4. Dynamics

5 minutes trailer

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Dynamics of rigid bodies

- M: placement in SE3
- ν: “spatial” velocity of SE3
  - $\dot{M} = \nu \times M$
- α: “spatial” acceleration in SE3
  - $\nu \in M^6 = se(3)$
  - $\alpha \in M^6 = se(3)$
  - $\alpha = \dot{\nu}$
- ϕ: “spatial” force in SE3
  - Power $P = < \phi \mid \nu > = {}^{A}\phi^{T}\,{}^{A}\nu \in R$
  - $\eta \in F^6$: momentum
- Y: “spatial” inertia in SE3
  - $\eta = Y\,\nu$
  - $\phi = Y\,\alpha$

Dynamics of rigid bodies

- Newton-Euler law of movements

$$\dot{h} = \phi$$

$$\boxed{Y\alpha + \nu \times Y\nu = \phi}$$

Dynamics of articulated bodies

- Dynamic equation of the robot

$$M(q)\dot{v}_q + h\left(q, v_q\right) + g(q) = \tau_q$$

- Actuation of the robot
  - Fixed manipulator: $\tau_q = \tau_m$
  - Floating robot: $\tau_q = \begin{bmatrix} 0 \\ \tau_m \end{bmatrix} = S^T \tau_m$
  - Robot in contact: : $\tau_q = S^T \tau_m + J^T \phi$

Contact dynamics

- Gauss principle: lowest movement variation

$$a = \min \| a - a_{free} \|$$

- Gauss principle with contact

$$a = \min \| a - a_{free} \| \quad \text{s.t.} \quad J\, a = 0$$

- Forces arise as Lagrange multipliers of the constraint

Optimization with constraint

- Linearly-Constrained Quadratic Program (LCQP)

$$\min_{x} ||Ax - b||^2$$
$$s.t\ \ Cx \leq d$$