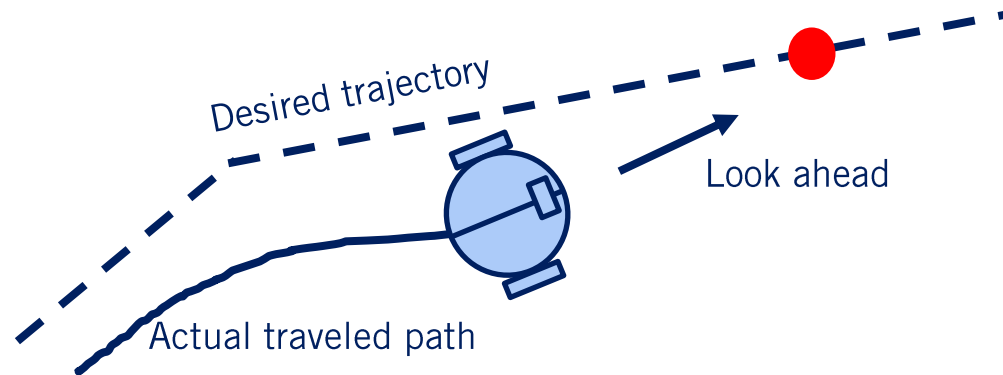# Geometric Steering Control I – Pure Pursuit

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Learning Objectives

- In this video, you'll
  - Define the concept of a geometric path tracking controller
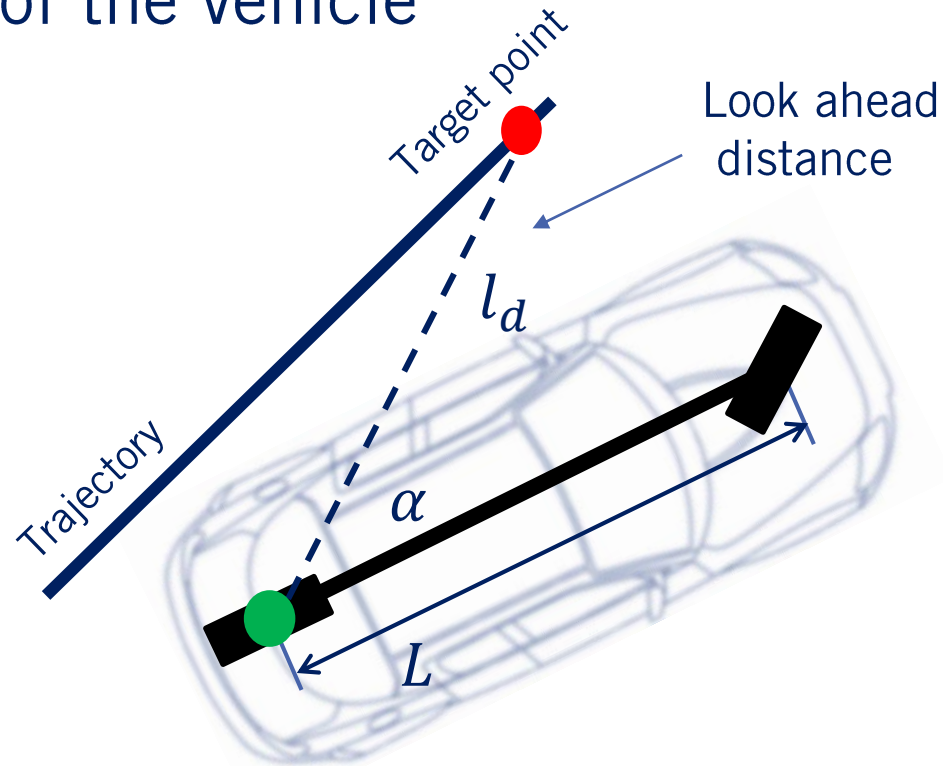 - Develop a pure pursuit controller for path tracking

Geometric path tracking

- One of the most popular classes of path tracking in robotics and autonomous vehicle
  - Exploits geometric relationship between the vehicle and the path resulting in compact control law solutions to the path tracking problem
  - Use of reference point on path to measure error of the vehicle, can be ahead of the vehicle

Desired trajectory

Look ahead

Actual traveled path

Pure pursuit

- Pure pursuit method consists of geometrically calculating the trajectory curvature
- Connect the centre of rear axle location to a target point on the path ahead of the vehicle
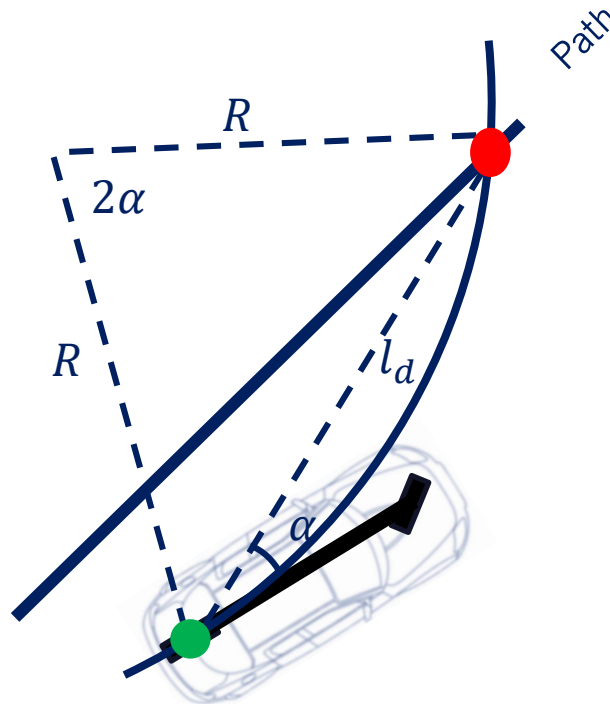
Pure pursuit - formulation

- Steering angle determined by target point location and angle between the vehicle's heading direction and lookahead direction.

- From the *law of sines*:

$$\frac{l_d}{\sin 2\alpha} = \frac{R}{\sin\left(\frac{\pi}{2} - \alpha\right)}$$

$$\frac{l_d}{2\sin\alpha\cos\alpha} = \frac{R}{\cos(\alpha)}$$

$$\frac{l_d}{\sin\alpha} = 2R$$

$$\kappa = \frac{1}{R} = \frac{2\sin\alpha}{l_d} \quad \text{Path curvature}$$
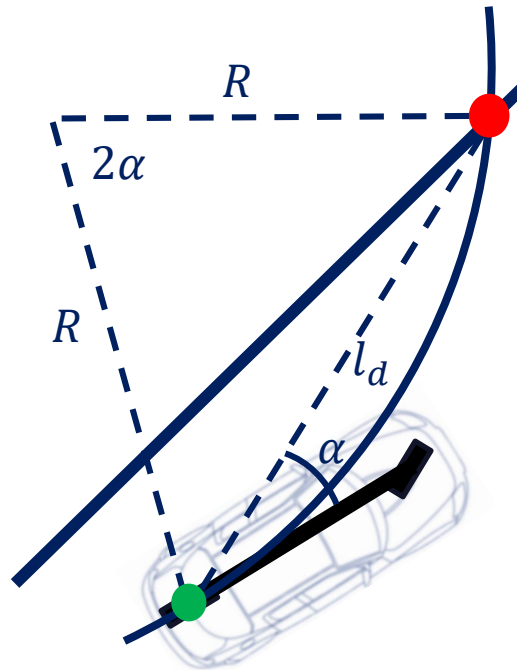
Pure pursuit - formulation

- Using the bicycle model the steering angle is calculated as:

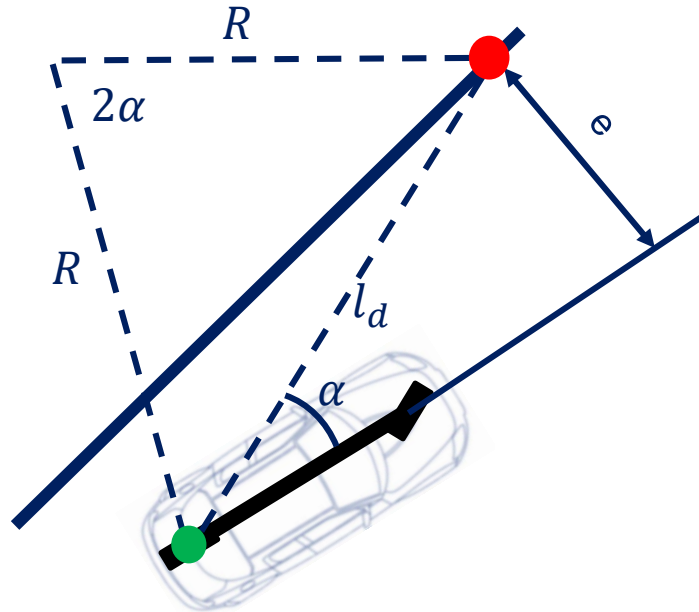$$\kappa = \frac{2 \sin \alpha}{l_d} \qquad \delta = \tan^{-1} \kappa L$$

$$\delta = \tan^{-1}\left(\frac{2L \sin \alpha}{l_d}\right)$$

Pure pursuit - formulation

- Crosstrack error (e) is defined here as the lateral distance between the heading vector and the target point so:

$$\sin \alpha = \frac{e}{l_d}$$

$$\kappa = \frac{2 \sin \alpha}{l_d}$$

$$\kappa = \frac{2}{l_d^2} e$$

Pure pursuit - formulation

- Pure pursuit is a proportional controller of the steering angle operating on a crosstrack error some look ahead distance in front of the vehicle

- The proportional gain $^2/_{l_d^2}$ can be tuned at different speeds (the l_d being assigned as a function of vehicle speed)
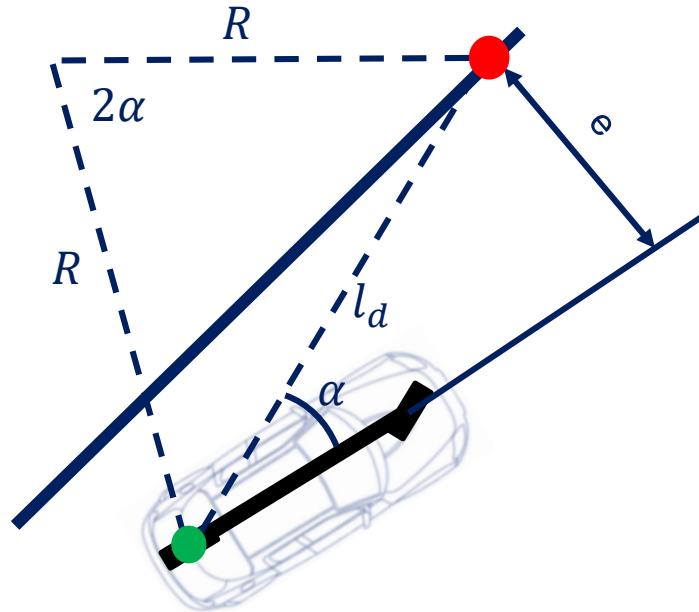
Pure pursuit - formulation

- Lookahead l_d is assigned as a linear function of vehicle speed: $l_d = K_{dd}v_f$

$$\delta = \tan^{-1}\left(\frac{2L\sin\alpha}{l_d}\right) \qquad \kappa = \frac{2}{l_d^2}e$$

$$\delta = \tan^{-1}\left(\frac{2L\sin\alpha}{K_{dd}v_f}\right)$$

Forward velocity

Summary

What we have learned from this lesson:
- The concept of geometrical path tracking and the pure pursuit method

What is next?
- A second geometrical path tracking method, the Stanley control approach