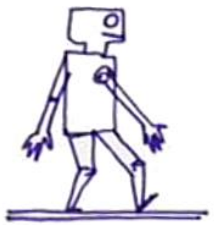
Kinematics and inverse kinematics

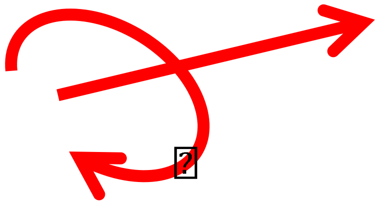
Nicolas Mansard

Gepetto
LAAS-CNRS & ANITI





Velocity is a field



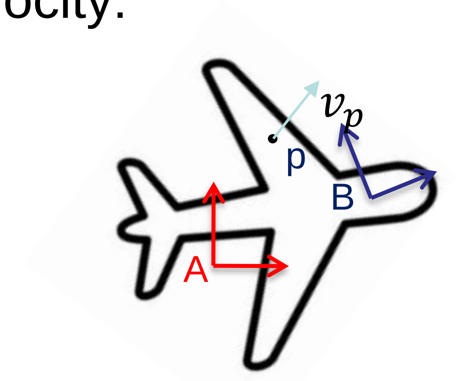
$$= \begin{bmatrix} v \\ \omega \end{bmatrix}$$

- Vector field defined by

$$v_A = v_B + \overrightarrow{AB} \times \omega$$

Spatial velocity

- Following the derivations of angular velocity:



The *spatial velocity* defines a vector field of linear velocities

- Spatial velocities are *transported* by SE(3)

Direct and inverse functions

- Direct geometry

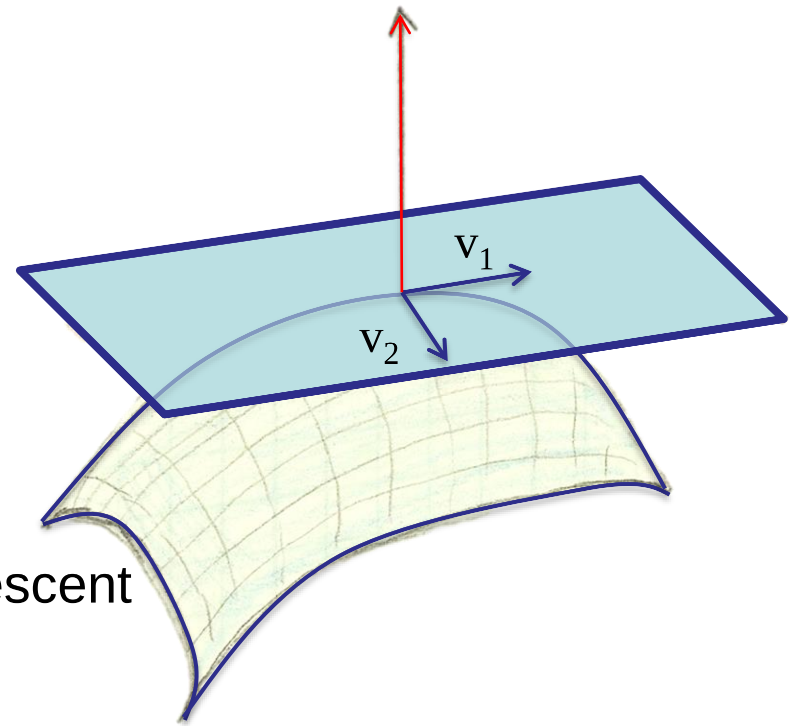
  h: q $\rightarrow$ h(q), C[1] continuous function

- Direct kinematics

  v: q, $\rightarrow$ v (q,) = J(q)

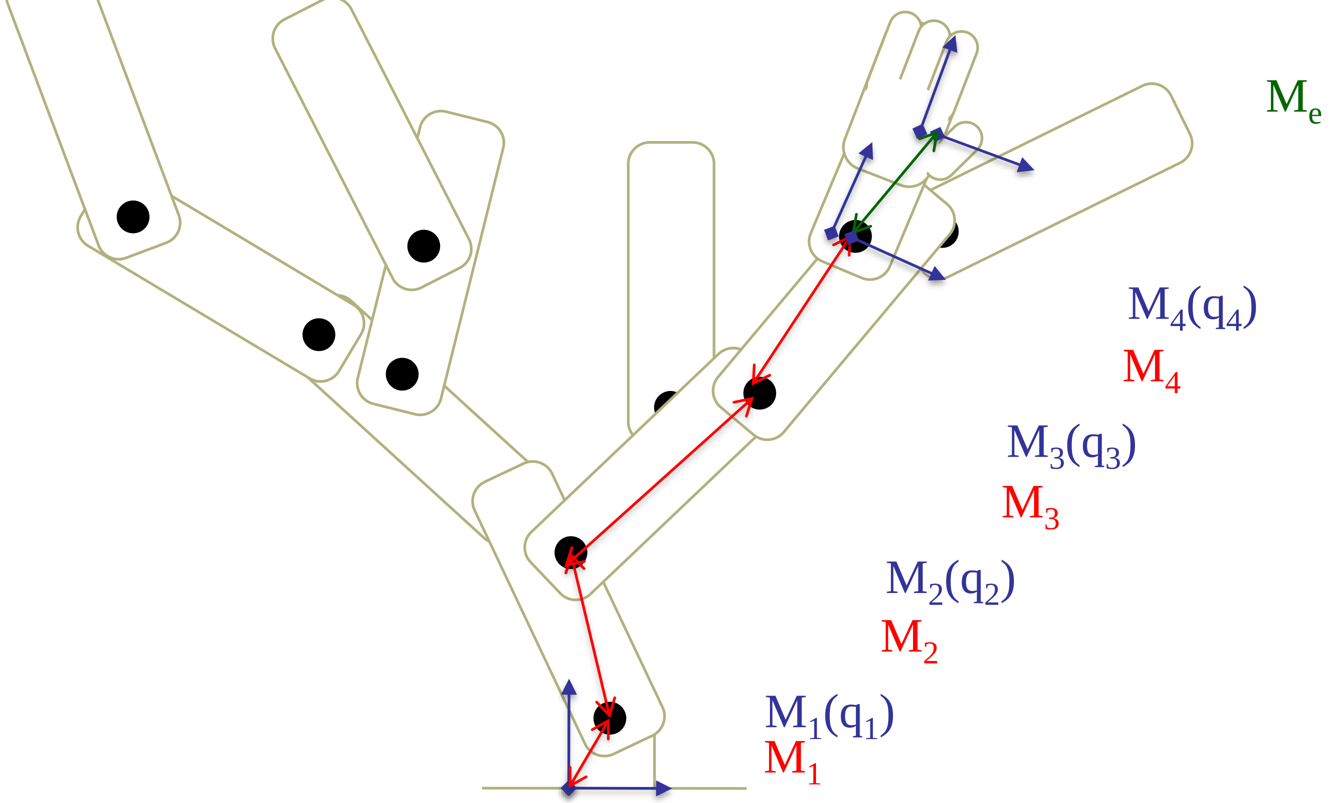
- Inverse geometry
  - Ill defined, singular points
  - Numerical inversion by Newton descent

- Integration of the descent
  - Robot trajectory
  - Quadratic problem at each step



Direct (forward) geometry

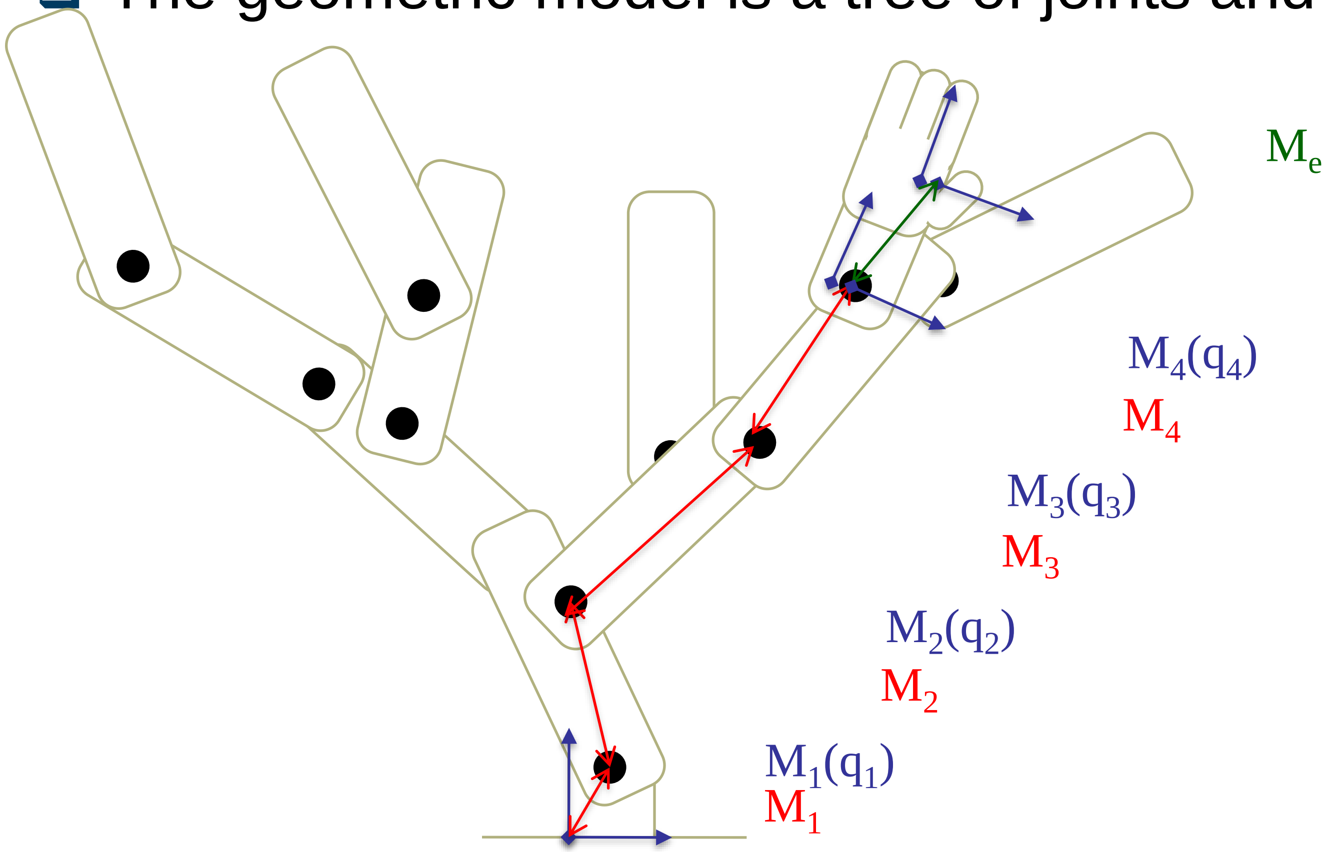
- The geometric model is a tree of joints and bodies



$$M(q) = M_1 \oplus M_1(q_1) \oplus M_2 \oplus \ \dots \oplus M_4 \oplus M_4(q_4) \ \oplus M_e$$

Direct (forward) kinematics

- The geometric model is a tree of joints and bodies



$$v_E = {}^0X_1\,{}^1v_1(q_1, v_{q1}) + {}^0X_2\,{}^2v_2(q_2, v_{q2}) + {}^0X_3\,{}^3v_3(q_3, v_{q3}) + \dots$$
$$= J_1(q_1)\;v_{q1} + J_2(q_2)\,v_{q2} + J_3(q_3)\;v_{q3} + \dots$$

Robot jacobian

- Transform joint velocities into Cartesian velocities

$$\dot{p} = J_3 \, v_q$$
$$\nu = J_6 \, v_q$$

- If we know the reference velocity v^* we want to see…

Search v_q so that $v = J \, v_q = v^*$

$$\min_{v_q} \left\| J \, v_q - v^* \right\|^2$$

A linear minimization solution

$$Ax = b$$

- If A is not invertible, two cases:
  - There is no way to *reach* b

$$x^* = \text{argmin } ||Ax-b||$$
$$= \{ x \text{, so that } ||Ax-b|| = \min ||Ax-b|| \}$$

  - There is many optimal solutions for x

$$x^* = \min \{ x \text{ so that } x = \text{argmin } ||Ax-b|| \}$$

$$\boxed{x^* = \min \text{ argmin } ||Ax-b||} = A^+ b$$

Control in the task space

- Configuration space VS task space

- Easy motion specification
- Reusability - versatility
- Deformation of the motion
- Sensor feedback