MS 221 — Homework Set (4)

(Partial Derivatives and The Chain Rule)

QUESTION 1

In the case of the function $f(x,\, y,\, z) = x^2 y - x y^2 z + z^3$, and the point $p = (x,\, y,\, z)$, calculate the following:

$$\frac{\partial f}{\partial x}(p), \quad \frac{\partial f}{\partial y}(p), \quad \frac{\partial f}{\partial z}(p), \quad \frac{\partial^2 f}{\partial x^2}(p), \quad \frac{\partial^2 f}{\partial x \partial y}(p) \quad \text{and} \quad \frac{\partial^2 f}{\partial y \partial x}(p)$$

QUESTION 2

If f is again the function given in Question 1 calculate

$$\frac{\partial f}{\partial y}(p) \quad \text{and} \quad \frac{\partial^2 f}{\partial x \partial y}(p) \quad \text{where the point} \quad p = (-1,\, 0,\, 3)$$

QUESTION 3

Given that $\dfrac{d}{du} \tan^{-1} u = \dfrac{1}{1+u^2}$ calculate

$$\frac{\partial}{\partial x} \tan^{-1}\left(\frac{y}{x}\right) \quad \text{and} \quad \frac{\partial}{\partial y} \tan^{-1}\left(\frac{y}{x}\right)$$

QUESTION 4

Consider the function

$$f(x,\, y) = \frac{xy}{x^2 + y^2} \quad \text{defined for all} \quad (x,\, y) \neq (0,\, 0).$$

In each of the following, investigate the behaviour of $f(p)$ as p approaches the origin:

(a) along the line $y = 2x$

(b) along the line $y = 3x$

(c) along any line $y = mx$.

What can be said about the existence or otherwise of $\lim_{p \to 0} f(p)$?

QUESTION 5

Consider the function

$$f(x,\,y) = \frac{x^2 y}{x^4 + y^2} \quad \text{defined for all} \quad (x,\,y) \neq (0,\,0).$$

In each of the following, investigate the behaviour of $f(p)$ as p approaches the origin:

(a) along any line $y = mx$

(b) along any parabola $y = mx^2$

What can be said about the existence or otherwise of $\lim_{p\to 0} f(p)$?

QUESTION 6

In the case of differentiable maps

$$\gamma : \boldsymbol{R} \to \boldsymbol{R}^3 : t \mapsto \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} \quad \text{and} \quad f : \boldsymbol{R}^3 \to \boldsymbol{R} : \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto f(x,\,y,\,z)$$

express the derivative $\dfrac{d}{dt} f(\,x(t),\,y(t),\,z(t)\,)$ in terms of the **Chain Rule**.

QUESTION 7

Let the point $\boldsymbol{p}$ and the curve γ be given by

$$\boldsymbol{p} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \gamma(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} t^2 - 4 \\ t \\ t^3 + 1 \end{bmatrix} \quad \forall\ t \in \boldsymbol{R}.$$

If the map $f : \boldsymbol{R}^3 \to \boldsymbol{R} : (x,\,y,\,z) \mapsto f(x,\,y,\,z)$ satisfies

$$\frac{\partial f}{\partial x}(\boldsymbol{p}) = -1, \quad \frac{\partial f}{\partial y}(\boldsymbol{p}) = 2, \quad \frac{\partial f}{\partial z}(\boldsymbol{p}) = 5.$$

calculate $\dfrac{d}{dt} f(\gamma(t))$ at $t = 1$.