DIFFERENTIAL COHOMOLOGY SEMINAR

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Differential cohomology theory wants to be a cohomology theory (so algebraic topology) that incorporates information about differentiable manifolds (so differential geometry). Various framework have been proposed to make this concept precise.

In this learning seminar we want to focus on a modern approach that uses ∞-categorical methods and particularly ∞-categorical sheaves. The first half introduces differential cohomology from an abstract higher categorical perspective. The second part aims to apply these definitions and results in various contexts.

1. List of Talks

Here is the list of talks, dates and speakers:

	Talk	Speaker	Day	Date
(1)	Motivation	Nima Rasekh	Wednesday	09.04.2025
(2)	History	Konrad Waldorf	Wednesday	30.04.2025
(3)	∞-Categorical Background I	Matthias Frerichs	Tuesday	06.05.2025
(4)	∞-Categorical Background II	Matthias Frerichs	Tuesday	13.05.2025
(5)	Definition I	Hannes Berkenhagen	Wednesday	21.05.2025
(6)	Definition II	Hannes Berkenhagen	Tuesday	27.05.2025
(7)	Examples and Operations	Alessandro Nanto	Wednesday	03.06.2025
(8)	Main Result via Deligne Cohomology	Alessandro Nanto	Tuesday	18.06.2025
(9)	Summary and Future Steps	Nima Rasekh	Wednesday	16.07.2025
(10)	Review and Talk Distribution	Nima Rasekh	Tuesday	14.10.2025
(11)	Differential K-Theory I	Matthias Ludewig	Wednesday	29.10.2025
(12)	Differential K-Theory II	Matthias Ludewig	Wednesday	05.11.2025
(13)	Twisted Cohomology Theories	Hannes Berkenhagen	Wednesday	12.11.2025
(14)	Twisted Diff. Cohomology Theories	Alessandro Nanto	Wednesday	26.11.2025
(15)	∞-Topoi and Cohesion	Matthias Frerichs	Wednesday	03.12.2025
(16)	Differential Cohomology and Cohesion	Nima Rasekh	Wednesday	17.12.2025
(17)	Differential Cohomology of Lie Groupoids	Christian Becker	Wednesday	07.01.2026

2. List of Topics

The first half of the seminar focuses on the theory of differential cohomology, while the second half focuses on examples and applications. Here is a more detailed breakdown of the topics, with relevant citations:

Part (I) Theory

The first part focuses on theory with the aim of understanding differential cohomologies and the fracture square.

(1) **Motivation** & **History:** Very broad historical overview of the rise of differential cohomology theories out of ordinary cohomologies [HS05a, SS08, Sti11], solution via the differential cohomology hexagon in the context of sheaves of spectra [ADH21, Section 2], [Deb23].

(2) **Proper History:** More detailed historical development with a focus on the need for more refined invariants in the study of bundles [HS05a, SS08, Sti11].

(3) **Categorical background:** The ∞-categorical approach to homotopy theory, analysis of accessible and presentable ∞-categories, definition of the ∞-category of spectra, relation to cohomology theories, Brown representability theorem [Lur17, Section 1], [Gro10].

(4) **Differential cohomology:** Reviewing the category of manifolds, ∞-categorical sheaves [Lur09, Section 6], definition of differential cohomology theories via sheaves of spectra, first relevant properties, first simple examples such as constant sheaves [ADH21, Section 2]

(5) **Examples and Operations:** Ring structure in differential cohomology via cup product [ADH21, Section 8] and fiber integration [ADH21, Section 9]

(6) **Main Result via Deligne Cohomology:** Definition, structures and properties of ordinary differential cohomology and Deligne cohomology and explicit fracture square [ADH21, Sections 6, 7], [BNV16].

Part (II) Applications

(7) **Differential K-Theory:** Definition, structures and properties of differential K-theory as a differential refinement of ku [ADH21, Section 7], [HS05b, SS08, BNV16].

(8) **Twisted Differential Cohomology Theories:** Definition and properties of twisted cohomology theories [AS04, ABG$^+$14, ABG18, Ros24] and their generalization to twisted differential cohomology [BN19].

(9) **Differential Cohomology in Cohesive ∞-Topoi:** Cohesive ∞-topoi and differential cohomology theories therein [Sch13].

(10) **Differential Cohomology of Lie Groupoids:** Compare various possible definitions of differential cohomology of Lie groupoids

References

[ABG$^+$14] Matthew Ando, Andrew J. Blumberg, David Gepner, Michael J. Hopkins, and Charles Rezk. An ∞-categorical approach to R-line bundles, R-module Thom spectra, and twisted R-homology. *J. Topol.*, 7(3):869–893, 2014.

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[ADH21] Araminta Amabel, Arun Debray, and Peter Haine. Differential cohomology: Categories, characteristic classes, and connections. *arXiv preprint*, 2021. arXiv:2109.12250.

[AS04] Michael Atiyah and Graeme Segal. Twisted K-theory. *Ukr. Mat. Visn.*, 1(3):287–330, 2004.

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[BNV16] Ulrich Bunke, Thomas Nikolaus, and Michael Völkl. Differential cohomology theories as sheaves of spectra. *J. Homotopy Relat. Struct.*, 11(1):1–66, 2016.

[Deb23] Arun Debray. Differential cohomology (encyclopedia article). *arXiv preprint*, 2023. arXiv:2312.14338.

[Gro10] Moritz Groth. A short course on ∞-categories. *arXiv preprint*, 2010. arXiv:1007.2925.

[HS05a] M. J. Hopkins and I. M. Singer. Quadratic functions in geometry, topology, and M-theory. *J. Differential Geom.*, 70(3):329–452, 2005.

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[Lur09] Jacob Lurie. *Higher topos theory*, volume 170 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 2009.

[Lur17] Jacob Lurie. Higher algebra. Available online, September 2017.

[Ros24] Jonathan Rosenberg. Twisted cohomology. *arXiv preprint*, 2024. arXiv:2401.03966.

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[SS08] James Simons and Dennis Sullivan. Axiomatic characterization of ordinary differential cohomology. *J. Topol.*, 1(1):45–56, 2008.

[Sti11] Andrew Jay Stimpson. *Axioms for Differential Cohomology*. ProQuest LLC, Ann Arbor, MI, 2011. Thesis (Ph.D.)–State University of New York at Stony Brook.