

**MS321 Algebra, tutorial 8, question 3**

3. Suppose that  $H$  is a normal subgroup of  $G$ . Show that  $G/H$  is abelian if and only if

$$g_1 g_2 g_1^{-1} g_2^{-1} \in H, \text{ for any } g_1, g_2 \in G.$$

Suppose  $g_1, g_2$  are two elements in  $G$ . Then

$$\begin{aligned} g_1 g_2 g_1^{-1} g_2^{-1} \in H &\Leftrightarrow g_1 g_2 (g_2 g_1)^{-1} \in H \\ &\Leftrightarrow g_1 g_2 \in H (g_2 g_1) \\ &\Leftrightarrow g_1 g_2 \in (g_2 g_1) H \quad (\text{since } H \triangleleft G) \\ &\Leftrightarrow (g_1 g_2) H = (g_2 g_1) H \\ &\Leftrightarrow g_1 H g_2 H = g_2 H g_1 H \end{aligned}$$

which is equivalent to  $G/H$  being abelian.