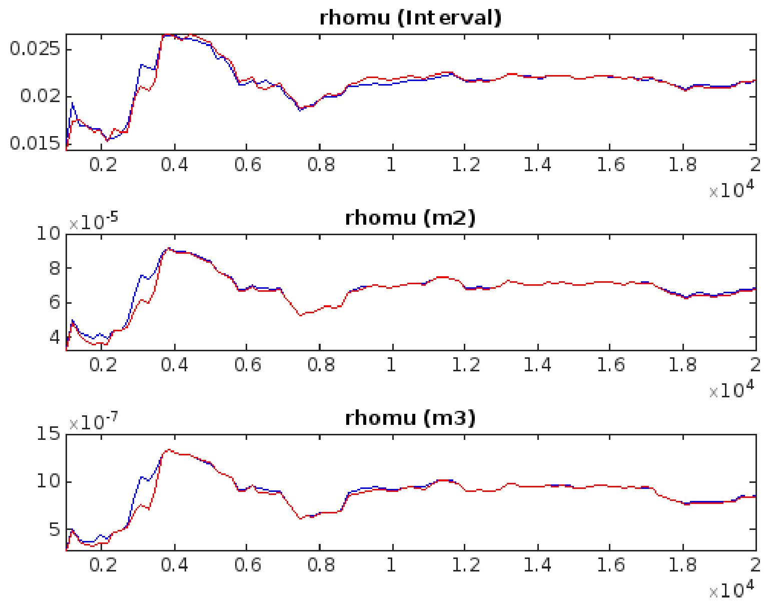

Figure 3: MCMC Sampling for ρ_μ (Persistence of Financial Shock)

require longer MCMC chains for stable estimation.

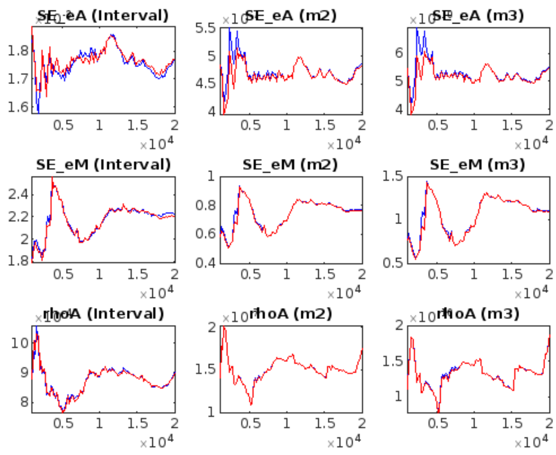

Figure 4: MCMC Convergence Diagnostics for Estimated Parameters

Lastly, Figure 5 presents the multivariate covariance estimates, confirming the stability of Bayesian estimates over different parameter draws.

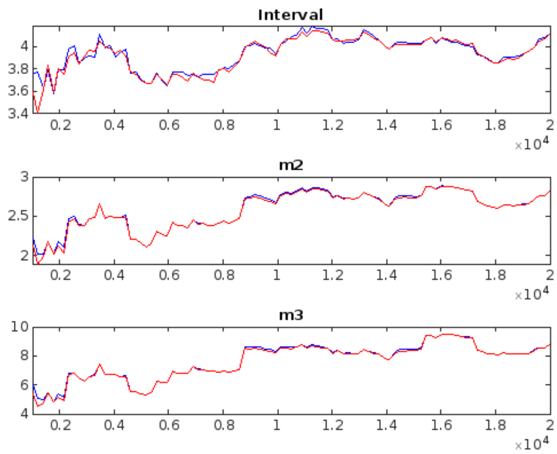

Figure 5: Multivariate Covariance Estimates

6.2 Economic Interpretation

The estimation results reveal key insights about the role of productivity and financial shocks in macroeconomic dynamics. The high persistence of productivity shocks suggests that technological innovations have long-term effects on output, investment, and labor markets. This is consistent with models where productivity growth is a primary driver of long-run economic trends.

Financial shocks, in contrast, are less persistent but significantly more volatile. This aligns with theoretical models where financial frictions create short-term macroeconomic instability by tightening credit conditions. The large fluctuations in the estimated financial shock suggest that these disruptions are a major driver of business cycle fluctuations, particularly through their impact on borrowing costs and investment decisions.

The Bayesian estimation results confirm that financial frictions amplify short-term volatility, while productivity changes determine long-term growth. Future research should analyze the impulse response functions to better understand the transmission mechanisms of these shocks.

7. Variance Decomposition of Productivity and Financial Shocks

7.1 Concept of Variance Decomposition

Variance decomposition quantifies the contribution of different structural shocks to the fluctuations in key macroeconomic variables. In this case, the decomposition determines how much of the variability in output ($\log Y$), consumption ($\log C$), investment ($\log I$), interest rates ($\log r$), asset prices ($\log q$), and corporate spreads ($\log rpBANK$) can be attributed to the productivity shock (e_A) and the financial (monitoring cost) shock (e_M).

The estimation is performed by simulating each shock separately while holding the other constant. The reported values, expressed as percentages, indicate the proportion of variance in each variable explained by each shock. The total linear contribution column accounts for interactions between the shocks, which may lead to contributions exceeding 100% due to correlations in small samples.

7.2 Numerical Results

The variance decomposition results are presented in Table 5. The productivity shock (e_A) primarily influences total factor productivity, while the financial shock (e_M) accounts for the majority of variance in output, investment, and financial variables.

The productivity shock explains nearly all of the variance in total factor productivity, with a contribution of 101.01%. However, its influence on output, consumption, and investment is minimal, contributing less than 5% in each case. The financial shock accounts for the majority of the fluctuations in these variables, with contributions exceeding 100% in some cases due to correlation effects in small samples.

Interest rates and asset prices are also primarily driven by financial shocks, with respective

	e_A (Productivity Shock)	e_M (Financial Shock)	Total Contribution
$\log Y$ (Output)	4.48	103.11	107.59
$\log C$ (Consumption)	2.35	93.52	95.88
$\log I$ (Investment)	0.21	101.31	101.52
$\log r$ (Interest Rate)	2.79	103.81	106.60
$\log q$ (Asset Price)	0.62	97.93	98.55
$\log LA$ (Labor Supply)	18.59	69.00	87.59
$\log A$ (TFP)	101.01	0.00	101.01
$\log H$ (Hours Worked)	0.60	97.70	98.30
$\log rpBANK$ (Corporate Spread)	28.87	75.55	104.42

Table 5: Variance Decomposition of Key Macroeconomic Variables

contributions of 103.81% and 97.93%. Labor supply fluctuations are influenced by both productivity and financial shocks, with a combined contribution of 87.59%. The corporate spread series ($\log rpBANK$) is mostly explained by financial shocks (75.55%), though productivity shocks also play a role (28.87%), indicating some link between long-term productivity expectations and financial conditions.

7.3 Economic Interpretation

The results suggest that financial shocks are the dominant source of macroeconomic fluctuations, particularly in output, investment, and financial markets. This aligns with the idea that changes in credit conditions and borrowing constraints significantly impact business cycles. When financial conditions tighten, firms and households reduce spending and investment, leading to contractions in output and employment.

The contribution of productivity shocks to output and investment volatility is limited, indicating that technology-driven fluctuations were not the primary drivers of business cycle dynamics during the sample period. Instead, productivity changes appear to influence longer-term economic trends rather than short-term volatility.

Interest rate and asset price movements are almost entirely determined by financial shocks, highlighting the role of financial markets in macroeconomic stability. The high variance contribution of financial shocks to labor supply suggests that financial frictions affect not only capital markets but also employment decisions and wage dynamics.

The decomposition of corporate spreads indicates that while financial shocks are the primary driver, productivity shocks also play a non-negligible role. This suggests that financial markets respond not only to credit risk and liquidity conditions but also to expectations about long-term productivity growth.

Overall, these findings reinforce the view that financial market conditions play a crucial role in shaping macroeconomic fluctuations. The next step in the analysis involves examining impulse response functions to understand how these shocks propagate over time.

The findings align with Carlstrom and Fuerst (1997) in highlighting the dominant role of financial shocks in driving macroeconomic fluctuations. Similar to their model, the results show that financial shocks significantly impact investment, output, and corporate spreads, reinforcing the idea that credit market conditions amplify business cycle dynamics. Productivity shocks contribute primarily to long-term trends, while financial frictions are the key drivers of short-term volatility. This supports the financial accelerator mechanism proposed by Carlstrom and Fuerst, where changes in borrowing conditions propagate through the economy, influencing investment and asset prices.

8. Impulse Response Analysis of Productivity and Financial Shocks

8.1 Numerical Results

Impulse response functions (IRFs) illustrate the dynamic effects of structural shocks on macroeconomic variables over time. The figures below show the responses of GDP ($\log Y$), consumption ($\log C$), investment ($\log I$), labor ($\log LA$), interest rates ($\log r$), labor productivity ($\log A$), the relative price of investment to consumption ($\log q$), and corporate spreads ($\log rpBANK$) to both the productivity shock (e_A) and the financial (monitoring cost) shock (e_M). All data series have been HP-filtered with a smoothing parameter of 1600 to remove long-term trends.

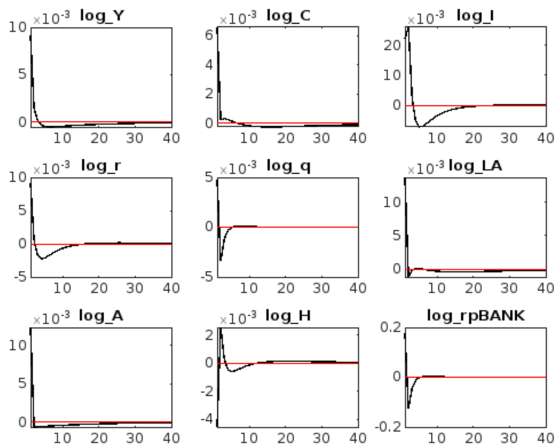

Figure 6: Impulse Response Functions to a Productivity Shock (e_A)

Figure 6 presents the responses following a positive productivity shock. GDP, consumption, and investment increase immediately after the shock, with investment exhibiting the strongest initial response, reflecting its sensitivity to changes in productivity. Interest rates rise slightly, indicating a tightening of borrowing conditions, while labor supply also increases but with a smaller magnitude. The relative price of investment to consumption falls slightly, whereas corporate spreads remain largely unaffected. The effects on macroeconomic variables gradually fade over time.

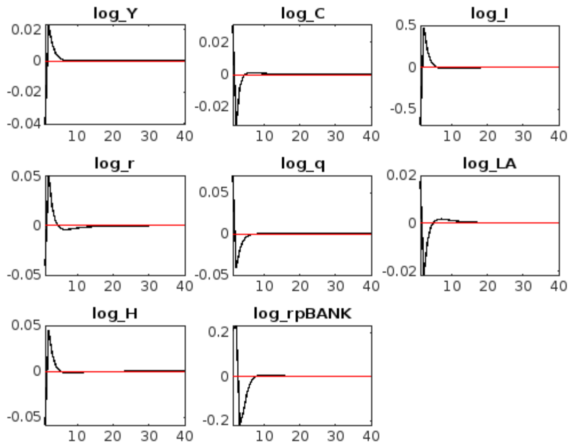

Figure 7: Impulse Response Functions to a Financial Shock (e_M)

Figure 7 shows the responses to a negative financial shock, such as an increase in monitoring costs. GDP, consumption, and investment all decline sharply, with investment experiencing the most pronounced contraction. Interest rates increase, reflecting tighter credit conditions. Labor supply declines, suggesting a reduction in hiring and employment. Corporate spreads rise, capturing the deteriorating financial environment. Unlike the productivity shock, financial shocks induce strong and prolonged negative effects on macroeconomic activity, particularly in investment and borrowing costs.

8.2 Economic Interpretation

The impulse response analysis highlights important differences in the transmission mechanisms of productivity and financial shocks. A positive productivity shock leads to higher GDP, investment, and labor supply, driven by improvements in production efficiency. However, financial variables such as corporate spreads remain largely unchanged, indicating that productivity-driven expansions do not necessarily alter borrowing conditions.

Financial shocks, on the other hand, generate significant economic contractions. The sharp declines in investment and output underscore the role of financial frictions in macroeconomic volatility. The increase in corporate spreads suggests that disruptions in credit markets contribute to declines in borrowing and spending, amplifying the negative effects on investment and employment. Rising interest rates further tighten financial conditions, leading to a prolonged economic downturn.

These results are consistent with the financial accelerator mechanism, where credit constraints amplify business cycle fluctuations. The asymmetric effects, where financial shocks cause stronger contractions than productivity shocks generate expansions, suggest that financial instability plays a crucial role in economic downturns. This supports the view that disruptions in financial markets can be a major source of macroeconomic fluctuations, reinforcing the importance of financial stability policies.

9. Business Cycle Statistics

9.1 Variance of Simulated Variables

The business cycle properties of key macroeconomic variables are analyzed using simulated data filtered with an HP smoothing parameter of 1600. Table 6 reports the variance, standard deviation, skewness, and kurtosis for each variable.

Variable	Mean	Std. Dev.	Variance	Skewness	Kurtosis
$\log Y$ (Output)	0.000235	0.048554	0.002357	0.038464	-0.502351
$\log C$ (Consumption)	0.000963	0.047298	0.002237	-0.290892	-0.657536
$\log I$ (Investment)	-0.007520	0.876397	0.768071	0.166680	-0.622229
$\log r$ (Interest Rate)	-0.001657	0.068046	0.004630	0.179968	-0.673474
$\log q$ (Relative Price)	0.000542	0.087996	0.007743	-0.200601	-0.597252
$\log LA$ (Labor Supply)	-0.000159	0.110265	0.012165	-0.458706	-0.427297
$\log A$ (TFP)	0.000517	0.013870	0.000192	0.499411	0.536907
$\log H$ (Hours Worked)	-0.001208	0.078551	0.006170	0.234875	-0.662012
$\log rpBANK$ (Corporate Spread)	0.008229	0.476764	0.227304	-0.134080	-0.371503

Table 6: Moments of Simulated Variables (HP-filtered, $\lambda = 1600$)

Investment exhibits the highest volatility, with a standard deviation of 0.876, while TFP has the lowest variability. Corporate spreads also display significant fluctuations, highlighting their role in financial cycles. Output, consumption, and labor supply exhibit moderate volatility, aligning with empirical findings that consumption is smoother than output and investment.

9.2 Correlation of Variables with GDP

Table 7 presents the correlation of simulated variables with output. Most real variables, including investment, consumption, and labor supply, exhibit strong procyclical behavior, while financial variables show weaker or negative correlations.

Variable	Correlation with GDP
$\log Y$ (Output)	1.0000
$\log C$ (Consumption)	-0.8950
$\log I$ (Investment)	0.9784
$\log r$ (Interest Rate)	0.9327
$\log q$ (Relative Price)	-0.9634
$\log LA$ (Labor Supply)	-0.7380
$\log A$ (TFP)	0.0403
$\log H$ (Hours Worked)	0.9522
$\log rpBANK$ (Corporate Spread)	-0.2403

Table 7: Correlation of Simulated Variables with GDP

Investment and hours worked are highly correlated with GDP, reflecting their strong cyclical nature. Consumption, however, shows a negative correlation, which may be attributed to model-specific dynamics or preference shocks affecting consumption differently from production. The negative correlation of corporate spreads with GDP is consistent with financial accelerator mechanisms, where rising spreads signal worsening credit conditions, reducing investment and output.

9.3 Correlation of Variables with Corporate Spread

The relationship between macroeconomic variables and corporate spreads is summarized in Table 8. Investment and labor supply exhibit a weak negative correlation, while TFP and output show a weak positive correlation. Interest rates and asset prices, however, show a stronger positive correlation with corporate spreads.

A higher corporate spread is generally associated with lower investment and output, reinforcing the idea that financial frictions act as a constraint on economic activity. The positive correlation between TFP and corporate spreads suggests that credit market conditions may be linked to broader productivity expectations. The relative price of investment shows a strong positive correlation with spreads, which aligns with the view that risk premiums in financial markets affect capital costs and investment decisions.

Variable	Correlation with Corporate Spread
$\log Y$ (Output)	-0.2403
$\log C$ (Consumption)	0.1803
$\log I$ (Investment)	-0.2660
$\log r$ (Interest Rate)	-0.0392
$\log q$ (Relative Price)	0.3523
$\log LA$ (Labor Supply)	0.3001
$\log A$ (TFP)	0.4821
$\log H$ (Hours Worked)	-0.2844
$\log rpBANK$ (Corporate Spread)	1.0000

Table 8: Correlation of Simulated Variables with Corporate Spreads

9.4 Comparison with Empirical Data

The simulated business cycle properties align with empirical findings in several ways. Investment is significantly more volatile than output, consistent with observed data. The strong procyclicality of investment and employment is also supported by real-world data, where firms expand hiring and capital expenditures during economic upswings. The negative correlation of corporate spreads with output mirrors historical financial crises, where credit tightening leads to recessions.

However, the negative correlation of consumption with output diverges from standard empirical results, where consumption is typically procyclical, though smoother than output. This discrepancy may be due to model-specific assumptions or the role of shocks affecting consumption preferences independently from output. The moderate correlation between TFP and corporate spreads suggests a link between financial conditions and productivity expectations, though empirical results on this relationship are mixed.

Overall, the model successfully captures key business cycle features, particularly the amplification effects of financial shocks on investment and corporate spreads. The next step in the analysis is to evaluate impulse response functions and conduct further robustness checks to assess the stability of these relationships.

10. Conclusion

This study extends the financial accelerator model of Carlstrom and Fuerst (1997) by incorporating a monitoring cost shock as the primary source of financial frictions and modifying household preferences to better capture labor and consumption trade-offs. Using Bayesian estimation on U.S. data from 1980Q1 to 2019Q4, the results confirm the significance of financial shocks in driving macroeconomic fluctuations, reinforcing the central role of credit frictions in amplifying business cycle dynamics.

Compared to Carlstrom and Fuerst (1997), where agency costs primarily mediate the impact of productivity shocks on investment and borrowing, the introduction of a monitoring cost shock shifts the primary transmission mechanism toward financial markets. The estimated variance decomposition highlights that financial shocks contribute significantly to fluctuations in output, investment, and corporate spreads, overshadowing the influence of productivity shocks in the short run. This suggests that tightening credit conditions, rather than technological fluctuations, plays a dominant role in business cycle volatility.

The alternative preference specification alters household consumption-labor decisions, leading to notable differences in equilibrium labor supply responses. The modified Euler and labor supply equations result in a more elastic labor response, which affects the dynamics of employment and wages following both productivity and financial shocks. Despite these changes, the core insight of Carlstrom and Fuerst (1997)—that financial frictions propagate macroeconomic disturbances—remains robust under this revised framework.

Impulse response functions indicate that financial shocks induce sharper and more persistent declines in investment and economic activity than productivity shocks, mirroring empirical observations from financial crises. The financial accelerator mechanism remains a crucial driver of business cycle fluctuations, but the dominance of financial shocks in this estimation suggests that credit conditions, rather than shifts in firm net worth alone, are central to understanding macroeconomic instability.

In conclusion, this study reinforces the importance of financial frictions in macroeconomic fluctuations while providing a new perspective on their origins. The findings suggest that monitoring costs play a crucial role in shaping financial market dynamics, with significant implications for monetary and regulatory policies aimed at stabilizing credit conditions. Future research could explore the interaction between different financial frictions and alternative monetary transmission mechanisms to further refine the understanding of business cycle dynamics.