MS115 Mathematics for Enterprise Computing
First in-class test - Answer ALL questions

Name	Student Id

1. In each part of this question, circle the number of **one** correct answer.

   Marking scheme for Question 1: $+3$ for each correct answer; -1 for each incorrect answer; 0 marks for no answer or an unclear answer.

   (a) Two sets A and B are equal when

       (i) $A \cap B \subseteq A$;

       (ii) $A \subseteq B$ and $B \subseteq A$;

       (iii) $A \cap B = \emptyset$;

       (iv) $A \cap B \neq \emptyset$.

   (b) Sets A, B and C are pairwise disjoint if

       (i) $A \cap B = C$;

       (ii) $A \cap B \cap C = \emptyset$;

       (iii) $A \cap B \cap C = (A \cap B) \cap C$;

       (iv) $A \cap B = \emptyset$, $B \cap C = \emptyset$ and $A \cap C = \emptyset$.

   (c) A *function* from a set A to a set B is a relation on A and B that

       (i) relates some element of A to some element of B;

       (ii) relates exactly one element of A to some element of B;

       (iii) relates each element of A to exactly one element of B;

       (iv) relates every element of A to more than one element of B.

   (d) A *function* from a set A to a set B is invertible if

       (i) $|A| \geq |B|$;

       (ii) no element of A is related to more than one element of B;

       (iii) every element of B is the image of exactly one element of A;

       (iv) every element of B is the image of at least one element of A.

   (e) The relation $\leq$ on the set of integers $\mathbb{Z}$ is

       (i) reflexive, symmetric and transitive;

       (ii) reflexive and symmetric, but not transitive;

       (iii) reflexive and transitive, but not symmetric;

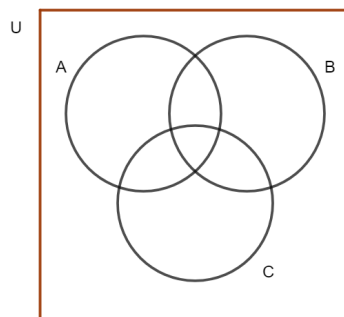 (iv) symmetric and transitive, but not reflexive.

2. Compute the truth table of the following compound proposition:
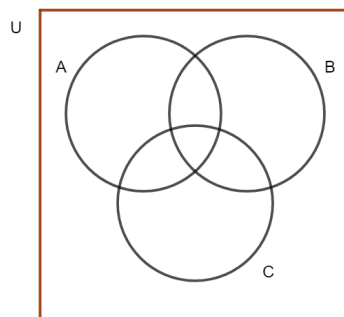
$$(P \Rightarrow Q) \vee (P \vee Q).$$

[10 marks]

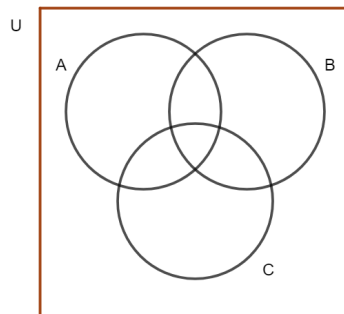3. In each case, shade the relevant region of the given Venn diagram:

(i) $(A \cap B) \cup C$

(ii) $(A \cap B) \cap \overline{C}$

(iii) $(\overline{A} \cap \overline{B}) \cap C$

[6 marks]

4. Consider two sets A and B within a universal set U. Determine the number of elements in $A \cap B$ given the following set cardinalities:
$$|U| = 80, \qquad |\overline{A}| = 50, \qquad |\overline{B}| = 40, \qquad |\overline{(A \cup B)}| = 30.$$
(You may find it helpful to sketch a Venn diagram) [6 marks]

5. For $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, consider the equivalence relation R on A defined as follows:
$$x R y \text{ exactly when } y - x = 3k \text{ for some } k \in \mathbb{Z}.$$
Write down the distinct equivalence classes that form a partition of A.

(Note: For $x \in A$, the equivalence class of x is the set $E_x = \{y \in A \mid y R x\}$).

[7 marks]

6. Consider the function
$$f(x) = \frac{x + 1}{2x - 2}$$

(i) Let its domain be the largest possible subset of $\mathbb{R}$. Describe this set.

(ii) Show that $\frac{1}{2}$ is not an element of the range of f.

[6 marks]