# Problem Set 5

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*Note: all figures are attached in the last page.*

**Problem 1:**
*Solution*

**5.10:**
This system is causal since it is mentioned. The region of convergence for this system is outside the largest pole and also includes the unit circle, both being conditions for stability. Hence, the system is both stable and causal. We can now look at the inverse system. From class, we saw that taking the inverse will swap the poles and zeros. This would make the largest zero (corresponding to $z = \infty$ ) from the figure, the largest pole we have to worry about. $\infty$ includes the unit circle, so it is stable. But since it's infinite, then it must not be causal. Hence, the inverse system is stable, but not causal.

**5.23:**
(a)
This is false. We can take a filter like ,

$$H(\omega) = 0.1 - e^{-j\omega}$$

This filter is an example of a causal filter. Where we have our formula ($\theta = 0$),

$$grd(1 - 0.1e^{-j\omega}) = \frac{(0.1)^2 - 0.1\cos\omega}{|1 - 0.1e^{-j\omega}|^2}$$

we can see that for some value when $\omega < \pi$, take $\omega = 0$. We have,

$$grd = \frac{(0.1)^2 - 0.1}{|1 - 0.1|^2} < 0$$

Hence, we can have negative values.

(b) This is also false since we can delay a filter by a positive constant to become causal and the delay of the resulting output will be the same as the the delay applied to it. A counter example for this would be the filter $H(z) = z^{-2} - 1$.

(c) This is true. We can evaluate this integral, since we know the relation between the phase and time delay. All the poles and zeros of the filter will meet the criteria that the poles and zeros on the real axis are between $\pm 1$. So we can evaluate the integral,

$$\int_0^{\pi} \tau d\omega = \theta(0) - \theta(\pi)$$

where we know now that $\theta(\pi) = \theta(0) = 0$. Hence, the integral is 0.

**5.34:**
We expect the output to be the second one, $y_2[n]$. If we look at the input signal, there are three frequency components (revealed more clearly in its Fourier transform). Applying the filter will take in frequencies less than $0.14\pi$ and scale them to 1.9, take in frequencies near $0.3\pi$ and scale them to 1.6, then zero frequencies at $0.5\pi$. So, we only care about group delays at $0.14\pi$ and $0.3\pi$. These are 40 and 80. Mapping these to the potential outputs, the first out put isn't possible because it doesen't have the same delay or scale factor. This is also true for the third and fourth. Hence, the second is the only feasible option.

**Problem 2:**
*Solution:*

See Figure 1 and 2 for plots of the impulse and group delay.

**Problem 3:**
*Solution:*

(a) See Figure 3.
(b) See Figure 4.
(c) See Figure 5/6. We see that the frequency range for $|z| > 0.77$ is larger than the other case.

**Problem 4:**
*Solution:*

(a) See Figure 6.
(b) See Figure 7/8/9.
(c) The impulse response here works for a larger frequency range than the 0.77 in Problem 2.
(d) Yes, they do. See Figures 10, 11, 12.

**Problem 5:**
*Solution:*

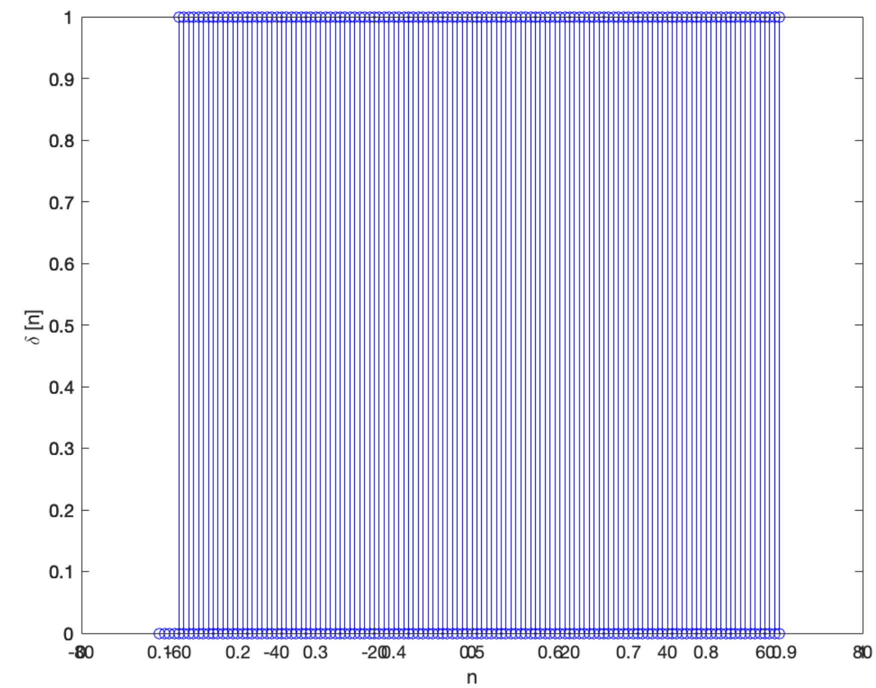


**Figure 1:** Matlab problem 1 (impulse)

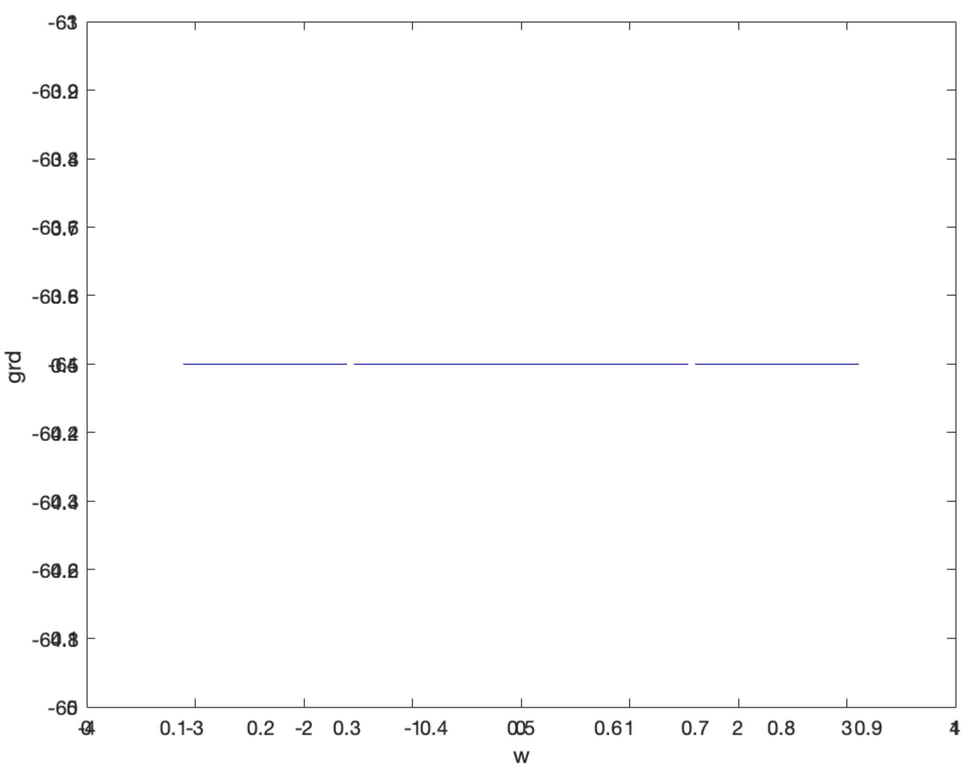


**Figure 2:** Matlab problem 1 (group delay)

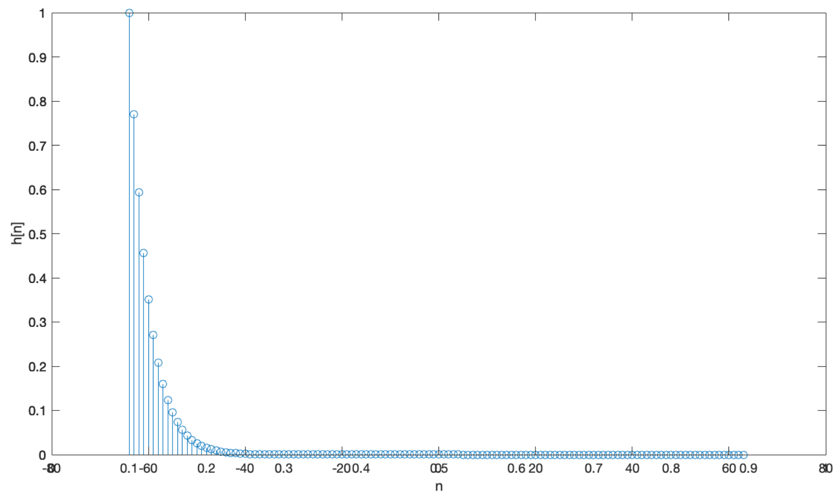


**Figure 3:** Matlab problem 2 (a)  (impulse response)

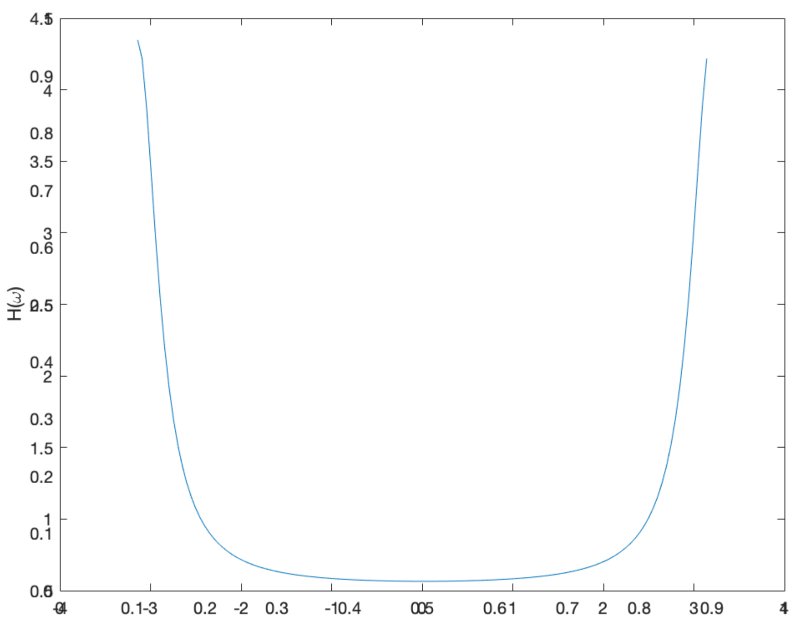


**Figure 4** Matlab problem 2 (b)  )

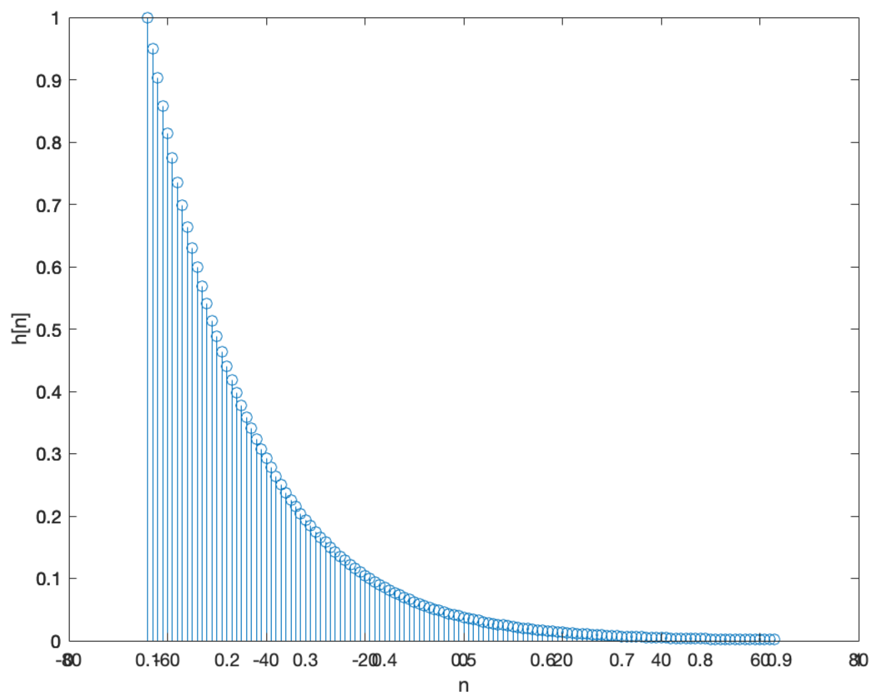


**Figure 5** Matlab problem 2 (c ) [h[n]]

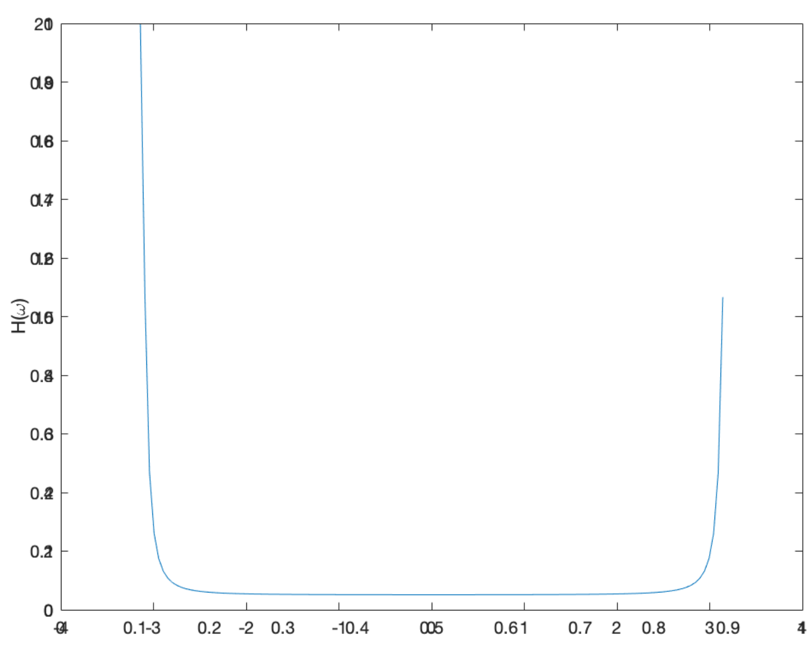


**Figure 6** Matlab problem 2 (d ) (freq response)

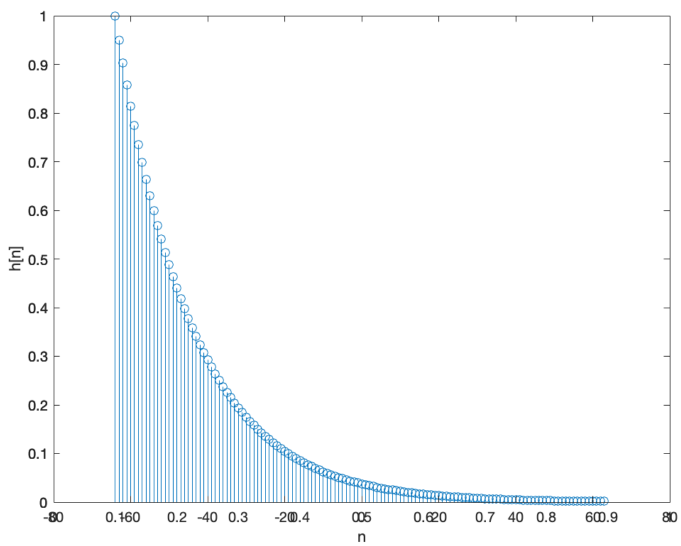


**Figure 7** Matlab problem 3(a ) (impulse response)

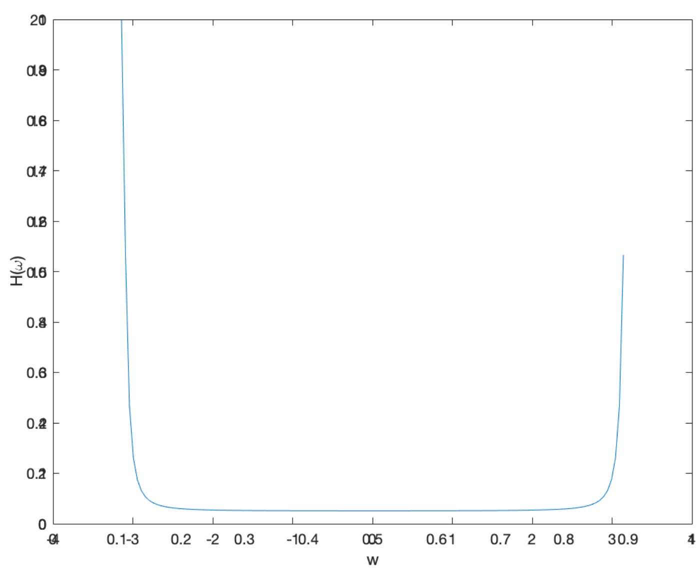


**Figure 8** Matlab problem 3(b ) (freq resp)

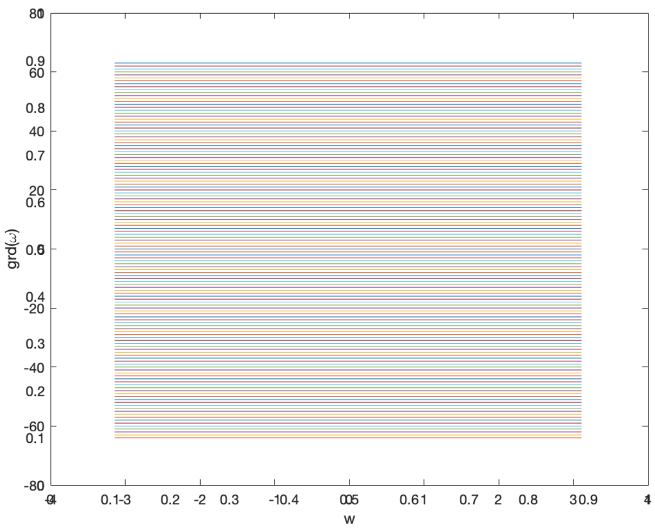

**Figure 9** Matlab problem 3(b ) (grd)

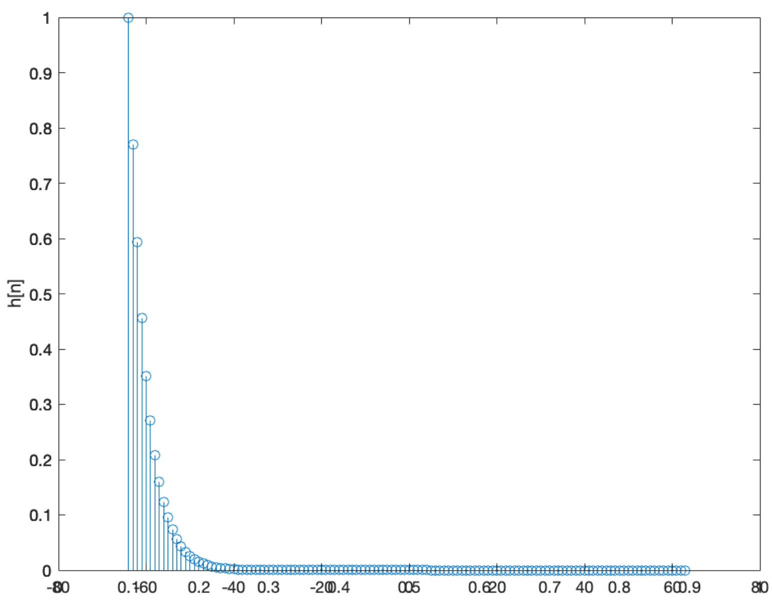

**Figure 10** Matlab problem 3(d ) (h[n])

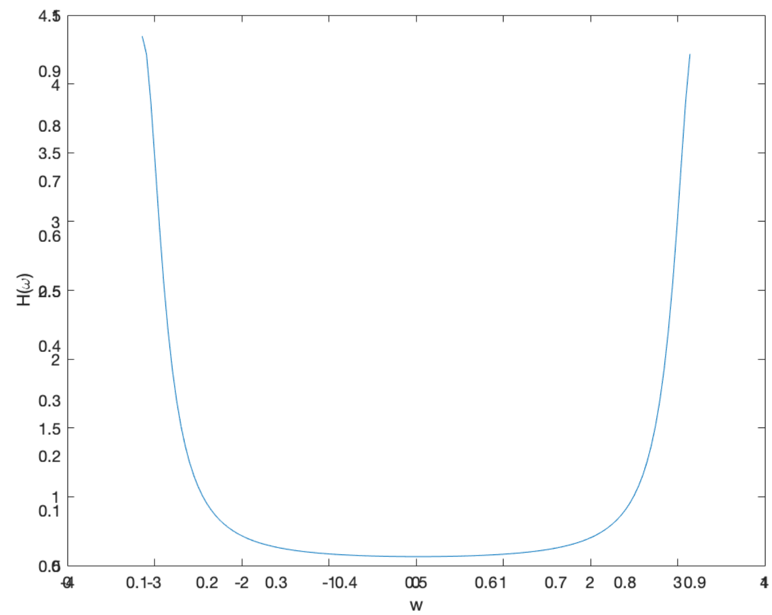


**Figure 11** Matlab problem 3(d ) (H(w))

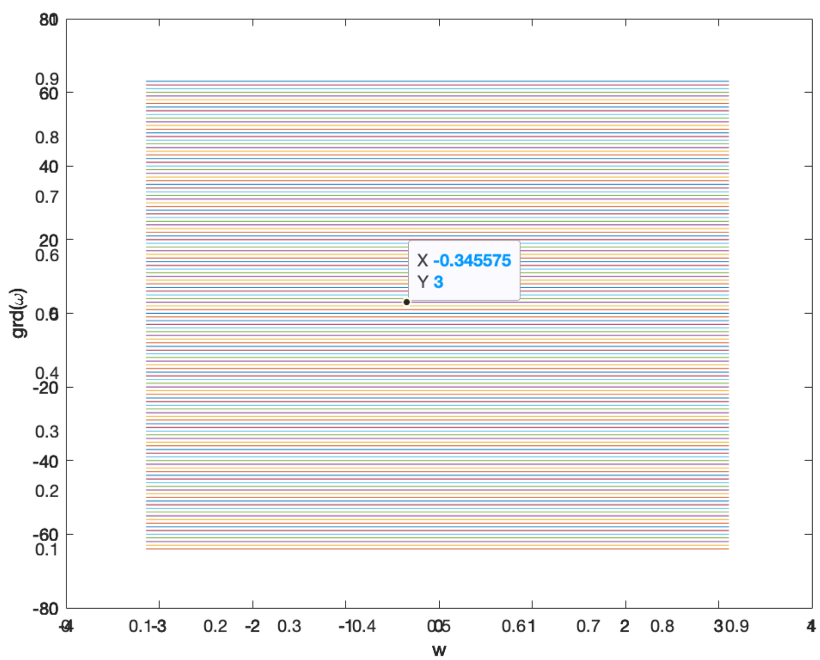


**Figure 12** Matlab problem 3(d ) (grd(w))