Geometry

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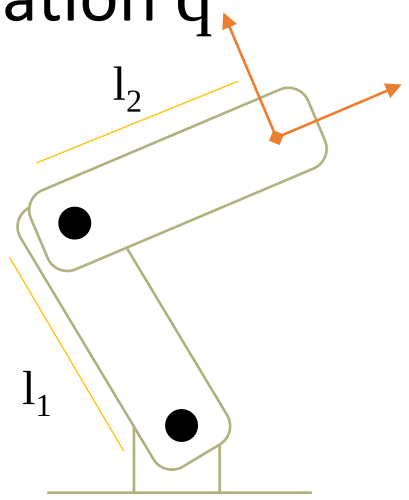
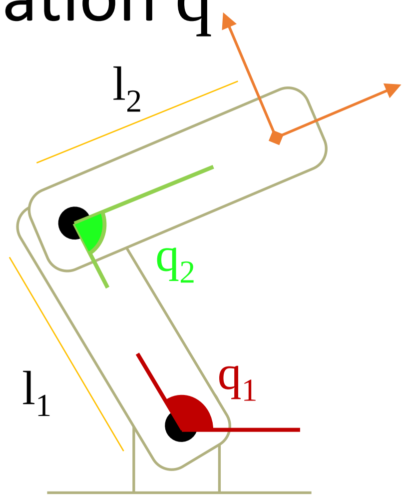
Geometry model

Robot configuration q



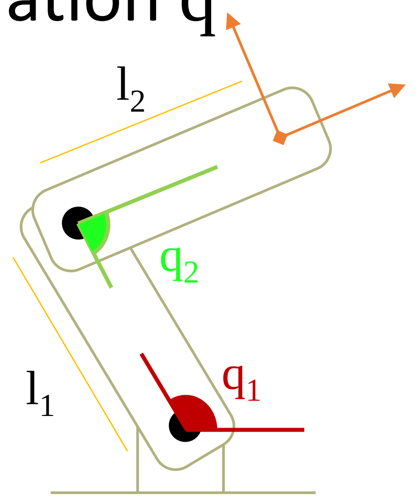
Geometry model

Robot configuration q



Geometry model

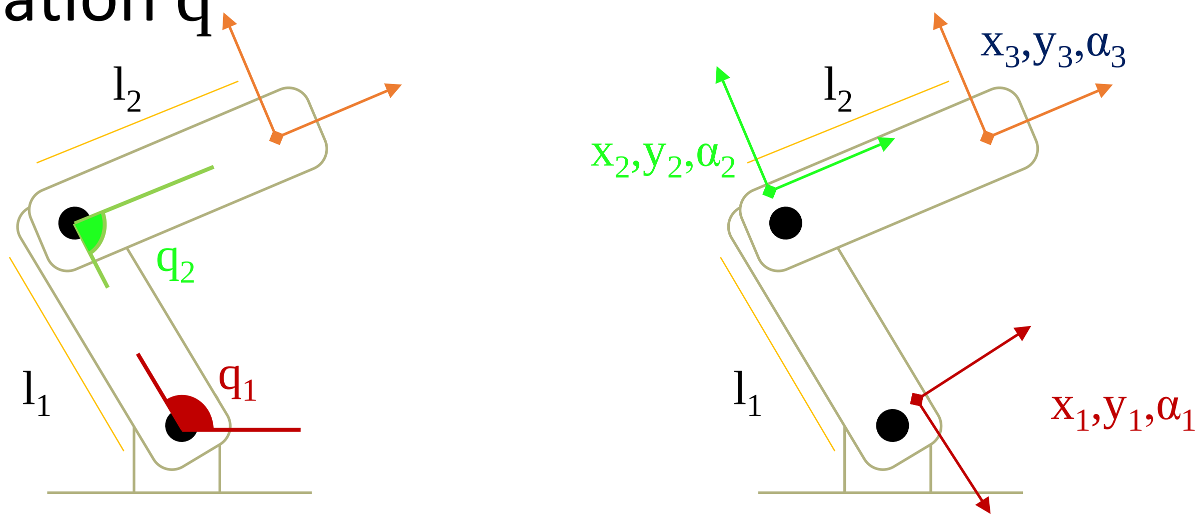
Robot configuration q



$$\begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1+q_2) \\ l_1 \sin(q_1) + l_2 \sin(q_1+q_2) \end{bmatrix}$$

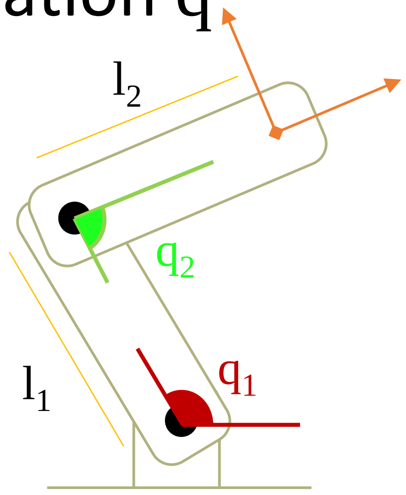
Geometry model

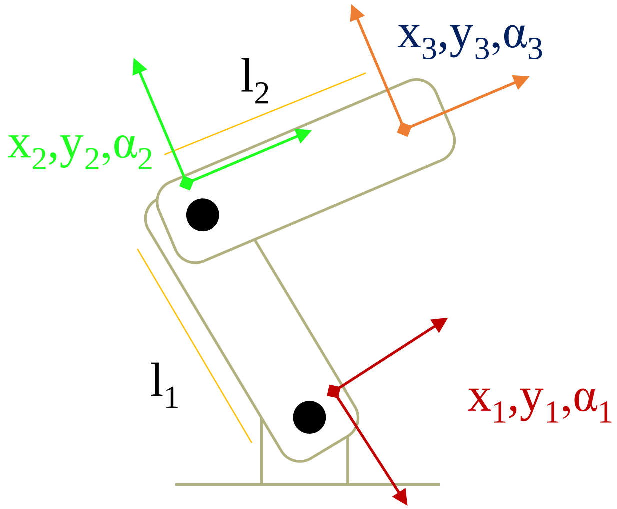
Robot configuration q



Geometry model

Robot configuration q





… such that

$$(x_1-x_2)^2+(y_1-y_2)^2 = cst$$
$$(x_2-x_3)^2+(y_2-y_3)^2 = cst$$

… etc …

Rotation

- Rotation matrices

$$R = \begin{pmatrix} r00 & r01 & r02 \\ r10 & r11 & r12 \\ r20 & r21 & r22 \end{pmatrix}$$

- Derivation of a matrix

$$\dot{R} = \cdots$$

- Representation of rotations
  - Matrices n=9, usable, non vector
  - Quaternion n=4, partly usable, vector, simple constraint
  - Angle vector n=3, non usable, vector, minimal unconstrained
  - ~~Euler angle / roll-pitch-yaw~~ don't use it

Angular velocity / Angle vector

- Formal definition

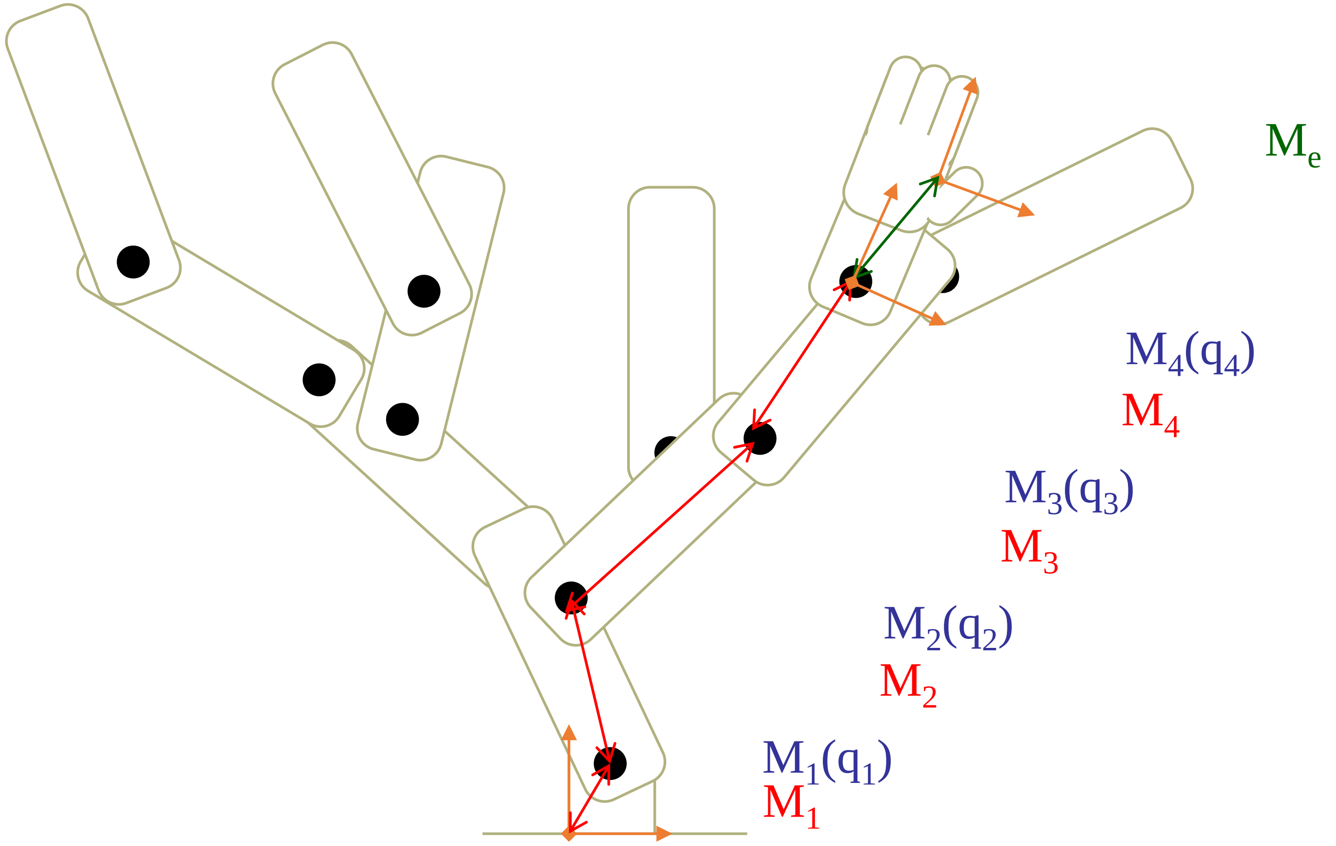
$$\dot{R} = \omega \times R$$

- From rotation to velocity
  - $R \to \omega$
- From velocity to rotation?
  - $\omega \to R$ … integrate

- Meaning of ω ?
- Angle axis representation

Direct geometry

- The geometric model is a tree of joints and bodies



$$M(q) = M_1 \oplus M_1(q_1) \oplus M_2 \oplus \ldots \oplus M_4 \oplus M_4(q_4) \oplus M_e$$

About representation of motion

$$M(q) = M_1 \oplus M_1(q_1) \oplus M_2 \oplus \ \dots \oplus M_4 \oplus M_4(q_4) \ \oplus M_e$$

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

Homogeneous matrix
… represents SE(3)

$$\dot{R} = \omega \times R$$

Canonical definition
of angular velocity

The geometric model is a tree of joints and bodies

What is $M \in SE(3)$

What is $\dot{M}$ (and $\dot{R}$)

Links with the differential geometry

Joint models

- Maps
  - From configuration space
  - To SE3 space

$$h(q) = {}^{k}M_{k+1}(q) \quad \in SE(3)$$

- For example, Revolute-Z is:

$$h(q) = \begin{bmatrix} \cos q & \sin q & 0 & 0 \\ -\sin q & \cos q & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & 0 & & 1 \end{bmatrix}$$

Kinematic model parametrization

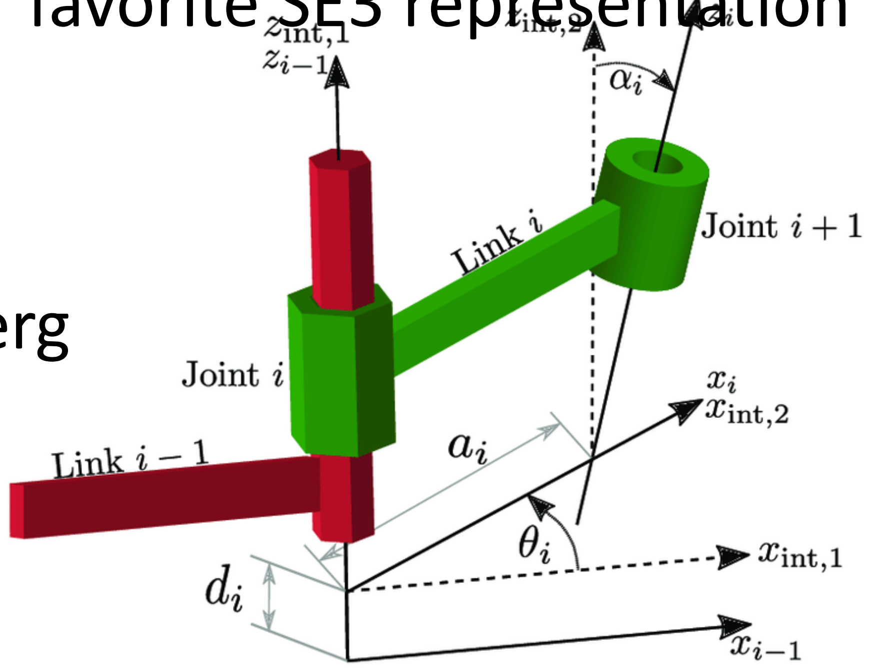
- Parent-to-child joint transformation

- Modern solution: with your favorite SE3 representation

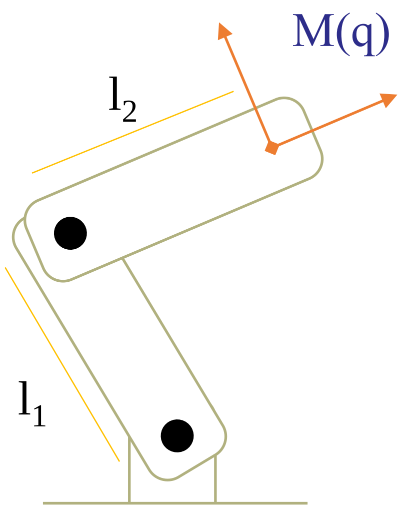
- Good-old days:

  with Denavit-Hartenberg

  minimal parameters



Direct geometry



From

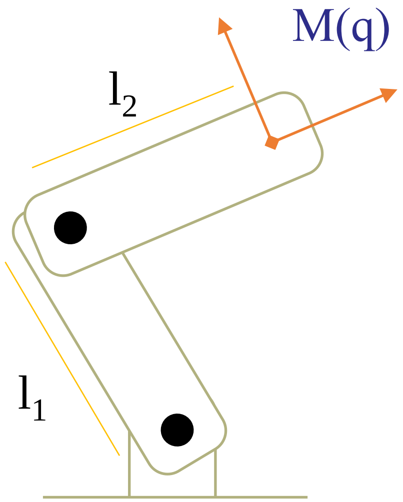
Robot configuration q

To

Effector cartesian placement M

$$M: q \rightarrow M(q)$$

Inverse geometry



Being given a M^* …

what is q such that $M(q) = M^*$

$$M^{-1}: M^* \rightarrow q=M^{-1}(M^*)$$

Numerical inversion of the geometry

- Computing analytically h^{-1} is difficult and tedious
- We can compute it numerically!

- Problem definition

$$search\ f(x) - f^{*} = 0$$

$$min\ \| f(x) - f^{*} \|^{2}$$