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Digital Communications

Homework #4

1. Suppose that a transmitter modulates a sinusoid by using an in-phase component consisting of a square wave that takes on values of either +1 or -1 and a quadrature component consisting of a square wave that takes on values of either +2 or -2. How is the resulting sinusoid modulated (amplitude, phase or frequency)? How many values can the (amplitude/phase/frequency) assume and what are they?

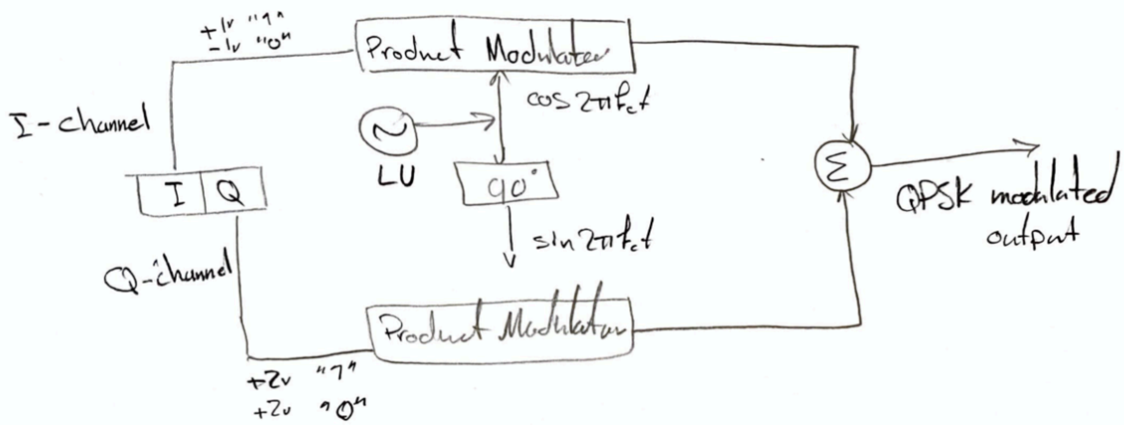
$\boxed{1}$ This can describe QPSK modulation. With NRZ coding of the message

I channel

$m(t) \rightarrow +1v \rightarrow$ logic 1 $\quad m(t) \rightarrow -1v \rightarrow$ logic 0

Q-channel

$m(t) \rightarrow +2v \rightarrow$ logic 1 $\quad m(t) \rightarrow -2v \rightarrow$ logic 0



Output: $s(t) = \pm \cos 2\pi f_c t - 2 \sin 2\pi f_c t$

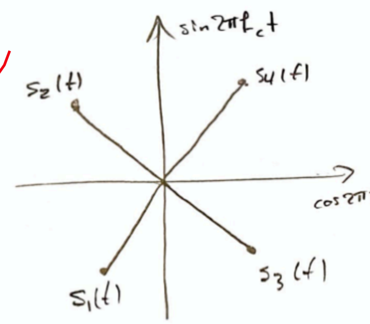
The values:

$m_0(t) = 00 \rightarrow S_0(t) = \sqrt{5} \cos[2\pi f_c t - 63.43]$

$m_1(t) = 01 \rightarrow S_1(t) = \sqrt{5} \cos[2\pi f_c t - 116.5]$

$m_2(t) = 10 \rightarrow S_2(t) = \sqrt{5} \cos[2\pi f_c t - 296.5]$

$m_3(t) = 11 \rightarrow S_3(t) = \sqrt{5} \cos[2\pi f_c t - 63.43]$



2. An analog voice signal is to be sampled, quantized and transmitted using BPSK modulation. The original analog signal has a bandwidth of 4kHz. A quantization SNR of 50dB is desired. The quantization SNR can be written as

$$\text{SNR}(dB) = 6.02n + 4.77 - 20\log 10\left(V_{\text{peak}}/V_{rms}\right)$$

where n is the number of bits used in the quantization process and $V_{\text{peak}}/V_{\text{rms}}$ is the peak-to-average voltage ratio of the signal (linear). If the peak to average power ratio of the analog signal of interest is 10dB, what is the bandwidth of the transmitted BPSK signal if a raised cosine pulse is used with roll-off factor 0.5 (assume infinitely long pulses)?

$\boxed{2}$ SNR = 50 dB

$10\log\left(\frac{V^2_{peak}}{V^2_{rms}}\right) = 10 \text{ dB} = 20\log\left(\frac{V_{peak}}{V_{rms}}\right)$

$SRR = 6.02n + 9.77 - 20\log\left(\frac{V_{peak}}{V_{rms}}\right)$

$\Rightarrow 50 - 9.77 + 10 = 6.02n \Rightarrow n = 9.174 \Rightarrow n = 10$

$BW = (1+r)\frac{nf_s}{2}$ ← sampling freq

↑ roll-off factor

$\Rightarrow BW = (1+0.5)\frac{80}{2}$ $\quad$ $BW = 60 \text{ kHz}$

-1

3.

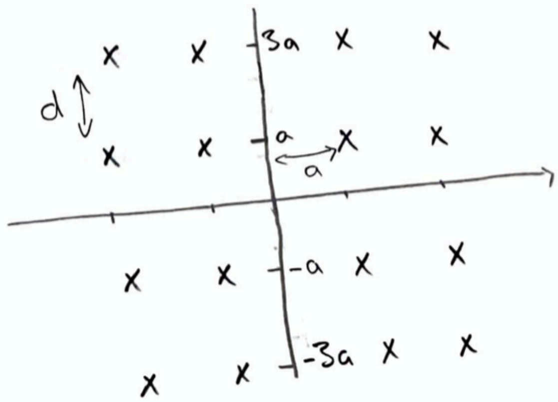
3.1. Draw the signal constellation diagram for a 16-QAM signal with maximum amplitude *A*

3.2. What is the average energy per symbol?

3.3. What is the distance between neighboring signal points in terms of the averaged energy per symbol?

$\boxed{3}$

(a) Constellation diagram of 16QAM



(b) Average energy per symbol

Answer should be in terms of A -1

$$E_s = \frac{1}{4}\left[(a^2+a^2)+(9a^2+a^2)+(a^2+9a^2)+(9a^2+9a^2)\right] = 10a^2$$

(c) The distance between neighboring points

$$d = 2a = 2\sqrt{0.1 E_s} = \sqrt{0.4 E_s}$$ -1

4.

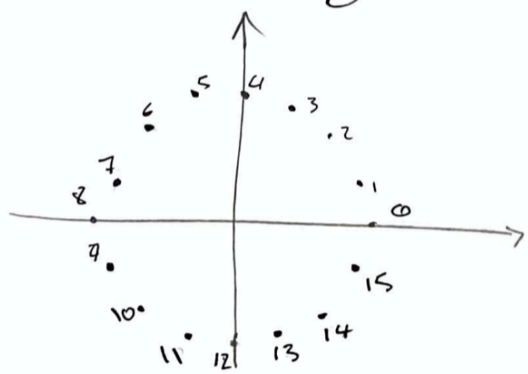
4.1. Draw the signal constellation diagram for a 16-PSK signal with maximum amplitude *A*

4.2. What is the average energy per symbol?

4.3. What is the distance between neighboring signal points in terms of the average energy per symbol?

4.4. Change the amplitude so that the average energy is the same as in 16-QAM presented in problem 3. How do the minimum distance between signal points compare?

4 Constellation diagram for 16PSK



In terms of A

→ Average energy per symbol $\sqrt{E_s}$

for each symbol $\sqrt{E_s}\{1, e^{j\frac{2\pi}{M}}, e^{j\frac{4\pi}{M}}, \ldots, e^{j\frac{2\pi(M-1)}{M}}\}$ -1

→ For the distance between neighboring points if we assume s_{16} to have $\sqrt{E_s}$ energy then distance can be approximated (via diagram above):

$2\sqrt{E_s}\sin(\frac{\pi}{16})$

→ For PSK the distance is $2\sqrt{E_s}\sin(\frac{\pi}{16})$ whereas in QAM it is $2\sqrt{\frac{E_b}{10}}$ which would result in a ratio of 1.62.

5. Explain the difference between coherent reception, non-coherent reception and differentially coherent reception.

Coherent systems refer to the situation where the receiver needs the phase information of the transmitter (the carrier phase) to recover the transmitted data at the receiving side. The phase information comes from either a reference from the transmitter or could be from a local oscillator at the receiving side. The oscillators used at both the transmitter and receiver are not phase-locked; therefore there is a phase difference between the reference signal transmitted and the reference signal used in the demodulator. This phase difference must be estimated and corrected. The receiver is more complicated and expensive. But it has better bit error rate of detection.

On the other hand, in non-coherent systems the receiver do not need the phase information the the transmitter carrier to recover the signal. This type of systems do not require expensive and complex carrier recovery circuit. But it has worse bit error rate of detection in comparison with the coherent systems.

In differentially coherent receiver a very simple estimate of the fading distortion is used whereas the coherent receiver uses the optimal estimate.

6.

6.1. Explain the trade-off between energy efficiency and bandwidth efficiency for MPSK and MASK.

6.2. Does this trade-off differ for MFSK?

$\boxed{6}$ Energy efficiency comparison

MPSK:

- At fixed E_b/N_0, increasing M degrades P_b (bit error rate)
- At fixed P_b, increaseing M increases E_b/N_0 requirement.

MFSK:

- At fixed E_b/N_0, increasing M can provide an improvement on P_b
- At fixed P_b, increasing M can provide a reduction in E_b/N_0 requirement

→ Therefore, MFSK is more energy efficient than MPSK

Bandwidth efficiency comparison:

In general, the bandwidth required to pass MPSK/MQAM signal is approximately given by $B = \frac{1}{T_s}$

and $R_b = \frac{\log_2 M}{T_s}$ = bit rate

Then bandwidth efficiency can be expressed as

$$\eta = \frac{R_b}{B} = \log_2 M \quad \text{(bits/sec/Hz)}$$

MFSK

- Bandwidth required to transmit MFSK signal is

$B = \frac{M}{2T}$ (adjacent frequencies need to be separated by $\frac{1}{2T}$ to maintain orthogonality)

- Bandwidth efficiency of MFSK signal

$$\eta = \frac{R_b}{B} = \frac{2 \log_2 M}{M} \quad \text{(bit/sec/Hz)}$$

→ Therefore, as M increases, bandwidth efficiency of MPSK/MQAM increases, but bandwidth efficiency of MFSK decrease.

7. Assume that a QPSK signal is used to send data at a rate of 30Mbps over a satellite transponder. The transponder has a bandwidth of 24MHz.

   7.1. If the satellite signal is to use a raised pulse shape, what is the roll-off factor r that is required?

   7.2. Could a rolloff factor *r* be found so that a 50-Mbps data rate could be supported?

$\boxed{7}$ data rate $R_b = 30$ Mbps

Bandwidth $= 24$ MHz

a) $BW = \dfrac{R_b(1+\alpha)}{\log_2 M}$

$M = 4$ for QPSK $\Rightarrow 24M = \dfrac{30M}{2}(\alpha+1)$

$\Rightarrow \alpha + 1 = 1.6 \Rightarrow$ roll off factor $\boxed{\alpha = 0.6}$

b) $BW = \dfrac{R_b}{2}(\alpha+1)$

For $R_b = 50$ Mbps $\Rightarrow 24M = \dfrac{50M}{2}(\alpha+1)$

$\Rightarrow \alpha + 1 = 0.96 \Rightarrow \alpha = -0.04 \rightsquigarrow$ but it can not be negative, so the data rate is not achievable.