MS115 Mathematics for Enterprise Computing Tutorial Sheet 1 Solutions

	P	Q	R	$P \lor Q$	$Q \vee R$	$(P \lor Q)$	$\vee R$	$P \vee (Q$	$\vee R)$		
	T	T	T	T	T	T		T			
	$\mid T \mid$	T	F	T	T	T		T			
	$\mid T \mid$	F	T	T	T	T		T			
1. <i>(i)</i>	$\mid T \mid$	F	F	T	F	T		T			
	F	T	T	T	T	T		T			
	F	T	F	T	T	T		T			
	F	F	T	F	T	T		T			
	F	F	F	F	F	F		F			
	P	\overline{Q}	R	$P \wedge Q$	$Q \wedge R$	$(P \wedge Q)$	$\wedge R$	$P \wedge (Q$	$\wedge R$)		
	T	T	T	T	T	T		T			
	$\mid T \mid$	T	F	T	F	F		F			
	T	F	T	F	F	F		F			
(ii)	T	F	F	F	F	F		F			
()	F	T	T	F	T	F		F			
	F	T	F	F	F	F		F			
	F	F	T	F	F	F		F			
	F	F	F	F	F	F		F			
	P	\overline{Q}	R	$Q \wedge R$	$P \lor Q$	$P \vee R$	$P \vee ($	$Q \wedge R$	$(P \lor$	$\overline{Q}) \wedge (P$	$(\vee R)$
	P	Q T	$R \over T$	$Q \wedge R$ T	$P \lor Q$ T	$P \lor R$ T	$P \vee ($	$Q \wedge R$	$(P \lor$	$Q) \wedge (P)$ T	$P \vee R$)
				-			$P \vee ($		$(P \lor$		$P \vee R$
	T	T	T	T	T	T		\overline{T}	$(P \lor$	T	$P \vee R$)
(iii)	T T	T T	T F	T F	$T \ T$	$T \ T$		T T	$(P \lor$	T T T T	$(V \setminus R)$
(iii)	$egin{array}{c} T \ T \ T \ T \ F \end{array}$	T T F	T F T	T F F	$T \ T \ T$	$T \ T \ T$		T T T	$(P \vee$	T T T T T	$P \vee R$
(iii)	$egin{array}{c} T \\ T \\ T \\ T \end{array}$	T T F F	T F T F	T F F	T T T	$T \ T \ T \ T$		T T T	$(P \lor$	T T T T	$P \vee R$
(iii)	$egin{array}{c} T \\ T \\ T \\ F \\ F \end{array}$	T T F T	T F T F T	T F F T	T T T T	T T T T		T T T T	$(P \lor$	T T T T T	$(V \vee R)$
(iii)	$\begin{bmatrix} T \\ T \\ T \\ F \\ F \end{bmatrix}$	T T F T T T	T F T F T F	T F F T T	T T T T T	T T T T T F		T T T T T	(<i>P</i> ∨	T T T T T F	$(V \lor R)$
(iii)	$egin{array}{c} T \\ T \\ T \\ F \\ F \end{array}$	T F T	T F T F T F F T F	T F F F T F F F F F F F F F F F F F F F	T T T T T F F	T T T T T F T F		T T T T T F F		T T T T F F	,
(iii)	T T T F F F	T F T T T T T T T T	$egin{array}{cccc} T & & & & & & & & & & & & & & & & & & $	T F F T F	T T T T T T	T T T T F		T T T T T F	$(P \lor \\ (P \land \\$	T T T T F F	$(Y \wedge R)$
(iii)	T T T F F F F	T F F T F F F	T F T F T F T F R	T F F T F F F	T T T T T F F	T T T T F T F		T T T T T F F F		T T T T F F F $Q) \lor (P$,
(iii)	T T T F F F T T	T T F T T F F T	T F T F T F T F T T T T T T T T T T T T	T F F T F F F T T T T	T T T T T F F T	T T T T F T F T F		T T T T F F T T		$ \begin{array}{c} T \\ T \\ T \\ T \\ F \\ F \\ \hline Q) \lor (P \\ T \end{array} $,
	T	$ \begin{array}{c} T \\ F \\ F \\ T \\ F \\ F \end{array} $	T F T F T F T F T F T F F F F F T F	T F F T F F T T	T T T T F F T T T T T T T T	T T T T F T F T		T T T T F F F T T T T		T T T T F F F T T T T T	,
(iii) (iv)	T	T T F T T F F T T F F	T F T F T F T F T	T F F T F F T T T	T T T T T F F T	T T T T F T F T F		T T T T F F F T T T T T		$ \begin{array}{c} T \\ T \\ T \\ T \\ F \\ F \\ \hline Q) \lor (P \\ T \\ T \end{array} $,
	T T T F F F T T T T T F F F F F F T T T F F F F F T	$\begin{array}{c} T \\ F \\ F \\ T \\ T \\ F \\ \end{array}$	T F T F T F T F T F T F T F F T F T F T	T F F F F F T T T T F	T T T T F F T	$egin{array}{cccc} T & T & T & T & T & T & T & T & T & T $	$P \wedge ($	T T T T T F F T		T T T T F F T	,
	T	$\begin{array}{c} T \\ F \\ F \\ T \\ T \\ F \\ F \\ \end{array}$	T F T F T F T F T F T F T T F T T T T T	T F F F F F T T T T T T	T T T T F F T	T T T T F T	$P \wedge ($	T T T T F F T		$ \begin{array}{c} T \\ T \\ T \\ T \\ F \\ F \\ \hline Q) \lor (P \\ \hline T \\ T \\ F \\ F \end{array} $,

Note. In class, we used the expression $P \vee Q \vee R$ without using brackets. This is valid because of the "associativity" property of the OR operator, established in (i).

	P	Q	not P	$P \Rightarrow Q$	$(\text{not } P) \vee Q$
	T	T	F	T	T
2.	T	F	F	F	F
	F	T	T	T	T
	F	F	T	T	T

3. (i) One example is the following:

P: Today is Friday.

Q: Tomorrow is not a school day.

Clearly $P \Rightarrow Q$ is true, but $Q \Rightarrow P$ is not true in every case: today being Saturday is a case where Q holds.

Another example is the following:

P: x is an even number.

Q: 2x is an even number.

Clearly $P \Rightarrow Q$ is true for every even number x: if x = 2k for some k then 2x = 2(2k), which is even.

However, $Q \Rightarrow P$ is not true in every case: 2(3) is even but 3 is not even.

- (ii) Considering the associated truth tables, we note that if $Q \Rightarrow P$ is not logically true, then we must be in the situation where Q is true and P is false. Moreover, if P is false, we are guaranteed that $P \Rightarrow Q$ is logically true.
- 4. (i) Great Danes are large dogs and I have a Great Dane, or I have lots of money and I do not have a Great Dane.
 - (ii) Here's an argument:
 - As $(P \wedge R) \vee (Q \wedge \text{not } R)$ is false, we know that $P \wedge R$ is false and $Q \wedge \text{not } R$ is false.
 - As $P \wedge R$ is false and P is true (we've assumed this), it follows that R is false.
 - As R is false, not R is true.
 - As $Q \wedge \text{not } R$ is false, it follows that Q is false. Thus R is false and Q is false.