Empirical potential

- Bohr & Mottelson Vol.1

$$U = V f(r) + V_{\ell s}\left(\frac{\boldsymbol{\ell}\cdot\boldsymbol{s}}{\hbar^2}\right) r_0^2 \frac{1}{r}\frac{d}{dr} f(r)$$

- Central part roughly follows shape of density

$$f(r) = \left[1 + \exp\left(\frac{r-R}{a}\right)\right]^{-1}$$

- Woods-Saxon form
- Depth $V = \left[-51 \pm 33 \left(\frac{N-Z}{A}\right)\right] \text{MeV}$

  \+ neutrons

  \- protons

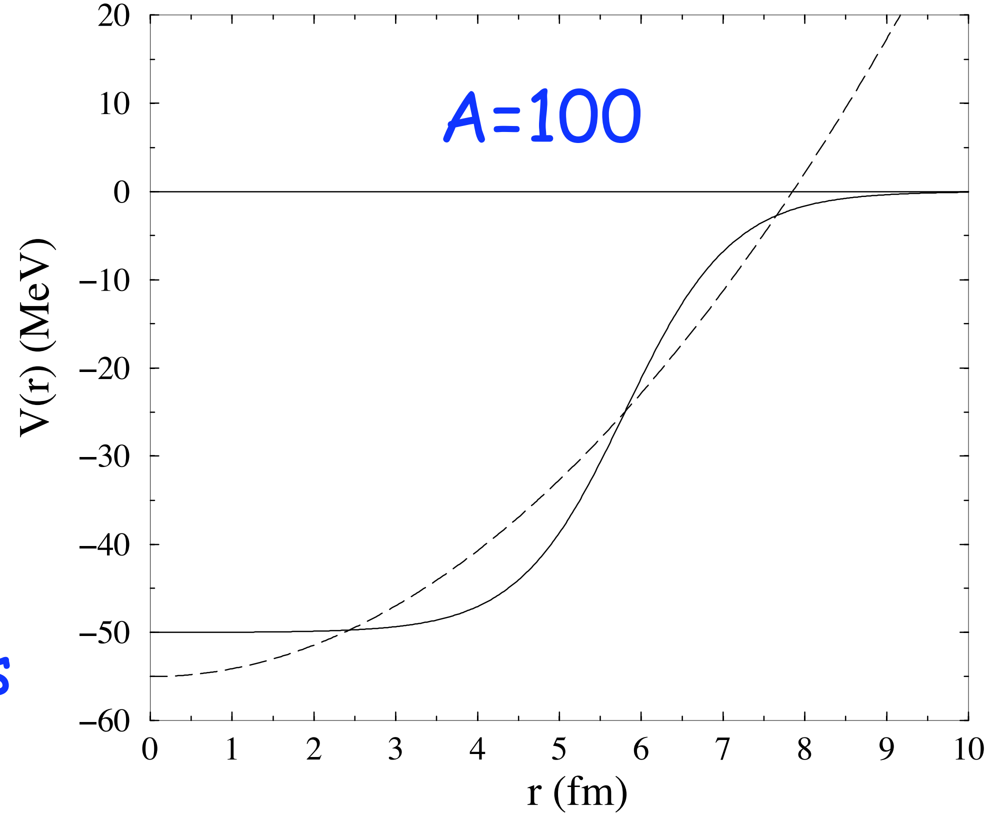
- radius $R = r_0 \, A^{1/3}$ with $r_0 = 1.27 \text{ fm}$
- diffuseness $a = 0.67 \text{ fm}$

Analytically solvable alternative

- Woods-Saxon (WS) generates finite number of bound states
- IPM: fill lowest levels ⇒ nuclear shells ⇒ magic numbers
- reasonably approximated by 3D harmonic oscillator

$$U_{HO}(r) = \frac{1}{2}m\omega^2 r^2 - V_0$$

$$H_0 = \frac{\boldsymbol{p}^2}{2m} + U_{HO}(r)$$



- Eigenstates in spherical basis

$$H_{HO}\left|n\ell m_\ell m_s\right\rangle = \left[\hbar\omega(2n+\ell+\tfrac{3}{2}) - V_0\right]\left|n\ell m_\ell m_s\right\rangle$$

Harmonic oscillator

- Filling of oscillator shells
- # of quanta $N = 2n + \ell$

N	n	ℓ	# of particles	"magic #"	parity
0	0	0	2	2	+
1	0	1	6	8	-
2	1	0	2		+
2	0	2	10	20	+
3	1	1	6		-
3	0	3	14	40	-
4	2	0	2		+
4	1	2	10		+
4	0	4	18	70	+

Need for another type of sp potential

- 1949 Mayer and Jensen suggest the need of a spin-orbit term
- Requires a coupled basis

$$|n(\ell s)jm_j\rangle = \sum_{m_\ell m_s} |n\ell m_\ell m_s\rangle \left(\ell\ m_\ell\ s\ m_s|\ j\ m_j\right)$$

- Use $\boldsymbol{\ell} \cdot \boldsymbol{s} = \frac{1}{2}(\boldsymbol{j}^2 - \boldsymbol{\ell}^2 - \boldsymbol{s}^2)$ to show that these are eigenstates

$$\frac{\boldsymbol{\ell} \cdot \boldsymbol{s}}{\hbar^2} |n(\ell s)jm_j\rangle = \tfrac{1}{2}\left(j(j+1) - \ell(\ell+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right) |n(\ell s)jm_j\rangle$$

- For $j = \ell + \frac{1}{2}$ eigenvalue $\frac{1}{2}\ell$
- while for $j = \ell - \frac{1}{2}$ $-\frac{1}{2}(\ell + 1)$
- so SO splits these levels! and more so with larger ℓ

Inclusion of SO potential and magic numbers

- Sign of SO?

$$V_{\ell s}\left(\frac{\boldsymbol{\ell}\cdot\boldsymbol{s}}{\hbar^2}\right)r_0^2\frac{1}{r}\frac{d}{dr}f(r)$$

$$V_{\ell s}=-0.44V$$

- Consequence for

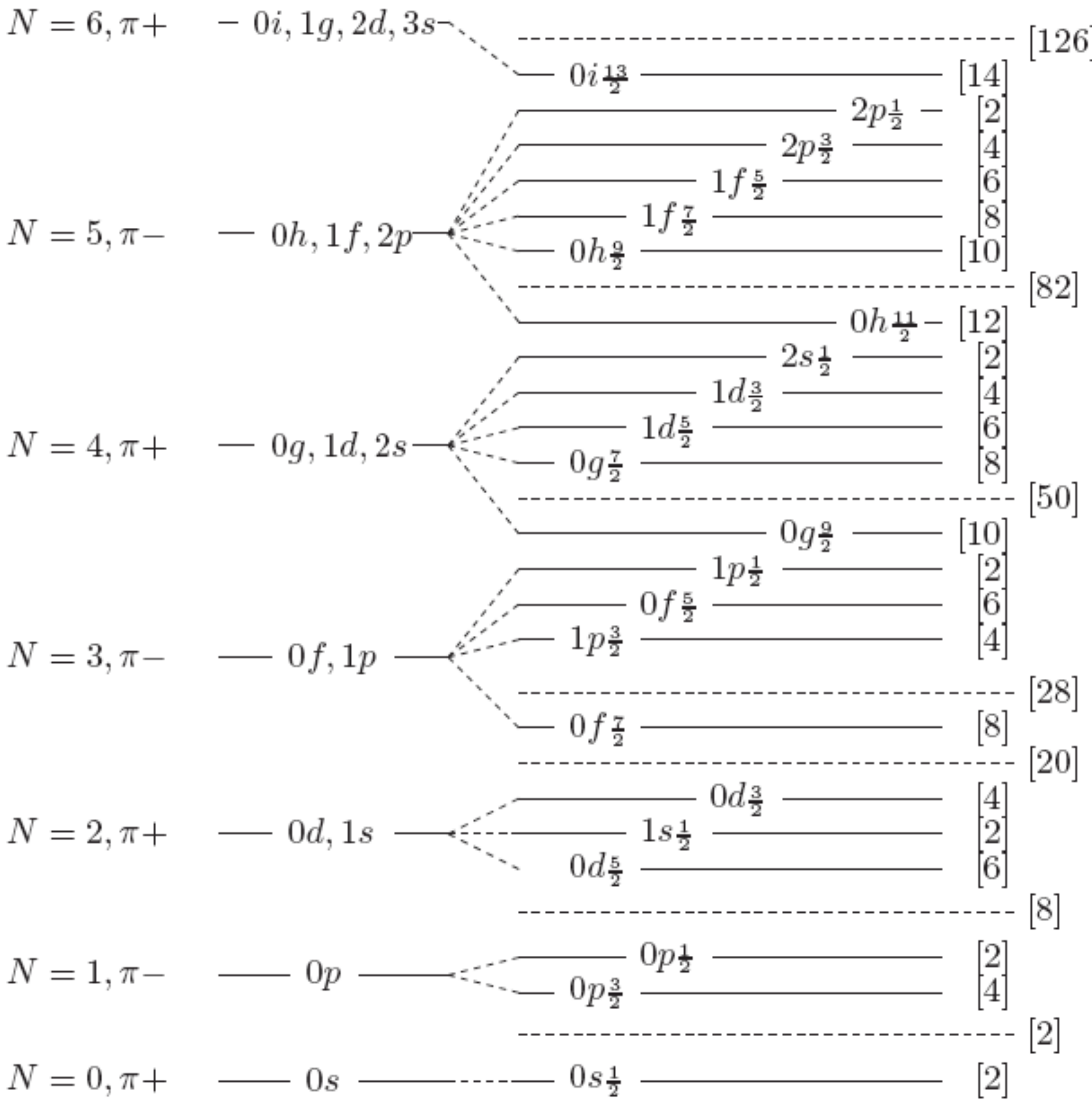
$$0f\tfrac{7}{2}$$

$$0g\tfrac{9}{2}$$

$$0h\tfrac{11}{2}$$

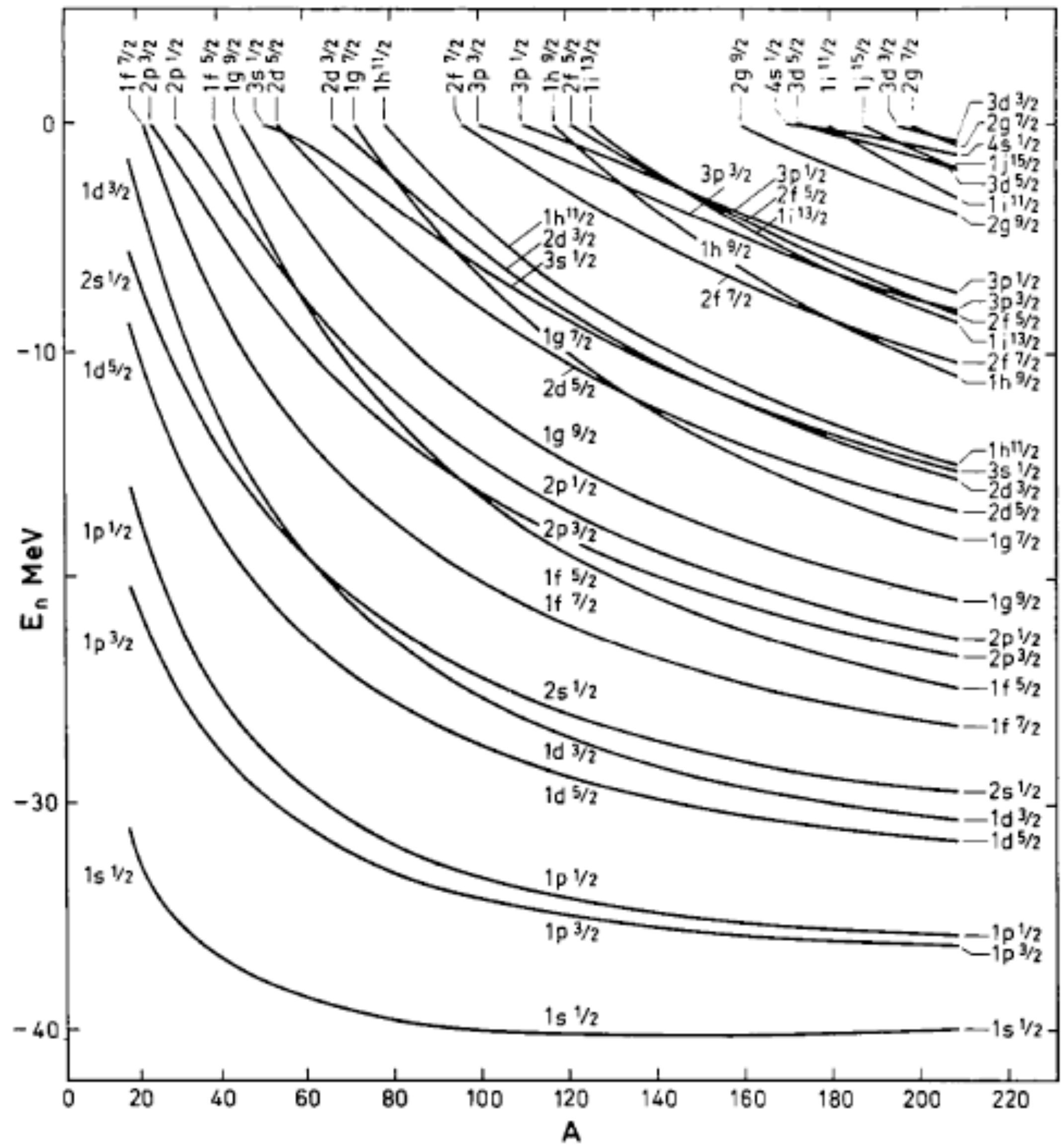
$$0i\tfrac{13}{2}$$

- Noticeably shifted
- Correct magic numbers!



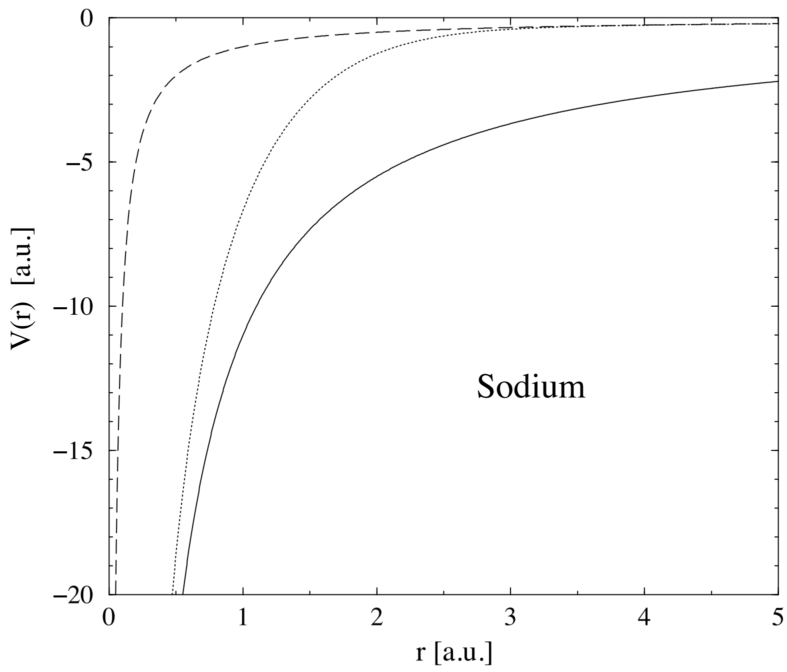
Neutron levels as a function of A

- Phenomenological!
- Calculated from Woods-Saxon plus spin-orbit



Effect of other electrons in neutral atoms

- Consider effect of electrons in closed shells for neutral Na
- large distances: nuclear charge screened to 1
- close to the nucleus: electron "sees" all 11 protons
- approximately:

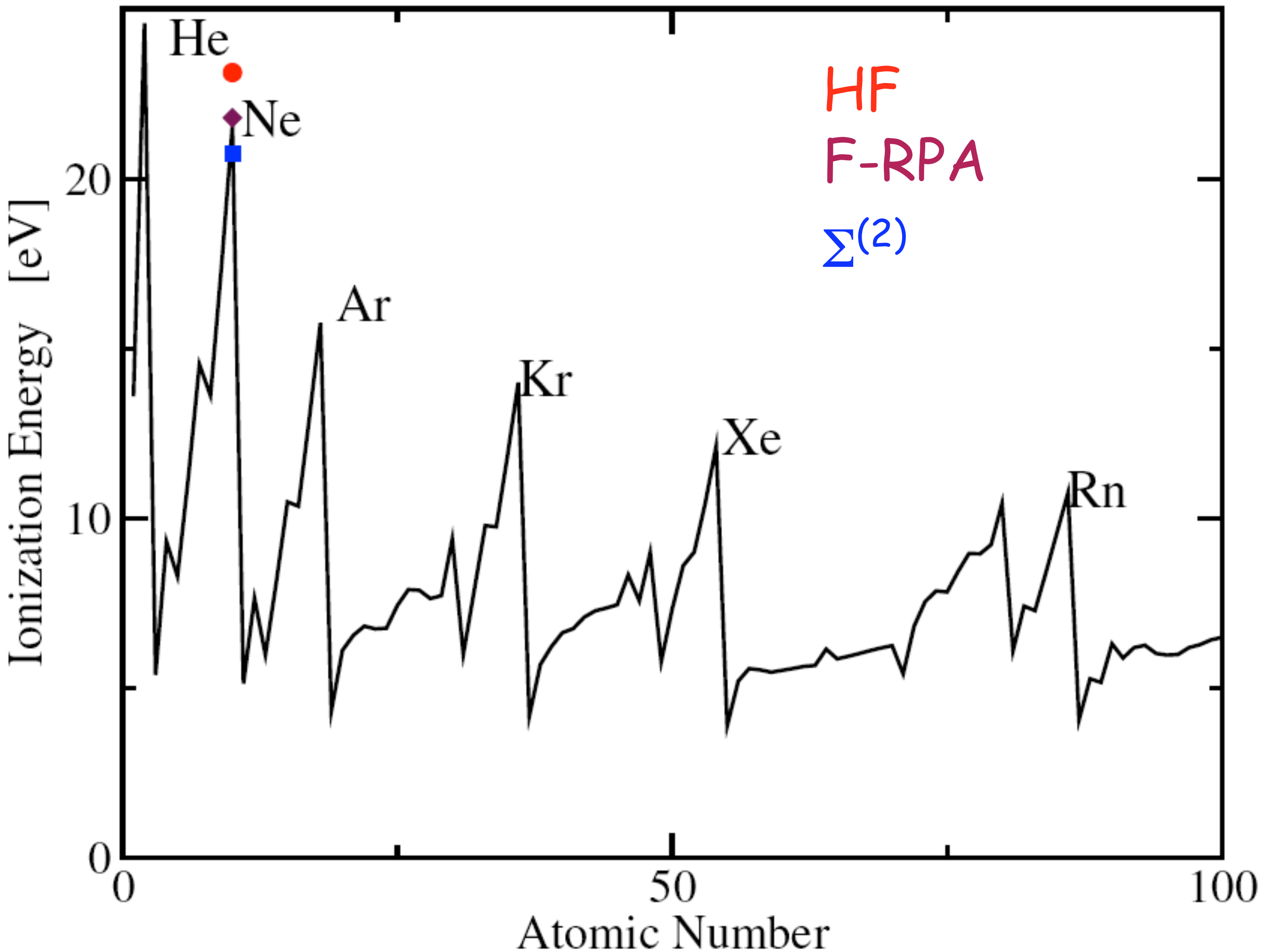


- lifts H-like degeneracy: $\varepsilon_{2s} < \varepsilon_{2p}$

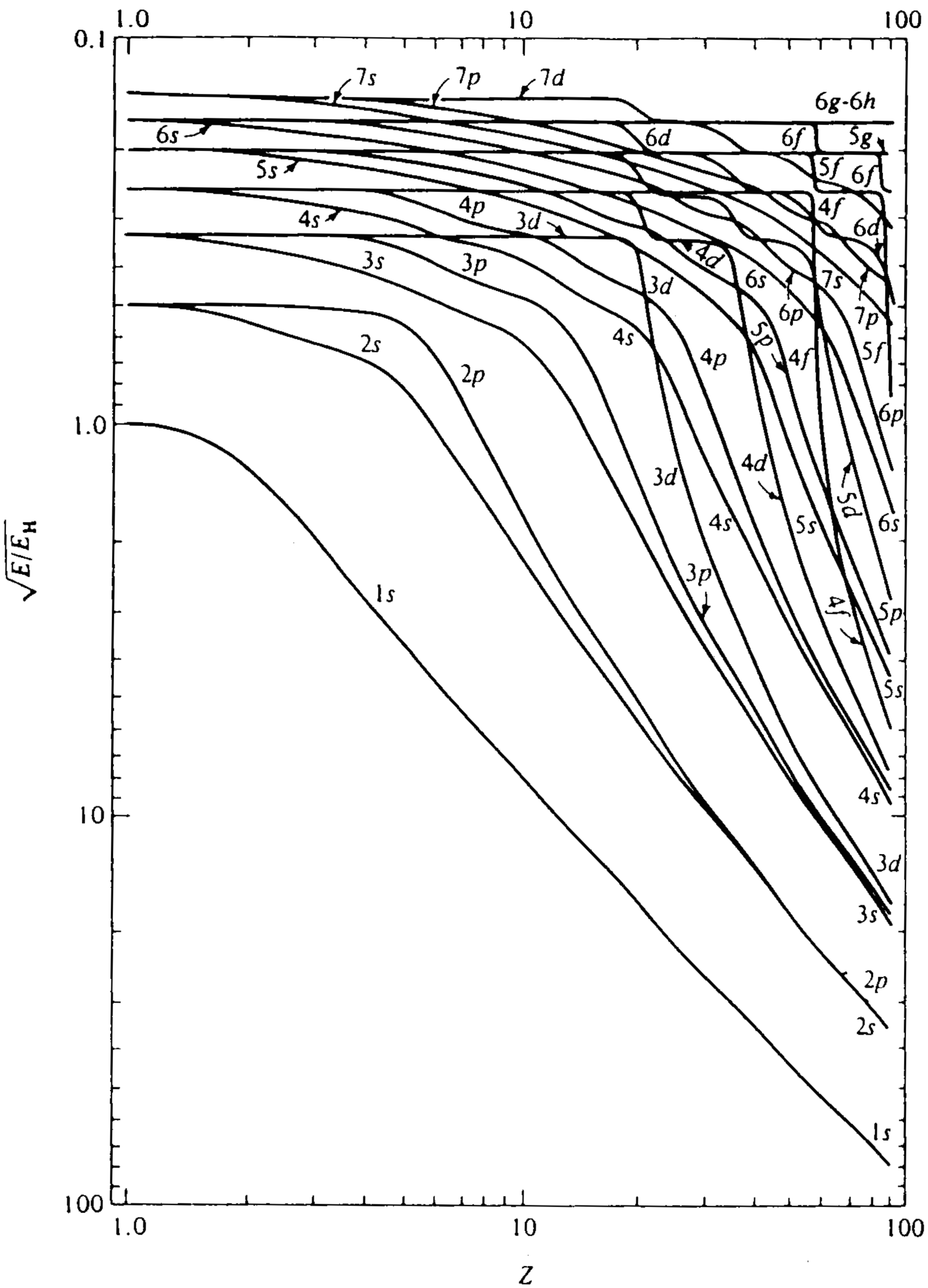
$$\varepsilon_{3s} < \varepsilon_{3p} < \varepsilon_{3d}$$

- "Far away" orbits: still hydrogen-like!

Periodic table

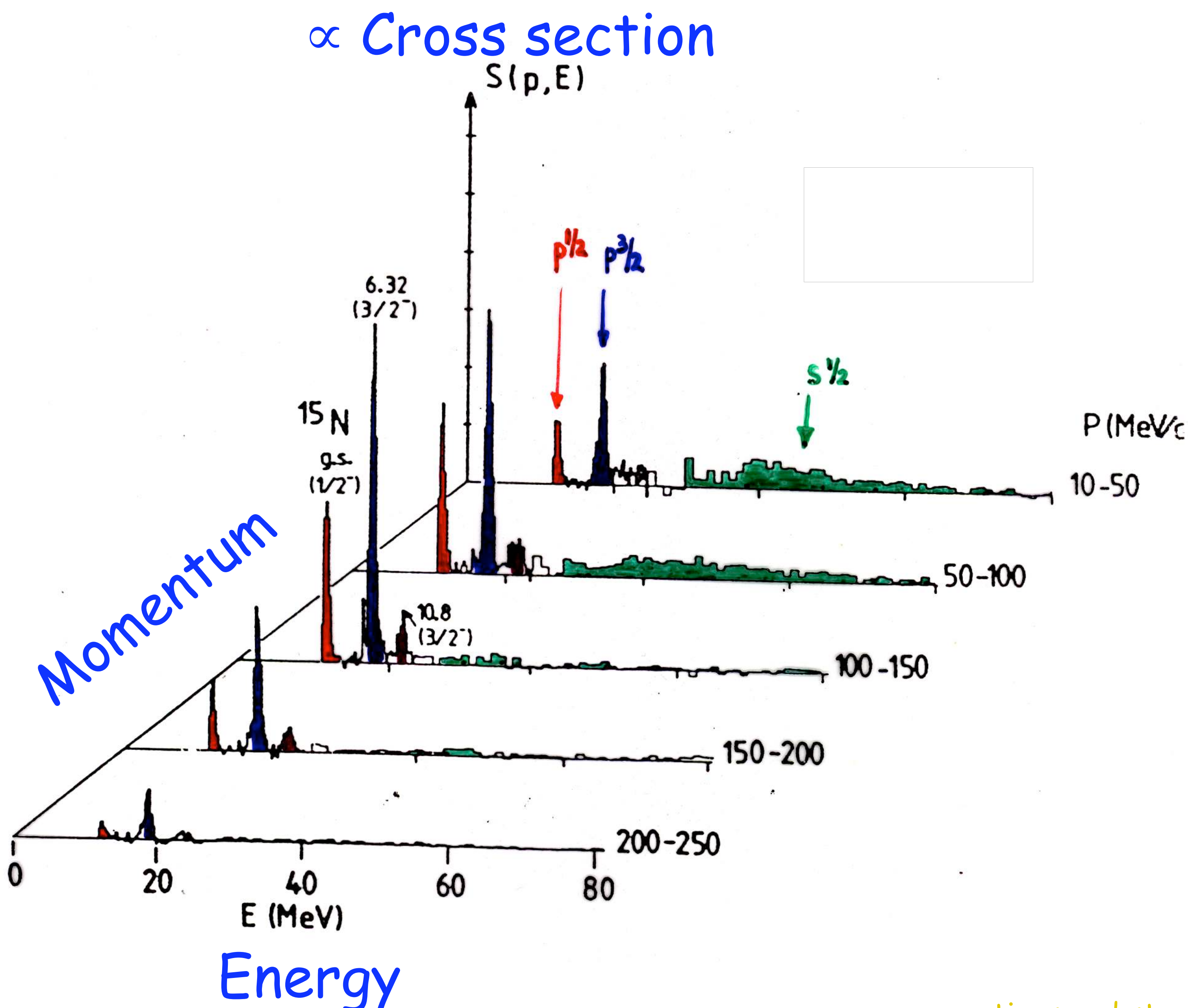


Level sequence (approximately)



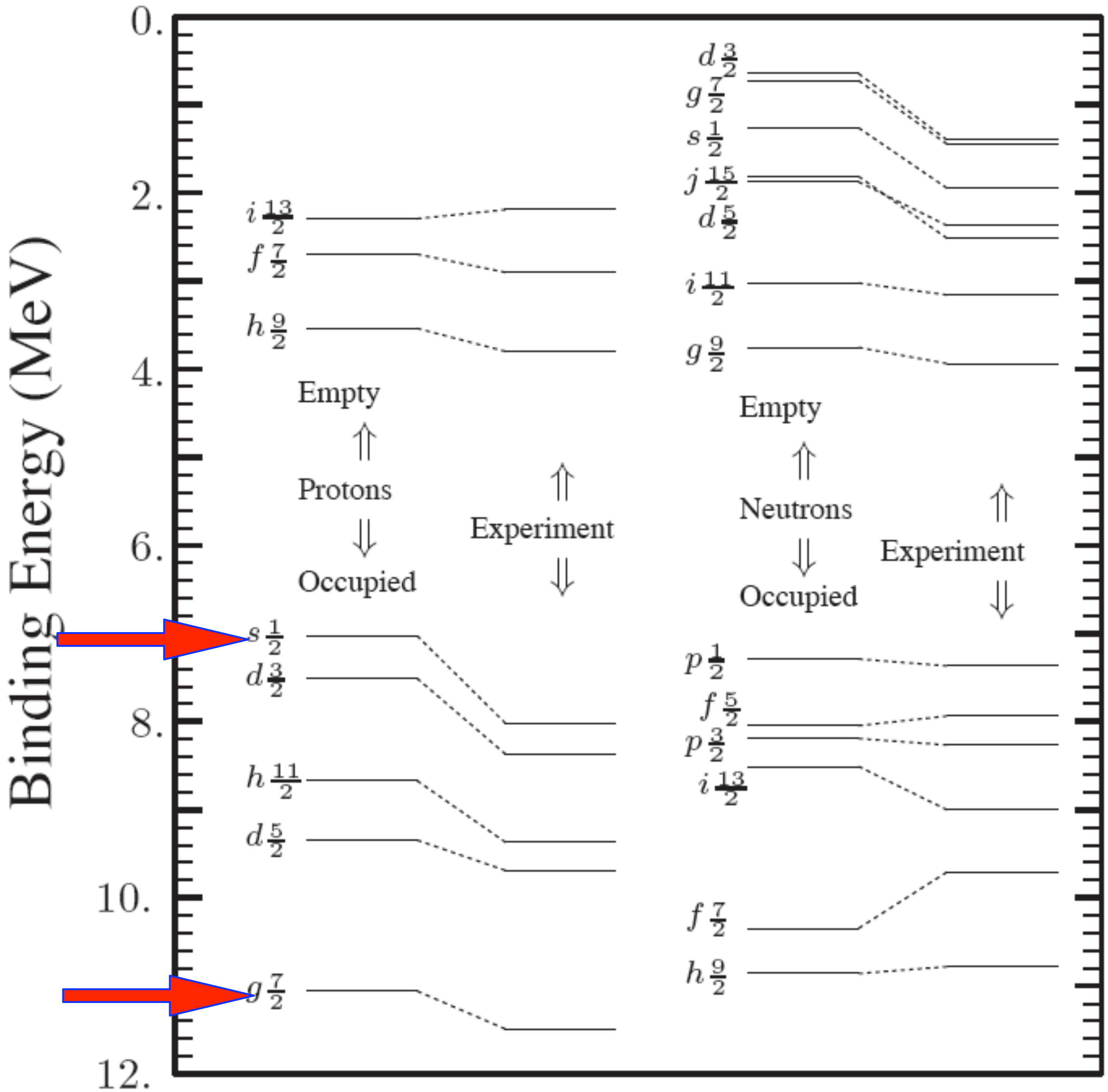
Mougey et al., Nucl. Phys. A335, 35 (1980)

$^{16}O(e,e'p)$



Consider

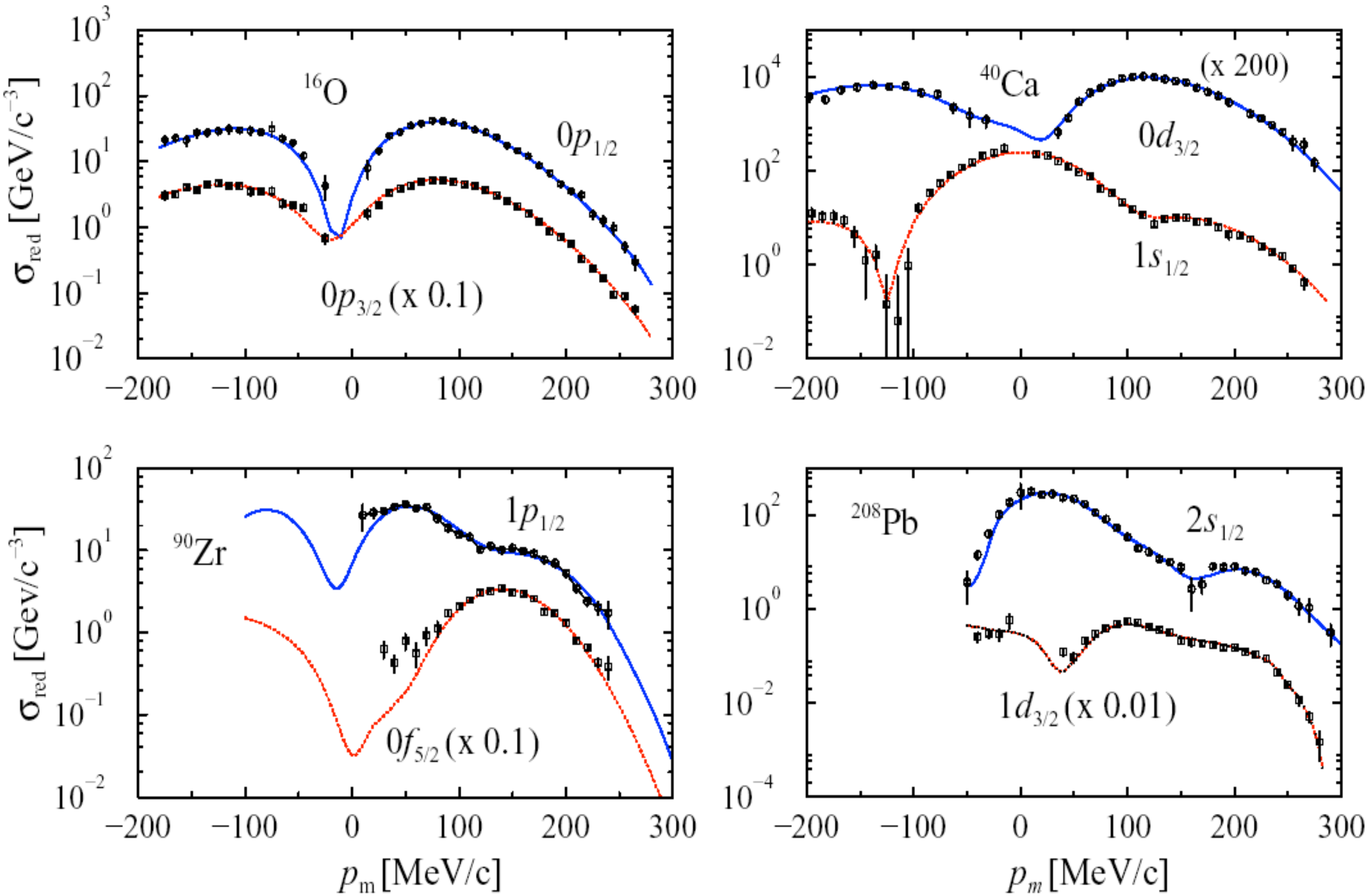
- ^{208}Pb sp levels



0f

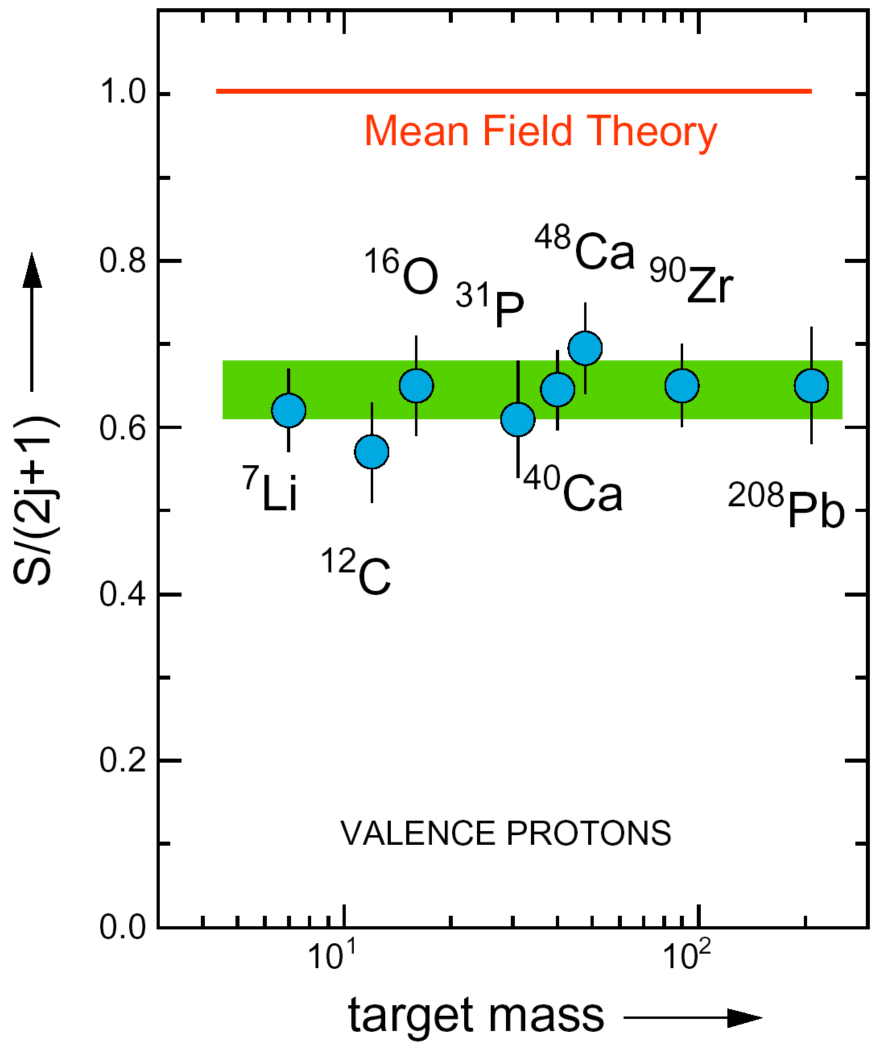
Momentum profiles for nucleon removal

- Closed-shell nuclei
- NIKHEF data, L. Lapikás, Nucl. Phys. A553, 297c (1993)



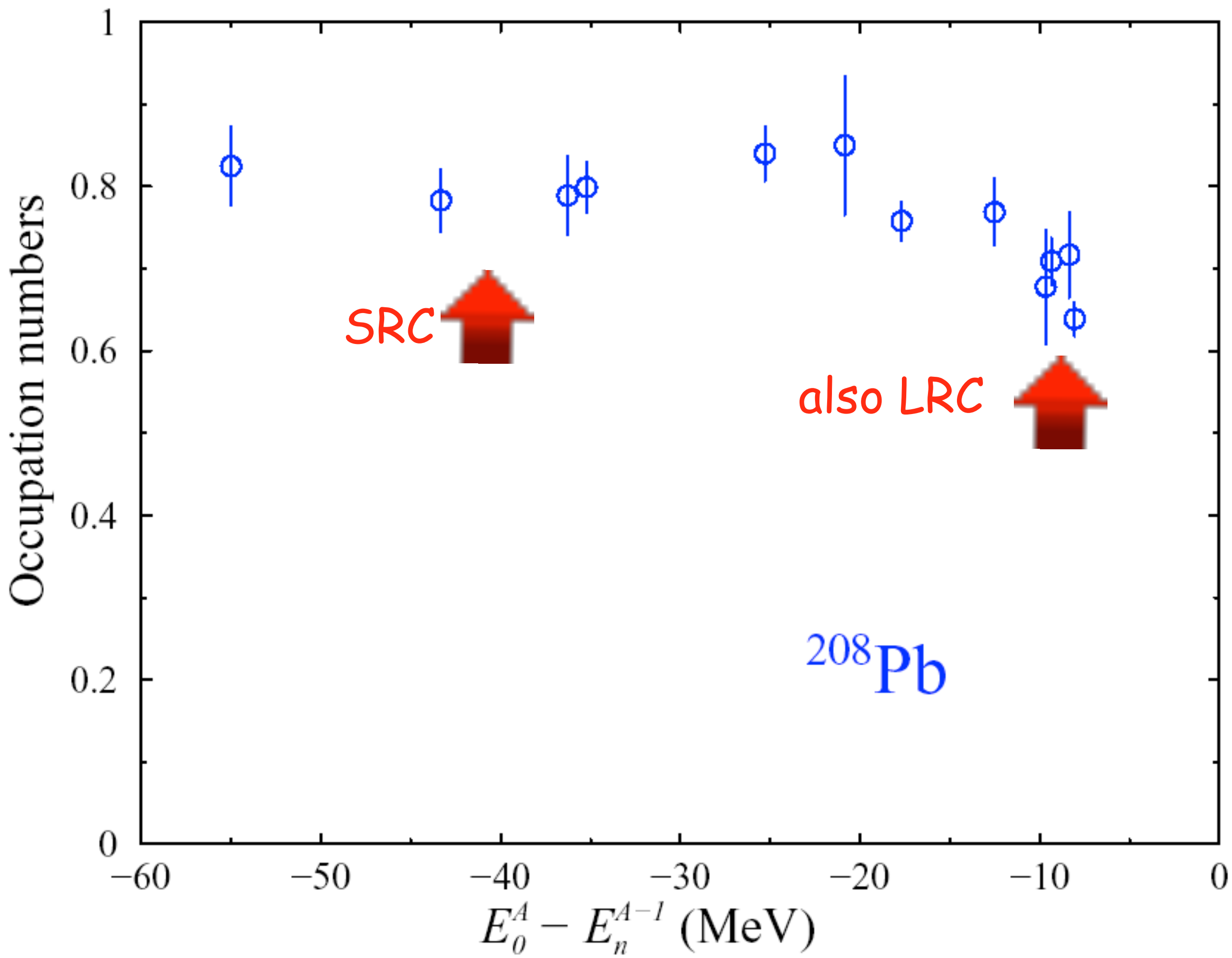
But...

- Spectroscopic factors substantially smaller than simple IPM

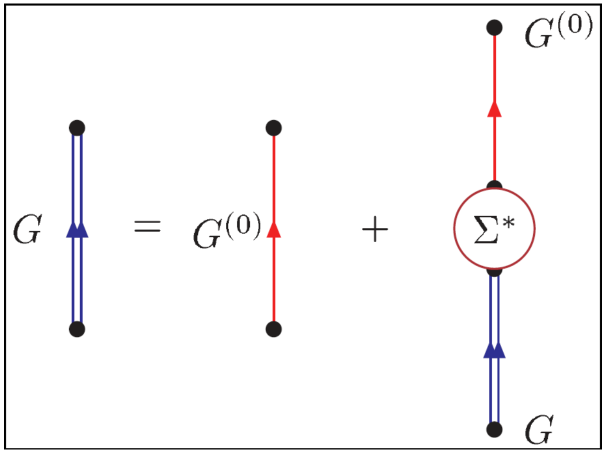


Recent Pb experiment

- 100 MeV missing energy
- 270 MeV/c missing momentum
- complete IPM domain



Propagator from Dyson Equation and "experiment"



Equivalent to ...

Schrödinger-like equation with: $E_n^- = E_0^A - E_n^{A-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1 ⇒ as extracted from (e,e′p) reaction

$$\frac{k^2}{2m}\phi^n_{\ell j}(k) + \int dq\; q^2\; \Sigma^*_{\ell j}(k,q;E_n^-)\;\phi^n_{\ell j}(q) = E_n^-\;\phi^n_{\ell j}(k)$$

Spectroscopic factor $\quad S^n_{\ell j} = \int dk\; k^2\; \left|\langle \Psi_n^{A-1} |\, a_{k\ell j}\, | \Psi_0^A \rangle\right|^2 < 1$

Dyson equation also yields $\quad \left[\chi^{elE}_{\ell j}(r)\right]^* = \langle \Psi^{A+1}_{elE} |\, a^\dagger_{r\ell j}\, | \Psi^A_0 \rangle$ for positive energies

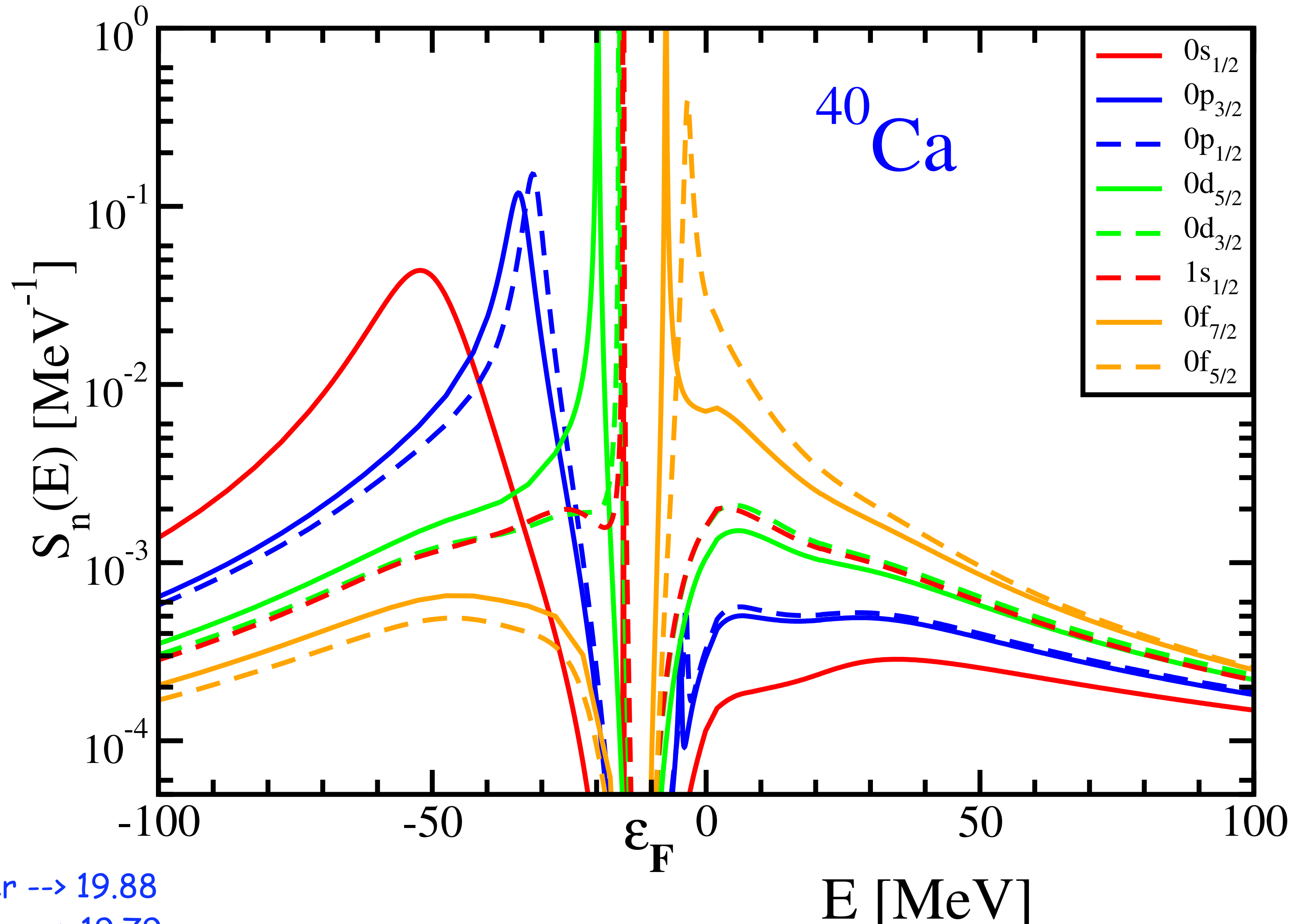
Elastic scattering wave function for protons or neutrons

Dyson equation therefore provides:

Link between scattering and structure data from **dispersion relations**

Spectral function for bound states

- [0,200] MeV —> constrained by elastic scattering data



proton number --> 19.88

neutron number -> 19.79

$S_{0d3/2}$= 0.76

$S_{1s1/2}$ = 0.78

0.15 larger than NIKHEF analysis!

reactions and structure

Quantitatively

- Orbit closer to the continuum —> more strength in the continuum
- Note "particle" orbits
- Drip-line nuclei have valence orbits very near the continuum

Table 1: Occupation and depletion numbers for bound orbits in ^{40}Ca. $d_{nlj}[0, 200]$ depletion numbers have been integrated from 0 to 200 MeV. The fraction of the sum rule that is exhausted, is illustrated by $n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$. Last column $d_{nlj}[0, 200]$ depletion numbers for the CDBonn calculation.

orbit	$n_{n\ell j}$ DOM	$d_{n\ell j}[0, 200]$ DOM	$n_{n\ell j} + d_{n\ell j}[\varepsilon_F, 200]$ DOM	$d_{n\ell j}[0, 200]$ CDBonn
$0s_{1/2}$	0.926	0.032	0.958	0.035
$0p_{3/2}$	0.914	0.047	0.961	0.036
$1p_{1/2}$	0.906	0.051	0.957	0.038
$0d_{5/2}$	0.883	0.081	0.964	0.040
$1s_{1/2}$	0.871	0.091	0.962	0.038
$0d_{3/2}$	0.859	0.097	0.966	0.041
$0f_{7/2}$	0.046	0.202	0.970	0.034
$0f_{5/2}$	0.036	0.320	0.947	0.036

- Phys. Rev. C90, 061603(R) (2014); arXiv:1410.2582