

# Multi-directional two-dimensional PCA with matching score level fusion for face recognition

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**Abstract** The traditional matrix-based feature extraction methods that have been widely used in face recognition essentially work on the facial image matrixes only in one or two directions. For example, 2DPCA can be seen as the row-based PCA and only reflects the information in each row, and some structure information cannot be uncovered by it. In this paper, we propose the directional 2DPCA that can extract features from the matrixes in any direction. To effectively use all the features extracted by the D2DPCA, we combine a bank of D2DPCA performed in different directions to develop a matching score level fusion method named multi-directional 2DPCA for face recognition. The results of experiments on AR and FERET datasets show that the proposed method can obtain a higher accuracy than the previous matrix-based feature extraction methods.

**Keywords** Face recognition · Matrix-based feature extraction · 2DPCA · Feature fusion

## 1 Introduction

Many feature extraction methods have been proposed for face recognition in the past few decades [1–3]. Generally, they can be divided into vector-based and matrix-based feature extraction methods. In the vector-based methods, the 2D image matrix must be previously transformed into

high-dimensional 1D vector. The PCA [4] and LDA [2] are two well-known vector-based feature extraction methods in face recognition. The matrix-to-vector operation they used will induce a large-scale scatter matrix. As a result, the computation of the eigen-vectors of the vector-based methods is very time-consuming. More importantly, this operation destroys the structural information embedding in the image matrix. The matrix-based methods do not need the matrix-to-vector operation and directly process image matrixes.

In 2004, the matrix-based PCA feature extraction method, i.e., two-dimensional PCA (2DPCA), was proposed by Yang et al. [5]. The 2DPCA directly extracts the features from the image matrix by projecting the image matrix along the projection axes that are the eigen-vectors of the image scatter matrix [6], and the projected result is the so-called feature matrix. As the scatter matrix of 2DPCA has the lower dimensionality than that of PCA, 2DPCA is computationally more efficient than PCA. In Ref. [7], the authors first indicate that 2DPCA essentially works in the row direction of images, and then propose the Alternative-2DPCA that works in the column direction of images. 2DPCA learns an optimal matrix from a set of training images reflecting information in each row of images. Similarly, the Alternative-2DPCA learns optimal matrix reflecting information in each column of images, which also can be viewed as the 2DPCA working in column direction. Xiong et al. [8] and Li et al. [9] extended one-dimensional LDA to 2DLDA. One of the common disadvantages of the naive matrix-based feature extraction methods, including 2DPCA, Alternative-2DPCA and 2DLDA, is that they extract feature from the image matrix in only one direction. To alleviate this problem, some researchers proposed bidirectional matrix-based feature extraction methods such as (2D)2LDA [10], (2D)2PCA [7]

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and 2DPCA plus 2DLDA [11]. For example, the (2D)2PCA simultaneously utilizes the 2DPCA and Alternative-2DPCA to find the projections on both sides of the image matrix independently. Y. Qi et al. proposed an iterative matrix-based feature extraction method, i.e., (2D)2PCALDA [12]. In this method, the 2DLDA working in the column direction is performed on the feature matrix of the 2DPCA working in the row direction.

All the previous matrix-based methods extract features from the matrix in one or two directions, i.e., row or column direction. The features extracted from one or two directions are always not sufficient for achieving high classification accuracy. It is reasonable to have the assumption that the vectors of the image matrix in different directions have different influences for accurate classification. In this study, we aim to develop a novel matrix-based feature extraction method that can extract features from the matrix in any direction, not only the row and column directions used in the previous methods. For achieving this object, we rotate the sample matrix in certain angle and perform 2DPCA on the rotated matrixes, which is equivalent to performing 2DPCA in the corresponding direction. We refer to this method as directional 2DPCA (D2DPCA). Since the D2DPCA can extract features from matrix in different directions, how to effectively fuse these features for improving classification accuracy should be well concerned. For solving this problem, we present Multi-directional 2DPCA (MD2DPCA) that uses the matching score level fusion framework to integrate several D2DPCA performed in different directions for face recognition. The implementation of our method is not complicated. The experimental results carried on the public face datasets indicate that MD2DPCA is potentially more powerful for face recognition than 2DPCA, Alternative-2DPCA and (2D)2PCA.

Although this paper focuses on improving 2DPCA, the idea and method could be useful for other matrix-based feature extraction methods. The rest of this paper is organized as follows: In Sect. 2, we briefly review the 2DPCA method. In Sect. 3, we formally present the D2DPCA and show the equivalence between the D2DPCA and a special block-based PCA. The MD2DPCA simultaneously using several D2DPCAs for face recognition is proposed in Sect. 4. In Sect. 5, the experiments on two public face datasets are given to evaluate the performance of our method. Finally, we offer the conclusion in Sect. 6.

## 2 2DPCA

Consider  $l$  training samples  $(A_1, A_2, \dots, A_l)$ , and each of them is an  $m$  by  $n$  image matrix. Projecting the training samples onto  $x$  yields a vector:

$$y_i = A_i x \quad (i = 1, 2, \dots, l) \quad (1)$$

To evaluate the goodness of a projection vector, the 2DPCA uses the trace of the scatter matrix. It tries to find the optimal vector  $x_{\text{opt}}^{2DPCA}$  such that

$$x_{\text{opt}}^{2DPCA} = \arg \max (x^T G^{2DPCA} x) \quad (2)$$

where  $G^{2DPCA}$  denotes the scatter matrix of 2DPCA:

$$G^{2DPCA} = \frac{1}{l} \sum_{i=1}^l (A_i - \bar{A})^T (A_i - \bar{A}) \quad (3)$$

$\bar{A}$  is the mean of all the training images. Generally, it is not enough to have only one optimal projection axis. The 2DPCA usually needs to select a set of projection axes,  $x_1, x_2, \dots, x_d$ , satisfying:

$$\begin{cases} x_1^{2DPCA}, x_2^{2DPCA}, \dots, x_d^{2DPCA} = \arg \max (x^T G^{2DPCA} x) \\ x_i^{2DPCA} x_j^{2DPCA} = 0 \quad (i, j = 1, 2, \dots, d \text{ and } i \neq j) \end{cases} \quad (4)$$

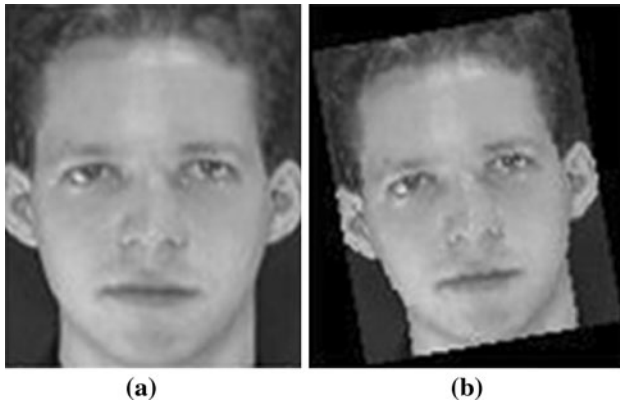
## 3 Directional 2DPCA

### 3.1 Feature extraction by directional 2DPCA

In this section, we aim to develop a novel matrix-based method i.e., directional 2DPCA (D2DPCA) that can extract the features from the sample matrix in any direction. This idea can be simply implemented by performing the naïve 2DPCA on the rotated sample matrixes which is obtained by rotating original sample matrix in certain angle. It is helpful to understand this trick by reviewing the implementation of the Alternative-2DPCA, because the implementation of the Alternative-2DPCA also can be viewed as the naïve 2DPCA performing on the rotated matrixes, whose rotation angle is  $\pi/2$ . We rotate each sample matrix with a certain angle  $w$  and refer to the rotated sample matrixes as  $r_w(A_1), r_w(A_2), \dots, r_w(A_l)$ . The new position of each pixel in the rotated image is obtained by using original position multiplication of the rotation matrix  $R(w)$  in Euclidean space. The rotation matrix is a matrix that is used to perform the rotation. Let the rotation angle be  $w$ , we have:

$$R(w) = \begin{bmatrix} \cos w & -\sin w \\ \sin w & \cos w \end{bmatrix} \quad (5)$$

For making the rotated image be matrix form, we fill its four corners with 0 pixel values. Figure 1 presents the example of rotated image with  $w = \pi/18$ . We note that the four extra corners give no influence for the final feature extraction results, because their components of the feature matrix also have 0 values.



**Fig. 1** **a** The original face image, **b** the rotated face image ( $w = \pi/18$ )

Then, we project the rotated samples,  $r_w(A_1), r_w(A_2), \dots, r_w(A_l)$ , along the vector  $x$ .

$$Y_i = r_w(A_i)x \quad (i = 1, 2, \dots, l) \quad (6)$$

We define the image scatter matrix of the D2DPCA as:

$$G_{D2DPCA} = \frac{1}{l} \sum_{i=1}^l (r_w(A_i) - \overline{r_w(A)})^T (r_w(A_i) - \overline{r_w(A)}) \quad (7)$$

where  $\overline{r_w(A)}$  denotes the mean of all the rotated training images. Similarly to 2DPCA, the D2DPCA also uses the trace of the image scatter matrix to characterize the criterion for choosing the projection.

$$J(x) = x^T G_{D2DPCA} x \quad (8)$$

The optimal projection  $x_{\text{opt}}^{D2DPCA}$  is chosen, when the criterion is maximized. We have:

$$x_{\text{opt}}^{D2DPCA} = \arg \max J(x) \quad (9)$$

$J(x)$  is maximized when the projection vector is the eigen-vector of  $G_{D2DPCA}$ . We use  $d'$  orthonormal eigen-vectors of  $G_{D2DPCA}$ , i.e.  $x_1^{D2DPCA}, x_2^{D2DPCA}, \dots, x_{d'}^{D2DPCA}$ , corresponding to the  $d'$  largest eigen-values. The D2DPCA also uses the feature matrix to present the feature extract result. For an image sample  $A$ , its feature matrix  $Y$  obtained by D2DPCA with angle  $w$  can be computed as following equation:

$$Y = r_w(A)X \quad (10)$$

where  $Y = [y_1, y_2, \dots, y_{d'}]$  and  $X = [x_1^{D2DPCA}, x_2^{D2DPCA}, \dots, x_{d'}^{D2DPCA}]$ . As we can see from above, the 2DPCA and the Alternative-2DPCA are two special cases of D2DPCA, when  $w = 0, \pi/2$ , respectively.

### 3.2 Image reconstruction by directional 2DPCA

In the D2DPCA method, the feature matrix and eigen-vectors can be combined to reconstruct the image in the

following way. We suppose the eigen-vectors corresponding to the first  $d'$  largest eigen-values of the image scatter matrix are used in D2DPCA. After the image samples are projected onto these axes, the resulting principal component vectors can be obtained by Eq. (10). Since  $x_1^{D2DPCA}, x_2^{D2DPCA}, \dots, x_{d'}^{D2DPCA}$  are orthogonal, it is easy to obtain the reconstructed image of the rotated sample  $r_w(A_i)$  by:

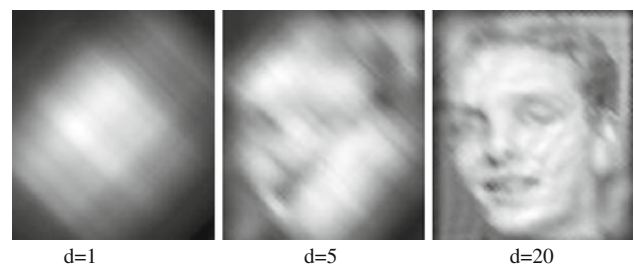
$$r_w(\hat{A}_i) = Y_i X^T \quad (11)$$

For getting the reconstructed sample, we can re-rotate  $r_w(\hat{A}_i)$  in the angle  $-w$ . We present some reconstructed images by D2DPCA with  $w = \pi/4$ .

In Fig. 2, the first reconstructed image contains most of the energy of the original image. The other two images add the detailed local information from different levels. Additionally, it is clear that all the images constructed by the D2DPCA emphasize the information in the diagonal direction, which is actually determined by the direction, in which the D2DPCA is performed. This example shows that the D2DPCA is apparently indicated to preserve structure information of image since it lies on sample matrixes in certain direction. The reconstructed image is an approximation for original image, if the selected number of the principal component vectors  $d' < s$ , where  $s$  is the total number of the eigen-vectors of the image scatter matrix of D2DPCA. If  $d' = s$ , the training image can be completely reconstructed by Eq. (11).

### 3.3 The explanation of directional 2DPCA from block-based PCA

When the D2DPCA is performed with angle  $w$ , we use the blocks  $r_w(X_i)_1^T, r_w(X_i)_2^T, \dots, r_w(X_i)_m^T$  to denote the  $m$  rows of the rotated and centered  $m$  by  $n$  matrixes, respectively. All the blocks have same size of  $1 \times n$ . Then, the scatter matrix of D2DPCA can be rewritten by:



**Fig. 2** Some reconstructed images based on D2DPCA with different number of eigen-vectors. The D2DPCA used in this example works on the matrix in the diagonal, because  $w$  is set to  $\pi/4$

$$\begin{aligned}
 G^{D2DPCA} &= \frac{1}{l} \sum_{i=1}^l \begin{bmatrix} r_w(X_i)_1^T \\ r_w(X_i)_2^T \\ \vdots \\ r_w(X_i)_m^T \end{bmatrix} \begin{bmatrix} r_w(X_i)_1^T \\ r_w(X_i)_2^T \\ \vdots \\ r_w(X_i)_m^T \end{bmatrix} \\
 &= \frac{1}{l} \sum_{i=1}^l \sum_{j=1}^m r_w(X_i)_j r_w(X_i)_j^T \quad (12)
 \end{aligned}$$

The D2DPCA is to compute the leading eigen-vectors of Eq. (12). When we choose each row of the rotated matrix as a block, there are  $l \times m$  blocks. For block-based PCA method, it views each block as a sample and performs PCA on all the samples. The scatter matrix of this special block-based PCA is same as  $G^{D2DPCA}$ . Therefore, if we consider each block of the rotated sample as a sample vector, D2DPCA is equivalent to the block-based PCA. That is, D2DPCA method is essentially to view each block as a sample. Similarly, 2DPCA also has this property. The difference is that the D2DPCA uses the rows of the rotated image, which are corresponding to the vectors in certain direction, as the block samples, and the 2DPCA uses the rows of the original matrix as the block samples. It explains that D2DPCA has greater flexibility in the division of the blocks than 2DPCA.

#### 4 Multi-directional 2DPCA

Simultaneously using the features extracted by the D2DPCA in different directions may be helpful for decreasing error classification accuracy. It is necessary to find a method to effectively fuse the features extracted by the D2DPCA in different directions for classification. In this section, we aim to combine a bank of D2DPCA in several different directions to develop a matching score level fusion method for face recognition, i.e., MD2DPCA. The process of the MD2DPCA is implemented by the following steps:

Step 1 Initialize an angle set:  $\{0, \pi/c, 2\pi/c, \dots, (c-1)\pi/c\}$ , where  $c$  is the number of the D2DPCA used in the fusion framework. Perform D2DPCA with each angle in the above set to obtain the projection vectors.

Step 2 In each D2DPCA, we project all the rotated training and testing samples along the projection vectors to obtain their feature matrix and then calculate the matching score between each testing sample and training sample using:  $s_{w,i,j} = \sum_{c=1}^d \|Y_i^c - Y_j^c\|$ , where  $Y_i^c$  and  $Y_j^c$  represent the  $c$ th projected feature vector of the feature matrix from sample  $A_i$  and  $A_j$ , respectively.  $\|Y_i^c - Y_j^c\|$

denotes the Euclidean distance between the two vector  $Y_i^c$  and  $Y_j^c$ .

Step 3 Calculate the final matching score of D2DPCA in all directions by:

$$s_{i,j} = \sum s_{w,i,j} \quad (13)$$

Step 4 Use nearest neighbor classifier for classification.

We also give the frameworks of the D2DPCA as the following Figure (Fig. 3).

#### 5 Experiments

The proposed MD2DPCA method is used for face recognition, and we carry experiments on two public face datasets. As the comparisons, three matrix-based PCA feature extraction methods, i.e., 2DPCA, Alternative-2DPCA and (2D)2PCA, also are performed on the datasets. The first face image dataset we used in the experiments is the AR dataset. It contains over 4000 face images of 126 people (70 men and 56 women) including frontal views of faces with different facial expressions, lighting conditions and occlusions. The face portion of each image is manually cropped and then normalized to the size of  $50 \times 40$ . We used only the images of these 120 subjects, and each subject has 26 images. Figure 4 shows the images of one subject. Thirteen image samples randomly selected from one class are used for training, and the remaining images for test.

We use four and six different directions in our methods, i.e.,  $c = 4, 6$  in step 1 of the MD2DPCA, respectively. Comparisons of recognition rates of our method and the other three methods, with different eigen-vector numbers ( $p = 6, 8, \dots, 20$ ), are shown in Table 1. It is noted that the (2D)2PCA obtains  $p$  eigen-vectors in both sides. For each D2DPCA used in MD2DPCA, every training sample is transformed to  $m \times p$ . It can be observed from Table 1 that our method always performs better than the others in all cases. Compared with the results from Table 1, the top recognition rate of 61.60% achieved by our method has 1.15, 1.28 and 2.11 percentage points higher than the

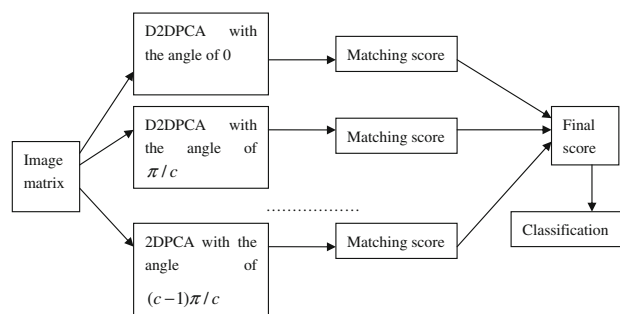


Fig. 3 The framework of MD2DPCA



**Fig. 4** Some images of one subject in AR dataset

**Table 1** Comparison of recognition rate (%) on AR dataset

Eigen-vector number	6	8	10	12	14	16	18	20	Average
2DPCA	57.69	58.78	59.55	60.32	50.26	60.45	60.38	60.38	59.31
Alternative-2DPCA	57.56	58.72	59.10	59.42	59.49	59.94	50.94	60.32	59.73
(2D)2PCA	57.44	58.65	59.36	59.10	59.49	59.29	59.29	59.42	59.42
Our method (MD2DPCA with 4 directions)	<b>58.08</b>	59.55	60.06	60.77	60.77	60.83	60.83	<b>60.77</b>	60.21
Our method (MD2DPCA with 6 directions)	<b>58.08</b>	<b>59.74</b>	<b>61.03</b>	<b>60.83</b>	<b>60.83</b>	<b>60.90</b>	<b>61.60</b>	<b>60.77</b>	<b>60.47</b>

Bold indicate the significance of highest accuracy values



**Fig. 5** Some images of one subject in FERET dataset

**Table 2** Average recognition rate (%) of different methods on FERET dataset

Training image number per class	2	3	4
2DPCA	47.12	52.66	55.20
Alternative-2DPCA	48.31	53.21	53.97
(2D)2PCA	47.70	52.36	55.45
Our method (MD2DPCA with 4 directions)	<b>49.36</b>	53.94	57.08
Our method (MD2DPCA with 6 directions)	48.93	<b>54.83</b>	<b>57.60</b>

Bold indicate the significance of highest accuracy values

highest recognition rate achieved by 2DPCA, Alternative-2DPCA and (2D)2PCA, respectively. Table 1 also shows the average recognition rates of all methods. The average recognition rate of 61.47% obtained by our method is also significantly higher than that of the others.

The FERET face database is also used in our experiments. It is a result of the FERET program, which was sponsored by the US Department of Defense through the DARPA Program [13, 14]. It has become a standard database for testing and evaluating state-of-the-art face



recognition algorithms. The proposed method was tested on a subset of the FERET database. This subset includes 1,400 images of 200 individuals (each individual has seven images). This subset involves variations in facial expression, illumination and pose. In our experiment, each original image was tightly cropped and resized to  $80 \times 80$  pixels. Some example images of one person are shown in Fig. 5. To examine how the recognition rate changes depending on the number of training samples of each class, we pick  $k = 2, 3, 4$  images from each subject at random for training. Remaining  $7-k$  images are employed for testing. Ten possible selections of  $k$  training images per class are chosen in the experiments, so the experiments are repeated 10 times with these selections. We use 20 eigen-vectors in our method, 2DPCA, Alternative-2DPCA and 2D2PCA.

Table 2 shows us the comparisons of our method and the other three methods on AR dataset. From Table 2, we can find our method with 6 directions has the highest average recognition rates of 54.83% and 57.60% with 3 and 4 training samples per class among all methods. For 2 training samples per class, our method with 4 directions achieves the highest average recognition rate of 49.36%.

## 6 Conclusion

The traditional matrix-based feature extraction methods can be performed only in row or column direction; some useful information in other directions cannot be extracted by them. In this paper, we present the D2DPCA to extract feature from the matrix in any direction, whose implementation is very simple. Then, we combine a bank of D2DPCA to develop a matching score level fusion method, i.e., MD2DPCA. The results of experiments on AR and FERET datasets show that the proposed method is potentially more powerful for classifying human face images than other previous matrix-based feature extraction methods. These results also prove not only are the features extracted in row or column direction useful for accurate

classification, but also the features extracted in other directions are helpful for improving accuracy.

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