Estimator-based Performance Enhancement for Feedback-based Optimization

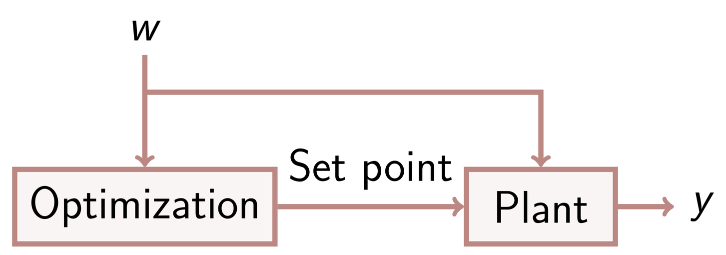
Niloufar Yousefi and John W. Simpson-Porco

The Edward S. Rogers Sr. Department of Electrical and Computer Engineering, University of Toronto

Background

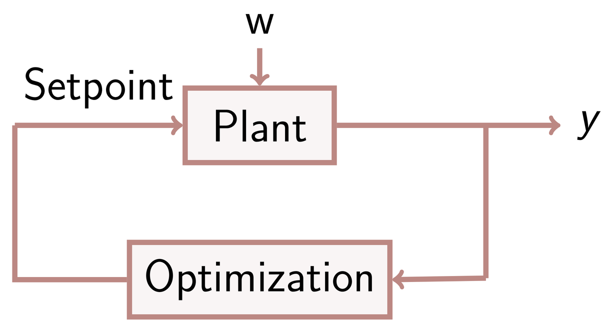
Offline Optimization

- Model-based
- Requires an estimate or measurement of the Disturbance



Feedback-based Optimization (Online Optimization)

- Operates autonomously without external set points
- More robust against uncertainties and model mismatches [Hauswirth '24]



Problem Formulation

Plant Model

- Considering the LTI system where w is a *constant* disturbance with unknown initial condition

$$\begin{aligned}\dot{x} &= Ax + Bu + Ew\\ \dot{w} &= 0\\ y &= Cx\end{aligned} \qquad (1)$$

- Considering A is Hurwitz the steady-state input-to-output map for fixed u and w is:

$$\bar{y} = \Pi_u \bar{u} + \Pi_w w$$

- Where $\Pi_u := -CA^{-1}B$ and $\Pi_w := -CA^{-1}E$.

Optimization Problem

- Want to drive the output and control input of (1) to the solution of the optimization problem:

$$\begin{aligned}\underset{\bar{y},\bar{u}}{\text{minimize}} \quad & f(\bar{u}) + g(\bar{y})\\ \text{subject to} \quad & \bar{y} = \Pi_u \bar{u} + \Pi_w w\end{aligned} \qquad (2)$$

- The optimization can be written:

$$\min_{\bar{u}} \quad f(\bar{u}) + g(\Pi_u \bar{u} + \Pi_w w)$$

- Solve optimization using:

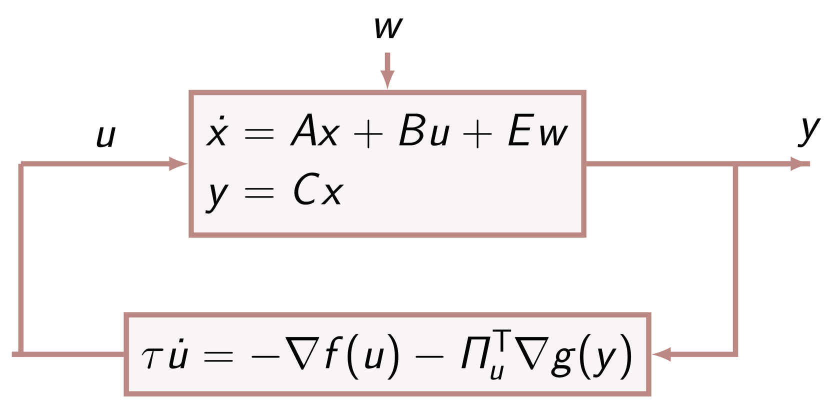
$$\tau \dot{u} = -\nabla f(u) - \Pi_u^{\mathsf{T}} \nabla g(\Pi_u u + \Pi_w w)$$

Closed-loop System

- **Replace** $\bar{y}$ with the *measured* output value of y

$$\tau \dot{u} = -\nabla f(u) - \Pi_u{}^{\mathsf{T}} \nabla g(y) \qquad (3)$$

Closed-loop is stable for sufficiently large τ [Menta '18]



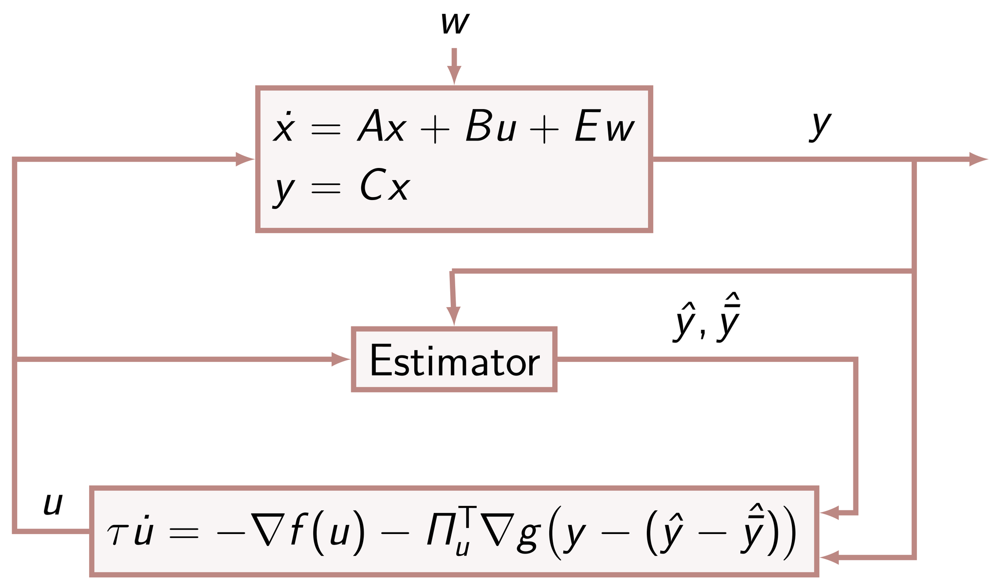
Motivation and Proposed Design

Motivation

- The **ideal** implementation assumes access to the **steady-state output** $\bar{y}$
- But $\bar{y}$ is **not directly measurable** and we only measure the $y(t)$

Goal

- Using further **model information** to produce an *estimate* $\hat{\bar{y}}$, which can be fed into the controller
- The estimator-enhanced feedback-based optimization **(EE-FBO)** design, replacing (3)



$$\begin{bmatrix}\dot{\hat{x}}\\ \dot{\hat{w}}\end{bmatrix} = \underbrace{\begin{bmatrix}A & E\\ \mathbb{0} & \mathbb{0}\end{bmatrix}}_{\mathbf{A}_{\text{aug}}} \begin{bmatrix}\hat{x}\\ \hat{w}\end{bmatrix} + \begin{bmatrix}B\\ \mathbb{0}\end{bmatrix} u + \mathbf{L}(y - \hat{y})$$

$$\hat{y} = \underbrace{\begin{bmatrix}C & \mathbb{0}\end{bmatrix}}_{\mathbf{C}_{\text{aug}}} \begin{bmatrix}\hat{x}\\ \hat{w}\end{bmatrix} \qquad \hat{\bar{y}} = \Pi_u u + \Pi_w \hat{w}$$

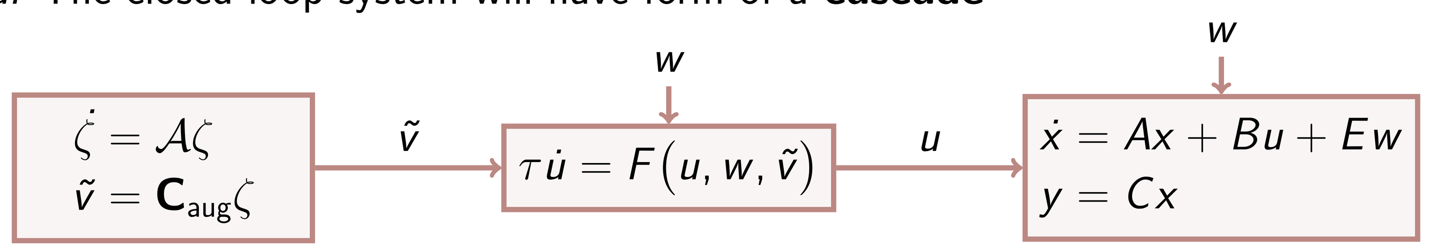
Stability and Convergence Rate

New closed-loop system possesses a unique globally exponentially stable equilibrium point with a corresponding output

No limitations are required on the tuning parameter τ

The convergence rate is limited by the convergence rate of the open-loop plant

Proof Idea: The closed-loop system will have form of a **cascade**



- Select $\mathbf{L}$ such that $\mathcal{A} = \mathbf{A}_{\text{aug}} + \mathbf{L}\mathbf{C}_{\text{aug}}$ is Hurwitz
- Define error variables $\tilde{x} := x - \hat{x}$, $\tilde{w} := w - \hat{w}$, $\tilde{y} := y - \hat{y}$, and consider $\tilde{\zeta} = \begin{bmatrix}\tilde{x} & \tilde{w}\end{bmatrix}^{\mathsf{T}}$

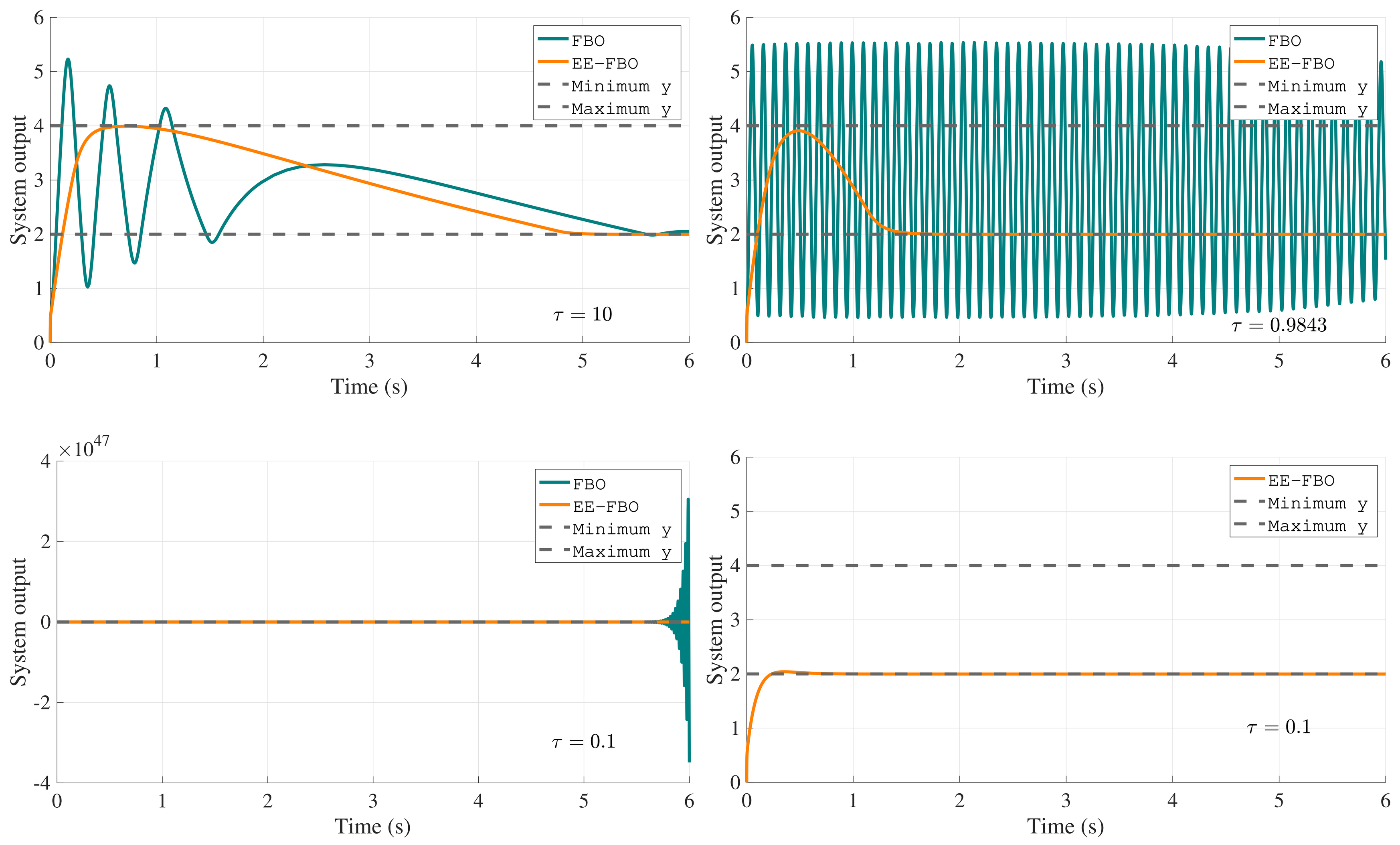
$$\dot{\tilde{\zeta}} = \mathcal{A}\tilde{\zeta}, \qquad \tilde{y} = \mathbf{C}_{\text{aug}}\tilde{\zeta} \qquad \hat{\bar{y}} = \Pi_u u + \Pi_w(w - \tilde{w})$$

$$\tau\dot{u} = -\nabla f(u) - \Pi_u^{\mathsf{T}} \nabla g(\tilde{y} + \hat{\bar{y}}) = -\nabla f(u) - \Pi_u^{\mathsf{T}} \nabla g(\Pi_u u + \Pi_w w + \underbrace{C\tilde{x} - \Pi_w \tilde{w}}_{:=\tilde{v}}) := F(u, \tilde{v})$$

Simulation Results

Stability and convergence rate of EE-FBO compared to FBO

- FBO becomes unstable by decreasing τ while EE-FBO remains stable



Conclusion

Summary

- The stability of the closed-loop system (1) and (3) depends on τ^*, which can be very large, leading to poor performance
- By incorporating **model information** and designing an estimator, the closed-loop remains stable **without limitations on** τ.

Future Work

- Extend the results to the **nonlinear plant** [Hauswirth '21]
- Explore alternative estimator designs that reduce reliance on model information in the design process

References

[Hauswirth '24] Hauswirth, A., He, Z., Bolognani, S., Hug, G., Dorfler, F. (2024). Optimization algorithms as robust feedback controllers. Annual Reviews in Control, 57:100941. DOI: 10.1016/j.arcontrol.2024.100941.

[Menta '18] Hauswirth, B., Bolognani, S., Hug, A., Dorfler, F., Menta, A. (2018). Stability of Dynamic Feedback Optimization with Applications to Power Systems. Fifty-sixth Annual Allerton Conference on Communication, Control, and Computing, held at Allerton House, Monticello, Illinois, October 2-5, 2018. ISBN: 9781538665961.

[Hauswirth '21] Hauswirth, B., Bolognani, S., Hug, A., Dorfler, F. (2021). Timescale Separation in Autonomous Optimization. IEEE Transactions on Automatic Control, 66(2):611-624. DOI: 10.1109/TAC.2020.2989274.

Contact: niloufar.yousefi@utoronto.ca