

# Geometric Steering Control I – Pure Pursuit

Course 1, Module 6, Lesson 2



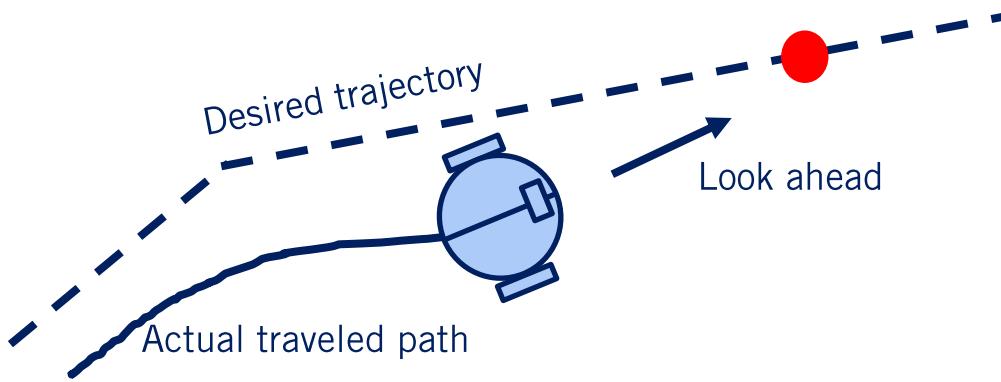
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# Learning Objectives

- In this video, you'll
  - Define the concept of a geometric path tracking controller
  - Develop a pure pursuit controller for path tracking

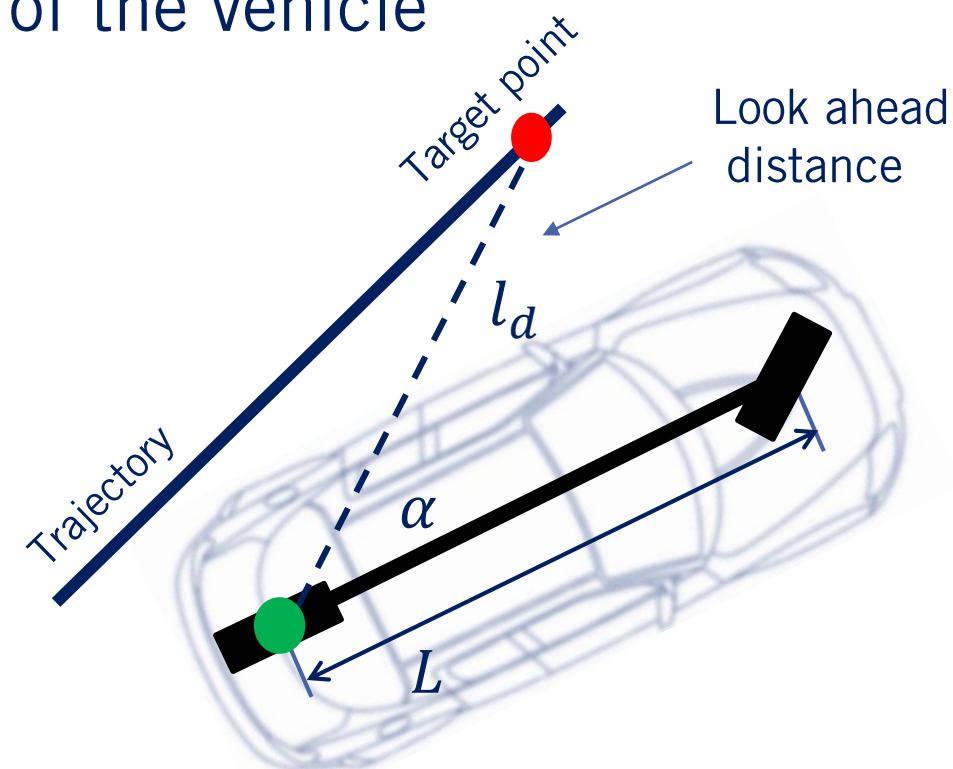
# Geometric path tracking

- One of the most popular classes of path tracking in robotics and autonomous vehicle
  - Exploits geometric relationship between the vehicle and the path resulting in compact control law solutions to the path tracking problem
  - Use of reference point on path to measure error of the vehicle, can be ahead of the vehicle



# Pure pursuit

- Pure pursuit method consists of geometrically calculating the trajectory curvature
- Connect the centre of rear axle location to a target point on the path ahead of the vehicle



# Pure pursuit - formulation

- Steering angle determined by target point location and angle between the vehicle's heading direction and lookahead direction.
- From the *law of sines*:

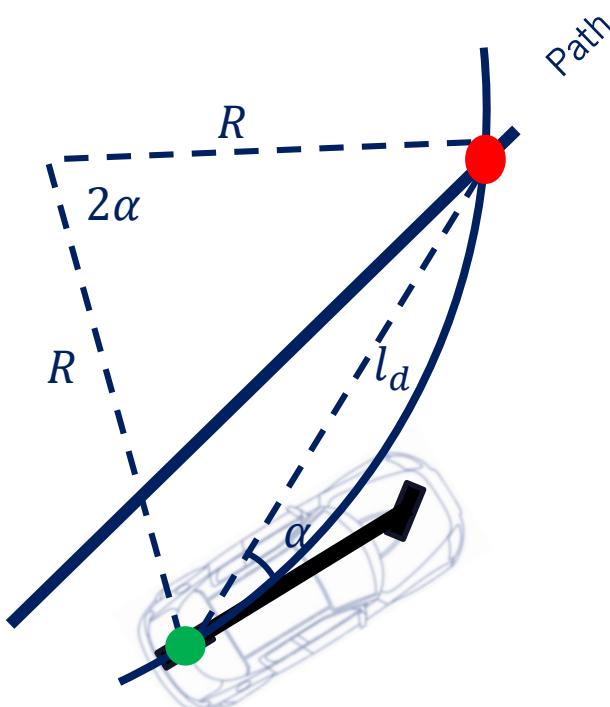
$$\frac{l_d}{\sin 2\alpha} = \frac{R}{\sin\left(\frac{\pi}{2} - \alpha\right)}$$

$$\frac{l_d}{2\sin \alpha \cos \alpha} = \frac{R}{\cos(\alpha)}$$

$$\frac{l_d}{\sin \alpha} = 2R$$

$$\kappa = \frac{1}{R} = \frac{2 \sin \alpha}{l_d}$$

Path curvature



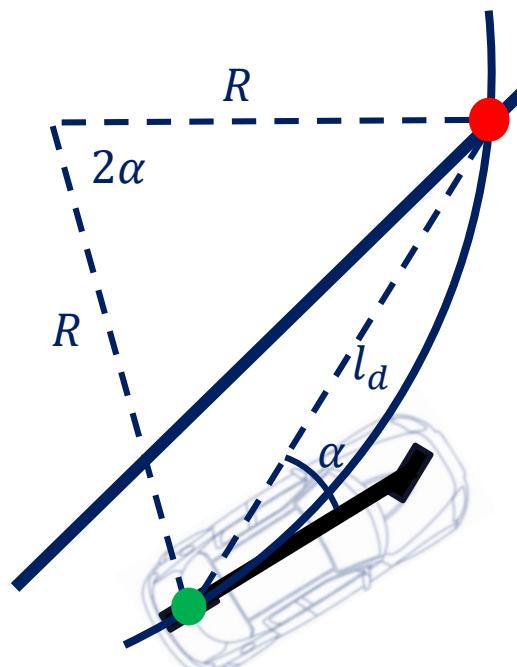
# Pure pursuit - formulation

- Using the bicycle model the steering angle is calculated as:

$$\kappa = \frac{2 \sin \alpha}{l_d} \quad \delta = \tan^{-1} \kappa L$$



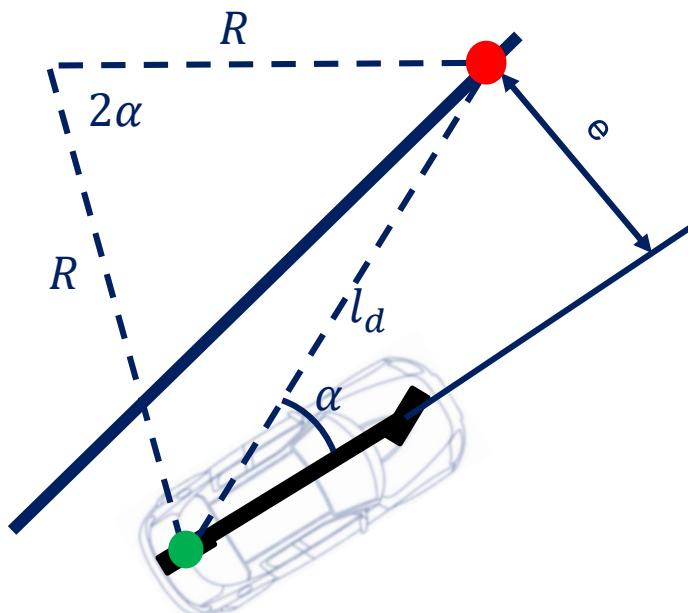
$$\delta = \tan^{-1} \left( \frac{2L \sin \alpha}{l_d} \right)$$



# Pure pursuit - formulation

- Crosstrack error ( $e$ ) is defined here as the lateral distance between the heading vector and the target point so:

$$\left. \begin{aligned} \sin \alpha &= \frac{e}{l_d} \\ \kappa &= \frac{2 \sin \alpha}{l_d} \end{aligned} \right\} \quad \kappa = \frac{2}{l_d^2} e$$



# Pure pursuit - formulation

- Pure pursuit is a proportional controller of the steering angle operating on a crosstrack error some look ahead distance in front of the vehicle
- The proportional gain  $^2/l_d^2$  can be tuned at different speeds (the  $l_d$  being assigned as a function of vehicle speed)

# Pure pursuit - formulation

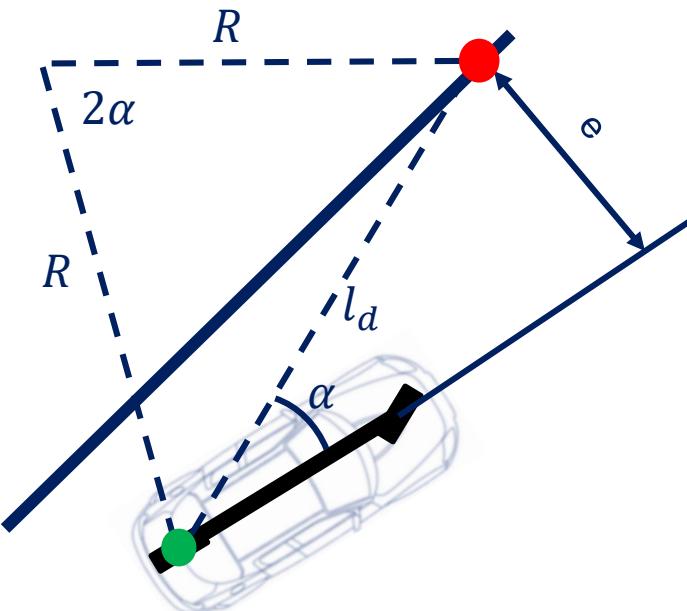
- Lookahead  $l_d$  is assigned as a linear function of vehicle speed:  $l_d = K_{dd} v_f$  

$$\delta = \tan^{-1} \left( \frac{2L \sin \alpha}{l_d} \right) \quad \kappa = \frac{2}{l_d^2} e$$



$$\delta = \tan^{-1} \left( \frac{2L \sin \alpha}{K_{dd} v_f} \right)$$

Forward velocity



# Summary

What we have learned from this lesson:

- The concept of geometrical path tracking and the pure pursuit method

What is next?

- A second geometrical path tracking method, the Stanley control approach