Euclid Study Advice

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1 Introduction

1.1 What is this guide?

This is a guide designed to help you prepare for the Euclid! I'm writing it because studying for the Euclid isn't like studying for most other math tests, and nobody ever really tells you how to approach studying for it. In a second pdf, I've included my writeup of the solutions to the 2019 Euclid. **These are not just copy/pastes of the website solutions**. Instead, I'll be talking about how I would approach those questions, and how you can come up with the solution, not just what the solution is. 2019 was actually the only Euclid I wrote! I did well enough to be allowed into the advanced courses in first year, and I made significant progress on both 9 and 10, so I hope my insights are valuable. In the document, you'll see me implement some of those strategies! Do notice that most of the advice in this guide is geared at the later, more difficult questions, because the first few are usually direct applications of things you've learned in a classroom.

1.2 Who is this for?

This guide is mainly directed at people who have little to no experience with contest math outside of the Waterloo contests and the AMC. If you've done the CMO or AIME or the IMO, or gone to any math camp, everything in this guide will be familiar. It might still be a good refresher, but don't expect anything new!

2 Studying Advice

2.1 Why is the Euclid different than a normal math test?

Anyone who's written a math contest can tell you it feels different from writing a normal math test - but why? What it ultimately boils down to is that Euclid questions aren't formulaic, and questions you see in a classroom are. An advanced functions worksheet might have you solve 15 questions like "Solve $\log_3(2x) + \log_3(x) = 2$", with the numbers swapped around. Knowing how to do one of these, you know how to do all of them. That isn't how the Euclid works; the whole point of contests like that is to ask questions that require creative, unique application of relatively basic math. Knowing how to do past questions isn't how you do current questions. Instead, you need to practice the skill of taking something you know, and using it creatively.

Every math class you've taken through school has worked in a relatively rote way. You learn new concepts, then you're taught how to solve some set of problems that can be solved by those tricks. Sometimes the problems will be disguised - lots of the time basic algebra gets all dressed up as a fancy word problem, but you're still just doing basic algebra. This is categorically different in the Euclid. No matter what, the last few questions - the ones that can bring you from "meh" to a $10k scholarship - require some original insight. They generally avoid rehashing tricks (except in some geometry problems). You will need to come up with the key insight, not just draw on something you've already done. This means you can't just study by learning how every past problem was solved - you need to practice coming up with the insight, because that's what you'll be doing on the day of the test.

2.2 How to Study

Colloquial wisdom is to do past contests, and colloquial wisdom is right. The only way to prepare for Euclid style questions is to actually look at past ones. But, if you haven't done math contests before, you will have *trouble* when you first start. That's ok! If you find it challenging, it means that you're learning. This is why, at first, I recommend not setting any time limits for yourself, and instead working as long as you need to to solve it. You will gain far more from working through the solution yourself than you will from just seeing how someone else solved it, so take your time! As you do it more, you will naturally get faster, but as the day of the contest approaches, start practicing with a time limit. This'll help you prepare for the actual time pressure on contest day - you don't want to know how to solve all 10 questions, then only get 6 solutions written down because you never practiced with a timer.

2.2.1 Getting Stuck

It will frequently happen that you get stuck on a question. I have details on how to approach questions you're stuck on in the next section, but right now know that being stuck is ok, so long as you're stuck in a productive way. You need to make sure that you're actively trying - or at least thinking about - new things. When you're staring at a problem, waiting for divine inspiration to strike, that's a good sign you need a break. When you first start the question, you can at least write down what you're given and what you're looking for, and start trying to connect it to topics or problems you know. You can break down the problem. Once you've started having ideas, then those ideas are exhausted, take a break; you will be stunned by how many problems you can solve after a walk in the sun and a snack. If you go through that process twice - Attempt, break,

attempt, break - and you still have absolutely nothing that looks like it could turn into a solution, that's a good time to take a look at the solution.

When you look at the solution, don't just go "Ah yes, this is how this is solved". Look at what you tried, see how you could've come up with their solution. If the solution they give makes no sense, ask a friend! It doesn't matter if you know how to solve past questions, it matters that you know how you could come up with them. Going through the solutions should be an involved process; it won't just be reading. You should be actively annotating and following along, and really understanding how you could've come up with this. At this point it's also worth mentioning - if you tried a question, failed to solve it, then understood the solution, that does not mean you could solve it into a contest. Don't fool yourself into thinking you'll do well if you just understand all the solutions! You need to be able to come up with them!

2.3 How to approach Euclid Questions

This section will be discussing abstract tips; if you want to see some of them implemented, check out the other pdf. This section is also mostly geared at the more difficult questions (Questions 7-10 or 8-10).

So, you see a question you've never done before, that doesn't even resemble anything you've practiced. What should you do? First of all, understand what the question is asking. This might sound stupid, but a lot of the later questions on the Euclid have pretty complicated setups. Parse through the notation and definitions, and try to rephrase/re-express what it's asking in a way that makes sense to you. Next, make a mental (or physical!) note of what concepts this question is making use of. The reason to do this is to get your brain primed in the right direction - you might not know how to do the question, but at least you'll know how to start. Once you have that, write down the information that the question gives you, in a format that makes sense.

Once you've done those things, you'll have a clear idea of what you're given, and what you're trying to get to. Once that happens, don't just sit, waiting for divine inspiration to strike. For the harder questions, the vast majority of the time you won't be able to figure out a full solution in your mind, or even a full outline. As soon as you have an idea for a rough approach, write it down, and start working on it. If while you're doing it, something else comes to mind, jot that down, try it after. It's ok if your idea isn't something that seems promising, because often incorrect or incompletes approaches give some hint towards a bigger solution - you'll see this in my solution for Question 5(a)! Trying things is so important, even when they don't work. Another important thing to keep in mind is even if some approach doesn't work out right away, it's not necessarily completely dead! If you try to do a geometry problem by showing two specific triangles are similar, and it turns out they aren't similar, that doesn't mean your final solution won't involve similar triangles. It just means you can't think about those specific triangles being similar.

2.3.1 Geometry Proof Advice

If it's a proof question, make sure you have an idea of why the statement you're trying to prove is true - often that idea can turn into the ultimate solution. For proofy geometry questions, often they provide a diagram (and if they don't, draw one). You can use that diagram to convince yourself the statement is true. To be very unambiguous here, **proving something in one case,**

about one specific diagram is not sufficient to solve the problem. However, understanding why it's true in that specific case can help you figure out what the proof is in general. For example, if two triangles are similar in the provided diagram, and that's helpful to understanding why stuff is true, maybe you can prove similarity in general. Whatever contributes to the statement being true in the diagram might be something you can prove in general. If it isn't obvious from one diagram, draw a different one with the same situation. It can also often be shockingly fruitful to add lines, for example to split a rectangle into triangles, because there are lots of nice theorems about triangles. No matter what, ALWAYS have a picture in your mind for a geometry question, and make sure you're convinced the thing you're proving is true.

2.3.2 Non-Geomtry Proof Advice

In non-geometry proofs, you still want to believe the statement you're proving is true, because it'll often happen that the intuitive belief will give some insights as to what the ultimate proof will look like. Later, we'll see a question where it asks us to prove that when some function outputs an integer, that integer is always divisible by 3. When you get a question like that and you have no clue how to start, pick some numbers! Sometimes, seeing when it's true can help us figure out why it's true. See my writeup of Question 9 in the second PDF to see how numerical examples were helpful in that case.

3 What you need to know

3.1 List of Topics

Make sure that you are familiar with all of the following . This is by no means a comprehensive list

- Exponents and Exponential Laws
- Logarithms and Logarithmic laws
- Remainder Theorem, Synthetic/Long division of polynomials
- Factoring polynomials
- Trigonometry - sin and cos laws, basic identities, solving trigonometric questions
- Geometry - Similar Triangles, Circle arc length and section area, angles of inscribed triangles, internal angles of polygons
- Function notation
- Basic Probability and Combinatorics/Counting Problems

3.2 Resources

If you found yourself unfamiliar with any of the topics listed here, you probably want to review them! I recommend Khan Academy and/or Paul's Online Math Notes. The former has videos covering every high school topic under the sun, and the latter is a set of excellent notes covering late high school and early university content. The Art of Problem solving also has a list of contest resources, including a formula sheet, which is going to be helfpul if you aren't familiar with geometry at the contest level. Note that not everything they recommend is necessary for the Euclid. If you're interested in writing other contests in the future, I also highly recommend their books, which you can buy on their website.

4 Test Day!

First of all, make sure that you get enough sleep, and enough to eat and drink. Cramming is less effective than getting enough sleep - this has been demonstrated in countless studies, and it's especially true of the Euclid. You can't memorize your way to a 90, it requires genuine cleverness, and that will come more easily if you're rested. Day of, do a final review of the formulas you've struggled with, and if there's anything you really struggle with it, say it to yourself out loud or under your breath, then as soon as the contest starts, write it down somewhere. Take a look at all of the questions before starting any of them, and make sure you understand all the later questions. You will need to spend time doing this anyways, and if you do it at the start, your subconscious will start to work on the hard questions, connecting them to things you've already seen.

Start with the questions you are most confident about, usually questions 1 and 2. Getting questions right is a nice ego boost, and that way you have the freebie points out of the way. From there, proceed to the question you have the clearest idea about. You do NOT need to go in order; time is a limited resource on the Euclid, and you should do the questions that take you the longest last, because the points per question are the same. If a question takes you 30 minutes, you're getting 1 point/3 minutes. If another, easier question takes you 3 minutes, you're getting 10 times the return on your time. If you have a thought about some question while working on a different question, **write it down**. If you forget it, you will be kicking yourself - keeping track of something incorrect is way better than not keeping track of something, forgetting something, and then wondering if it would've been correct.

You got this! I really, genuinely wish you the best. If you find yourself really struggling to remember something, take a couple deep breaths. You have hours for the contest, and 30 seconds to calm down won't hurt anything, and when you're calmer, you'll have an easier time remembering the important stuff you forgot. You worked hard!

5 Summary

So, to review, you should practice contests, untimed at first, then timed. Avoid looking at solutions until you've hit an insurmountable wall (at least a few hours of effort split over 2 sessions). Your goal in studying isn't to figure out how past problems were solved, but rather to figure out the solutions yourself. The reason for this is that even though tricks might be repeated between contests, knowing all the tricks isn't enough - you need to be able to creatively apply them. Most people haven't practiced that skill, and solving problems without looking at the solution is how you do it. When doing geometry proofs, try to get a visual intuition from provided diagrams, and always fill in as much information as you can. When doing non-geometry proofs, try your best to gain an intuition for why the statement is true - numerical examples can really help with this, and so can reducing the problem to a simpler or smaller case. Proving a single case of a problem is not sufficient to complete the question! If something asks you to prove a fact about all right triangles, using one as an example isn't enough.

When you see a new question you don't know how to approach, try to find basic links between it and concepts you know, or questions you've already done. If you can reduce the problem to a simpler case, solving that might give insight to the original question.

On the day of the test, hydrate well, eat enough, get enough sleep, and relax. If you get stuck on an "easy" question - or any question really - leave it and move to a question you have more ideas for. If you're struggling to remember something, take a deep breath and calm down; it will come to you. Do the questions you have the best ideas for first, and do the hardest questions last; if you have an idea for a question you aren't working on, write it down. Good luck, you got this!!!