# Estimator-Enhanced Feedback-based Optimization with Applications to Power Systems

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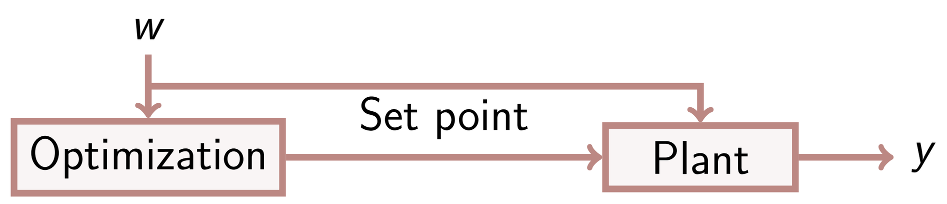
## Background

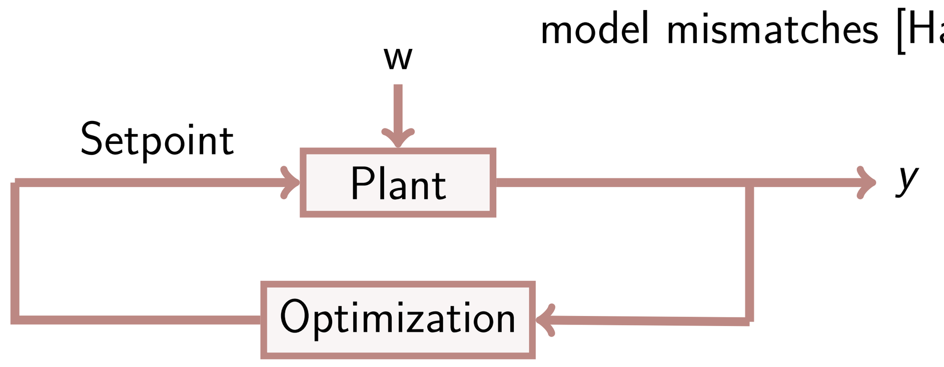
### Offline Optimization

- Model-based
- Requires an estimate or measurement of the Disturbance




### Feedback-based Optimization (Online Optimization)

- Operates autonomously without external set points
- More robust against uncertainties and model mismatches [Hauswirth '24]




## Problem Formulation

### Plant Model

- Consider the stable LTI system, $w$ is *constant*.

$$\dot{x} = Ax + Bu + Ew$$
$$\dot{w} = 0$$
$$y = Cx$$

- Steady-state input-to-output map for fixed $u$ and $w$ is:
- Where $\Pi_u := -CA^{-1}B$ and $\Pi_w := -CA^{-1}E$.

$$\bar{y} = \Pi_u \bar{u} + \Pi_w w$$

### Optimization Problem

- Want to drive the output and input of the plant to the solution of the optimization:

$$\underset{\bar{y},\bar{u}}{\text{minimize}} \quad f(\bar{u}) + g(\bar{y})$$
$$\text{subject to} \quad \bar{y} = \Pi_u \bar{u} + \Pi_w w$$

- The optimization can be written:

$$\min_{\bar{u}} \quad f(\bar{u}) + g(\Pi_u \bar{u} + \Pi_w w)$$

- Solve optimization using:

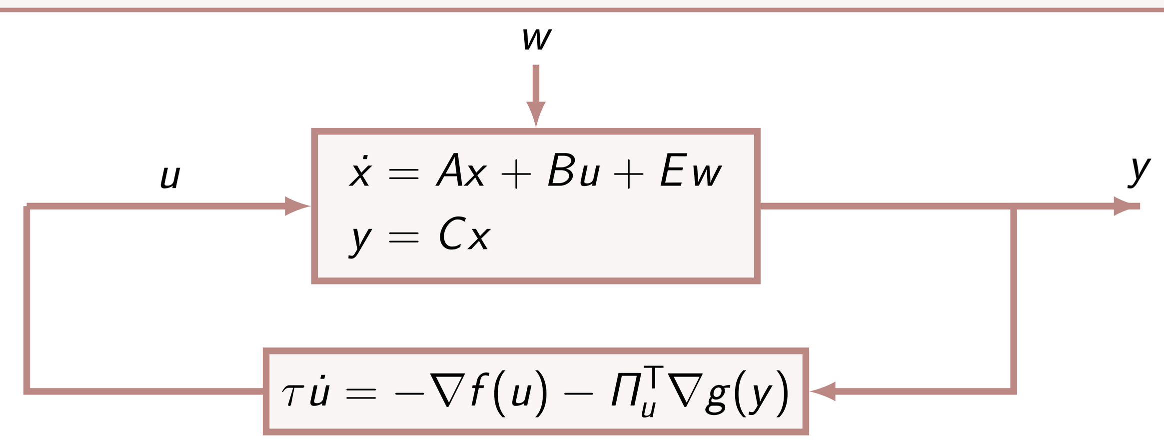
$$\tau \dot{u} = -\nabla f(u) - \Pi_u^{\mathsf{T}} \nabla g(\Pi_u u + \Pi_w w)$$

### Closed-loop System

- **Replace** $\bar{y}$ with the *measured* output value of $y$

$$\tau \dot{u} = -\nabla f(u) - \Pi_u{}^{\mathsf{T}} \nabla g(y)$$

**Closed-loop is stable for sufficiently slow controller** [Menta '18]



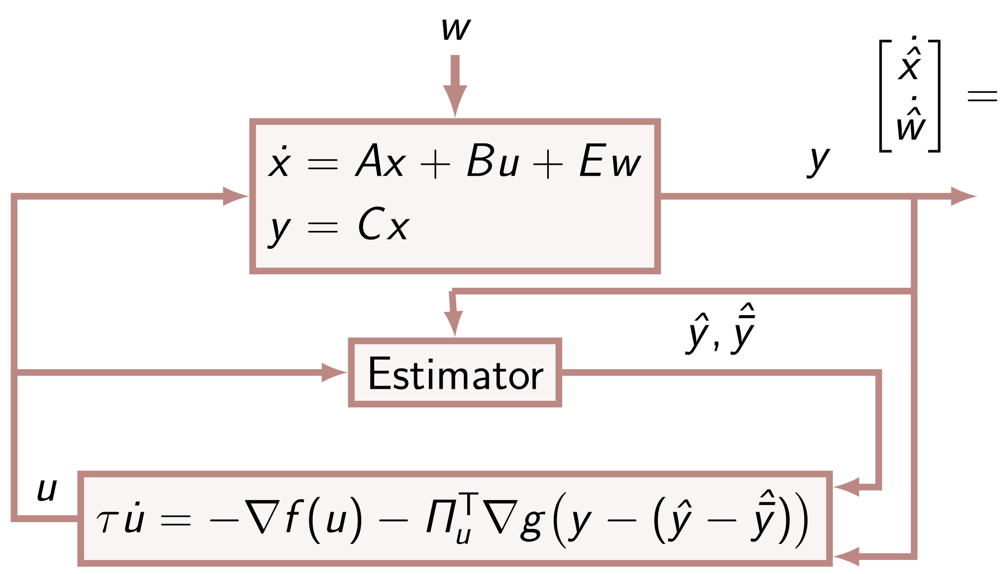

## Motivation and Proposed Design

### Motivation

- The **ideal** implementation assumes access to the **steady-state output** $\bar{y}$
- But $\bar{y}$ is **not directly measurable** and we only measure the $y(t)$

### Goal

- Use further **model information** to produce *estimate* $\hat{\bar{y}}$, then fed into the controller
- The estimator-enhanced feedback-based optimization **(EE-FBO)** design



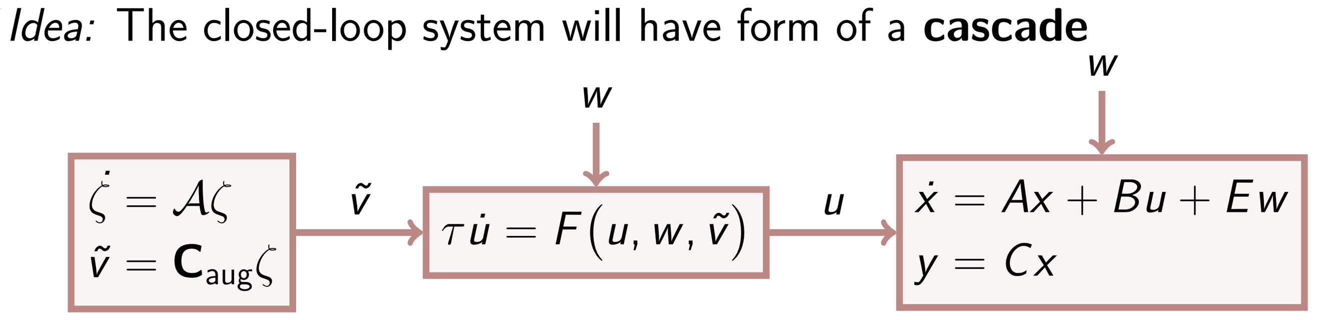

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{w}} \end{bmatrix} = \underbrace{\begin{bmatrix} A & E \\ \mathbb{0} & \mathbb{0} \end{bmatrix}}_{\mathbf{A}_{\text{aug}}} \begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix} + \begin{bmatrix} B \\ \mathbb{0} \end{bmatrix} u + \mathbf{L}(y - \hat{y})$$

$$\hat{y} = \underbrace{\begin{bmatrix} C & \mathbb{0} \end{bmatrix}}_{\mathbf{C}_{\text{aug}}} \begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix}$$

$$\hat{\bar{y}} = \Pi_u u + \Pi_w \hat{w}$$

**Stable under fast controller and estimator tuning; convergence is then limited by the plant dynamics.**

*Proof Idea:* The closed-loop system will have form of a **cascade**



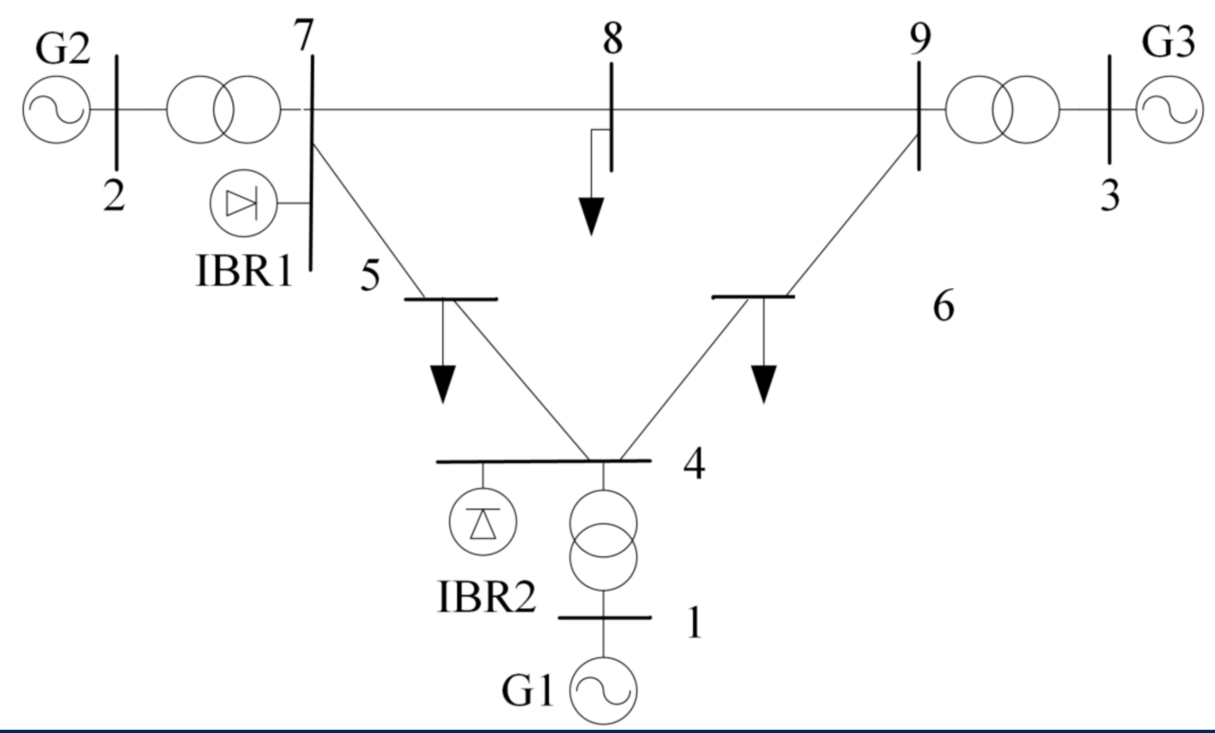

- Select $\mathbf{L}$ such that $\mathcal{A} = \mathbf{A}_{\text{aug}} + \mathbf{L}\mathbf{C}_{\text{aug}}$ is Hurwitz
- Define error variables $\tilde{x} := x - \hat{x}$, $\tilde{w} := w - \hat{w}$, $\tilde{y} := y - \hat{y}$, and consider $\tilde{\zeta} = \begin{bmatrix} \tilde{x} & \tilde{w} \end{bmatrix}^{\mathsf{T}}$

$$\dot{\tilde{\zeta}} = \mathcal{A}\tilde{\zeta}, \qquad \tilde{y} = \mathbf{C}_{\text{aug}}\tilde{\zeta} \qquad \hat{\bar{y}} = \Pi_u u + \Pi_w (w - \tilde{w})$$

$$\tau \dot{u} = -\nabla f(u) - \Pi_u^{\mathsf{T}} \nabla g(\tilde{y} + \hat{\bar{y}}) = -\nabla f(u) - \Pi_u^{\mathsf{T}} \nabla g(\Pi_u u + \Pi_w w + \underbrace{C\tilde{x} - \Pi_w \tilde{w}}_{:=\tilde{v}}) := F(u, \tilde{v})$$

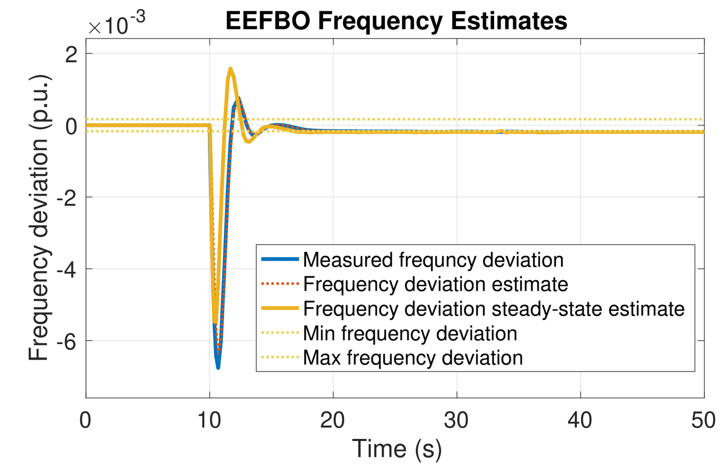
## Power System Application

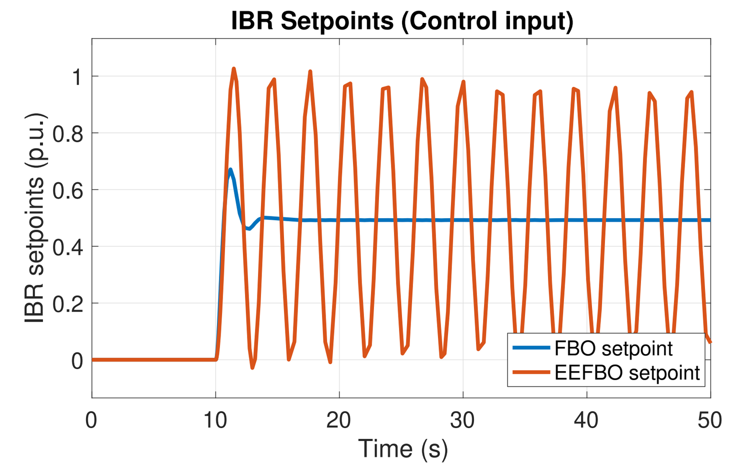
Using the system frequency response (SFR) model, design EE-FBO to compute IBR setpoints that rapidly restore frequency within operational limits

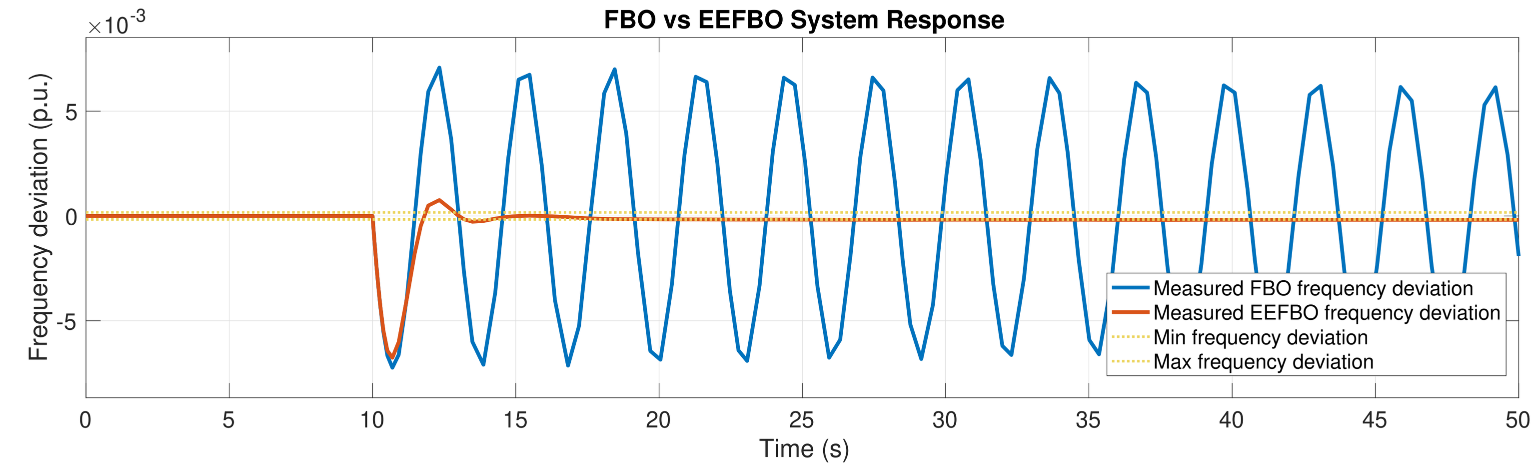



## Simulation Results

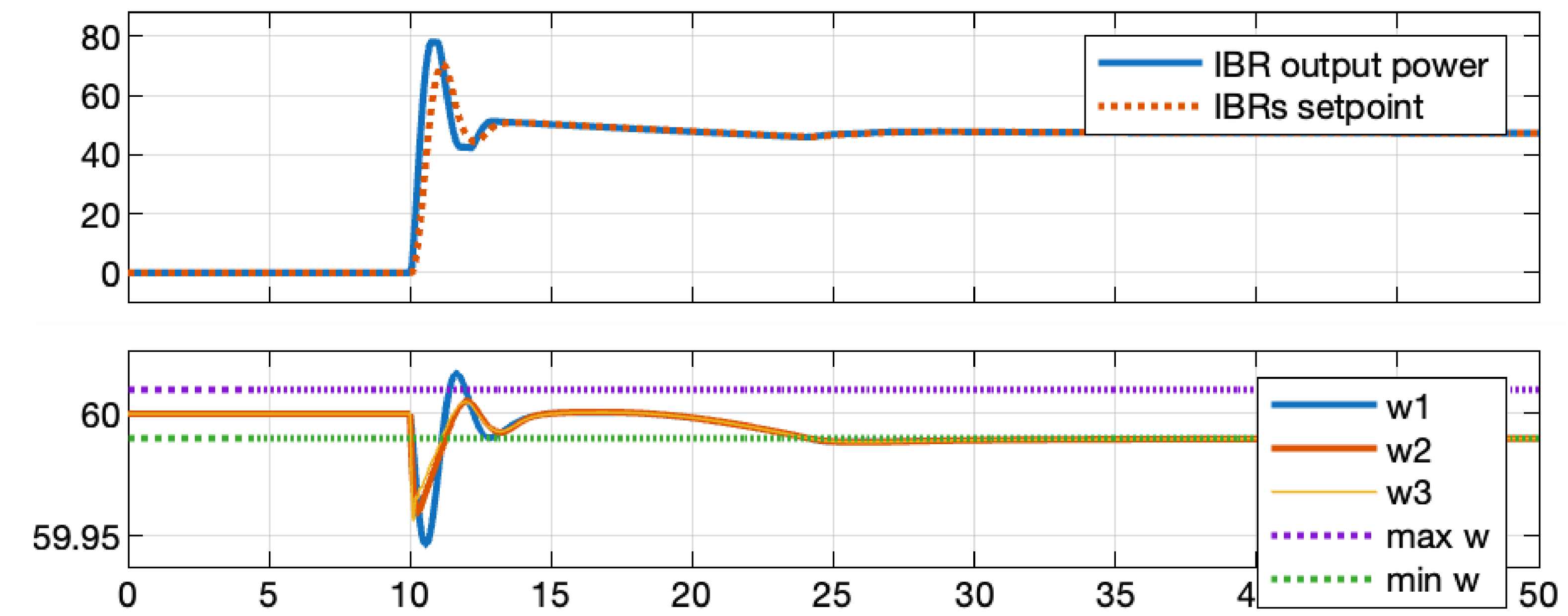
- FBO becomes unstable by decreasing $\tau$ while EE-FBO remains stable




- Real model simulation with small $\tau$ with EEFBO.




## Conclusion

### Summary

- The stability of the closed-loop system depends on the controller's speed, which can be very slow, leading to poor performance
- By incorporating **model information** and designing an estimator, the closed-loop remains stable **without limitations**.

### Future Work

- Extend the results to the **nonlinear plant** [Hauswirth '21]
- Explore alternative estimator designs that reduce reliance on model information in the design process

## References

[Hauswirth '24] Hauswirth, A., He, Z., Bolognani, S., Hug, G., Dorfler, F. (2024). Optimization algorithms as robust feedback controllers. Annual Reviews in Control, 57:100941. DOI: 10.1016/j.arcontrol.2024.100941.

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