D MS115
· We can use truth tables to prove
De Morgan's hows
$\text{P not}(P \land Q) = \text{not} P \vee \text{not} Q$
(i) not (PVQ) = not P 1 not Q
Let's prove (?):
PQ not P not Q PrQ not (PrQ) not Printe TFFFT FFTT FFTT FTT
F F T T F T T
Hence $not(P \cap Q) \equiv not P \vee not Q$ $logically equivalent to$
oo exercise.
· Per the tutorial sheet (1), a proposition is logically true if it is true in all cases
eg. the proposition P v not P is logically true:

P not P P v not P
T F T Recall: The conditional operator P=>Q
states that if P is true
then Q is also true. It has truth table PQPDQ TTFF FF For 2 given propositions P and Q, we will usn't to prove that P > Q is logically true. Why? Then we know that if P is true, then Q is true. How do we show P > Q is logically true for 2 given propositions P/2 Q?

Strotogy: Show the case where
P is true & Q is false count occur. This gives as 3 methods of orgunent: Direct argument: Assume P is true, show that Q is true.

· Contrapositive argument » Assume that Q is talse, show that P is talse (Recall: the contrapositive of P => Q
is the logically equivalent statement
that not Q => not P « Proof by contradiction s Assume P is true and Q is false and derive a contradiction. Examples: An integer is a whole number that its zero, positive or negotive (0, ±1, ±2,000) An integer x is even if there is

on integer x such that

An integer y is odd if y = 2n + (for some integer n Direct argument to show that x and y odd integers >> xy is odd · Let x = 2n+1 and y = 2m+1 for some integers In and m Then xy = (2n+1)(2m+1)= 4nm + 2n + 2m + 1= 2(2nm + n + m) + 1

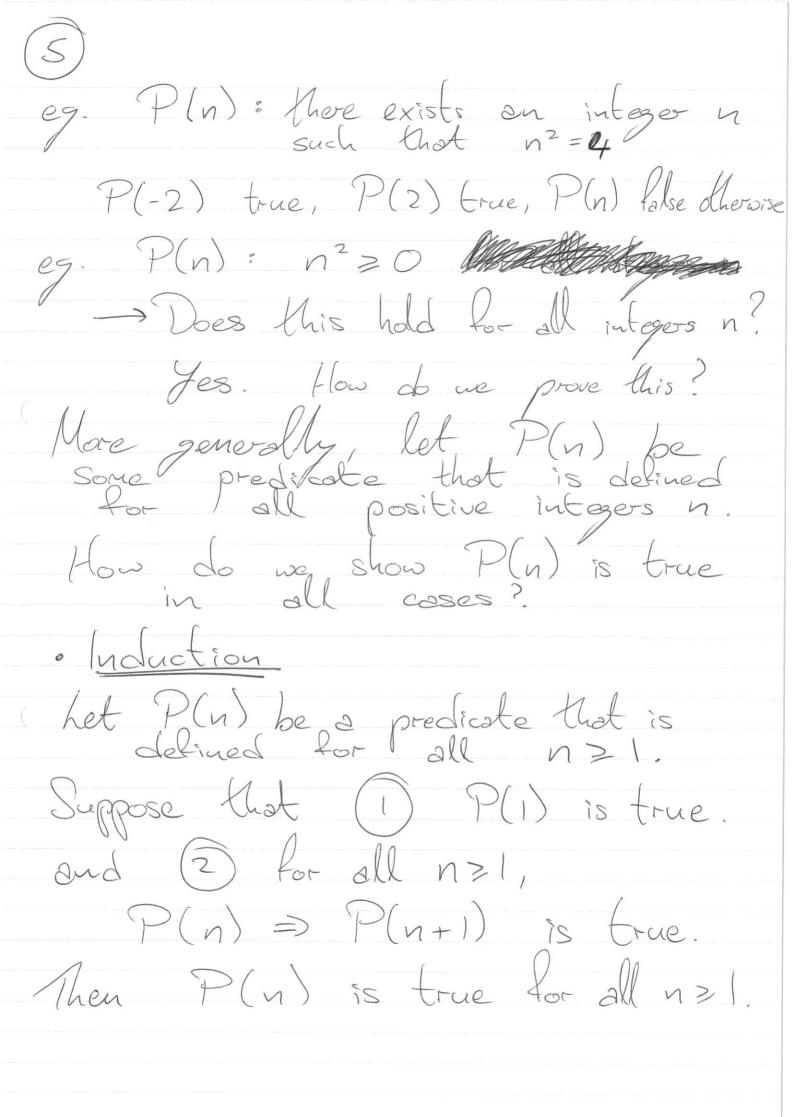
Contrapositive argument:

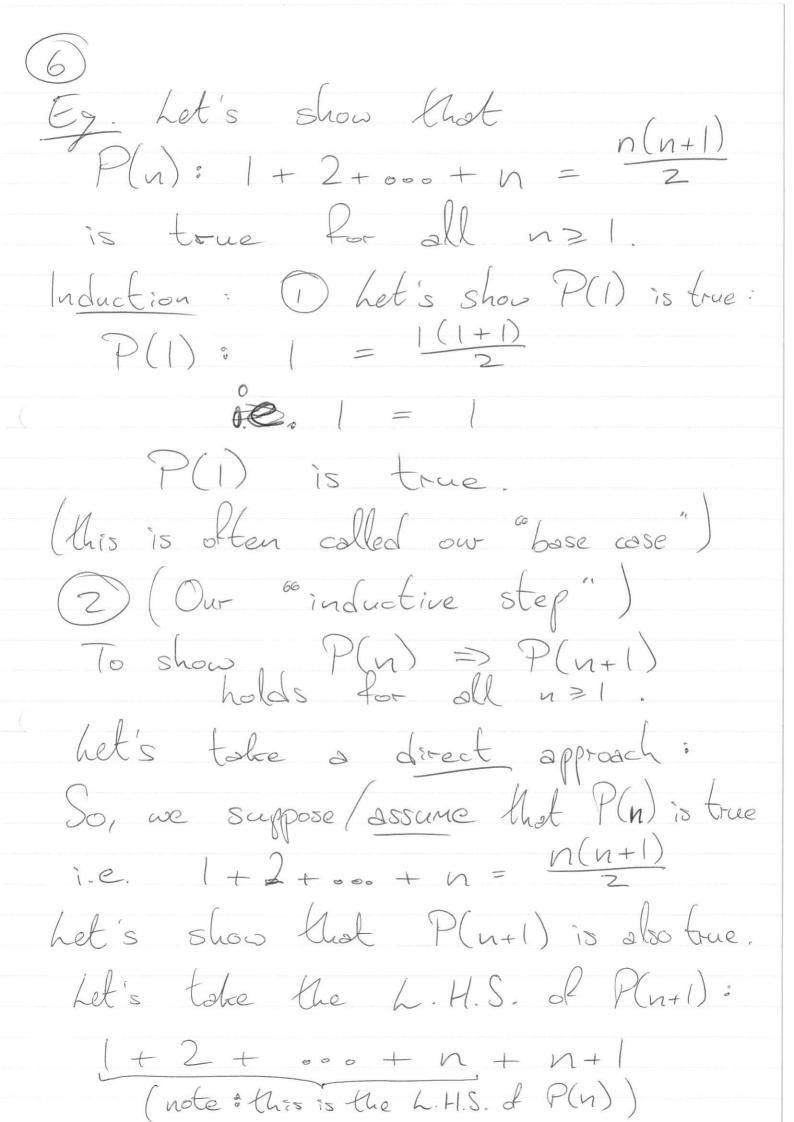
het's show that x^2 odd $\Rightarrow x$ odd

by showing x even $\Rightarrow x^2$ even

whet x = 2k for some integer

1 2 (1) (21) = $2(2k^2)$ Then $x^2 = (2k)(2k) = 2(2k^2)$ · Prod by contradiction read a proof of the fact that x is not a fraction if $x^2 = 2$. -> read a proof that there are infinitely many prime numbers. A statement that contains variables can either be true or false depending on the value of the variables. Notwal application: while (9 < 10)
These statements are called predicates. When we fix the value of the variable we have a proposition that false. eg. P(n) on is an integer greater than 3 P(-1) is false, P(3) is talse, P(n) is true for all n > 3





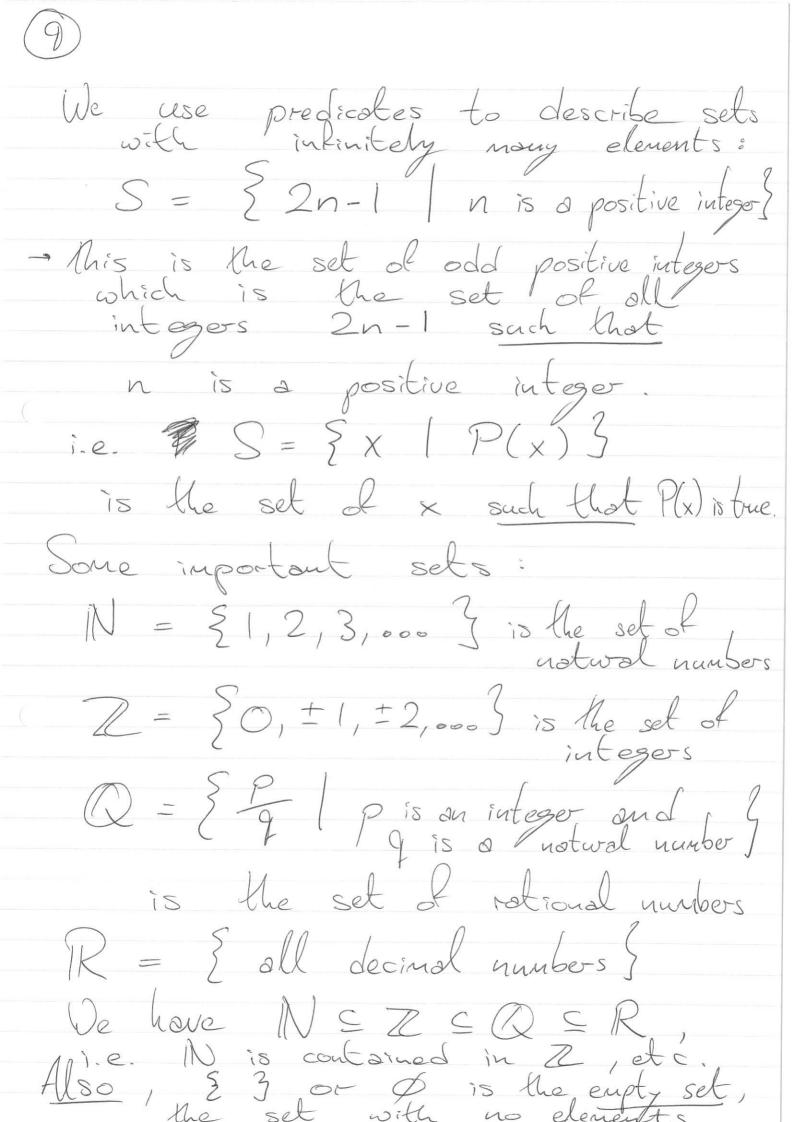
Thus 1+2+000 + n+1 $=\frac{n(n+1)}{2}+n+1, 2s P(n) is true$ = n(n+1) + 2(n+1) $= \frac{(n+1)(n+2)}{2}$ This is the RHS of P(n+1), i.e. P(n+1) is true. Hence P(n) is true for all n 21. Eg. 2 Let's prove that

7n-1 is divisible by 6

Induction

Our P(n) is 6 | 7n-1 so P(1) is 6/7-1=6: true. 2) Assume P(n) is true, i.e. 617^-1. het's show P(n+1) is true, i.e. 6/7ⁿ⁺¹-1

het's show 7"-1 is a multiple of 6: $7^{n+1}-1=7^n,7-1$ $= (7^{1}-1).7+6$ As P(n) is true, $7^n-1=6k$ for some kHence $7^{n+1}-1=(7^n-1)7+6$ = (6k) 7 + 6= 6(7k) + 6= 6(7k+1).P(n+1) is true. Thus P(n) is true for all n21. Hence Sets For us, a set will mean a collection of objects, collect elements. We can list the elements and use curly brackets to show we're dealing with a set, eg. the set S might be $S = \frac{2}{2}1, 2, 4$



 $\frac{3}{3} = \frac{3}{2} \times \left[\times \neq \times \right]$ We say a set A is a subset
of A is contained in B, i.e. Xe A => xe B. if x is on element of A, then x is on element of B When we two sets equal?
When they have the Frame elements.
So, we can show two sets , are equal by showing ASB and BSA. eg. let A = \frac{2}{n} n is an odd integer \frac{5}{2} I B = { n | n is an odd integer }. Then A = B (Why? hook back) · We'll set set operations next.