# Tire Slip & Modeling

Course 1, Module 4, Lesson 7

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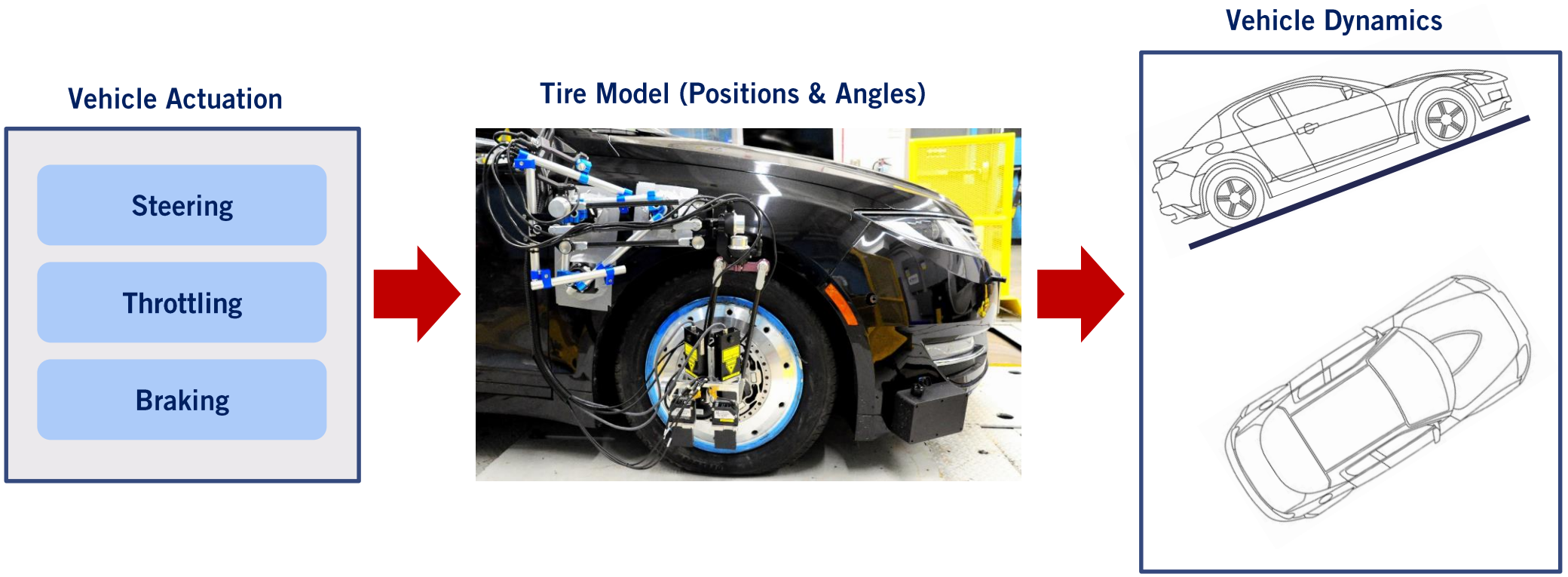
# Learning Objectives

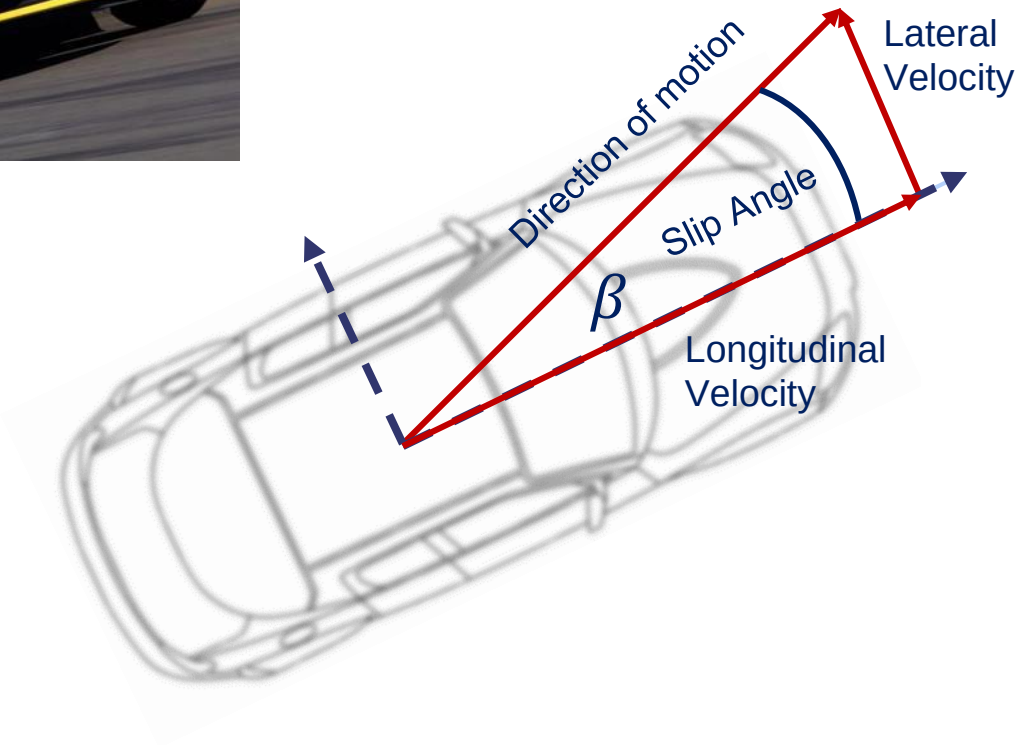
- Study tire slip angle and slip ratio in more detail
- Define tire models that capture forces produced by tires

# Importance of Tire Modeling

- The tire is the interface between the vehicle and road



# Vehicle Slip Angle

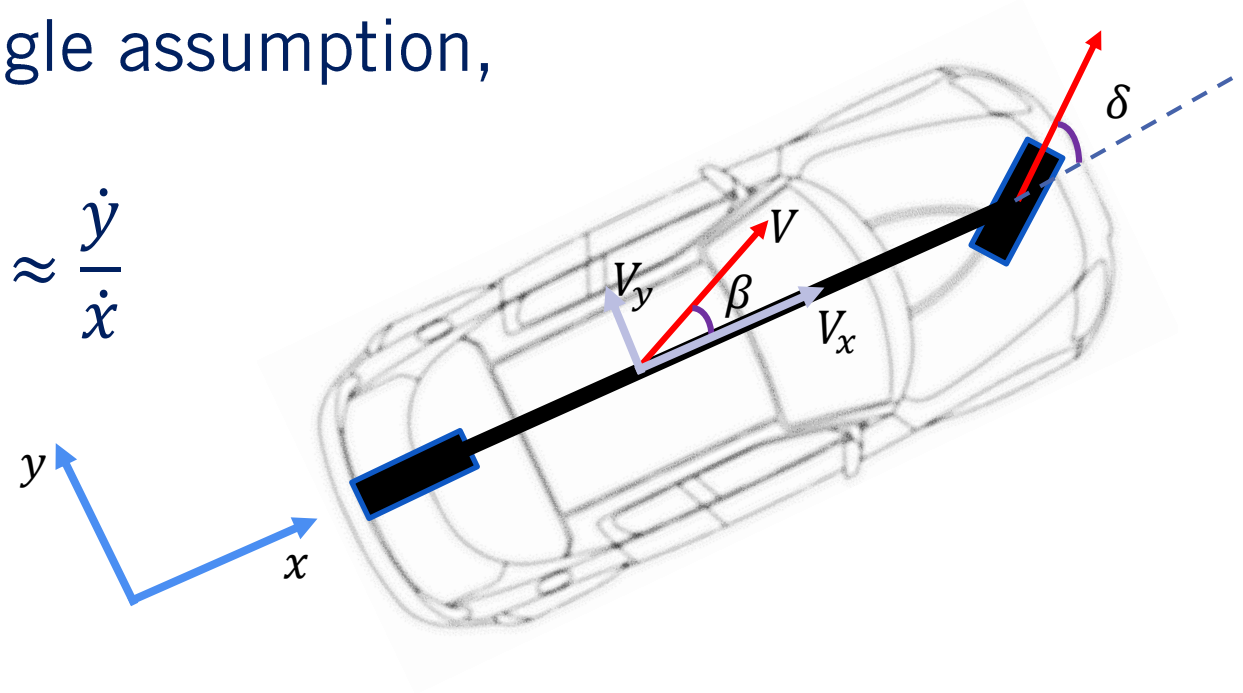
# Vehicle (Bicycle) Slip Angle

- Slip angle

$$\beta = \tan^{-1}\frac{V_y}{V_x} = \tan^{-1}\frac{\dot{y}}{\dot{x}}$$

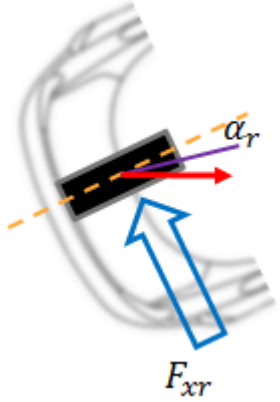
- Using small angle assumption,

$$\beta \approx \frac{\dot{y}}{\dot{x}}$$



# Tire Slip Angles

- Tire slip angle is the angle between the direction in which a wheel is pointing and the direction in which it is actually travelling



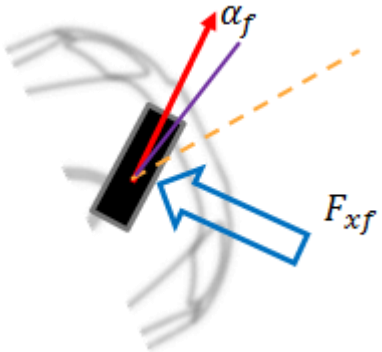

Rear tire slip angle

$$\alpha_r = -\beta + \frac{l_r \dot{\psi}}{V}$$

yaw rate

vehicle slip angle

forward velocity



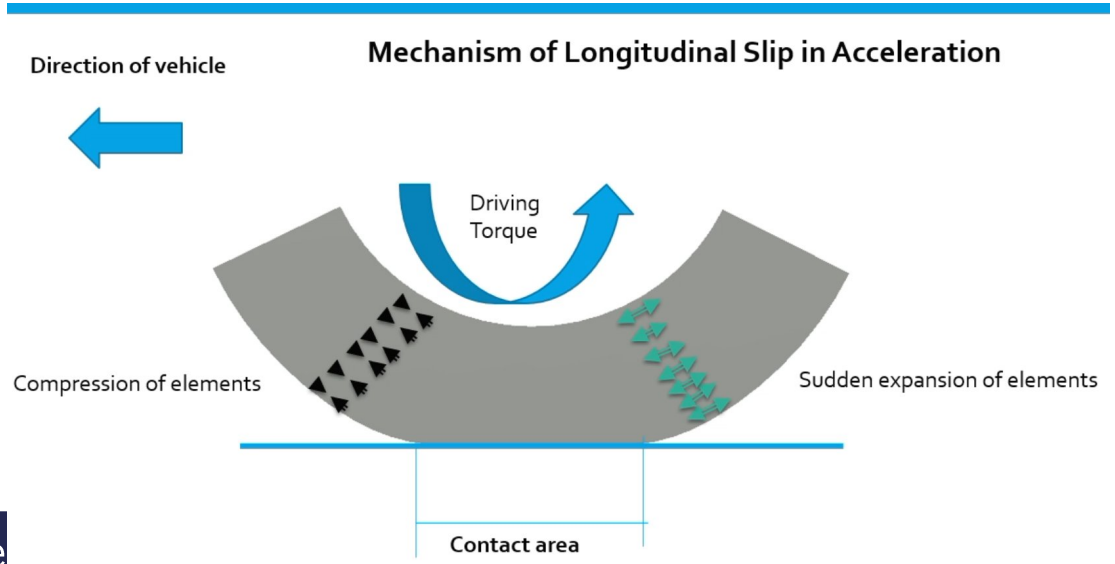

Front tire slip angle

$$\alpha_f = \delta - \beta - \frac{l_f \dot{\psi}}{V}$$

steering angle

# Slip Ratios

- Longitudinal slip (also called slip ratio)




wheel angular speed

tire effective radius

$$s = \frac{w\, r_e - V}{V}$$

vehicle forward velocity

$wr_e < V$ Wheels are deceleratio braking

Stress builds up and gets released , during the release of elements there is slip.

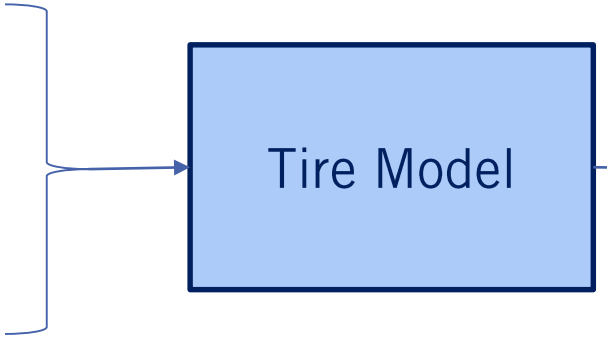
$wr_e > V$ Wheels are spinning, this happens during acceleration, especially in low friction driving (icy road)

$wr_e = 0$ Wheels are locked, this happens during heavy or panic braking where the vehicle loses its desired traction

# Tire Modeling

**Inputs to the tire model**

Tire Slip Angle
Slip Ratio
Normal Force
Friction Coefficient
Camber Angle
Tire properties




**Outputs of the tire model**

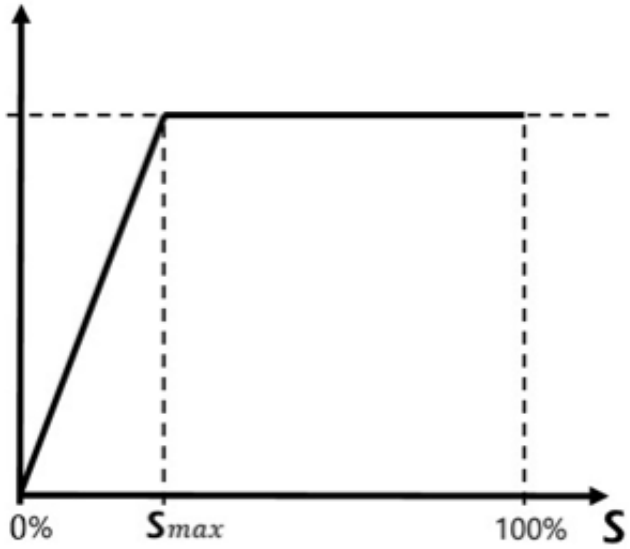
Lateral Force
Longitudinal Force
Self-Aligning Moment
Rolling Resistance Moment
Overturning Moment

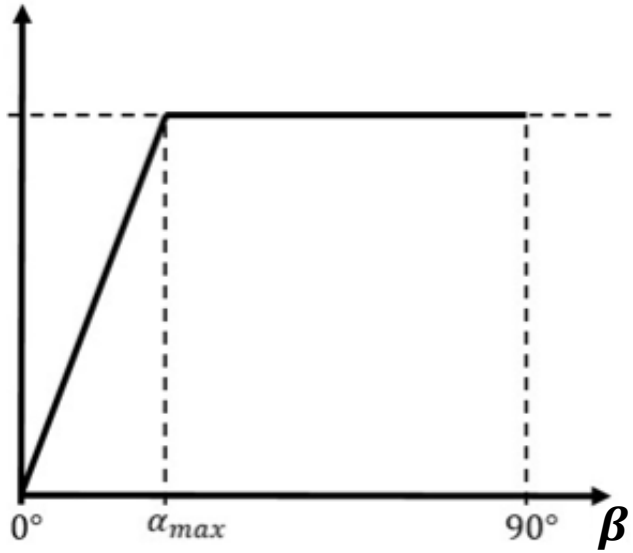
# Tire Modeling

- Analytical - Brush, Fiala, Linear
  - Tire physical parameters are explicitly employed
  - Low precision, but simple
- Numerical
  - Look up tables instead of mathematical equations
  - No explicit mathematical form
  - Geometry and material property of tire are considered
- Parameterized – Linear, Pacejka, Dugoff
  - Need experiments for each specific tire
  - Formed by fitting model with experimental data
  - Match experimental data very well
  - Used widely for vehicle dynamics simulation studies and control design

# Linear Tire Model

- Assumption: the relationship between slip angle and force is linear
  - Piecewise linear curves: $F(x) = \begin{cases} Cx & if\ |x| < x_{max} \\ F_{max} & if\ |x| \geq x_{max} \end{cases}$



# Pacejka Tire Model

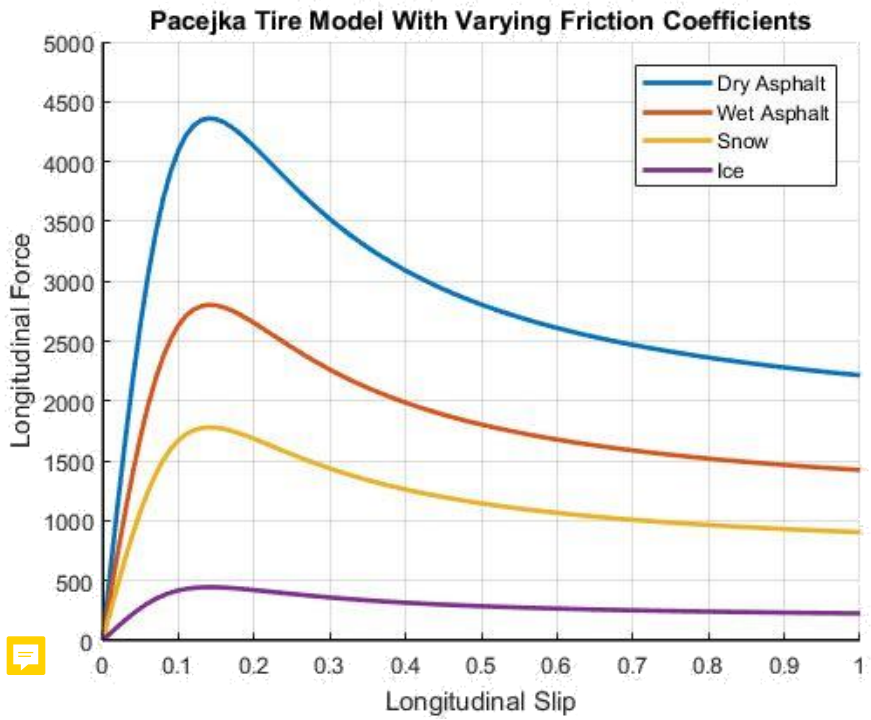
- Also called Magic Formula tire model
  - Widely used in model-based control development.

tire vertical force

$$F(x, F_z) = D \sin(C \tan^{-1}(Bx - E(Bx - \tan^{-1}(Bx)))) \mu F_z$$

$x$ could be either slip ratio or slip angle (in tire modeling)

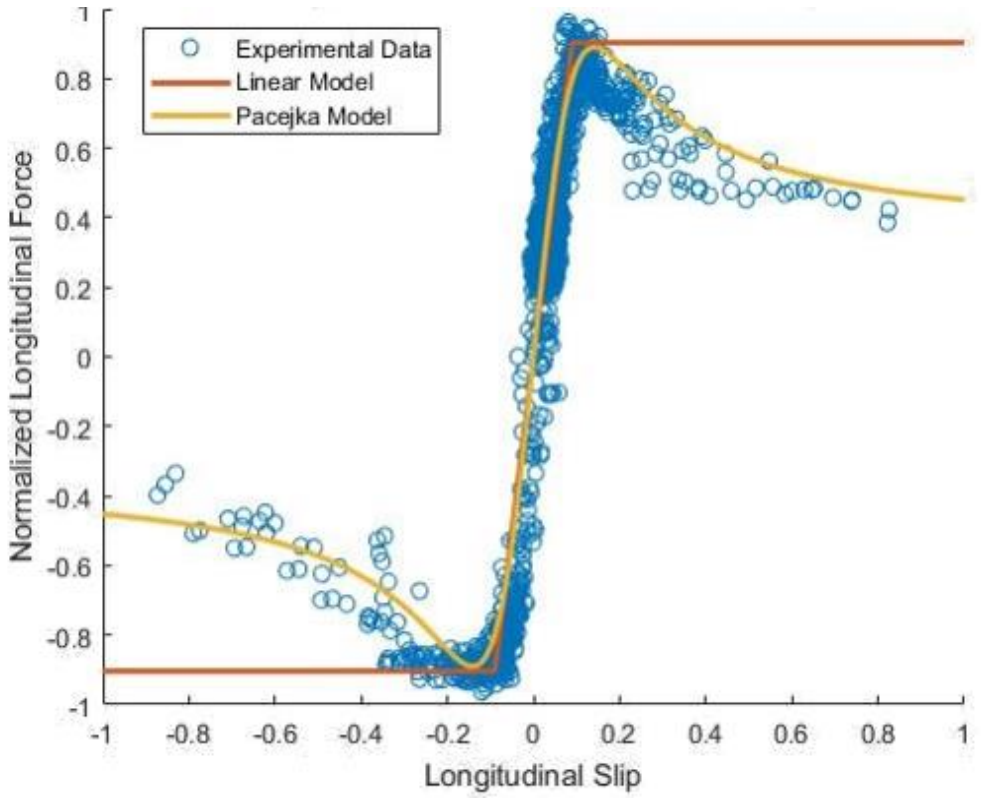
road friction coefficient



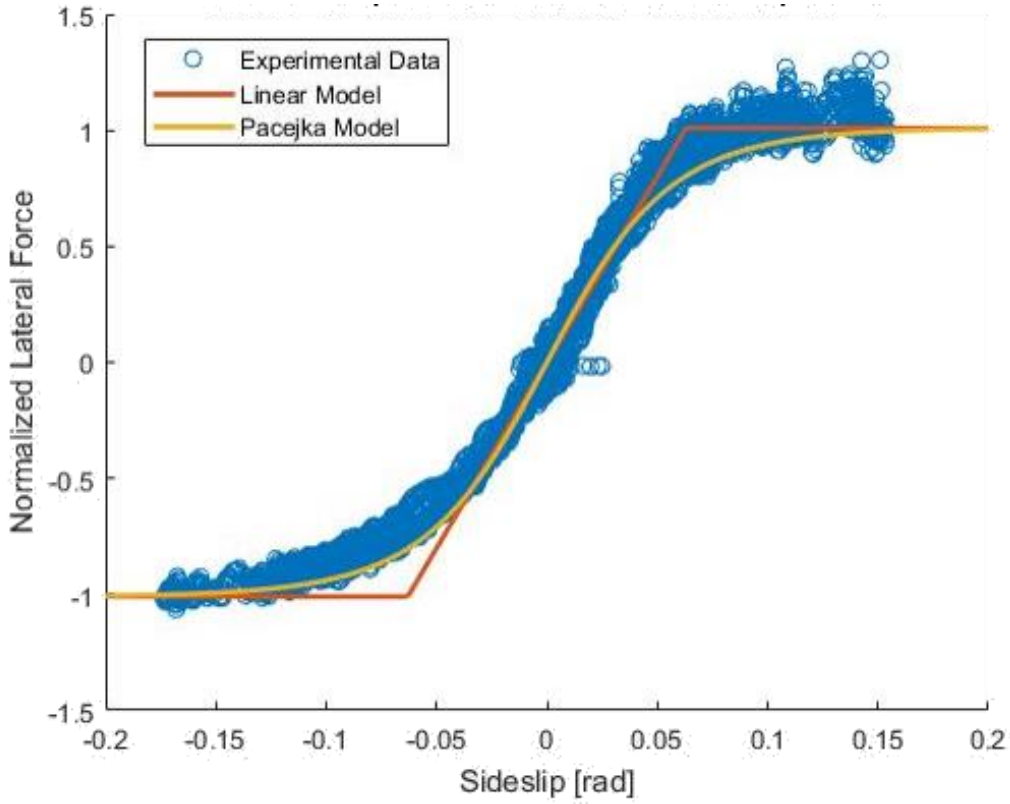

B – Stiffness Factor
C – Shape Factor
D – Peak Factor
E – Curvature Factor

# Forces vs Slips

## Normalized Longitudinal Force vs. Slip Ratio




## Normalized Lateral Force vs. Slip Angle



# Lesson Summary

What we have learned from this lesson:

- The role of tire in vehicle dynamics
- The terminology used in tire modelling such as slip angle and slip ratio
- The linear and Pacejka tire models

# Module Summary

What we have learned from this module:

- Kinematic and dynamic modeling of vehicles
- Kinematic bicycle model
- Lateral and longitudinal dynamic modeling
- Actuator and tire subsystem modeling

What is next?

- The basics of  controller design and its application to vehicle longitudinal control