

Tire Slip & Modeling

Course 1, Module 4, Lesson 7



UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

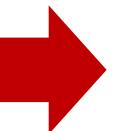
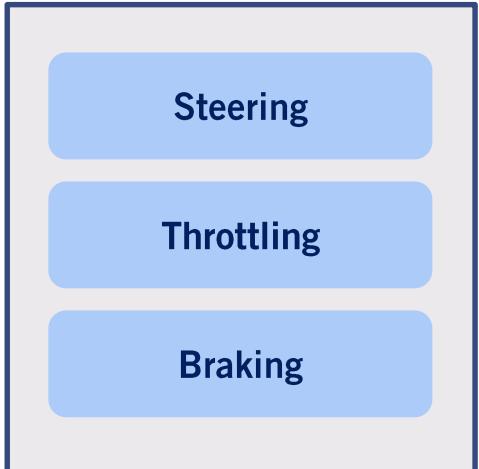
Learning Objectives

- Study tire slip angle and slip ratio in more detail
- Define tire models that capture forces produced by tires

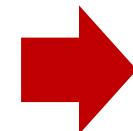
Importance of Tire Modeling

- The tire is the interface between the vehicle and road

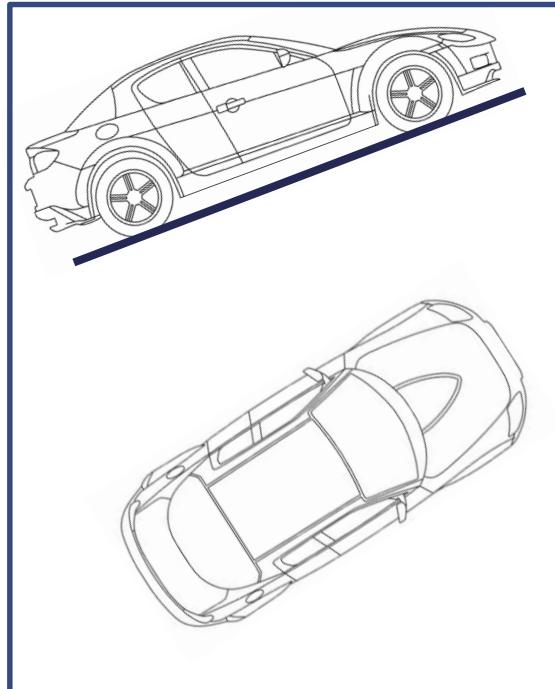
Vehicle Actuation



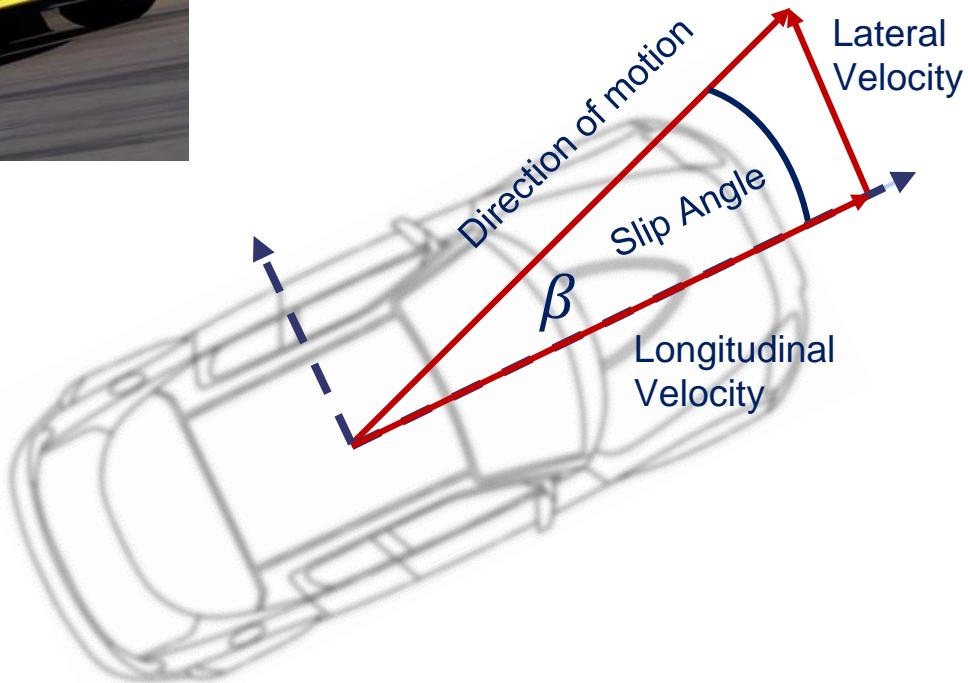
Tire Model (Positions & Angles)



Vehicle Dynamics



Vehicle Slip Angle



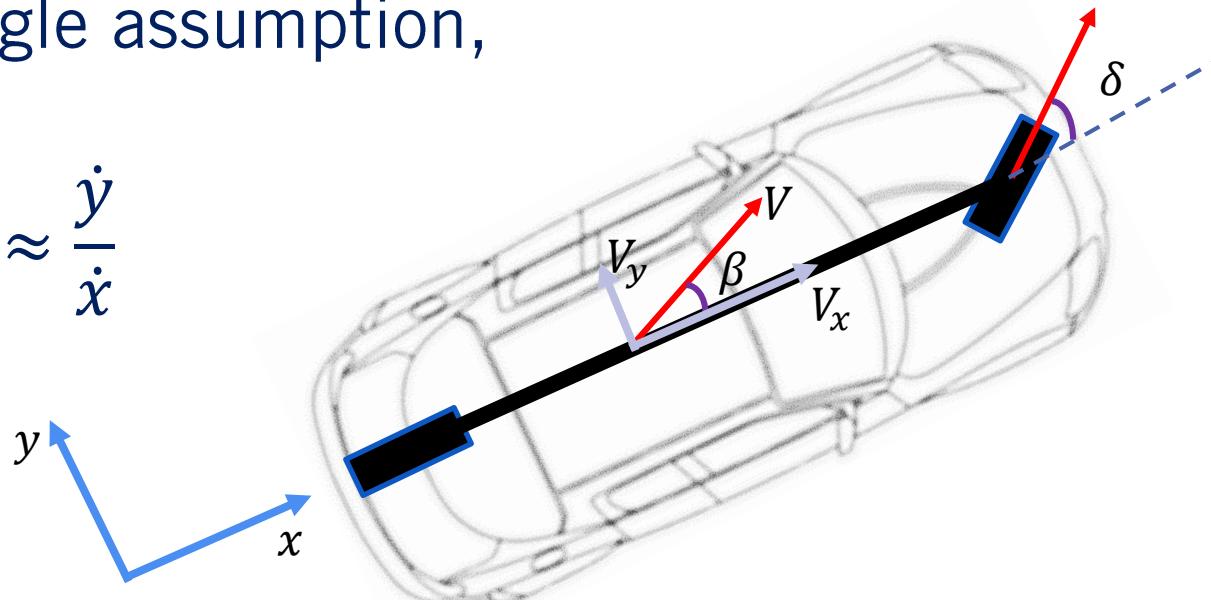
Vehicle (Bicycle) Slip Angle

- Slip angle

$$\beta = \tan^{-1} \frac{V_y}{V_x} = \tan^{-1} \frac{\dot{y}}{\dot{x}}$$

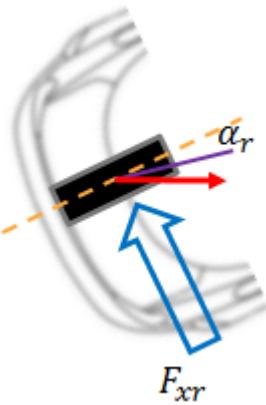
- Using small angle assumption,

$$\beta \approx \frac{\dot{y}}{\dot{x}}$$

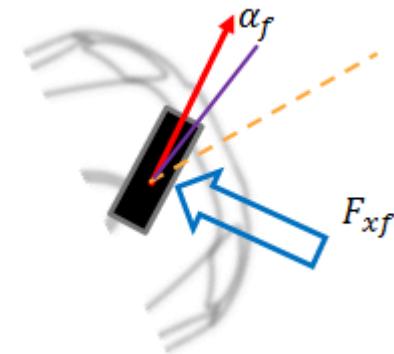


Tire Slip Angles

- Tire slip angle is the angle between the direction in which a wheel is pointing and the direction in which it is actually travelling



Rear tire slip angle



Front tire slip angle

$$\alpha_r = -\beta + \frac{l_r \dot{\psi}}{V}$$

vehicle slip angle

forward velocity

yaw rate

$$\alpha_f = \delta - \beta - \frac{l_f \dot{\psi}}{V}$$

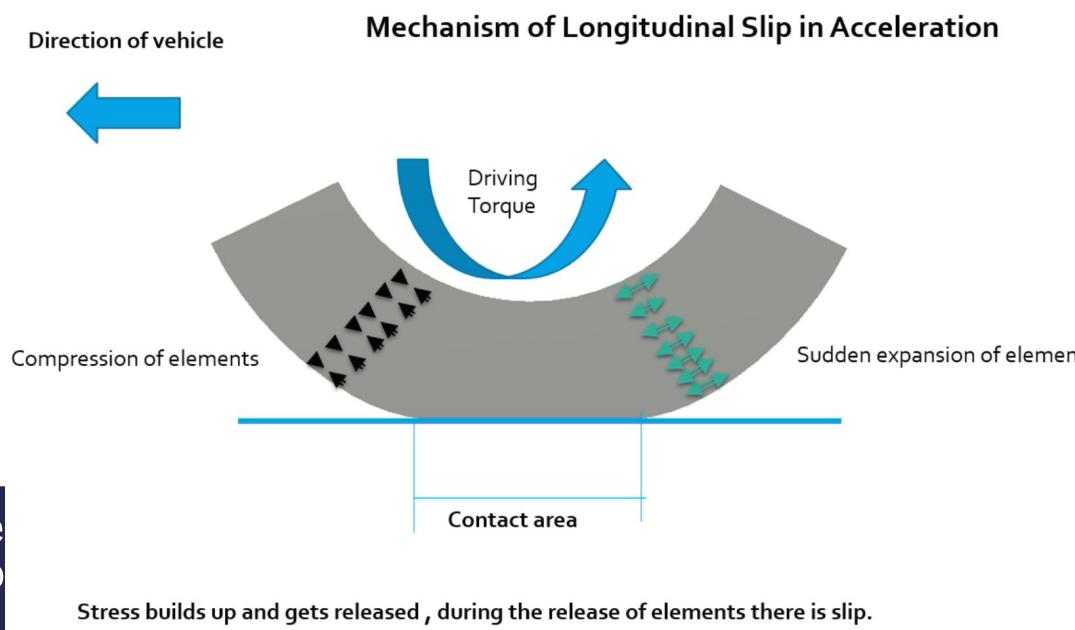
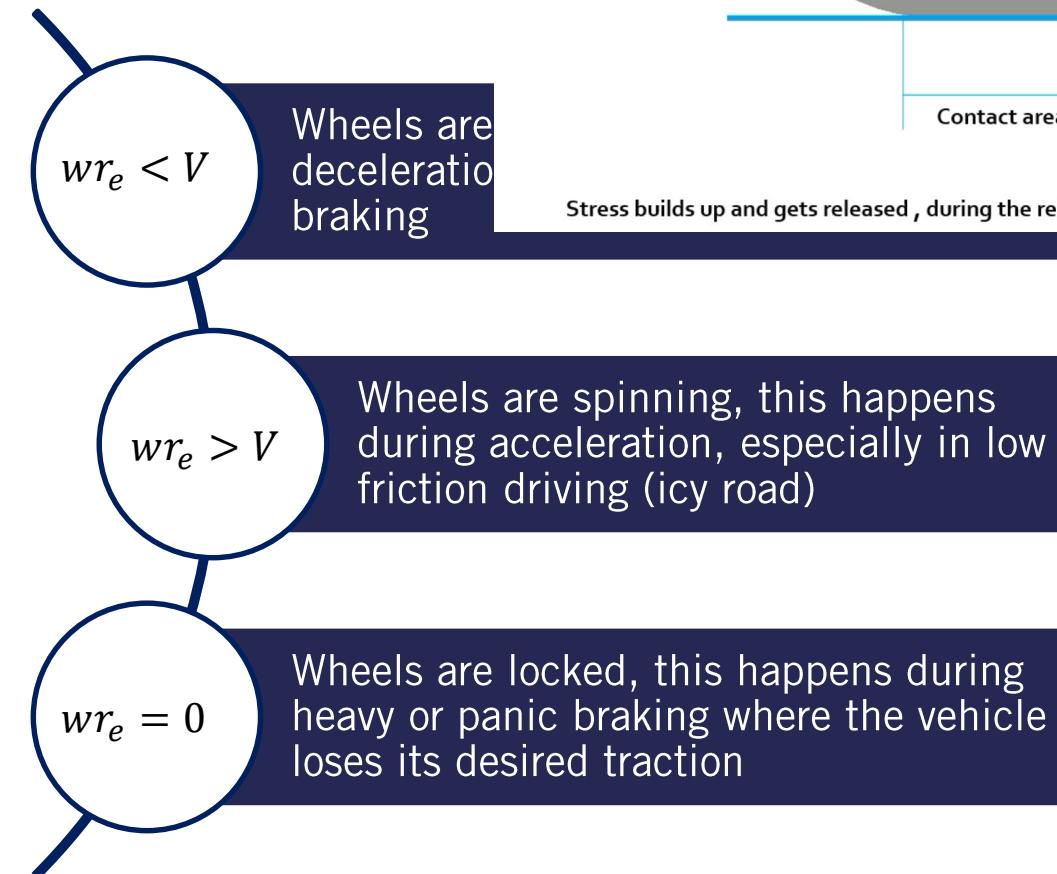
steering angle

Slip Ratios

- Longitudinal slip (also called slip ratio)

$$S = \frac{w r_e - V}{V}$$

wheel angular speed tire effective radius
vehicle forward velocity



Stress builds up and gets released , during the release of elements there is slip.

Tire Modeling

Inputs to the tire model

- Tire Slip Angle
- Slip Ratio
- Normal Force
- Friction Coefficient
- Camber Angle
- Tire properties



Outputs of the tire model

- Lateral Force
- Longitudinal Force
- Self-Aligning Moment
- Rolling Resistance Moment
- Overturning Moment

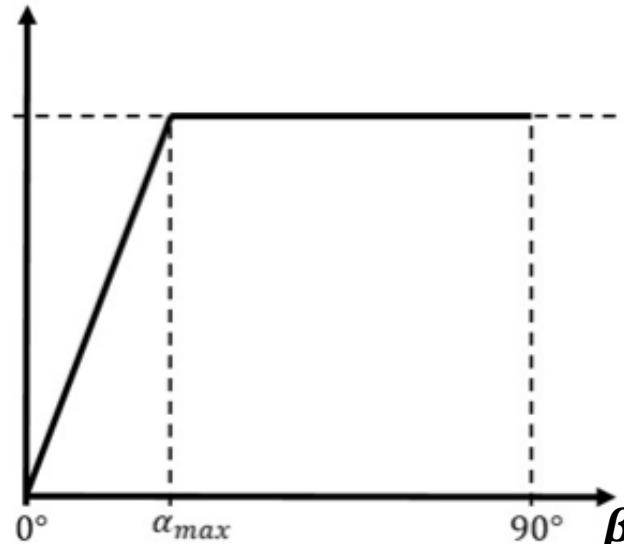
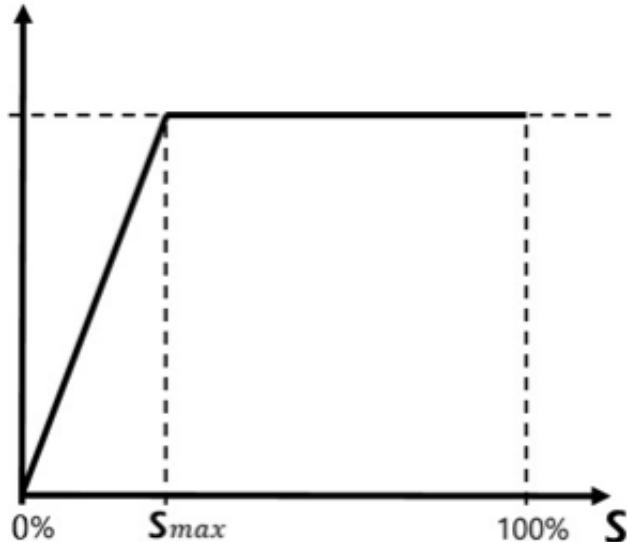
Tire Modeling

- Analytical - Brush, Fiala, Linear
 - Tire physical parameters are explicitly employed
 - Low precision, but simple
- Numerical
 - Look up tables instead of mathematical equations
 - No explicit mathematical form
 - Geometry and material property of tire are considered
- Parameterized – Linear, Pacejka, Dugoff
 - Need experiments for each specific tire
 - Formed by fitting model with experimental data
 - Match experimental data very well
 - Used widely for vehicle dynamics simulation studies and control design

Linear Tire Model

- Assumption: the relationship between slip angle and force is linear

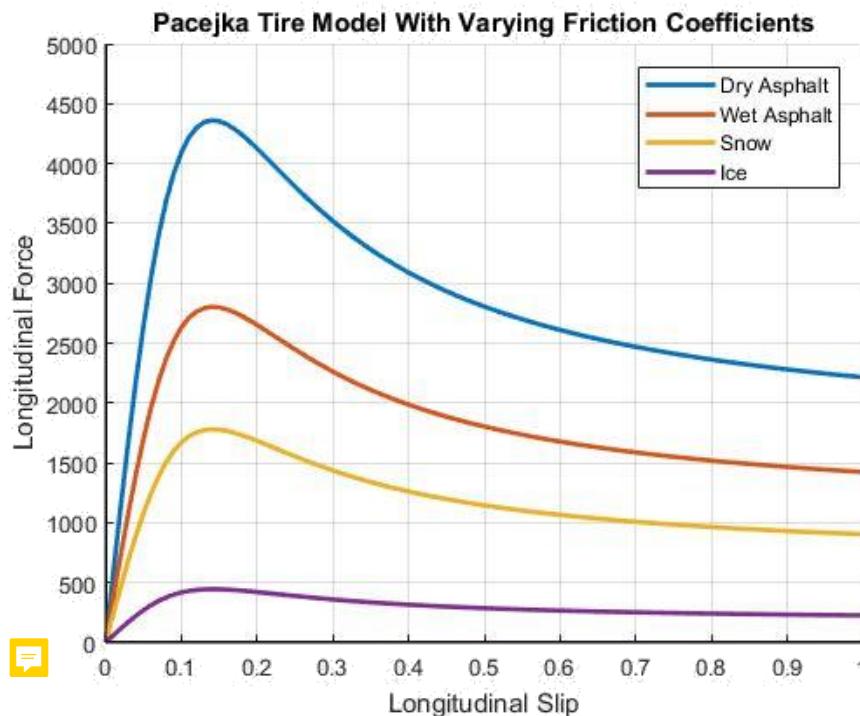
- Piecewise linear curves: $F(x) = \begin{cases} Cx & \text{if } |x| < x_{max} \\ F_{max} & \text{if } |x| \geq x_{max} \end{cases}$



Pacejka Tire Model

- Also called Magic Formula tire model
 - Widely used in model-based control development.

$$F(x, F_z) = D \sin(C \tan^{-1}(Bx - E(Bx - \tan^{-1}(Bx)))) \mu F_z$$



x could be either slip ratio or slip angle (in tire modeling)

tire vertical force

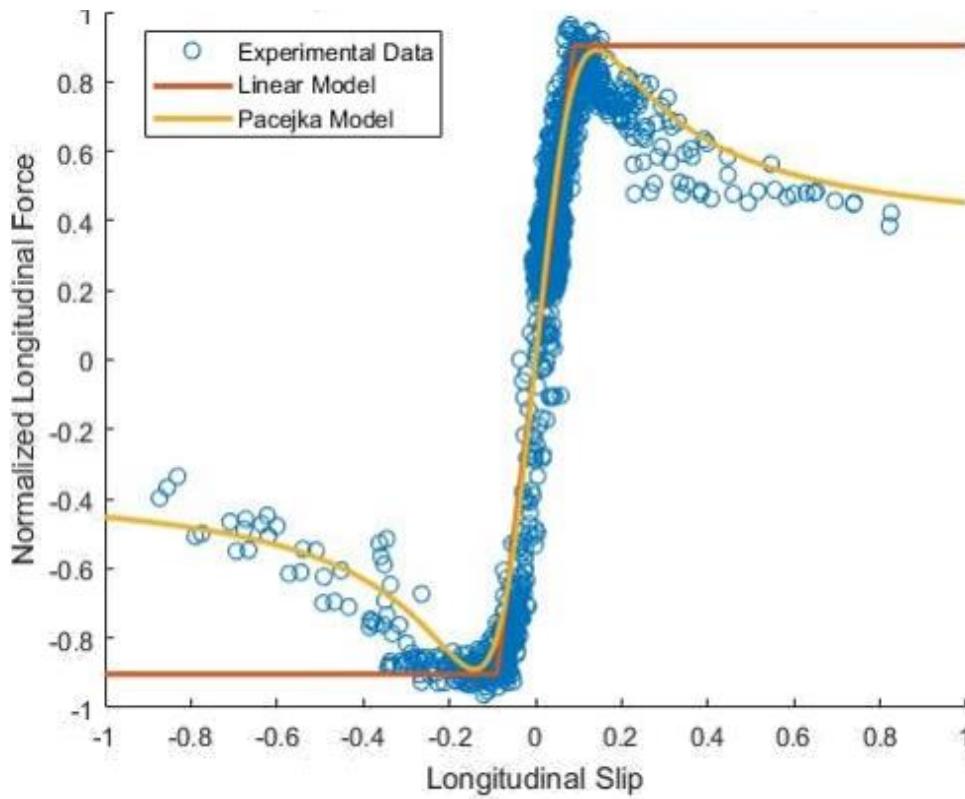
road friction coefficient

B – Stiffness Factor
C – Shape Factor
D – Peak Factor
E – Curvature Factor

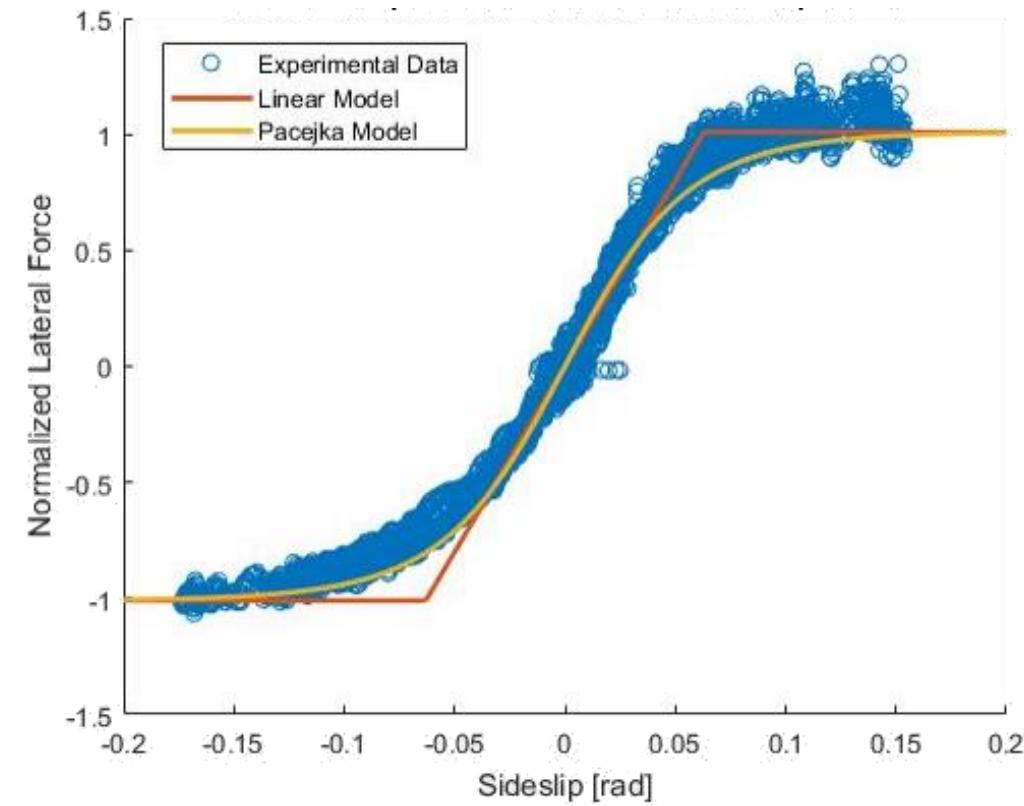


Forces vs Slips

Normalized Longitudinal Force vs. Slip Ratio



Normalized Lateral Force vs. Slip Angle



Lesson Summary

What we have learned from this lesson:

- The role of tire in vehicle dynamics
- The terminology used in tire modelling such as slip angle and slip ratio
- The linear and Pacejka tire models

Module Summary

What we have learned from this module:

- Kinematic and dynamic modeling of vehicles
- Kinematic bicycle model
- Lateral and longitudinal dynamic modeling
- Actuator and tire subsystem modeling

What is next?

- The basics of controller design and its application to vehicle longitudinal control