Problem Set 6

Nima Leclerc (nleclerc@seas.upenn.edu)

PHYS 532 (Quantum Mechanics II)
School of Engineering and Applied Science
University of Pennsylvania

May 6, 2021

Problem 1:
Solution:

We consider a Hydrogen atom in its ground state $(n, l, m) = (1, 0, 0)$ placed in a capacitor. When the field

$$\mathbf{E} = \begin{cases} 0 & t < 0 \\ \mathbf{E}_0 e^{-t/\tau}, t > 0 \end{cases}$$

with $\mathbf{E}_0 = E_0 e^{-t/\tau}\hat{z}$. We can compute the probability for the atom to be in either $(n, l, m) = (2, 1, 0), (2, 1, 1), (2, 1, -1), (2, 0, 0)$. For this, we invoke time dependent perturbation theory to the first order to solve for the probability P_{if},

$$P_{if} = \frac{1}{\hbar^2} | \int_{t_0}^{t} dt' e^{\frac{i}{\hbar}(E_f - E_i)t'} V_S^{fi}(t') dt' |^2$$

with V_S^{fi} as the matrix element corresponding to time-dependent stark shift. We have that, $V_S = eE_0 e^{-t/\tau} z$. Hence,

$$V_S^{fi} = E_0 e^{-t/\tau} \langle f | z | i \rangle$$

Hence, our expression becomes (with $\omega_{if} = \frac{E_f - E_i}{\hbar}$)

$$P_{if} = \frac{|E_0|^2 |\langle f|z|i\rangle|^2}{\hbar^2} | \int_0^t dt' e^{i\omega_{if} t'} e^{-t'/\tau} dt' |^2 = \frac{|E_0|^2 |\langle f|z|i\rangle|^2}{\hbar^2} | \frac{\tau - e^{-\frac{t}{\tau} + it\omega_{if}}}{1 - i\tau\omega_{if}} |^2$$

Hence, we need to compute $\langle f|z|i\rangle = \langle n'l'm_l'|z|nlm\rangle$ for each transition. From parity, we know that $\langle 200|z|100\rangle = 0$. Moving forward, we need to explicitly compute the transitions for between the ground state and the 3 2p states. Here, we have

$$\langle 210|z|100\rangle = \frac{1}{4\sqrt{2}\pi a_0^4}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \int_0^{\infty} drr^3 e^{-r/a_0} e^{-r/2a_0}(r^2\cos^2\theta)$$

$$= \frac{1}{2\sqrt{2}a_0^4}\int_0^{\pi} d\theta \sin\theta\cos^2\theta \int_0^{\infty} drr^4 e^{-3r/2a_0} = \frac{1}{3\sqrt{2}a_0^4}\int_0^{\infty} drr^4 e^{-3r/2a_0} = \frac{1}{3\sqrt{2}a_0^4}(\frac{8a_0^5}{81}) = \frac{8a_0}{243\sqrt{2}}$$

$$\langle 211|z|100\rangle = -\frac{1}{8\pi a_0^4}\int_0^{2\pi} d\phi e^{-i\phi}\int_0^{\pi} d\theta \sin^2\theta\cos\theta \int_0^{\infty} drr^4 e^{-3r/2a_0} = 0$$

Likewise, we get $\langle 21-1|z|100\rangle = 0$ due to the integration over ϕ. Hence, the only non-zero transition we have corresponds to the matrix element $\langle 210|z|100\rangle$. The corresponding transition probabilities are given by,

$$P_{(100)\to(210)} = \frac{32|E_0|^2 a_0^2}{(243)^2\hbar^2}|\frac{\tau - e^{-\frac{t}{\tau}+it\omega_{if}}}{1-i\tau\omega_{21}}|^2$$

$$P_{(100)\to(211)} = P_{(100)\to(21-1)} = P_{(100)\to(200)} = 0$$

with $\omega_{21} = \frac{3(13.6eV)}{4\hbar}$.