$$\lim_{x \to \pi} \frac{\sin^2 x}{1+\cos^3 x} = \frac{0}{0}$$

HOP:

$$\lim_{x \to \pi} \frac{2\cancel{\cos} x \cancel{\sin x}}{-3\cos^{\cancel{2}} x \cancel{\sin x}} = \frac{2}{-3\cos x} = \frac{2}{3}$$

$$\lim_{x\to 1}(1-x)\tan\frac{\pi}{2}x = 0\times\infty$$

$$t = 1-x, \quad x = 1-t, \quad x\to 1 \Leftrightarrow t\to 0$$

$$= \lim_{t\to 0} t\cdot\tan\frac{\pi}{2}(1-t)$$

$$= \lim_{t\to 0} t\cdot\tan\left(\frac{\pi}{2}-\frac{\pi t}{2}\right)$$

$$= \lim_{t \to 0} t \cdot \cot\left(\frac{\pi t}{2}\right)$$

$$= \lim_{t \to 0} t \cdot \frac{1}{\tan \frac{\pi t}{2}} = \frac{t}{\tan \frac{\pi t}{2}}$$

$$\xRightarrow{\times \frac{\pi}{2} / \pi/2} \lim_{t \to 0} \left( \frac{\frac{\pi t}{2}}{\tan \frac{\pi t}{2}} \times \frac{2}{\pi} \right)$$

$$\lim_{u \to 0} \frac{u}{\tan u} = 1$$

$$\Longrightarrow \lim_{t \to 0} \left(1 \times \frac{2}{\pi}\right) = \frac{2}{\pi}$$

$$\lim_{x \to 0} \frac{\sqrt{1-\cos^2 x}}{x} = \frac{0}{0}$$

$$1-\cos^2 x = \sin^2 x \Rightarrow$$

$$= \lim_{x \to 0} \frac{\sqrt{\sin^2 x}}{x} = \frac{|\sin x|}{x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \Longrightarrow \lim_{x \to 0^-} \frac{|\sin x|}{0^-} = -1 , \lim_{x \to 0^+} \frac{|\sin x|}{0^+} = +1$$

$$\lim_{n\to+\infty} \frac{\sqrt{n+\sqrt{n+\sqrt{n}}}}{n} = \frac{\infty}{\infty}$$

$$\lim_{u\to a} \sqrt{u+\sqrt{u}} \simeq \sqrt{\sqrt{u}}$$

$$= \lim_{n\to+\infty} \frac{\sqrt{\sqrt{\sqrt{n}}}}{n} = \frac{n^{\frac{1}{8}}}{n^{1}} \quad \frac{1}{8}<1 \quad = \boxed{0}$$

$$\lim_{x\to 1} \frac{\cos\frac{\pi}{2}x}{1-x} = \frac{0}{0}$$

$$t = 1 - x \Rightarrow x = 1 - t \,,\; x \to 1 \Leftrightarrow t \to 0$$

$$= \lim_{t\to 0} \frac{\cos\frac{\pi}{2}(1-t)}{t} = \frac{\cos\left(\frac{\pi}{2} - \frac{\pi t}{2}\right)}{t}$$

$$\cos x = -\sin\left(x - \frac{\pi}{2}\right) \Rightarrow$$

$$\lim_{t\to 0} \frac{\cancel{-}\sin\left(\cancel{-}\frac{\pi t}{2}\right)}{t} \times \frac{\frac{\pi}{2}}{\pi/2}$$

$$\lim_{t \to 0} \frac{\sin\left(\frac{\pi t}{2}\right)}{\underbrace{\frac{\pi t}{2}}_{1}} \times \frac{\pi}{2} = \frac{\pi}{2}$$

$$\lim_{n \to +\infty} \frac{\sqrt[3]{27n^6 - 4n}}{4n^2} =$$

$$\xrightarrow{\text{بجوار}} \lim_{n \to +\infty} \frac{\sqrt[3]{27n^6}}{4n^2} = \frac{3n^2}{4n^2} \overset{2=2}{=} \frac{3}{4}$$

$$\lim_{n\to\infty} \frac{1+2+\dots+n}{3n^2} =$$

$$\xrightarrow[\text{توان}]{\div} \lim_{n\to\infty} \frac{n^1}{3n^2} \quad 1<2 \longrightarrow = 0$$

$$\lim_{n\to\infty} \frac{1^2+2^2+\dots+n^2}{4n^3} =$$

$$\xrightarrow{\text{توان} \div} \lim_{n\to\infty} \frac{n^2}{4n^3} \quad 2<3 \longrightarrow = 0$$

$$\lim \frac{\sqrt[3]{1-x/3}-\sqrt[4]{1+x/5}}{1-\sqrt{1-x/2}}$$

$$\lim_{x\to 0} \frac{\sqrt[3]{1-x/3}-\sqrt[4]{1+x/5}}{1-\sqrt{1-x/2}}$$

0?

$$\lim_{n \to \infty} \left( \frac{n+5}{n+4} \right)^{n+4}$$

$$= \lim_{n \to \infty} \left( \frac{1}{n+4} + \frac{n+4}{n+4} \right)^{n+4}$$

$$= \lim_{n \to \infty} \left( 1 + \frac{1}{n+4} \right)^{n+4}$$

$$t = n+4, \quad n \to \infty \iff t \to \infty$$

$$\lim_{t \to \infty} \left( 1 + \frac{1}{t} \right)^{t} = e$$

$$\lim_{x \to 0} \frac{\sqrt{1-x} - \sqrt{1+x}}{x} =$$

$$\lim_{x \to 0} \frac{\left(\cancel{1} - \frac{x}{2}\right) - \left(\cancel{1} + \frac{x}{2}\right)}{x}$$

$$= \lim_{x \to 0} \frac{-\cancel{2}x/\cancel{2}}{x} = -1$$

$$\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{x^2 - 6x + 5} = \frac{0}{0}$$

HOP:

$$\lim_{x \to 1} \frac{\frac{1}{3\sqrt[3]{x}}}{2x - 6} = \frac{1}{3(2-6)} = \frac{1}{-12}$$

$$
f(x) = \begin{cases} 2ax^3 + bx - 3 & x < 1 \\ x^3 - x + 4a & 1 \leq x < 2 \\ ax - 2b & x \geq 2 \end{cases}
$$

$a , b = ?$ تا تابع پیوسته

باشد

$$f(1) = 1^3 - 1 + 4a = 4a$$

$$\lim_{x \to 1^-} f(x) = 2a + b - 3$$

$$\lim_{x \to 1^-} = f(1) \Rightarrow 2a + b - 3 = 4a$$

$$\Rightarrow b - 3 = 2a$$

$$f(2) = 2a - 2b$$

$$\lim_{x \to 2^-} f(x) = 2^3 - 2 + 4 | a$$

$$\lim_{x \to 2^-} = f(2) \Rightarrow$$

$$\cancel{2}a - 2b = \overset{6}{\cancel{8 - 2}} + \overset{2}{\cancel{4}} | a$$

$$\Rightarrow -2b = 6 + 2a$$

$$\Rightarrow \begin{cases} -2b = 6 + 2a \\ b - 3 = 2a \end{cases}$$

$-$

$$-3b + 3 = 6 \Rightarrow -3b = 3$$

$$b = -1$$

$$\overbrace{b}^{-1} - 3 = 2a \Rightarrow a = -2$$

$$\lim_{n\to\infty} n\sqrt{n^2+1} - n$$

$$\lim_{n\to\infty} n\left(\sqrt{n^2+1} - 1\right)$$

$$\lim_{n\to\infty} \infty\left(\sqrt{\infty^2+1} - 1\right)$$

$$\lim_{n\to\infty} \infty(\infty - 1) = \infty$$

if $\tanh x = \frac{13}{14}$ $\quad x > 0$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$1 - \frac{169}{196} = \frac{1}{\cosh^2 x} \Rightarrow \frac{27}{196} = \frac{1}{\cosh^2 x}$$

$$\cosh^2 x = \frac{196}{27} \Rightarrow \cosh x = \left|\frac{14}{3\sqrt{3}}\right|$$

$$x > 0 \Rightarrow \cosh x > 0$$

$$\Rightarrow \boxed{\cosh x = \frac{14}{3\sqrt{3}}}, \quad \boxed{\operatorname{sech} x = \frac{3\sqrt{3}}{14}}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \Rightarrow \cosh x \cdot \tanh x = \sinh x$$

$$\Rightarrow \sinh x = \frac{\cancel{14}}{3\sqrt{3}} \times \frac{13}{\cancel{14}} = \frac{13}{3\sqrt{3}}, \quad \operatorname{csch} x = \frac{3\sqrt{3}}{13}$$

$$\coth x = \frac{1}{\tanh x} \xRightarrow{\tanh x = \frac{13}{14}} \coth x = \frac{14}{13}$$