Syllabus

Nima Rasekh

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Summary

Category theory was designed as an abstract formalism for mathematics. It has since found applications in computer science, physics, chemistry, and biology. This course will introduce the basic concepts of category theory and explore its applications in these fields, based on student interest.

Prerequisites

1. Some familiarity with abstract mathematics (e.g. proofs, sets, functions, relations).
2. One course in mathematics (e.g., linear algebra, abstract algebra, real analysis, topology).

Pure Category Theory Background

Part of the following material will be covered as lectures in the first weeks of the course.

1. Categories
2. Functors
3. Natural transformations
4. Yoneda lemma
5. Adjunctions
6. Limits and colimits

Pure Category Theory References

1. S. Mac Lane, *Categories for the working mathematician*, second edition, Graduate Texts in Mathematics, 5, Springer, New York, 1998
2. E. Riehl, *Category theory in context*, Aurora Dover Modern Math Originals, Dover, Mineola, NY, 2016
3. M. Brandenburg, *Einführung in die Kategorientheorie. Mit ausführlichen Erklärungen und zahlreichen Beispielen.* 2nd revised edition. Heidelberg: Springer Spektrum, 2017

Student Talks

Each participant will give one talk addressing four points:

1. Background material related to the application.
2. Relevant categorical background.
3. How category theory is applied in this particular context.
4. Whether the categorical application is in fact useful.

Application Topics and References

Here is a list of topics along with sample papers. This note is not exhaustive, and we can explore other topics and students based on student interest.

Pure Mathematics

1. (1) Algebra (Lawvere theory, monoidal category theory)
   - B. Pareigis, *Kategorien und Funktoren*, Mathematische Leitfäden, Teubner, Stuttgart, 1969
   - E. Riehl, *Categorifying cardinal arithmetic*, talk notes, https://math.jhu.edu/ eriehl/arithmetic-condensed.pdf
2. (2) Topology (Pointless topology)
   - S. Mac Lane and I. Moerdijk, *Sheaves in geometry and logic*, corrected reprint of the 1992 edition, Universitext, Springer, New York, 1994
3. (3) Algebraic topology (fundamental groupoid)
   - T. Tom Dieck, *Algebraic topology*, EMS Textbooks in Mathematics, Eur. Math. Soc., Zürich, 2008
4. (4) Analysis (Metric spaces as enriched categories)
   - E. Riehl, *Categorical homotopy theory*, New Mathematical Monographs, 24, Cambridge Univ. Press, Cambridge, 2014

Statistics and Probability

1. (5) Probability (Probability monad)
   - M. Giry, *A categorical approach to probability theory*, in Categorical aspects of topology and analysis (Ottawa, Ont., 1980), pp. 68–85, Lecture Notes in Math., 915, Springer, Berlin-New York,
   - J. Asilis, *Probability Monads: A motivated treatment of monads and their algebras*, https://www.math.harvard.edu/media/JulianAsilisThesis.pdf
2. (6) Statistics (Markov processes)
   - J. C. Baez, B. Fong and B. S. Pollard, *A compositional framework for Markov processes*, J. Math. Phys. **57** (2016), no. 3, 033301, 30 pp.
   - T. Fritz, *A synthetic approach to Markov kernels, conditional independence and theorems on sufficient statistics*, Adv. Math. **370** (2020), 107239, 105 pp.
3. (7) Entropy (axiomatic entropy)
   - J. C. Baez and T. Fritz, *A Bayesian characterization of relative entropy*, Theory Appl. Categ. **29** (2014), No. 16, 422–457
   - T. Leinster, *Entropy and diversity—the axiomatic approach*, Cambridge Univ. Press, Cambridge, 2021

4. (8) Causal inference (Categorical causal models)
   - B. Fong, *Causal Theories: A Categorical Perspective on Bayesian Networks*, https://arxiv.org/abs/1301.6201
   - E. F. Rischel, S. Weichwald, *Compositional abstraction error and a category of causal models*, https://proceedings.mlr.press/v161/rischel21a.html
   - B. Jacobs, A. Kissinger and F. Zanasi, *Causal inference via string diagram surgery: a diagrammatic approach to interventions and counterfactuals*, Math. Structures Comput. Sci. **31** (2021), no. 5, 553–574

Logic

1. (9) Set theory (elementary topos theory)
   - S. Mac Lane and I. Moerdijk, *Sheaves in geometry and logic*, corrected reprint of the 1992 edition, Universitext, Springer, New York, 1994
2. (10) Logic (categorical logic)
   - J. Lambek and P. J. Scott, *Introduction to higher order categorical logic*, reprint of the 1986 original, Cambridge Studies in Advanced Mathematics, 7, Cambridge Univ. Press, Cambridge, 1988
3. (11) Type theory (categorical semantics of type theory)
   - B. Jacobs, *Categorical logic and type theory*, Studies in Logic and the Foundations of Mathematics, 141, North-Holland, Amsterdam, 1999

Computer Science

1. (12) Machine learning (backpropagation as functor)
   - B. Fong, D. I. Spivak and R. Tuyéras, *Backprop as functor: a compositional perspective on supervised learning*, in *2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*, [13 pp.], IEEE, NJ
2. (13) Deep learning (Compositional Deep Learning)
   - B. Gavranović, Compositional Deep Learning, https://arxiv.org/abs/1907.08292
3. (14) Functional programming (monads)
   - B. Ahrens, K. Wullaert, *Category Theory for Programming*, arXiv preprint, https://arxiv.org/abs/2209.01259
4. (15) Data bases (algebraic databases)
   - D. I. Spivak, *Functorial data migration*, Inform. and Comput. **217** (2012), 31–51
   - P. Schultz, D. I. Spivak and R. Wisnesky, *Algebraic model management: a survey*, Lect. Notes Comput. Sci. 10644, 56–69 (2017)
5. (16) Automata theory (coalgebra)
   - B. Jacobs and A. Silva, *Automata Learning: A Categorical Perspective*, Lect. Notes Comput. Sci. 8464, 384–406 (2014)
6. (17) Dynamical Systems (compositional dynamical systems)
   - B. Fong, P. Sobociński and P. Rapisarda, *A categorical approach to open and interconnected dynamical systems*, in Proceedings of the 31st Annual ACM-IEEE Symposium on Logic in Computer Science (LICS 2016), 10 pp., ACM, New York
7. (18) Game theory (semantics)
   - D. Pavlovic, *A semantical approach to equilibria and rationality*, in Algebra and coalgebra in computer science, 317–334, Lecture Notes in Comput. Sci., 5728, Springer, Berlin

Physics

1. (19) Classical mechanics (open systems)
   - J. C. Baez, D. Weisbart and A. M. Yassine, *Open systems in classical mechanics*, J. Math. Phys. **62** (2021), no. 4, Paper No. 042902, 24 pp.
2. (20) Thermostatics (open systems)
   - J. C. Baez, O. Lynch and J. Moeller, *Compositional thermostatics*, J. Math. Phys. **64** (2023), no. 2, Paper No. 023304, 16 pp.
3. (21) Electrical circuits (open systems)
   - J. C. Baez, B. Coya and F. Rebro, *Props in network theory*, Theory Appl. Categ. **33** (2018), Paper No. 25, 727–783
4. (22) Field theory (Topological field theory)
   - R. Cohen (notes by E. Malm): *MATH 283: Topological Field Theories (2008)*, lecture notes https://ericmalm.net/ac/projects/math283-w08/math283-w08-n.pdf
5. (23) Quantum mechanics (Categorical quantum mechanics)
   - S. Abramsky and B. Coecke, *Categorical quantum mechanics*, in Handbook of quantum logic and quantum structures—quantum logic, 261–323, Elsevier/North-Holland, Amsterdam,

Chemistry

1. (24) Chemical Reactions (Petri nets or reaction networks)
   - J. C. Baez and B. S. Pollard, *A compositional framework for reaction networks*, Rev. Math. Phys. **29** (2017), no. 9, 1750028, 41 pp.
2. (25) Organic Chemistry (Dagger categories)
   - E. Gale, L. Lobski and F. Zanasi, *A categorical model for organic chemistry*, Theoret. Comput. Sci. **1032** (2025), Paper No. 115084, 37 pp.

Biology

1. (26) Biochemistry (open systems)
   - R. Aduddell, J. Fairbanks, A. Kumar, P. S. Ocal, E. Patterson, and B. T. Shapiro, *A compositional account of motifs, mechanisms, and dynamics in biochemical regulatory networks*, Compositionality **6** (2024), no. 2, 30 pp.
2. (27) Genetics (ologs)
   - *Y. Wu*, *Gene ologs: a categorical framework for Gene Ontology*, arXiv Preprint, https://arxiv.org/abs/1909.11210

Other Applications

1. (28) Linguistics (Frobenius algebras)
   - M. Sadrzadeh, S. Clark and B. Coecke, *The Frobenius anatomy of word meanings I: subject and object relative pronouns*, J. Logic Comput. **23** (2013), no. 6, 1293–1317
2. (29) Assembly Planning (String diagrams)
   - J. Master, E. Patterson, S. Yousfi, and A. Canedo, *String diagrams for assembly planning*, in Diagrammatic representation and inference, 167–183, Lecture Notes in Comput. Sci. Lecture Notes in Artificial Intelligence, 12169 , Springer, Cham,