DIFFERENTIAL COHOMOLOGY SEMINAR 5

TALK BY NIMA RASEKH

In this talk we summarize what we covered until now and discuss possible future directions.

1. Summary

We established that in the modern point of view a differential cohomology theory is an ∞-categorical sheaf on the site of manifolds valued in the ∞-category of spectra. This approach naturally leads to several relevant questions, that were the focus of our talks:

(1) What kind of theoretical framework is needed to work with such a theory?
(2) How does this framework relate to other approaches to differential cohomology?
(3) How can we construct differential cohomology theories in this framework?
There are also questions, we did not address, but could be focus of future talks:
(4) What are concrete benefits of this approach in contrast to others?

2. Theoretical framework

We saw that the theoretical framework is fundamentally that of presentable ∞-categories, stable ∞-categories, and ∞-categorical sheaves, as developed by Lurie [Lur09], among others.

As it primarily a background, for what follows we will take it for granted, and refer to the relevant sources.

3. Relation to other approaches

A lot of historical development of differential cohomology theories has focused on constructing them via specific data, and concretely the input often consists of an ordinary cohomology theory and some geometric data. This perspective is completely absent in the definition we just gave, so how can we reconcile them? This is the central theme of the *fracture square*. This already came up so let us quickly summarize. Let $\mathcal{C}$ be a presentable ∞-category.

Definition 3.1. A pre-sheaf F is called $\mathbb{R}$-*invariant* if, for any manifold M, the projection $M \times \mathbb{R} \to M$ induces an equivalence $F(M) \to F(M \times \mathbb{R})$. Denote by $\mathcal{PSh}_{\mathbb{R}}(\mathcal{M}\mathrm{fd}, \mathcal{C})$ the full sub-category of $\mathbb{R}$-invariant pre-sheaves, and $\mathcal{S}\mathrm{hv}_{\mathbb{R}}(\mathcal{M}\mathrm{fd}, \mathcal{C})$ the full sub-category of $\mathbb{R}$-invariant sheaves.

Recall from [Ber25] that the restriction $\mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd}, \mathcal{C}) \to \mathcal{S}\mathrm{hv}(\mathcal{E}\mathrm{uc}, \mathcal{C})$.

Lemma 3.2. *A sheaf F is $\mathbb{R}$-invariant if and only if every smooth map $\mathbb{R}^n \to \mathbb{R}^m$ induces an equivalence $F(\mathbb{R}^m) \to F(\mathbb{R}^n)$.*

Proof. The terminal map $\mathbb{R}^n \to \mathbb{R}^0$ is the composition of projections $M \times \mathbb{R} \to M$, therefore if F is a $\mathbb{R}$-invariant sheaf, $F(\mathbb{R}^0) \to F(\mathbb{R}^n)$ is an equivalence. For any map $f : \mathbb{R}^n \to \mathbb{R}^m$, there is a commutative diagram

$$\begin{tikzcd} F(\mathbb{R}^0) \arrow[r, "\simeq"] \arrow[dr, "\simeq"'] & F(\mathbb{R}^m) \arrow[d, "f^*"] \\ & F(\mathbb{R}^n) \end{tikzcd} \tag{3.3}$$

implying f^* is an equivalence.

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On the other hand, let F be a sheaf such that, for every smooth map f of Euclidean manifolds, f^* is an equivalence. Let M be a smooth manifold and $\mathcal{O}$ a differentiably good open cover (see [Nan25, Theorem 5.4]), then $\{U \times \mathbb{R} | U \in \mathcal{O}\}$ is a differentiably good open cover of $M \times \mathbb{R}$ and there is a commutative square

$$\begin{tikzcd} F(M) \arrow[r, "\simeq"] \arrow[d] & \lim_{U \in \mathcal{I}(\mathcal{O})} F(U) \arrow[d, "\simeq"] \\ F(M \times \mathbb{R}) \arrow[r, "\simeq"] & \lim_{U \in \mathcal{I}(\mathcal{O})} F(U \times \mathbb{R}) \end{tikzcd} \tag{3.4}$$

The horizontal maps are equivalences by the sheaf condition, while the right-vertical map is an equivalence because $U \times \mathbb{R} \to U$ is a smooth map of Euclidean manifolds, we then conclude that $F(M) \to F(M \times \mathbb{R})$ is an equivalence. □

Recall from [Nan25] that the left adjoint $\flat : \mathcal{C} \to \mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd}, \mathcal{C})$ to the global section functor Γ has the following description: Given an object $C \in \mathcal{C}$, $\flat C$ maps M to $C^{\mathrm{Sing} M}$, where $\mathrm{Sing} M$ denotes the (smooth) singular simplicial set of M.

Corollary 3.5. *The functor* $\flat : \mathcal{C} \to \mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd}, \mathcal{C})$ *factors through* $\mathcal{S}\mathrm{hv}_{\mathbb{R}}(\mathcal{M}\mathrm{fd}, \mathcal{C})$. *The adjunction*

$$\begin{tikzcd} \mathcal{C} \arrow[r, shift right, "\flat"'] & \mathcal{S}\mathrm{hv}_{\mathbb{R}}(\mathcal{M}\mathrm{fd}, \mathcal{C}) \arrow[l, shift right, "\Gamma"'] \end{tikzcd} \tag{3.6}$$

is an equivalence.

Proof. Since every Euclidean manifold has contractible underlying homotopy type, $\flat C$ maps every map f of Euclidean spaces to an equivalence. By Equation (3.2), we conclude that $\flat C$ is $\mathbb{R}$-invariant.

Let F be a $\mathbb{R}$-invariant sheaf, set $C := F(\mathbb{R}^0)$, then $F|_{\mathcal{E}\mathrm{uc}}$ is equivalent to the constant pre-sheaf with value C (see [Nan25, Remark 2.7]), which is $\flat C|_{\mathcal{E}\mathrm{uc}}$, therefore $F \simeq \flat C$. □

Theorem 3.7. *There is a 3-terms adjunction*

$$\begin{tikzcd} \mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd}, \mathcal{C}) \arrow[r, shift left=2, "\Pi_\infty"] \arrow[r, shift right=2, "\Gamma"'] & \mathcal{C} \arrow[l, "\flat" description] \end{tikzcd} \tag{3.8}$$

where Γ *is the global sections function, i.e. evaluation at* $\mathbb{R}^0$, $\flat$ *is the locally constant sheaf functor,* Π_∞ *is in the proof below. There is an induced 3-terms adjunction*

$$\begin{tikzcd} \mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd}, \mathcal{C}) \arrow[r, shift left=2, "L_{\mathrm{hi}}"] \arrow[r, shift right=2, "R_{\mathrm{hi}}"'] & \mathcal{S}\mathrm{hv}_{\mathbb{R}}(\mathcal{M}\mathrm{fd}, \mathcal{C}) \arrow[l] \end{tikzcd} \tag{3.9}$$

where $L_{\mathrm{hi}}(F)$, *resp.* $R_{\mathrm{hi}}(F)$, *is the locally constant sheaf associated to* $\Pi_\infty(F)$, *resp.* $\Gamma(F) = F(\mathbb{R}^0)$.

Proof. Since the composition $\mathcal{C} \xrightarrow{\flat} \mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd}, \mathcal{C}) \to \mathcal{S}\mathrm{hv}(\mathcal{E}\mathrm{uc}, \mathcal{C})$ is the constant functor, it has as left adjoint the colimit functor $\mathcal{S}\mathrm{hv}(\mathcal{E}\mathrm{uc}, \mathcal{C}) \to \mathcal{C}$. Define $\Gamma_\sharp$ as the pre-composition of the colimit functor with $-|_{\mathcal{E}\mathrm{uc}}$. The induced adjunction comes from composing the adjunction in Equation (3.8) with the adjoint equivalence

$$\begin{tikzcd} \mathcal{C} \arrow[r, shift left=2, "\flat"] \arrow[r, shift right=2, "\flat"'] & \mathcal{S}\mathrm{hv}_{\mathbb{R}}(\mathcal{M}\mathrm{fd}, \mathcal{C}) \arrow[l, "\Gamma" description] \end{tikzcd} \tag{3.10}$$

□

Remark 3.11. In particular, $\mathcal{S}\mathrm{hv}_{\mathbb{R}}(\mathcal{M}\mathrm{fd}, \mathcal{C})$ is both a reflective and coreflective sub-category of $\mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd}, \mathcal{C})$. Reflectivity could also be deduced from the definition of $\mathbb{R}$-invariant sheaves as local objects with respect to the class of projections $M \times \mathbb{R} \to M$.

Definition 3.12. Let $\mathcal{C}$ be a category, $\mathcal{D} \subseteq \mathcal{C}$ a full sub-category. An object C is *right orthogonal to* $\mathcal{D}$ if $\mathrm{Hom}_{\mathcal{C}}(D, C)$ is contractible, for every $D \in \mathcal{D}$. Denote by $\mathcal{D}^\perp$ the full sub-category of objects right orthogonal to $\mathcal{D}$.

Definition 3.13. A right orthogonal sheaf to $\mathcal{S}\mathrm{hv}_{\mathbb{R}}(\mathcal{M}\mathrm{fd}, \mathcal{C})$ is called *pure*.

Lemma 3.14. *F is pure if and only if $\Gamma(F)$ is terminal.*

Proof. By Equation (3.5), every $\mathbb{R}$-invariant sheaf is of the form $\flat C$, therefore F is pure if and only if $\mathrm{Hom}_{\mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd},\mathcal{C})}(\flat C, F) \simeq \mathrm{Hom}_{\mathcal{C}}(C, \Gamma(F))$ is contractible, for all C, which is equivalent to $\Gamma(F)$ being terminal. $\square$

Definition 3.15. Given a sheaf F, denote by $\mathrm{Def}(F)$ the fiber of the unit $F \to L_{\mathrm{hi}}(F)$.

Definition 3.16. A sheaf is *pure* if the value at the point is the terminal spectrum. We denote the full subcategory of pure sheaves by $\mathcal{S}\mathrm{hv}_{\mathrm{pure}}(\mathcal{M}\mathrm{fd})$.

Similarly, the category of pure sheaves is closed under limits and colimits, so we have the following result:

Theorem 3.17. *There is a diagram of adjunctions*

$$\begin{tikzcd} \mathcal{S}\mathrm{hv}_{\mathrm{pure}}(\mathcal{M}\mathrm{fd}) \arrow[r, hook, shift left=2] \arrow[r, hook', shift right=2] & \mathcal{S}\mathrm{hv}(\mathcal{M}\mathrm{fd}) \arrow[l, shift left=4, "\mathrm{Def}"'] \arrow[l, "\mathrm{Cyc}"] \end{tikzcd}$$

Remark 3.18. The notational choice is not coincidental. The left adjoint to Cyc is precisely the functor Def, where the domain is restricted to pure sheaves.

Notice we obviously have the following result.

Lemma 3.19. *A sheaf is* trivial *if it is $\mathbb{R}$-invariant and pure.*

So, pure and $\mathbb{R}$-invariant sheaves are "disjoint". Even better, they cover everything, which is the gist of the fracture square.

Remark 3.20. Notice in both adjunction diagrams there is one fully faithful functor that is not named and we will abuse notation and directly consider objects in the full subcategory as objects in the larger category.

Theorem 3.21. *Let E be a differential cohomology theory. Then the following is a pullback square*

$$\begin{tikzcd} E \arrow[r] \arrow[d] & \mathrm{Cyc}E \arrow[d] \\ L_{hi}E \arrow[r] & L_{hi}\mathrm{Cyc}E \end{tikzcd}$$

In fact we can further expand this pullback square to several other pullback squares:

$$\begin{tikzcd} \Sigma^{-1}L_{hi}\mathrm{Cyc}E \arrow[r] \arrow[d] & R_{hi}E \arrow[r] \arrow[d] & 0 \arrow[d] \\ \mathrm{Def}E \arrow[r] \arrow[d] & E \arrow[r] \arrow[d] & \mathrm{Cyc}E \arrow[d] \\ 0 \arrow[r] & L_{hi}E \arrow[r] & L_{hi}\mathrm{Cyc}E \end{tikzcd}$$

We can restructure this diagram to more resemble the fracture square, which is the following:

$$\begin{tikzcd} & R_{hi}(E) \arrow[rr] \arrow[dr] & & L_{hi}(E) \arrow[dr] & \\ \Sigma^{-1}L_{hi}\mathrm{Cyc}E \arrow[ur] \arrow[dr] & & E \arrow[ur] \arrow[dr] & & L_{hi}\mathrm{Cyc}E \\ & \mathrm{Def}(E) \arrow[rr] \arrow[ur] & & \mathrm{Cyc}E \arrow[ur] & \end{tikzcd}$$

The proof of all these results has as of yet been postponed.

4. Construction of differential cohomology theories

Having advanced theory is of course interesting, but not enough, we also want explicit examples. Indeed we saw in past weeks several ways to explicitly construct differential cohomology theories.

Example 4.1. Given a spectrum (cohomology theory) E, we can define the constant sheaf, which is by definition a differential cohomology theory. Even better, it is an $\mathbb{R}$-invariant sheaf, and every $\mathbb{R}$-invariant sheaf is obtained this way.

Notice in this case the pure part is trivial, meaning we get the following, somewhat trivial, diagram of pullback squares:

$$\begin{tikzcd} 0 \arrow[r] \arrow[d] & E \arrow[r] \arrow[d] & 0 \arrow[d] \\ 0 \arrow[r] \arrow[d] & E \arrow[r] \arrow[d] & 0 \arrow[d] \\ 0 \arrow[r] & E \arrow[r] & 0 \end{tikzcd}$$

This means we have a very good understanding of $\mathbb{R}$-invariant sheaves, using classical algebraic topology. However, of course these examples have no geometric content, so we want to go beyond them.

Example 4.2. Given a sheaf of abelian groups, we can construct the differential cohomology theory given by post-composing with the functor that takes an abelian group A to the associated Eilenberg-MacLane spectrum $H(A)$.

Example 4.3. There is a non-trivial way to extend a functor from abelian groups to spectra to one from chain complexes to spectra, which is called the *stable Dold-Kan embedding*. Hence, again via post-composition, every sheaf of chain complexes gives rise to a differential cohomology theory.

Example 4.4. An important example of the previous example are truncated forms. Given $k \geq 0$, $\Omega^{\geq k}$ is a sheaf that associates to a manifold M the chain complex of k-truncated forms on M. Using the previous example we hence get a differential cohomology theory which we also denote $\Omega^{\geq k}$.

Notice, if $k > 0$ then $\Omega^{\geq k}$ is in fact pure, and also not trivial, which means it cannot be $\mathbb{R}$-invariant.

The approach we used until now helps us generate examples that either pure or $\mathbb{R}$-invariant, but we would also like to mix them. Here we use the fracture square. The fracture square is not only an abstract theoretical tool, it also gives us very explicit ways to construct differential cohomology theories, using methods closer to the historical approaches. Concretely, we now have the following results:

Corollary 4.5. *A differential cohomology theory is uniquely determined by the following three pieces of data:*

(1) An $\mathbb{R}$-invariant sheaf $E_{\mathbb{R}}$, which is equivalently a spectrum, or equivalently a cohomology theory.
(2) A pure sheaf E_{pure}.
(3) A morphism of spectra $E_{\mathbb{R}} \to L_{hi}E_{\text{pure}}$.

More specifically, we often face the situation that we have a fixed cohomology theory E, and want to find a lift of sorts.

Definition 4.6. Let E be a cohomology theory. A *differential refinement of E* is a differential cohomology $\hat{E}$, such that $L_{hi}\hat{E} \simeq E$.

Putting the definition and corollary together, we get the final result to generate examples of interest.

Corollary 4.7. *Let E be a cohomology theory (spectrum). The differential refinement $\hat{E}$ is uniquely determined by the following pieces of data:*

(1) A pure sheaf E_{pure}.
(2) A morphism of spectra $E \to L_{hi}E_{\text{pure}}$.

These results suggests the following algorithm for constructing differential cohomology theories of interest:

(1) Pick a pure sheaf E_{pure}, that encodes geometric data of interest.
(2) Compute the spectrum $L_{hi}E_{\text{pure}}$.
(3) Then for an arbitrary spectrum E, every map of spectra $E \to L_{hi}E_{\text{pure}}$ gives rise to a differential refinement of E.

Let us implement this algorithm in practice. As we saw, the key example a pure sheaf is $\Omega^{k\geq}$, the sheaf of k-truncated forms. We now have the following result due to [BNV16].

Theorem 4.8. *Let $\Omega^{k\geq}$ be the pure sheaf of k-truncated forms. Then, $L_{hi}\Omega^{k\geq} \simeq H\mathbb{R}$.*

Example 4.9. Let E be a spectrum with a map of spectra $E \to H\mathbb{R}$. Then there is a differential refinement, $\hat{E}$, given via the pullback square

$$\begin{tikzcd} \hat{E} \arrow[r] \arrow[d] & \Omega^{k\geq} \arrow[d] \\ E \arrow[r] & H\mathbb{R} \end{tikzcd}$$

Example 4.10. Let us see an example of the example. Let $E = H\mathbb{Z}$, i.e. singular cohomology, and $H\mathbb{Z} \to H\mathbb{R}$ the evident inclusion. Then the resulting differential refinement of singular cohomology recovers *Deligne cohomology* $\hat{\mathbb{Z}}[l]$. It has the property that the k-th sheaf cohomology group recovers the classical Deligne cohomology group $\hat{H}^k(M)$, however at other degrees we get trivial groups. This means the classical Deligne cohomology groups sit inside short exact sequences.

So, in hindsight the fact that historically people have been looking at maps into $H\mathbb{R}$ is not a coincidence, every differential cohomology theory whose pure part is $\Omega^{k\geq}$ is obtained this way.

Question 5. Given the example, here is a natural question one might wonder. Is there a single differential cohomology theory, such that its k-th sheaf cohomology group recovers the classical Deligne cohomology group $\hat{H}^k(M)$ for all $k \geq 0$? It seems the answer would have to be no, because one single cohomology theory would not decompose into a collection of short exact sequences.

6. Future Directions

For future talks we can consider the following topics.

6.1. **Fancy Definition of Cohomology Groups.** As we saw in Section 5, we might wonder whether it is possible to recover all Deligne cohomology groups out of one differential cohomology.

While this might not be possible with the naive regular definition of cohomology groups, there appears to be an alternative definition due to Bunke–Gepner [BG21], which is able to recover all Deligne cohomology groups, via a refined method.

Understanding this approach can help us understand ways to extract cohomological data in a non-formal way that is more helpful in geometrically motivated applications.

6.2. **Further Examples of Differential Cohomology Theories.** We already saw Deligne cohomology as a differential refinement of singular cohomology. However, many other cohomologies also admit differential refinements, that merit further study.

(1) We could look at differential K-theory. It is understood as the differential refinement of ku, the K-theory spectrum. Following the algorithm we want:
   - A pure sheaf $\Omega^{\geq k}(-; \mathbb{C}[u^{\pm 1}])$.
   - The computation $L_{hi}\Omega^{\geq k}(-; \mathbb{C}[u^{\pm 1}]) \simeq H\mathbb{C}[u^{\pm 1}]$.
   - A map of spectra $ku \to H\mathbb{C}[u^{\pm 1}]$, which is the *Chern character*.

   This was studied with classical means by Hopkins–Singer [HS05].

(2) There are further differential refinements, such as *differential algebraic K-theory* [BG21] or *differential complex cobordism* [BSSW09], however, they seem to have been studied before the sheaf-theoretic framework was developed.

6.3. **Abstract Proof and further Applications of Fracture Square.** One question is whether we want to explicitly go through the proof of the fracture square, and the fact that $\mathbb{R}$-invariant sheaves are precisely spectra. This involves understanding advanced aspects of sheaf theory, such as *recollements*.

The essential step in the proof is the following very technical result.

Proposition 6.1. *Assume we have the following data and assumptions:*

(1) A Grothendieck site $(\mathcal{C}, J)$, such that $\mathcal{C}$ has a terminal object.
(2) A stable ∞-category $\mathcal{T}$.
(3) The inclusion functor $\Delta\colon \mathcal{T} \to \mathrm{Shv}_{\mathcal{T}}(\mathcal{C}, J)$ is fully faithful and admits a left adjoint L_{const}.

Then the following holds:

(1) The full subcategory of $\mathrm{Shv}_{\mathcal{T}}(\mathcal{C}, J)$ consisting of sheaves P, such that $P(*)$ is the point admits a left adjoint $L^{\perp}\colon \mathrm{Shv}_{\mathcal{T}}(\mathcal{C}, J) \to \mathrm{Shv}_{\mathcal{T}}(\mathcal{C}, J)^{\perp}$.

(2) *For every sheaf P in $\mathcal{S}\mathrm{hv}_{\mathcal{T}}(\mathcal{C}, J)$, there is a pullback square*

$$\begin{tikzcd} P \arrow[r] \arrow[d] & L^{\perp}P \arrow[d] \\ L_{const}P \arrow[r] & L_{const}L^{\perp}P \end{tikzcd},$$

inside $\mathcal{S}\mathrm{hv}_{\mathcal{T}}(\mathcal{C}, J)$, where the inclusions are left implicit.

This is a very general result. We use it for the following specific case:

Lemma 6.2. *Let $(\mathcal{E}\mathrm{uc}, J)$ be the Euclidean site and $\mathcal{S}\mathrm{p}$ the stable ∞-category of spectra. Then the inclusion functor $\Delta\colon \mathcal{S}\mathrm{p} \to \mathcal{S}\mathrm{hv}_{\mathcal{S}\mathrm{p}}(\mathcal{E}\mathrm{uc}, J)$ is given by the constant presheaf functor.*

In other words, the constant presheaf is already a sheaf. This directly implies that Δ is fully faithful and that it admits a left adjoint via colimit. So we can directly apply the result above to get the fracture square.

Beyond looking at the proof, we can also look at manifestations of this general result in the context of other sites, such as:

(1) The site of topological spaces.
(2) The site of coarse spaces.
(3) The site of diffeological spaces.
(4) The site of Lie groupoids.

Concretely, we can pursue the following questions:

(1) Do these Grothendieck sites satify the assumptions of the proposition?
(2) If yes, what does the existence of a fracture square imply in these cases?
(3) If not, what are are the obstructions inherent to the theory?

6.4. **Twisted (differential) cohomology.** Twisted cohomology theories further generalize cohomology theories. They have been refined to a differential version by Bunke–Nikolaus [BN19]. Given their myriad applications, both twisted cohomology and its differential version merit a careful analysis.

6.5. **Applications to Physics.** Twisted differential cohomology theories have found concrete applications in physics, which we should explore.

6.6. **Further Differential Refinements of Cohomological Notions.** Twisted K-theory has known connections to the concept of *loop group representations* [FHT11]. While twisted K-theory has successfully been lifted to a differential cohomology theory, the loop group representations have not. This is an interesting open problem that should be further explored.

6.7. **Differential Cohomology in Cohesive ∞-Topoi.** The collective work of Grady, Sati, Schreiber, et al. has focused on developing differential cohomology theories in the context of cohesive ∞-topoi and study their applications to physics [Sch13, FSS24], which merits further study.

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