MS321 Tutorial 2

1. Show that any permutation in S_n can be expressed as a product of the elements from the set of transpositions of adjacent numbers

$$\{(1,2),(2,3),(3,4),\ldots,(n-1,n)\}.$$

(Example: For $n=4$, the transpositions are $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$, while the transpositions of adjacent numbers are $\{(1,2),(2,3),(3,4)\}$)
(Hint: We already know that any permutation in S_n can be expressed as a product of general transpositions. Now write a general transposition in terms of transpositions of adjacent numbers.)

2. Express $(135)(264)$ as a product of transpositions of adjacent numbers.

3. Can elements of S_3 be expressed as a product just involving the two 3-cycles $(1,2,3)$ and $(1,3,2)$? How about S_n and the set of all three cycles?

4. In how many ways can the permutation $(1,2,3,4)$ be factored as a product of three transpositions?