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Digital Communications

Homework #6

Question 1

(1)

$$\overline{E}_s = \frac{1}{8}\left(4a^2 + 4(2a^2)\right) = \frac{1}{2}a^2 + a^2 = a^2\left(\frac{3}{2}\right)$$

$$= \frac{3}{2}a^2$$

$$d_{i,i+1} = a \quad \Rightarrow d = \sqrt{\frac{2E_s}{3}}$$

$$P_e \approx 2Q\left(\frac{d}{\sqrt{2N_0}}\right) = 2Q\left(\frac{\sqrt{\frac{2}{3}E_s}}{\sqrt{2N_0}}\right) = 2Q\left(\sqrt{\frac{E_s}{3N_0}}\right)$$

$$E_b \log(M) = E_s \rightarrow E_s = 3E_b$$

$$\Rightarrow P_e = 2Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Comparing the plot with 8PSK, as follows, shows that the BER for the modulation scheme of the question is worse than 8PSK. This is not surprising as this the modulation with the constellation diagram shown in the question does not appear to be a popular one and if it had a better probability of error, people would probably use that instead of 8PSK.

To achieve the same probability of error, 8PSK requires a smaller SNR.

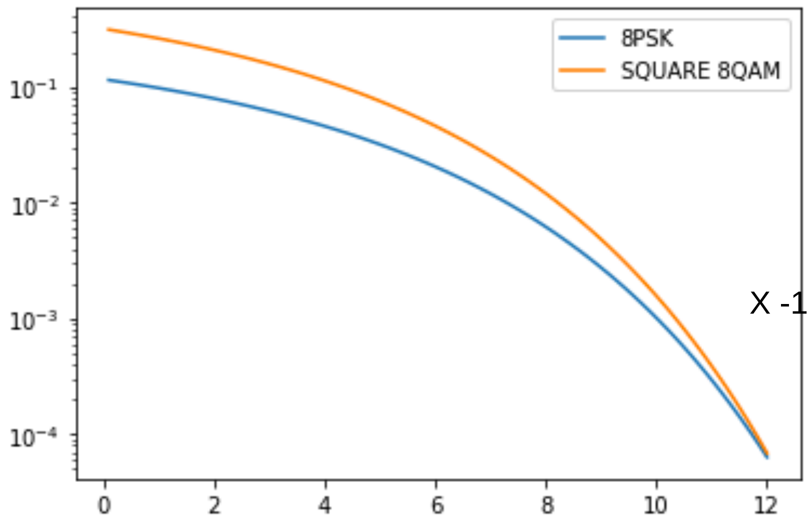
```
def calc8psk(ebn0dB):
  ebn0 = 10 ** (ebn0dB / 10)
  return (2/3) * qfunc(np.sqrt(6 * ebn0 * (np.sin(np.pi/8) ** 2)))

def calc8qam(ebn0dB):
  ebn0 = 10 ** (ebn0dB / 10)
  return 2.0 * qfunc(np.sqrt(ebn0))
```

```
ebn0dBs = np.linspace(0.1, 12, 100)

fig, ax1 = plt.subplots()
ax1.plot(ebn0dBs, calc8psk(ebn0dBs))
ax1.plot(ebn0dBs, calc8qam(ebn0dBs))
ax1.set_yscale('log')
ax1.legend(['8PSK', 'SQUARE 8QAM'])
```

```
<matplotlib.legend.Legend at 0x7f02e6faeda0>
```



X -1

Question 2

$$P_e = \frac{1}{2}\int_{-\infty}^{V_T} \frac{1}{\sqrt{2}\,\sigma} e^{\frac{-\sqrt{2}|r_0 - S_{01}|}{\sigma}} dr_0 + \frac{1}{2}\int_{V_T}^{\infty} \frac{1}{\sqrt{2}\,\sigma} e^{-\frac{\sqrt{2}|r_0 - S_{02}|}{\sigma}} dr_0$$

$\Rightarrow$ if we put $\frac{\partial P_e}{\partial V_T} = 0 \Rightarrow V_T = \frac{S_{01} + S_{02}}{2}$ ← Optimal threshold (similar to Gaussian noise)

$$P_e = F\left(\frac{S_{01} - S_{02}}{2\sigma_0}\right) = F\left(\sqrt{\frac{(S_{01} - S_{02})^2}{4\sigma_0}}\right)$$

CDF of Laplacian

If we use matched filter, the SNR will be $\left(\frac{S}{N}\right)_{out} = \frac{[S_{01} - S_{02}]^2}{\sigma_0^2} = \frac{2E_d}{N_0}$

then

$$P_e = F\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

In BPSK: $s_1(t) = A\cos(w_c t)\Big|_0^T \qquad s_2(t) = -A\cos(w_c t)\Big|_0^T$

$s_d(t) = 2A\cos(w_c t)\Big|_0^T$

$E_d = \int_0^T s_d^2(t)\,dt = 2A^2 T \Rightarrow P_e = F\left(\sqrt{\frac{E_d}{2N_0}}\right) = F\left(\sqrt{\frac{2A^2T}{2N_0}}\right) = F\left(\sqrt{\frac{A^2T}{N_0}}\right)$

$E_b = \frac{1}{2}\int_0^T s_1^2(t)\,dt + \frac{1}{2}\int_0^T s_2^2(t)\,dt = \frac{1}{2}A^2T \Rightarrow P_e = F\left(\sqrt{\frac{2E_b}{N_0}}\right)$

X -1

For BPSK:

Gaussian: $P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \xrightarrow{\frac{E_b}{N_0}(dB) = 5} \boxed{0.006}$

Laplacian: $P_e = F\left(\sqrt{\frac{2E_b}{N_0}}\right) \longrightarrow$ ~~0.147~~ $\boxed{0.0033}$

by standard definition

by HW definition of Laplace.

Question 3

Noise: $F = 5\,dB$

/ Microwave signal: $f = 2\,GHz$

transmit power: $10\,mW$

identical tx / rx antennas w/ 2 dB gain

8PSK

(*) Max data rate? → over 10 km with target BER 10^{-5}

Free space propagation

Antenna temperature: $T_{AR} = 240K$

point-to-point (no fading).

(3) $$\underset{13.15}{\frac{E_b}{N_o}} = \underset{-20}{P_T} + \underset{2}{G_{AT}} + \underset{2}{G_{AR}} - \underset{118.4}{L_P} - \underset{-198.97}{N_o} - R_b$$

$10 \log_{10}(0.01) = -20$ ← P_T

unknown ← R_b

$T_{sys} = T_{AR} + T_e$

$T_e = T_o (F - 1)$ ← From the slides it was not clear if F should be in dB. Looking at the Wikipedia page for "Noise Figure" it suggests that it shouldn't!

$F = 3.162$ ← for 5dB

$\Rightarrow T_e = 290(3.162 - 1) \approx 627$

$\Rightarrow T_{sys} = 917$

$\rightarrow N_o = k T_{sys} \Rightarrow N_o = -228.6 + 10 \log_{10}(917) = -198.97$

$L_P(dB) = 20 \log_{10}\left(\frac{4\pi d}{\lambda}\right) = 20 \log_{10}\left(\frac{4\pi(10,000)}{0.15}\right) = 118.4$

$\lambda = 0.15$ for $f = 2GHz$

For 8PSK → $P_e \simeq Q\left(\sqrt{\frac{0.88 E_b}{N_o}}\right)$

↳ To achieve $P_e = 10^{-5}$ we use inverse Q function: $\frac{E_b}{N_o} = \frac{(\text{InvQ}(P_e))^2}{0.88}$

$\Rightarrow \frac{E_b}{N_o} = 20.66 \Rightarrow \frac{E_b}{N_o} \approx 13.15\,dB$

$\Rightarrow R_b = -\frac{E_b}{N_o} + P_T + G_{AT} + G_{AR} - L_P - N_o$

$= -13.15 - 20 + 2 + 2 - 118.4 + 198.97$

≈ 51.42 ← Which I assume is in dB
So $R_b \approx 138675.$ in regular unit.