- 1 **Title**: Enhancing Cardiovascular Monitoring: A Non-Linear Model for Characterizing RR
- 2 Interval Fluctuations in Exercise and Recovery.
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Abstract

- 20 This work aimed to develop and validate a novel non-linear model to characterize RR 21 interval (RRi) time-dependent fluctuations throughout a rest-exercise-recovery protocol, 22 offering a more precise and physiologically relevant representation of cardiac autonomic 23 responses than traditional HRV metrics or linear approaches. Using data from a cohort of 24 272 elderly participants, the model employs logistic functions to capture the non-stationary 25 and transient nature of RRi time-dependent fluctuations, with parameter estimation 26 achieved via Hamiltonian Monte Carlo. Sobol sensitivity analysis identified baseline RRi 27 (α) and recovery proportion (c) as the primary drivers of variability, underscoring their 28 critical roles in autonomic regulation and resilience. Validation against real-world RRi data demonstrated robust model performance ($R^2 = 0.868$, $CI_{95\%}[0.834, 0.895]$ and Root Mean 29 30 Square Error [RMSE] = 32.6 ms, CI_{95%}[30.01, 35.77]), accurately reflecting autonomic 31 recovery and exercise-induced fluctuations. By advancing real-time cardiovascular 32 assessments, this framework holds significant potential for clinical applications in 33 rehabilitation and cardiovascular monitoring in athletic contexts to optimize performance 34 and recovery. These findings highlight the model's ability to provide precise, 35 physiologically relevant assessments of autonomic function, paving the way for its use in 36 personalized health monitoring and performance optimization across diverse populations.
- 37 **Keywords**: Heart Rate Variability, Exercise Physiology, Autonomic Nervous System,
- 38 Cardiovascular System, Models, Theoretical, Logistic Models.

Introduction

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41 The human cardiovascular system exhibits intricate dynamic responses to physical exertion, 42 reflecting the complex interplay between the autonomic nervous system (ANS) and cardiac 43 function. Understanding these time-dependent fluctuations is crucial for assessing 44 physiological adaptation to exercise, optimizing athletic performance, and evaluating 45 cardiovascular health¹⁻³. R-R intervals (RRi), representing the beat-to-beat time intervals 46 between successive heartbeats, provide a direct, high-resolution reflection of cardiac electrical activity. Unlike aggregated measures of heart rate variability (HRV), which 47 48 summarize autonomic activity over longer periods and can mask transient fluctuations, RRi 49 analysis offers a granular, beat-to-beat perspective on autonomic modulation during exercise and recover v^{4-7} . 50 51 This granular perspective is particularly relevant in dynamic exercise scenarios, where 52 rapid shifts in autonomic balance occur, and in specific populations such as older adults, 53 where age-related changes in autonomic function may influence cardiac responses^{2,3,8}. 54 Analyzing RRi allows for examining immediate cardiac responses to exercise-induced 55 stress, providing valuable insights into the efficiency and adaptability of the cardiovascular 56 system. 57 While many studies have investigated cardiovascular responses to exercise using quasi-58 stationary protocols, simplifying analysis by minimizing non-stationarities^{9,10}, these 59 approaches may not fully capture physiological responses' dynamic and continuous nature 60 during real-world activities. Although traditional linear methods like time-series analysis 61 and linear regression have been employed to model RRi behavior¹¹, they often fall short in 62 capturing the complex, non-linear time-dependent fluctuations of RRi transitions, 63 particularly during periods of intense exertion and the subsequent recovery phase 12. This 64 limitation is significant because the ANS undergoes rapid and non-linear shifts between 65 parasympathetic withdrawal and sympathetic activation during exercise, resulting in 66 intricate RRi fluctuations that linear models cannot adequately represent by their nature 13. 67 These rapid autonomic adjustments, including vagal tone and sympathetic outflow changes, contribute to the non-linear patterns observed in RRi data^{4,5}. Consequently, these simplified 68

- 69 models may miss critical physiological information related to cardiovascular adaptation,
- such as the speed and extent of recovery¹⁴.
- Model-based approaches, particularly those employing exponential functions, have been
- widely used to estimate heart rate and RRi recovery time constants after exercise 15-20.
- While these models provide valuable insights into recovery kinetics, they often focus on
- specific phases of the exercise-recovery cycle. They may not fully capture the continuous
- 75 transitions in RRi from rest to exercise and back to baseline. Furthermore, these models
- often rely on simplifying assumptions about the underlying physiological mechanisms,
- 77 which may limit their ability to represent individual variability across different exercise
- 78 intensities and populations accurately. Other models, like advanced non-linear approaches,
- have been developed to address the limitations of linear methods like decision tree-based
- 80 ensemble algorithms and convolutional neural networks^{21–23}. More advanced techniques,
- 81 such as non-linear mode decomposition^{24,25}, dynamical modeling^{26,27}, and the explicit
- 82 consideration of non-autonomous dynamics^{28,29}, have also been applied to analyze
- 83 physiological time series.
- 84 However, many of these existing non-linear models' lack of a direct link to underlying
- physiological processes is a significant limitation. While they may provide a better fit to the
- 86 observed data, they often lack clear physiological interpretability, limiting their clinical
- 87 utility and hindering a deeper understanding of the mechanisms driving RRi changes^{30–32}.
- 88 Furthermore, few models are designed to capture the continuous, beat-to-beat transitions in
- 89 RRi throughout the entire rest-exercise-recovery cycle while simultaneously providing
- 90 physiologically meaningful parameters that can explain individual variability across diverse
- 91 exercise intensities, durations, and populations³³. This gap hinders a comprehensive
- 92 understanding of how individuals adapt to exercise and how these adaptations might differ
- based on age, fitness level, or underlying health status. For example, understanding how
- 94 RRi time-dependent fluctuations differ between trained athletes and sedentary individuals
- 95 during and after exercise could provide valuable insights for personalized training programs
- 96 and rehabilitation strategies.
- 97 Therefore, this paper introduces a novel non-linear model designed to characterize the
- 98 continuous RRi transitions from rest to exercise and recovery. This model aims to address

the limitations of existing approaches by (1) accurately capturing the non-linear time-dependent fluctuations of RRi fluctuations throughout the entire rest-exercise-recovery cycle, providing a more complete picture of cardiovascular responses to exercise, and (2) providing physiologically interpretable parameters that reflect the underlying autonomic mechanisms, allowing for a more mechanistic understanding of individual adaptations. By focusing on these key aspects, this model offers a more detailed and physiologically relevant understanding of cardiovascular adaptation to exercise compared to traditional HRV metrics, with potential applications in personalized exercise prescription, performance monitoring, and clinical assessment of cardiovascular health.

Methods

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Data collection and preprocessing

To further assess the proposed model's performance, real-world RRi data were analyzed in addition to the synthetic data generated through simulation. The dataset consisted of 272 participants who underwent a validated exercise protocol encompassing rest, exercise, and recovery phases within a single, continuous measurement session².

Subjects

115 Participants were recruited from a local community. Subjects were included if (i) they were 116 aged 60 years or older; (ii) were permanently residing in the Magallanes and Chilean 117 Antarctic region; (iii) had a percentage greater than 60% on the Karnofsky Performance 118 Status Scale, which allowed us to work with older people who had a state of autonomy 119 necessary to carry out the study tests; (iv) absence of the following diagnosis: diabetic 120 neuropathy; use of pacemakers; clinical depression; cognitive or motor disability; and 121 dementia. The exclusion criteria were: (i) consumption of beta-blockers during the study, 122 (ii) taking drugs or stimulant substances within 12 hours before the cardiac assessment, and 123 (iii) having some degree of motor disability that prevented participants from moving 124 around. No participants met the exclusion criteria. This dataset was derived from a cohort 125 participating in the FONDECYT Project No. 11220116, funded by the Chilean National 126 Association of Research and Development (ANID). Ethical approval was granted by the

- 127 Ethics Committee of the University of Chile (ACTA No. 029-18/05/2022) and the Ethics
- 128 Committee of the University of Magallanes (No. 008/SH/2022).

Exercise protocol

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- The exercise protocol consisted of the continuous measurement of RRi before, during, and
- immediately after the 2-minute step test, which is a part of the Senior Fitness Test
- protocol³⁴. This functional cardiorespiratory test required each subject to march in place as
- many times as possible for 2 minutes. The participants were monitored throughout the
- assessment using cardiovascular measures (i.e., heart rate and blood pressure) to prevent
- adverse events during the exercise protocol. The evaluation protocol was estimated to last
- approximately 20 minutes for each subject. None of the participants expressed discomfort
- during the evaluation. Continuous heart rate data, including RRi, were collected using the
- Polar Team2 system (Polar®) application, capable of capturing dynamic fluctuations
- associated with varying exercise intensities and recovery.

Preprocessing of RRi data

- Preprocessing steps were conducted to remove artifacts and ectopic heartbeats, with less
- than 3% of data excluded following established guidelines³⁵. The preprocessed RRi data
- were then aggregated into time intervals to facilitate analysis, allowing the examination of
- acute exercise responses and post-exercise recovery patterns.
- 145 This real-world dataset provided a critical context for validating the model's predictive
- 146 capability against observed physiological responses, offering a robust foundation for
- understanding RRi time-dependent fluctuations under physical activity conditions.

Parameter Estimation

- 149 Parameter estimation was performed using Hamiltonian Monte Carlo (HMC) with the No-
- U-Turn Sampler (NUTS) to explore the parameter space³⁶. The parameters α , β , c, λ , ϕ , τ ,
- and δ were estimated by sampling from the posterior distribution, which was constructed
- from observed RRi data and model predictions.

The gradient of the log-likelihood function for each parameter was computed during estimation using the brms R package (v2.21.0), which employs the Stan probabilistic programming language. Convergence of the HMC chains was assessed using standard diagnostics, including R-hat values, kept below 1.01 for all parameters³⁷, and effective sample sizes targeted at a minimum of 1,000 for each parameter³⁸. Trace plots were inspected to confirm stable mixing. These diagnostics collectively confirmed reliable posterior estimates for each parameter.

The fitting process utilized five Markov Chain Monte Carlo (MCMC) chains, each consisting of 10,000 iterations with a burn-in period of 5,000 iterations, resulting in 25,000 post-warmup samples.

To enhance the exploration of parameter space, we performed a two-stage analysis: We assessed parameter values at the individual level, which we then used to estimate population-level parameters. This hierarchical structure enables us to capture individual variability through subject-level random effects while estimating group-level effects across all parameters, thus providing estimates of subject- and population-level model parameters.

Individual-level analysis

Firstly, each subject's RRi data $RRi_{i,t}$ was standardized against his mean $R\bar{R}i_t$ and standard deviation S_{RRi_t} to improve convergence and exploration of the posterior distribution. The standardized RRi data $y_{i,t}$ for each time point t was computed as:

$$y_{i,t} = \frac{RRi_{i,t} - R\bar{R}i_{t}}{S_{RRi_{i}}}$$
 (1)

- This standardization allowed the model to focus on relative changes in RRi time-dependent fluctuations independent of individual baseline differences.
- 175 The model for each subject i was then specified in terms of standardized RRi data $y_{i,t}$:

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$$y_{i,t} = \alpha_i + \frac{\beta_i}{1 + e^{\lambda_i \cdot (t - \tau_i)}} + \frac{-c_i \cdot \beta_i}{1 + e^{\phi_i \cdot (t - \tau_i - \delta_i)}} + \epsilon_{i,t}$$
 (2)

- where α_i , β_i , c_i , λ_i , ϕ_i , τ_i , δ_i are the individual-specific model parameters and $\epsilon_{i,t} \sim$
- 178 $\mathcal{N}(0, \sigma^2)$ is the residual error term at each time point t.
- Afterwards, we transformed the estimated α and β parameters back to the original RRi
- scale, ensuring a physiologically meaningful interpretation. The transformation for each
- subject i is given by:

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$$\alpha_i^{\text{RRi}} = \alpha_i \cdot S_{\text{RRi}_i} + \bar{\text{RRi}}_i \beta_i^{\text{RRi}} = \beta_i \cdot S_{\text{RRi}_i}$$
 (3)

183 Group-level analysis

- After obtaining the posterior distribution for each subject's parameters, each parameter's
- mean $(\theta^{\text{ obs}})$ and standard error (ϵ) were calculated. These estimates were then used as input
- data to create a univariate hierarchical model, capturing variability at both the subject and
- group levels. The modeling process is described as follows:
- 188 For each subject i, we estimated an interdependent stochastic process in which the true
- parameter $\theta_{k,i}$, with $k \in \{\alpha, \beta, c, \lambda, \phi, \tau, \delta\}$ with their corresponding standard error $\epsilon_{k,i}$ was
- used to model the observed parameter $\theta_{k,i}^{\text{obs}}$ as:

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$$\theta_{k,i}^{\text{obs}} \sim \mathcal{N}(\theta_{k,i}, \epsilon_{k,i}) \qquad (4)$$

192 Then, the true parameter $\theta_{k,i}$ was further modeled as:

$$\theta_{k,i} \sim \mathcal{N}\left(\mu_k + b_{k,i}, \sigma_k^2\right) \tag{5}$$

- where μ_k is the group-level mean for parameter k, $b_{k,i}$ represents the subject-level random
- effect for the subject i on parameter k and σ_k^2 is the residual variance for the parameter k.
- The subject-level effects $b_{k,i}$ were assumed to be distributed as $b_{k,i} \sim \mathcal{N}(0, \sigma^2)$, with σ
- being the standard error of the subject-level effect.

Model Performance

- 199 The primary statistical performance metrics, estimated for each subject, included R², root
- 200 mean square error (RMSE), and mean absolute percentage error (MAPE), estimated for

- 201 each subject. Bootstrap resampling across each metric was performed to estimate the mean
- performance of the model and corresponding quantile-based 95% CI.
- Also, residual analysis were conducted to evaluate the model's accuracy in capturing RRi
- 204 time-dependent fluctuations. Residuals were defined as the difference between observed
- and predicted RRi values. These residuals were analyzed for temporal structure and partial
- autocorrelation to ensure that no systematic patterns remained in the errors. This indicates
- 207 that the model has sufficiently captured the underlying time-dependent fluctuations of the
- 208 RRi response to exercise.

Model parameters sensitivity

- 210 Once a model that described RRi behavior in response to exercise was obtained, an
- assessment of the proportion of the variance explained by each model parameter was then
- 212 computed.

- We implemented a Sobol sensitivity analysis using Monte Carlo simulations to assess the
- sensitivity of model parameters influencing RRi over time. Sobol index (S_{ind}) provide a
- 215 measure of the proportion of the contribution of each parameter to the variance in RRi at
- 216 each time point, and it was selected for its robustness in handling non-linear and non-
- 217 monotonic relationships, which are intrinsic to RRi time-dependent fluctuations in response
- 218 to exercise 39 .
- To compute S_{ind} , 1000 Monte Carlo simulations were conducted, each involving 1000
- randomly sampled parameter sets (1,000,000 model runs). For each set of parameters, RRi
- were calculated at each time point t across a range from 0 to 20 minutes at intervals of 0.1
- 222 minutes. The 95% CI parameter values estimated from HMC-NUTS were then used as
- input ranges for S_{ind} computation. Finally, the mean values of S_{ind} over the 20-minute time
- span for each model parameter were estimated and reported, with their corresponding 95%
- 225 CI using a normal approximation based on estimated standard errors (SE).

Results

Problem characterization

RRi time-dependent fluctuations in response to exercise tend to follow a U-shaped form. The initial decrease in RRi is associated with exercise onset and an increased heart rate. After exercise cessation, an opposite increase in RRi is observed, associated with the cardiovascular recovery phase. In both cases, the drop and recovery phases occur at different rates; some individuals experience a quick recovery in RRi after exercise; however, in some others, this slope is less steep. Additionally, the new baseline reached following exercise cessation is often below the RRi baseline before exercise.

These hallmarks of RRi time-dependent fluctuations in response to exercise highlight the complex and non-linear behavior of the cardiovascular response in the context of rest and exercise conditions. Figure 1 shows an example of RRi record data.

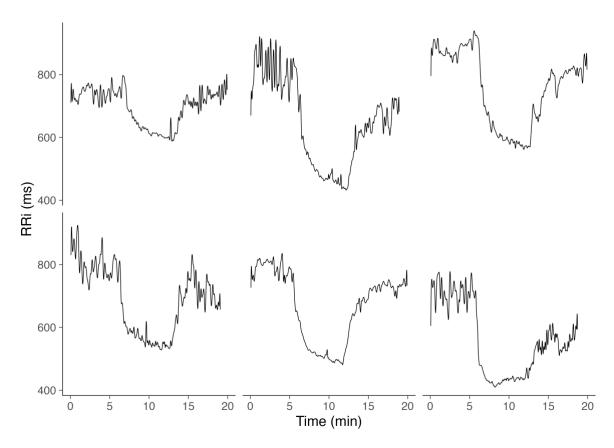


Figure 1. Example data of RRi recordings of 6 subjects over a 20-minute rest-exercise-

- recovery protocol in a sample of elderly individuals. The subject-level data shows the inter-
- 241 individual variability of RRi time-dependent fluctuations in response to exercised-induced
- 242 cardiovascular stress, with similar behavior and recovery trajectories over time.

Model construction

- 244 The process of deriving the final equation for modeling RRi fluctuations was guided by an
- 245 iterative exploration of mathematical functions capable of capturing the observed time-
- 246 dependent fluctuations. Initially, exponential and logarithmic functions were considered
- 247 due to their simplicity and broad applicability in describing temporal changes. Exponential
- 248 functions were hypothesized to capture the rapid initial adaptations of RRi post-exercise
- onset. In contrast, logarithmic functions were explored for their capacity to describe
- asymptotic behaviors observed in some physiological variables.
- However, neither approach successfully reproduced the non-linear and bidirectional nature
- of the RRi fluctuations. While effective at modeling monotonic decay or growth,
- exponential functions could not account for the observed sigmoidal transitions. Similarly,
- 254 logarithmic functions, with their inherent monotonicity, failed to represent the plateauing
- behavior seen in real-world data.
- We shifted to logistic functions to address these limitations, which inherently model
- sigmoidal transitions. Logistic functions introduce parameters for growth rate and inflection
- 258 point, allowing for precise control over the shape and timing of the transition between
- 259 dynamic states. By using two coupled logistic functions, one representing the initial
- decrease in RRi and a second inverted logistic function describing the recovery phase, we
- achieved a model structure that could flexibly reproduce the observed non-linear variations.
- 262 This approach provided a biologically plausible representation, with parameters that
- directly correspond to identifiable physiological features, such as the rate of adaptation and
- recovery, the time to peak response, and the extent of deviation from baseline. The logistic
- 265 function framework emerged as the optimal solution after systematic testing and evaluation
- against empirical data, ensuring that the model accurately captured the qualitative and
- quantitative aspects of RRi time-dependent fluctuations.

The mathematical model proposed to characterize the RRi response to exercise and recovery is defined by Equation 6.

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$$\operatorname{RRi}(t) = \alpha + \frac{\beta}{1 + e^{\lambda(t-\tau)}} + \frac{-c \cdot \beta}{1 + e^{\phi(t-\tau-\delta)}} \tag{6}$$

- 271 This model includes two logistic functions representing the RRi time-dependent
- 272 fluctuations across exercise and recovery phases. The first logistic term models the
- 273 decrease in RRi during exercise, where the parameter β denotes the magnitude of this
- decline. The rate of decrease is governed by λ , while τ represents the onset of the RRi
- decrease or the time the physiological shift begins.
- 276 The second logistic term accounts for RRi recovery post-exercise. Here, c scales the
- 277 magnitude of recovery relative to the initial decline represented by β , capturing the
- 278 proportion of the decline regained during recovery. The rate at which RRi returns to
- baseline is controlled by ϕ , and δ indicates the lag following the cessation of exercise,
- 280 marking the beginning of recovery.
- Additionally, the time-dependent fluctuations of RRi in response to physical exertion can
- be represented as a linear combination of a baseline RRi α and two logistic functions
- denoted as $f_1(t)$ and $f_2(t)$. The function $f_1(t)$ models the initial decay in RRi following
- 284 the initiation of exercise while $f_2(t)$ characterizes the recovery phase after exercise
- cessation.
- 286 Essentially, the fundamental structure of both logistic functions can be expressed as:

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$$f(t) = \frac{a_1}{1 + e^{a_2(t - a_3)}} \tag{7}$$

- In this equation, a_1 represents the asymptotic value the logistic function approaches, which
- can be either positive (indicating an increase) or negative (indicating a decrease). For $f_1(t)$,
- 290 this parameter is specified as β , indicating the absolute change in RRi at the onset of
- 291 exercise. In contrast, for $f_2(t)$, a_1 is parametrized as $-c \cdot \beta$, where c denotes the proportion
- of change relative to the initial drop indicated by β . This parametrization ensures that, after
- 293 the initial decline, the second logistic function facilitates the return of RRi toward the
- 294 baseline value α .

The parameter a_2 defines the rate at which the specified increase or decrease occurs. This rate parameter is expressed on a logarithmic scale; to convert it to a percentage change per unit of time, it can be scaled as $1 - \exp(a_2)$.

The parameter a_3 serves as an activation threshold, causing the value within the exponential function, and consequently, the value in the denominator, to increase significantly until reaching a_3 . Beyond this point, the denominator approaches 1, allowing the logistic function to attain the asymptotic level determined by the numerator. Figure 2 illustrates the behavior of the model constituents.

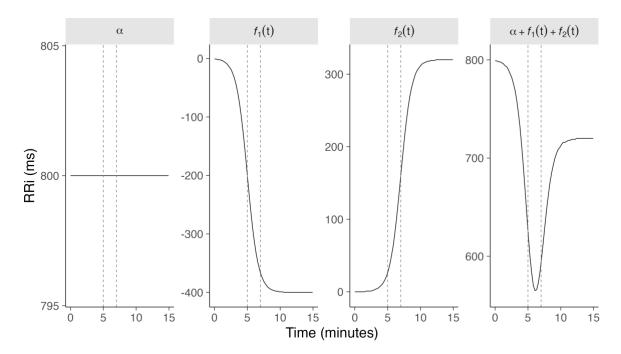


Figure 2. The RRi time-dependent fluctuations in response to exercise are expressed as a linear combination of model constituents based on the baseline RRi α and two logistic functions, denoted $f_1(t)$ and $f_2(t)$, respectively. The vertical dashed lines represent the time at which the exercise and recovery onset given by $\tau = 5$ and $\delta = 2$.

Sample characteristics

The sample used to assess RRi time-dependent fluctuations consists of 272 subjects selected from a local community of elderly individuals. The sample characteristics can be seen in Table 1

Characteristic	Overall	Female	Male
Sex		217 (79.8%)	55 (20.2%)
Age	71.14 ± 6.03	70.73 ± 6.27	72.73 ± 4.7
SBP (mm hg)	130.23 ± 17.07	129.58 ± 17.37	132.8 ± 15.69
DBP (mm hg)	77.1 ± 9.58	76.68 ± 9.83	78.75 ± 8.4
MAP (mm hg)	94.81 ± 10.69	94.31 ± 10.95	96.76 ± 9.45
PP (mm hg)	53.14 ± 14.07	52.9 ± 14.26	54.05 ± 13.38
BMI	30.66 ± 5.43	30.7 ± 5.64	30.53 ± 4.53
Weight (kg)	75.06 ± 14.23	73.88 ± 14.09	79.69 ± 13.95
Height (cm)	156.56 ± 9.18	155.29 ± 8.46	161.55 ± 10.24

Table 1. Sample characteristics from which continuous RRi monitoring data was collected during the rest-exercise-rest protocol. Data is presented as Mean ± standard deviation (SD).

SBP, systolic blood pressure; DBP, diastolic blood pressure; MAP, mean arterial pressure; PP, pulse pressure; BMI, body mass index.

An initial graphical exploration of RRi time-dependent fluctuations (see Figure 3) indicates a clear drop in RRi around the 5-7 minutes, associated with exercise-induced cardiovascular stress. However, greater variability across individuals in post-exercise

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recovery can be observed.

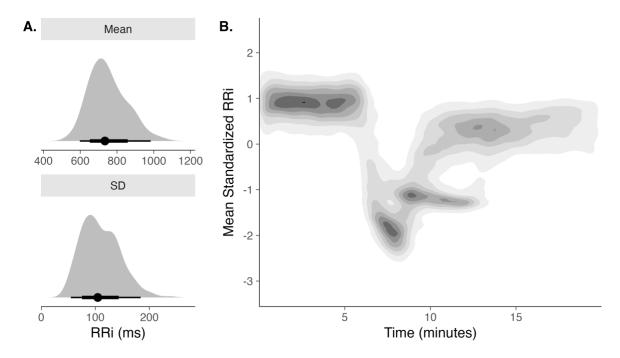


Figure 3. (A) Mean and SD from each subject's RRi recordings were used for the standardization process. (B) 2D kernel density of standardized RRi dynamics over time from a sample of individuals subjected to the rest-exercise-rest protocol. Darker colors indicate greater probability density. The contrary can be said about lighter colors.

Parameter estimation

Priors

Given the parameters that reproduced the observed RRi patterns in exercise and rest conditions, priors were chosen based on physiological constraints and the graphical visualization of standardized RRi data. Hence, ensuring the identifiability of model parameters by constraining the parameter space to plausible values to improve model convergence and parameter exploration. The prior distributions were defined as follows:

$$\alpha \sim \mathcal{N}(1,0.5)$$

$$\beta \sim \mathcal{N}(-2.5,0.5) \text{ with } \beta \leq 0$$

$$c \sim \mathcal{N}(0.8,0.2) \text{ with } c \geq 0$$

$$\lambda \sim \mathcal{N}(-2,0.5) \text{ with } \lambda \leq 0$$

$$\phi \sim \mathcal{N}(-2,0.5) \text{ with } \phi \leq 0$$

$$\tau \sim \mathcal{N}(5,0.5) \text{ with } \tau \geq 0$$

$$\delta \sim \mathcal{N}(5,0.5) \text{ with } \delta \geq 0$$

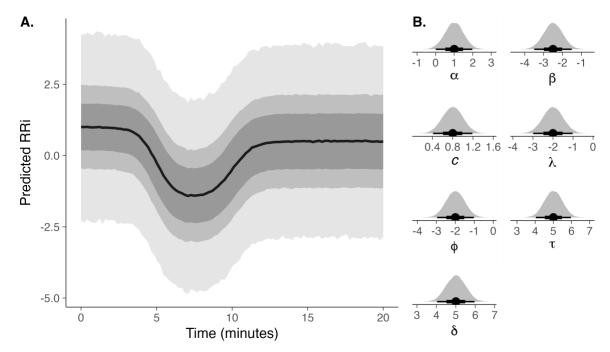


Figure 4. (A) Simulated standardized RRi time-dependent fluctuations based on prior parameter distributions, illustrating predicted RRi responses to exercise. Shaded areas represent 95%, 80%, and 60% quantile CI, offering insight into expected physiological variability across parameters. (B) Prior distributions and 95% CI were used to generate predictions based on physiological constraints and graphical visualization of standardized RRi data.

Parameter estimates

Once subject-level RRi data was fitted using the proposed model in Equation 2, a population-parameter value was estimated based on the proposed group-level methodology.

The estimated parameter values can be seen in Table 2

Parameter	Estimate ¹	SE ¹	Lower ²	Upper ²
α	861.78	5.73	850.57	872.85
β	-345.49	7.41	-359.81	-330.97
С	0.84	0.01	0.82	0.86

Parameter	Estimate ¹	SE ¹	Lower ²	Upper ²
λ	-3.05	0.06	-3.16	-2.94
φ	-2.60	0.06	-2.71	-2.48
τ	6.71	0.05	6.61	6.81
δ	3.24	0.10	3.05	3.44
σ	27.57	0.57	26.45	28.70

Table 2. Population-parameter values estimated from group-level analysis. ¹ Estimates and SE are computed as the posterior distribution's median and median absolute deviation, respectively; ² Lower and Upper bounds from the quantile-based CI_{95%} of the posterior distribution.

350 In Figure 5, the model parameter's posterior distribution can be observed.

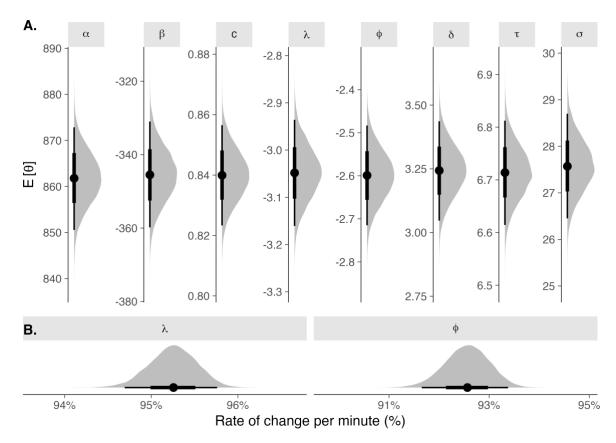


Figure 5. (A) Posterior probability distributions of the expectation for each population-

parameter estimate $(E[\theta])$ with quantile-based 95% CI. (B) Transformed rate parameters into a percentage scale using the $1 - \exp(\theta)$ transformation.

Model evaluation

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Model performance

indices remained insignificant (see Figure 6).

357 Estimated through bootstrapped resampling, relative statistical performance metrics suggest 358 that the model tends to deviate by 3.4% (CI_{95%}[3.06, 3.81]) from the observed RRi data. This is equivalent to a 32.6 ms in the RRi scale (CI95%[30.01, 35.77]). Additionally, the 359 360 bootstrapped R² indicates that the model explains 0.868 (CI_{95%}[0.834, 0.895]) of the total 361 variance observed in RRi. Residuals analysis showed that the estimated partial correlation function (ACF) from the 362 model residuals indicates a correlation among non-explained errors greater than 0.1 up to 363 364 the 5th lag. However, the partial ACF is significant (CI-wise) and strictly positive or 365 negative until the second lag. Correlations among model residuals against other time

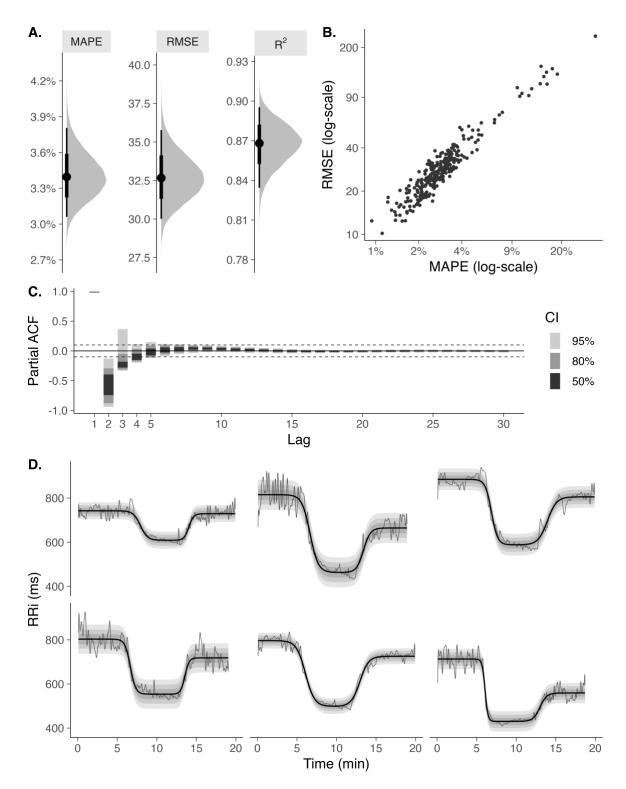


Figure 6. Individual-level performance metrics. (A) Bootstrapped MAPE and RMSE are statistical metrics of relative and absolute model deviance from observed RRi. (B) Individual-level estimates of model performance and the relationship between them. (C) Partial autocorrelation function (ACF) of model residuals with corresponding quantile-

based CI. (D) Example data with model estimates of RRi fluctuations and corresponding quantile-based CI initially displayed.

Model parameters sensitivity

Sobol sensitivity analysis reveals that the parameter α exerts the most substantial influence on the model's output, followed by parameters c and δ . In contrast, parameters β , λ , and ϕ demonstrate relatively minor effects, with some values crossing zero, indicating negligible influence within the tested parameter ranges.

Individual perturbation of each parameter highlighted that RRi time-dependent fluctuations are sensitive to the baseline RRi parameter, α . Conversely, the rate parameters for the initial decay during exercise, λ , and the recovery post-exercise, ϕ , show lower sensitivity, suggesting that they are not primary sources of variation in predicted RRi trajectories when assessed in isolation. The results of the sensitivity analysis are in Table 3.

Parameter	Estimate ¹	SE ¹	Lower ²	Upper ²
α	0.61329	0.01756	0.57887	0.64771
β	0.06651	0.00286	0.06090	0.07212
С	0.18939	0.00815	0.17342	0.20536
λ	0.00147	0.00007	0.00133	0.00161
ϕ	0.00160	0.00008	0.00144	0.00176
τ	0.04982	0.00172	0.04645	0.05319
δ	0.07896	0.00239	0.07428	0.08364

Table 3. Estimated S_{ind} of model parameters. ¹ Estimates and SE are computed as mean and standard deviation of Monte Carlo samples, respectively. Each parameter's S_{ind} reflects a uniform variation from the 95% CIs of the estimated parameter values.

Discussion

To our knowledge, this study represents the first attempt to develop a non-linear model specifically designed to capture RRi time-dependent fluctuations continuously across a complete rest-exercise-rest protocol. Previous studies have either focused on aggregate HRV indices or utilized simplified linear or exponential models, which are insufficient to describe the complex, non-stationary transitions observed during and after exercise⁴⁰. By employing a combination of logistic functions, our model uniquely accounts for the gradual shifts in autonomic regulation denoted by RRi time-dependent fluctuations, offering a detailed and physiologically relevant representation of cardiac dynamics. This continuous modeling framework integrates exercise-induced RRi decline and post-exercise recovery within a single unified structure, bridging a critical gap in the current literature. Such an approach advances our understanding of cardiovascular responses and opens new avenues for real-time monitoring and intervention in clinical and athletic settings.

The proposed model demonstrates a precise capacity to reproduce RRi dynamics. Its combination of logistic functions captures the key transitions of cardiac response, the initial decline during exercise, and the subsequent recovery. This design accommodates the inherent non-linearity and non-stationarity of RRi time-dependent fluctuations, overcoming the limitations of linear models and exponential functions commonly used in prior studies^{8,41}.

Compared to previous research, our findings align with efforts to capture nonlinear dynamics in HRV to understand cardiac responses during exercise¹². Similarly, previous studies have shown that dynamic fluctuations in RRi can serve as critical indicators of cardiorespiratory fitness^{7,8}. This supports the need for models to address the complexity of cardiovascular responses during physical stress⁸. However, while many existing models focus primarily on linear metrics or aggregate HRV measures, our study provides a high-resolution analysis of RRi time-dependent fluctuations that enhances interpretability and application across diverse fitness levels and exercise intensities. Critically, many model-based approaches, particularly those employing exponential functions, have been used to estimate time constants of heart rate and RRi recovery after exercise^{15–20}. These models often focus on characterizing the recovery phase and may not capture the continuous

transitions from rest to peak exercise and subsequent recovery. Our model, by contrast, provides a unified framework for modeling the entire rest-exercise-recovery cycle, allowing for the estimation of parameters that reflect both the exercise-induced changes in RRi and the subsequent recovery dynamics. This continuous modeling approach provides a more comprehensive picture of cardiovascular response to exercise than models focusing solely on recovery kinetics.

The flexibility of the logistic components allows for physiologically interpretable parameters, such as baseline RRi (α) and recovery proportion (c), which directly correlate with intrinsic cardiac function and autonomic recovery capacity, respectively. These features position the model as a robust framework for investigating the cardiovascular system's dynamic adaptation to physical stressors. For example, prior studies have highlighted the inadequacy of linear HRV metrics in capturing transient autonomic shifts⁴²; our results align with this critique, demonstrating the advantages of modeling RRi directly.

Prior studies have examined cardiorespiratory interactions using both deterministic and stochastic approaches. Deterministic models have demonstrated that respiration-driven heart rate fluctuations exhibit structured, predictable behavior, suggesting an underlying regulatory mechanism of autonomic control^{43,44}. Conversely, stochastic models emphasize the role of random variability in these interactions, accounting for inherent physiological fluctuations⁴⁵. Our non-linear model aligns with the deterministic perspective by employing logistic functions to characterize time-dependent RRi fluctuations while also incorporating inter-individual variability. Although this model does not explicitly integrate stochastic noise, it captures structured autonomic responses. Future work could explore the incorporation of stochastic elements to further enhance its applicability in more variable physiological conditions.

Unlike prior research that aggregates HRV measures or applies simple decay models, our approach directly models RRi changes, offering richer physiological insight. For instance, commonly utilized exponential decay models for post-exercise recovery are used but fail to incorporate the transition dynamics observed during exercise itself⁴⁶. By integrating exercise and recovery phases, our model provides a more comprehensive view of autonomic regulation. Furthermore, it's important to note that the traditional

- "sympathovagal balance" hypothesis, which posits a reciprocal relationship between sympathetic and parasympathetic activity, may be oversimplified, especially during exercise⁹. Recent evidence suggests that parasympathetic control can remain active even during periods of high sympathetic activation. By capturing the continuous time-dependent fluctuations of RRi, our model may provide insights into these complex interactions, potentially revealing nuances in autonomic control that are not captured by simpler models
- 453 that assume a strict sympathovagal balance.
- Moreover, the sensitivity of parameters such as λ (decay rate) and ϕ (recovery rate) was
- found to be relatively low, suggesting that the model is robust to variability in these rates
- 456 while remaining sensitive to key physiological parameters (α and c). This robustness makes
- 457 it suitable for individualized monitoring and population-level analyses, offering versatility
- in its application across different use cases.
- The Sobol sensitivity analysis revealed that baseline RRi (α) and recovery proportion (c)
- 460 are the primary drivers of model output variance, emphasizing their physiological
- 461 importance. These findings are consistent with prior research, which identified baseline
- 462 cardiac function as a determinant of cardiovascular health and recovery proportion as a
- 463 marker of autonomic resilience¹⁴.
- However, the Sobol method assumes parameter independence, which may overlook
- interactions common in biological systems $^{47-49}$. For example, the interplay between λ and
- 466 c, which dictates the rate and magnitude of recovery, is likely critical but remains
- 467 unexplored in the current framework. Future studies could explore Bayesian sensitivity
- 468 analysis or variance decomposition methods that account for parameter
- interdependence^{50,51}. Furthermore, more advanced techniques, such as non-linear mode
- 470 decomposition^{24,25}, dynamical modeling^{26,27}, and the explicit consideration of non-
- autonomous dynamics^{28,29}, offer powerful tools for analyzing physiological time series.
- While these methods can capture complex dynamics, our model provides a more direct link
- 473 to physiological interpretation through its parameters related to specific aspects of
- autonomic control. Future work could investigate how these approaches could be combined
- or compared to enhance our understanding of RRi time-dependent fluctuations.

This model demonstrates significant potential for practical applications in clinical and athletic settings. In clinical contexts, it could aid in tailoring cardiovascular rehabilitation protocols by monitoring autonomic recovery in real-time, ensuring safe and effective exercise regimens for at-risk populations⁵². This aligns with previous research, highlighting the importance of individualizing rehabilitation programs to optimize recovery^{52–54}.

- The model could guide training strategies in athletic settings, particularly for interval training, where determining optimal recovery periods is crucial. Similar findings suggest that precise monitoring of RRi time-dependent fluctuations can prevent overtraining and enhance performance^{55,56}. The model's ability to integrate real-time data from wearable devices further enhances its applicability in dynamic, uncontrolled environments, enabling field-based monitoring and feedback⁵⁷.
- While the model presents substantial advances, it has limitations that warrant consideration. First, the assumption of uniform parameter sampling in sensitivity analysis, while practical, may not fully capture the variability observed in populations with extreme autonomic profiles⁴. Empirical distributions, or Bayesian priors, could improve parameter estimation and enhance the model's applicability to diverse populations⁵¹. Bayesian inference could be a valuable extension of this work, particularly dynamic Bayesian inference^{58–60}, specifically designed to model time-evolving dynamics. This approach could allow for the incorporation of prior knowledge about individual physiological characteristics and provide more robust estimates of the model parameters.
 - Another limitation lies in the demographic composition of the sample, which consisted exclusively of elderly individuals. While this population provides valuable insights into age-specific cardiovascular time-dependent fluctuations, the findings may not fully generalize to younger populations, whose autonomic responses to exercise and recovery differ significantly due to higher baseline vagal tone, greater cardiac plasticity, and distinct metabolic profiles^{61,62}. Previous studies have demonstrated that younger individuals exhibit faster autonomic recovery and greater adaptability during physical exertion compared to older populations^{62,63}. This suggests that the parameter estimates derived from this model may vary across age groups^{62,63}. Future research should validate the model in more diverse cohorts, including younger adults and athletes, to ensure broader applicability and to

explore potential age-dependent modifications of the model's parameters. This would enhance its utility in clinical and athletic contexts, where age diversity is a critical factor^{62,63}.

Furthermore, the uneven sex ratio in our sample (79.8% female, 20.2% male) is another limitation that should be addressed in future studies. Sex differences in autonomic control have been reported⁶¹, and this imbalance could have influenced our results. Future research should strive for a more balanced sex ratio to minimize potential bias and explore sexspecific differences in RRi time-dependent fluctuations during exercise and recovery. This study did not explicitly consider environmental and psychological factors like temperature, stress, or sleep quality. Future work could integrate these variables into the model, enhancing its robustness and applicability across varied real-world scenarios. This aligns with calls for more integrative modeling approaches in cardiovascular research^{53,55,56}.

Conclusion

In summary, this study presents a novel non-linear model for RRi time-dependent fluctuations, capturing the complex and transient autonomic responses during rest-exercise-recovery protocols, overcoming the limitations of traditional autonomic metrics. The model emphasizes their critical roles in reflecting autonomic regulation and resilience by identifying baseline RRi and recovery proportion as the dominant contributors to variability. Validated across a cohort of elderly participants, the model demonstrates robust performance in real-time cardiovascular assessments, offering significant potential for clinical applications such as rehabilitation and monitoring in at-risk populations and athletic contexts like fatigue management and performance optimization. While the model's applicability is currently constrained by its focus on elderly individuals, future validation in younger cohorts and under diverse environmental conditions will enhance its generalizability and utility. This work establishes a foundational framework for advancing personalized cardiovascular health monitoring and intervention.

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Author Contributions

- 709 Conceptualization, MC-A; Data curation, MC-A; Investigation, MC-A, DM-C;
- 710 Methodology, MC-A, NMD; Supervision, CN-E; Formal analysis, MC-A; Visualization,
- 711 MC-A; Writing-original draft, MC-A, CN-E, DM-C; Writing-review & editing, MC-A,
- 712 CN-E, DM. All authors have read and agreed to the published version of the manuscript.

713 **Data Availability Statement**

- The authors will make the data supporting this article's conclusions available without
- reservation. If any data is required, please request it from the corresponding author of this
- 716 work.

717 Competing Interests Statement

- The authors declare that this research was conducted without any commercial or financial
- relationships that could be construed as potential conflicts of interest.

720 **Funding**

721 This work was funded by ANID Proyecto Fondecyt Iniciación N°11220116.