Inverse geometry

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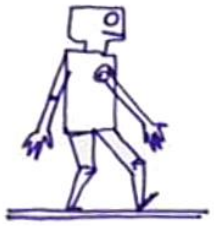
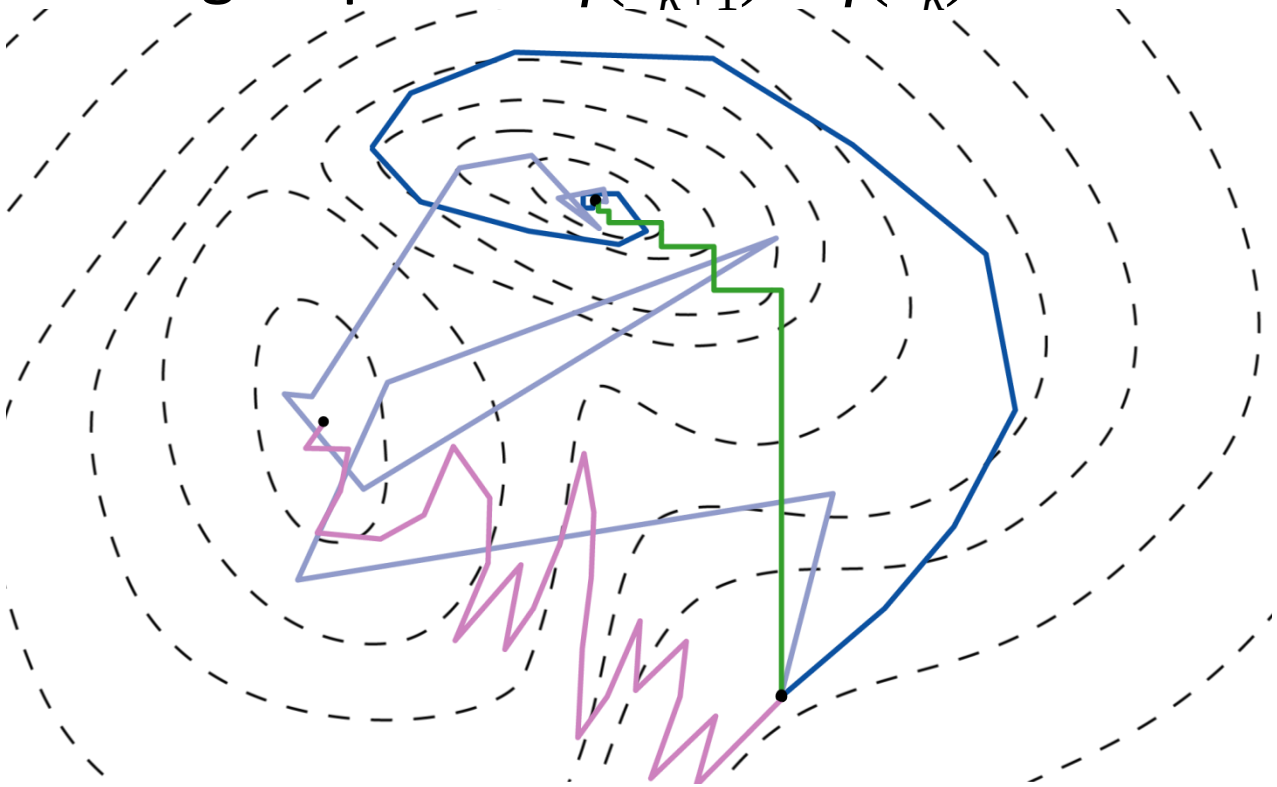


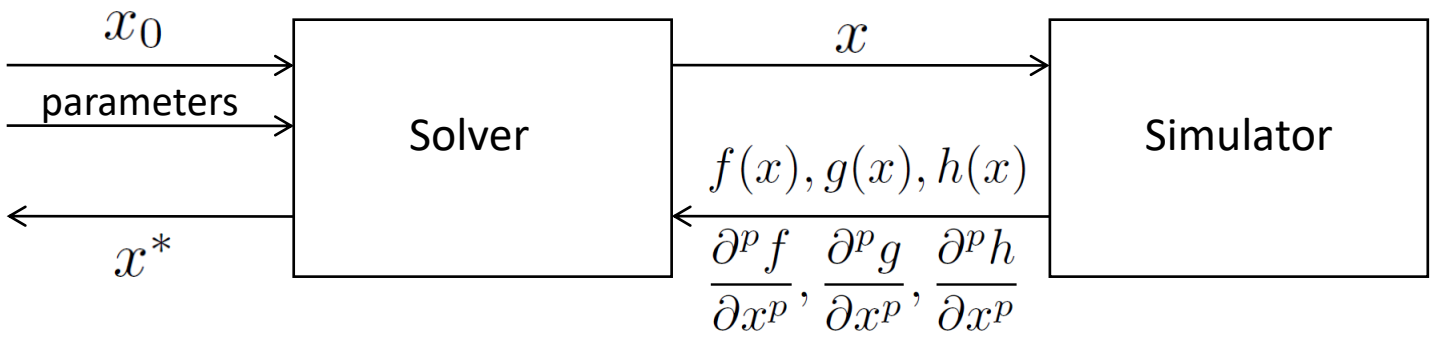
Follow the slope

- Decreasing sequence: $f(x_{k+1}) < f(x_k)$



Problem specifications

- Problem specification
  - Computing *f(x)* is easy
  - We can derivate *f:* $x \rightarrow f(x)$
  - We know the distance to the reference value



Convergence rate

	linear	superlinear	quadratic
0	1.	1.	1.
1	1.207106781186547524400844	1.207106781186547524400844	1.5
2	1.310660171779821286601267	1.345177968644245874001408	1.417893218813452475599156
3	1.362436867076458167701478	1.396954663940882755101619	1.414220332308854580746306
4	1.388325214724776608251583	1.410761782686652590061675	1.414213562396011063892029
5	1.401269388548935828526636	1.413638265758687972345020	1.414213562373095048801952
6	1.407741475461015438664163	1.414131377142465466450736	1.414213562373095048801689
7	1.410977518917055243732926	1.414203289219266351007820	
8	1.412595540645075146267307	1.414212420911558526824592	
9	1.413404551509085097534498	1.414213448226941396603980	
10	1.413809056941090073168094	1.414213551996171989510988	
11	1.414011309727803239103546	1.414213561508351460527464	
12	1.414112436050449143952618	1.414213562306576311242133	
13	1.414162999211772096377153	1.414213562368343710404578	
14	1.414188280721722894470766	1.414213562372778292908548	
15	1.414200921547403971636228	1.414213562373075251558368	
16	1.414207241960249510218958	1.414213562373093884257964	
17	1.414210402166672279510324	1.414213562373094984104816	
18	1.414211982269883664156006	1.414213562373095045396590	
19	1.414212772250778178360193	1.414213562373095048631434	
20	1.414213167311936613580941	1.414213562373095048793582	
21	1.414213364913226509309970	1.414213562373095048801321	
22	1.414213463643160779055829	1.414213562373095048801673	
23	1.414213513008127913928759	1.414213562373095048801689	
81	1.4142135		

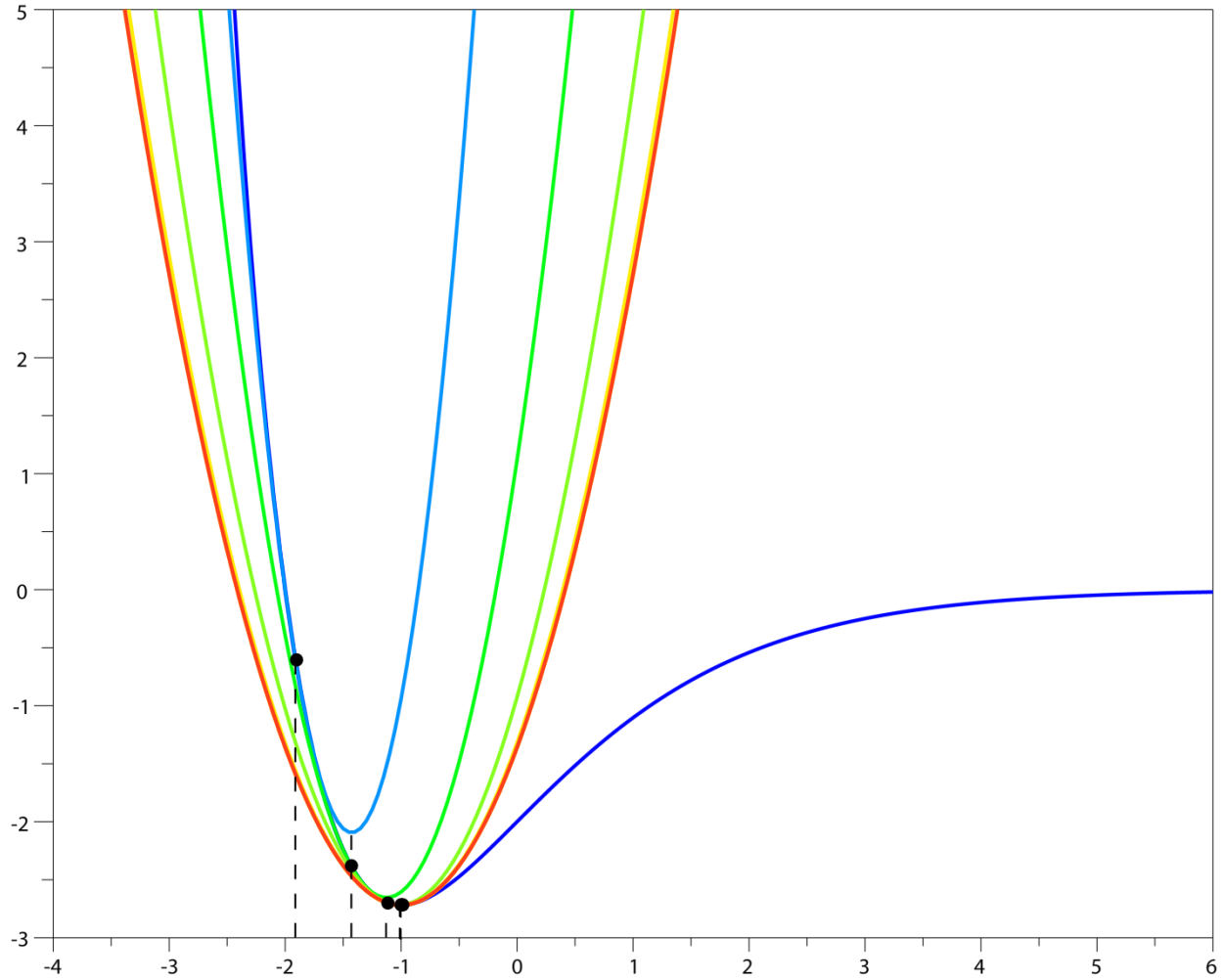
$p=2$

$$r_{k+1} \sim \alpha\, r_k\,, \qquad 0<\alpha<1$$

$$\frac{r_{k+1}}{r_k^{\,p}} \underset{+\infty}{\longrightarrow} \alpha \qquad 0<\alpha<1,\ p>1$$

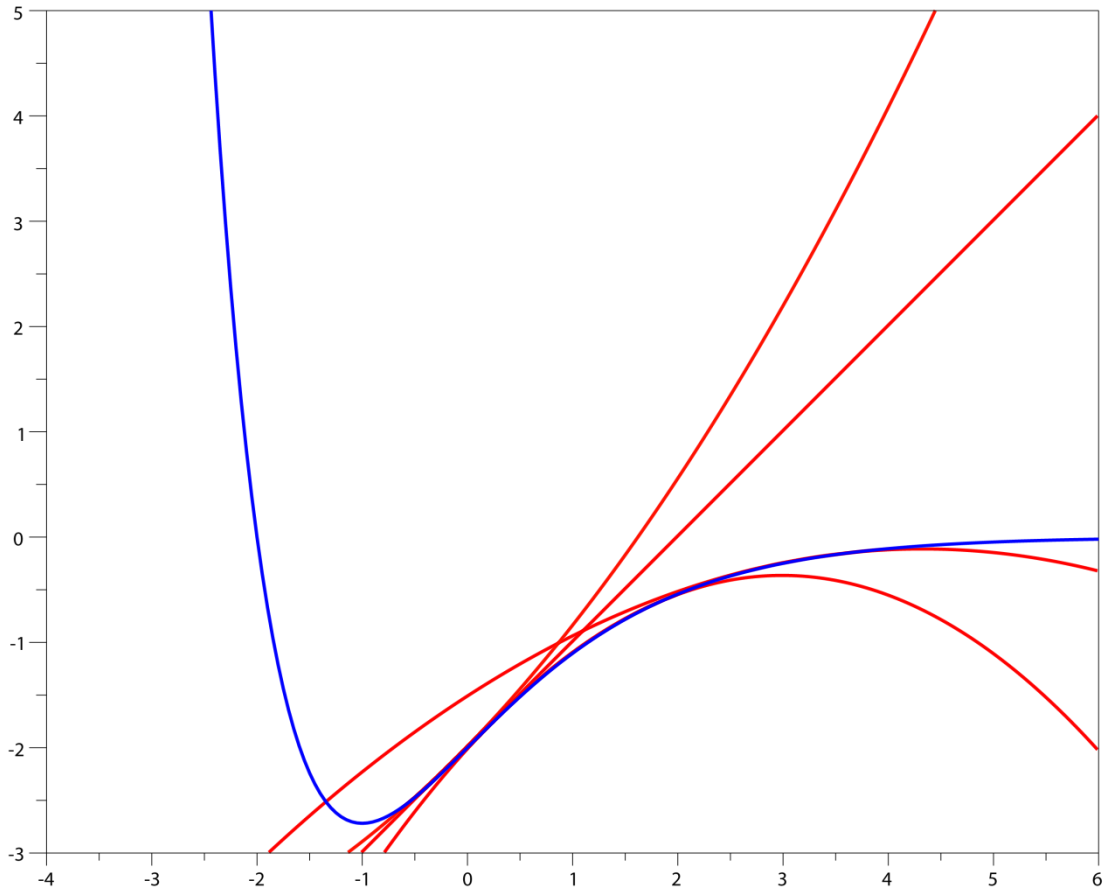
Newton method (unconstrained)

x0=-1.9
x1=-1.4263158
x2=-1.1274228
x3=-1.0144015
x4=-1.0002045
x5=-1.00000004
x6=-1.

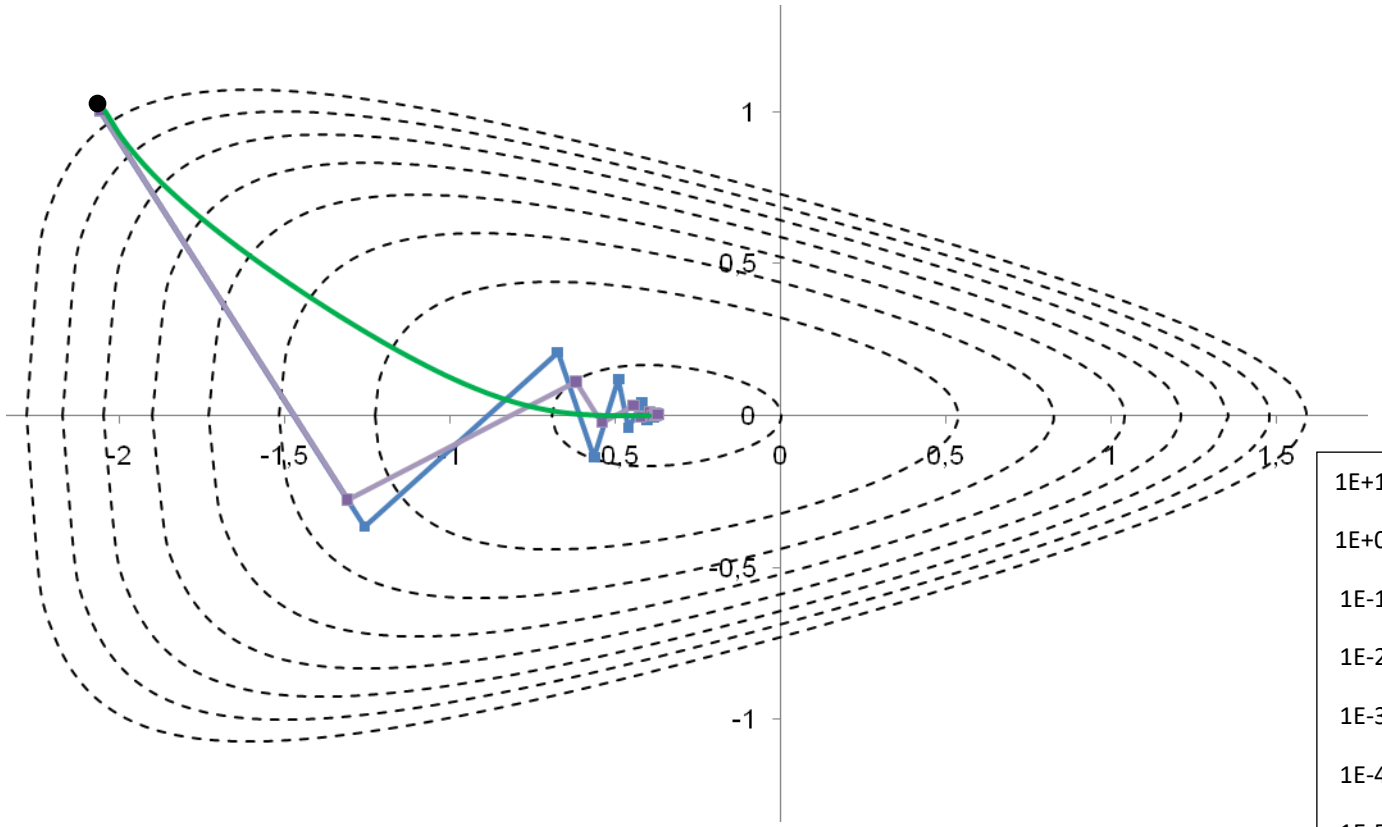


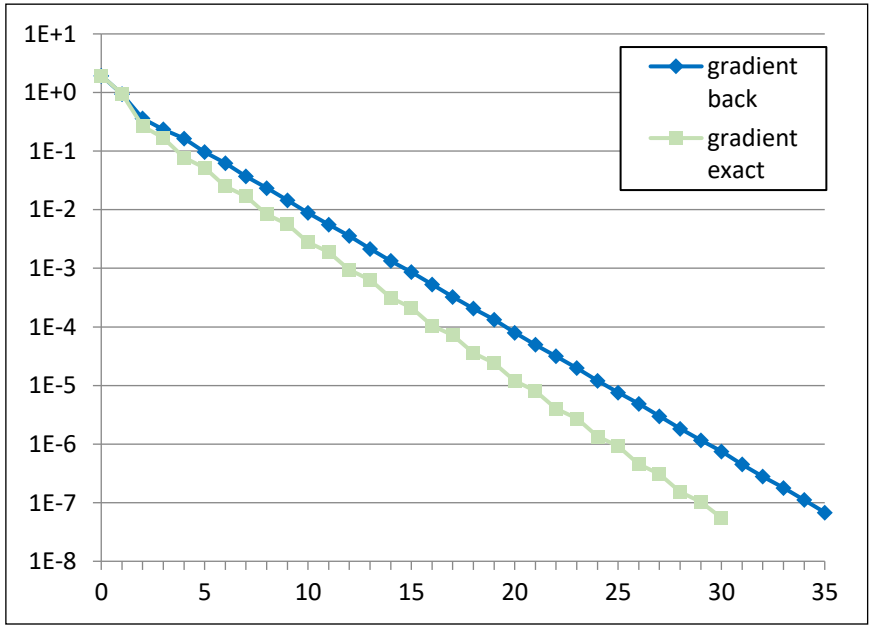
Newton method (unconstrained)

- Ill-conditionned hessian or non positive hessian
- Regularization requested



Gradient descent





Gradient descent

- The nemesis: the Rosenbrock banana function

$$f(x, y) = (1 - x)^2 + 100 * (y - x^2)^2$$

$$(x^*, y^*) = (1, 1)$$

$$p^* = 0$$