DIFFERENTIAL COHOMOLOGY SEMINAR

NIMA RASEKH

Differential cohomology theory wants to be a cohomology theory (so algebraic topology) that incorporates information about differentiable manifolds (so differential geometry). Various framework have been proposed to make this concept precise.

In this learning seminar we want to focus on a modern approach that uses ∞-categorical methods and particularly ∞-categorical sheaves. The first half introduces differential cohomology from an abstract higher categorical perspective. The second part aims to apply these definitions and results in various contexts.

1. List of Talks

Here is the list of talks, dates and speakers:

	Talk	**Speaker**	**Day**	**Date**
(1)	Motivation	Nima Rasekh	Wednesday	09.04.2025
(2)	History	Konrad Waldorf	Wednesday	30.04.2025
(3)	∞-Categorical Background I	Matthias Frerichs	Tuesday	06.05.2025
(4)	∞-Categorical Background II	Matthias Frerichs	Tuesday	13.05.2025
(5)	Definition I	Hannes Berkenhagen	Wednesday	21.05.2025
(6)	Definition II	Hannes Berkenhagen	Tuesday	27.05.2025
(7)	Operations	Alessandro Nanto	Wednesday	03.06.2025
(8)	Examples	Konrad Waldorf	Tuesday	18.06.2025
(9)	Invariance and Localizations	Christian Becker	Tuesday	24.06.2025
(10)	Fracture Square	Matthias Ludewig	Tuesday	01.07.2025
(11)			Wednesday	09.07.2025
(12)			Wednesday	16.07.2025

2. List of Topics

Here is a more detailed breakdown of the topics, with relevant citations:

(1) **Motivation** & **History:** Very broad historical overview of the rise of differential cohomology theories out of ordinary cohomologies [HS05, SS08, Sti11], solution via the differential cohomology hexagon in the context of sheaves of spectra [ADH21, Section 2], [Deb23].

(2) **Proper History:** More detailed historical development with a focus on the need for more refined invariants in the study of bundles [HS05, SS08, Sti11].

Part (I) Theory

The first part focuses on theory with the aim of understanding differential cohomologies and the fracture square.

(3) **Categorical background:** The ∞-categorical approach to homotopy theory, analysis of accessible and presentable ∞-categories, definition of the ∞-category of spectra, relation to cohomology theories, Brown representability theorem [Lur17, Section 1], [Gro10].

(4) **Differential cohomology:** Reviewing the category of manifolds, ∞-categorical sheaves [Lur09, Section 6], definition of differential cohomology theories via sheaves of spectra, first relevant properties, first simple examples such as constant sheaves [ADH21, Section 2]

(5) **Operations on Differential Cohomologies:** Ring structure in differential cohomology via cup product [ADH21, Section 8] and fiber integration [ADH21, Section 9]

Date: Summer Semester 2025.

(6) **The Main Result in Examples:** Definition, structures and properties of ordinary differential cohomology, differential K-theory [ADH21, Section 7], possibly other examples (such as differential algebraic K-theory, differential complex cobordism, ...) . Here we take the eventual results (the differential cohomology hexagon) for granted and look how it concretely manifests in those cases of interest.

(7) **$\mathbb{R}$-Invariant Sheaves and Localizations:** Proving $\mathbb{R}$-invariant sheaves are trivial [ADH21, Section 4] and how to study arbitrary sheaves via $\mathbb{R}$-invariant ones [ADH21, Section 5]

(8) **Fracture Square:** recollements, fracture squares, recovering differential cohomology hexagon [ADH21, Section 6], [BG16, Lur18].

Part (II) Applications

This part will mostly be determined later. You can also make suggestions!

(9) **Characteristic Classes:** Characteristic classes of differential cohomology theories [ADH21, Section 14], [BNV16], [Deb23, Section 2]

(10) **Lifting Chern-Weil homomorphism:** [ADH21, Section 13, Section 15]

(11) **Applications in Physics:** Applications to string theory [Fre02].

(12) **Other Applications:** ...

References

[ADH21] Araminta Amabel, Arun Debray, and Peter Haine. Differential cohomology: Categories, characteristic classes, and connections. *arXiv preprint*, 2021. arXiv:2109.12250.

[BG16] Clark Barwick and Saul Glasman. A note on stable recollements. *arXiv preprint*, 2016. arXiv:1607.02064.

[BNV16] Ulrich Bunke, Thomas Nikolaus, and Michael Völkl. Differential cohomology theories as sheaves of spectra. *J. Homotopy Relat. Struct.*, 11(1):1–66, 2016.

[Deb23] Arun Debray. Differential cohomology (encyclopedia article). *arXiv preprint*, 2023. arXiv:2312.14338.

[Fre02] Daniel S. Freed. K-theory in quantum field theory. In *Current developments in mathematics, 2001*, pages 41–87. Int. Press, Somerville, MA, 2002.

[Gro10] Moritz Groth. A short course on ∞-categories. *arXiv preprint*, 2010. arXiv:1007.2925.

[HS05] M. J. Hopkins and I. M. Singer. Quadratic functions in geometry, topology, and M-theory. *J. Differential Geom.*, 70(3):329–452, 2005.

[Lur09] Jacob Lurie. *Higher topos theory*, volume 170 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 2009.

[Lur17] Jacob Lurie. Higher algebra. Available online, September 2017.

[Lur18] Jacob Lurie. Spectral algebraic geometry. https://www.math.ias.edu/ lurie/papers/SAG-rootfile.pdf, February 2018.

[SS08] James Simons and Dennis Sullivan. Axiomatic characterization of ordinary differential cohomology. *J. Topol.*, 1(1):45–56, 2008.

[Sti11] Andrew Jay Stimpson. *Axioms for Differential Cohomology*. ProQuest LLC, Ann Arbor, MI, 2011. Thesis (Ph.D.)–State University of New York at Stony Brook.