# Collision-free dynamics

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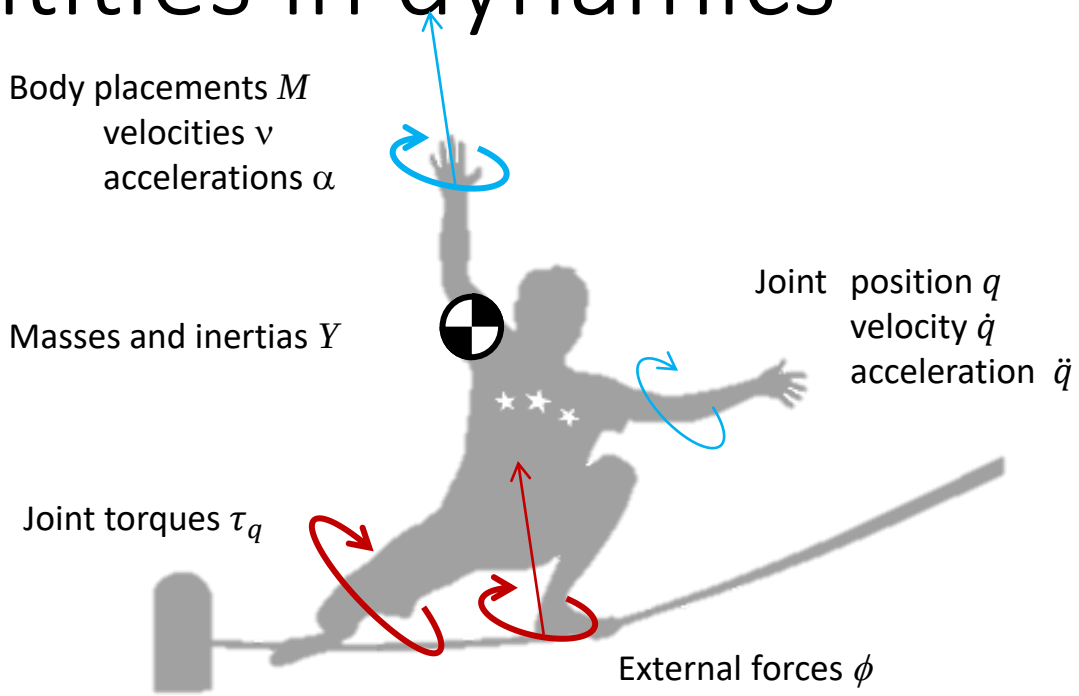
# Quantities in dynamics




| Inertia | Coriolis | Gravity | Forces |
|---|---|---|---|
| $Y\,\alpha \quad +$ | $\nu \times Y\,\nu$ | $+\,g$ | $= \phi$ |
| $M(q)\ddot{q} +$ | $b(q,\dot{q})$ | $+\,g(q)$ | $= \tau_q$ |

# Direct/inverse dynamics

- Inverse dynamics

$$M(q)\ddot{q} + b(q,\dot{q}) + g(q) = \tau_q$$

  - Control equation: from desired movement to needed torque

- Direct dynamics

$$\ddot{q} = M(q)^{-1}(\tau_q - b(q,\dot{q}) - g(q))$$

  - Simulation equation: from current state and torque, to acceleration

# Dynamics of articulated bodies

- Dynamic equation of the robot

$$M(q)\dot{v}_q + h(q, v_q) + g(q) = \tau_q$$

- Actuation of the robot
  - Fixed manipulator: $\tau_q = \tau_m$
  - Floating robot: $\tau_q = \begin{bmatrix} 0 \\ \tau_m \end{bmatrix} = S^T \tau_m$
  - Floating in contact: : $\tau_q = S^T \tau_m + J^T \phi$

# Dynamics for simulation

- Complementarity problem

$$\ddot{q} = M^{-1}(\tau - b + J^T f)$$

$$J\ddot{q} + \dot{J}\dot{q} \geq 0 \quad \perp \quad f \geq 0$$

*no penetration* *no pulling*

*one or the other*

- Equivalent to a principled QP

$$\min_{\ddot{q}} \left\| \ddot{q} - \ddot{q}_{free} \right\|_M \text{ s.t. } \quad J\ddot{q} + \dot{J}\dot{q} \geq 0$$