

Advanced Vehicle Control Methods

Course 1, Module 6, Lesson 4



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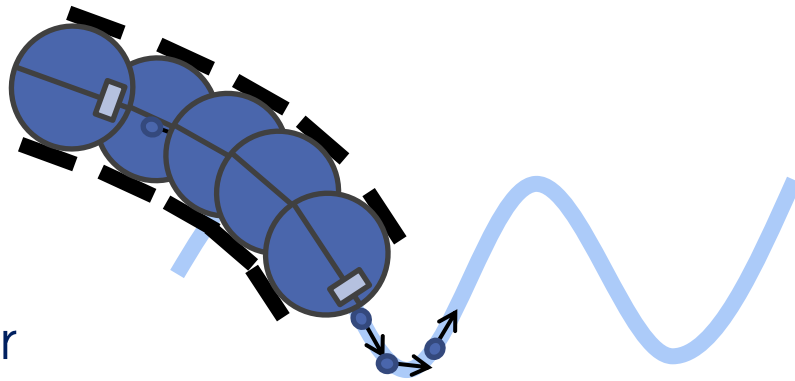
Learning Objectives

In this video, we will...

- Describe the MPC architecture and the concept of receding horizon control
- Formulate an MPC optimization problem for both linear and nonlinear models
- Apply MPC to joint longitudinal and lateral vehicle control

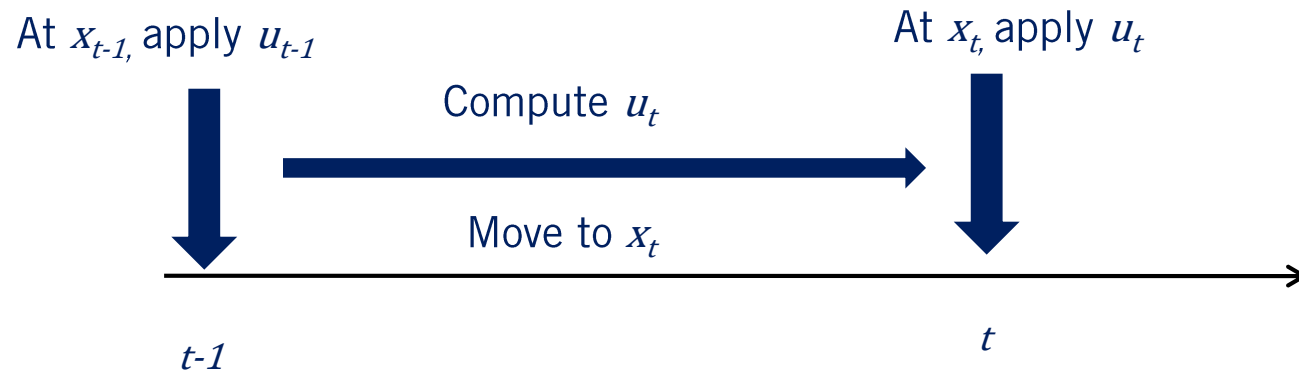
Model predictive control

- Model predictive control (MPC)
 - Numerically solving an optimization problem at each time step
 - Receding horizon approach
- Advantages of MPC
 - Straightforward formulation
 - Explicitly handles constraints
 - Applicable to linear or nonlinear models
- Disadvantages of MPC
 - Computationally expensive

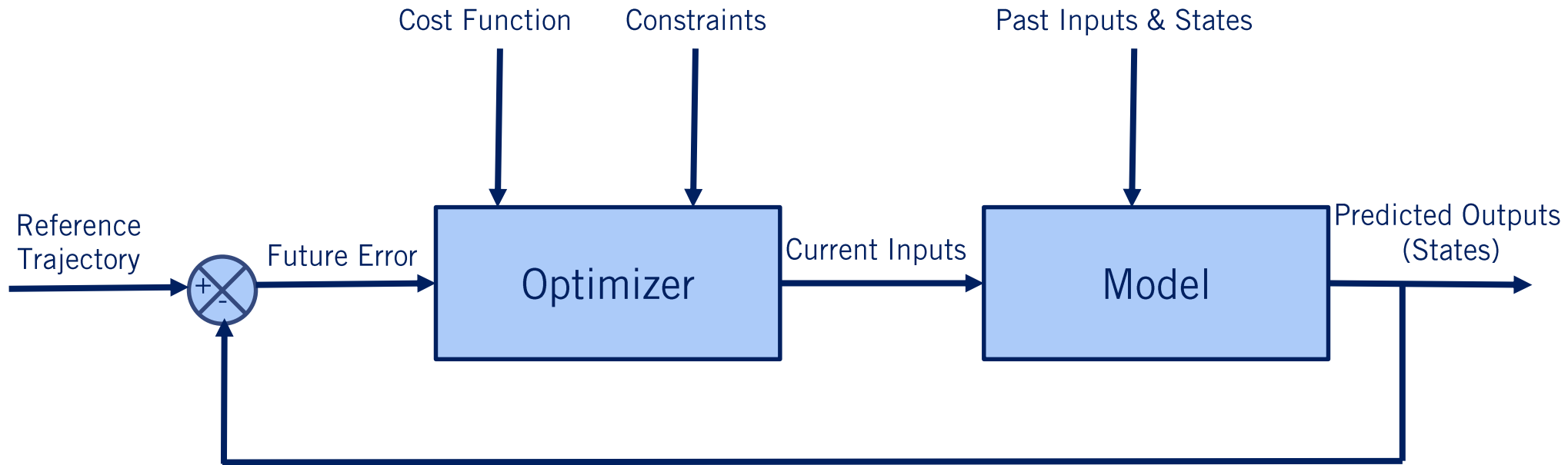


Receding horizon Control

- Receding Horizon Control Algorithm
 - Pick receding horizon length (T)
 - For each time step, t
 - Set initial state to predicted state, x_t
 - Perform optimization over finite horizon t to T while traveling from x_{t-1} to x_t
 - Apply first control command, u_t from optimization at time t



MPC structure



Linear MPC formulation

- Linear time-invariant discrete time model:

$$x_{t+1} = Ax_t + Bu_t$$

- MPC seeks to find control policy U

$$U = \{u_{t|t}, u_{t+1|t}, u_{t+2|t}, \dots\}$$

- Objective function - regulation:

$$J(x(t), U) = \sum_{j=t}^{t+T-1} x_{j|t}^T Q x_{j|t} + u_{j|t}^T R u_{j|t}$$

- Objective function - tracking:

$$\delta x_{j|t} = x_{j|t,des} - x_{j|t} \quad J(x(t), U) = \sum_{j=t}^{t+T-1} \delta x_{j|t}^T Q \delta x_{j|t} + u_{j|t}^T R u_{j|t}$$

Linear MPC SOLUTION

- Unconstrained, finite horizon, discrete time problem formulation:

$$\min_{U \triangleq \{u_{t|t}, u_{t+1|t}, \dots\}} J(x(t), U) = x_{t+T|t}^T Q_f x_{t+T|t} + \sum_{j=t}^{t+T-1} x_{j|t}^T Q x_{j|t} + u_{j|t}^T R u_{j|t}$$

$$s.t. \quad x_{j+1|t} = A x_{j|t} + B u_{j|t}, \quad t \leq j \leq t + T - 1$$

- Linear quadratic regulator, provides a closed form solution
 - Full state feedback: $u_t = -Kx_t$
 - Control gain K is a matrix
 - Refer to supplemental materials

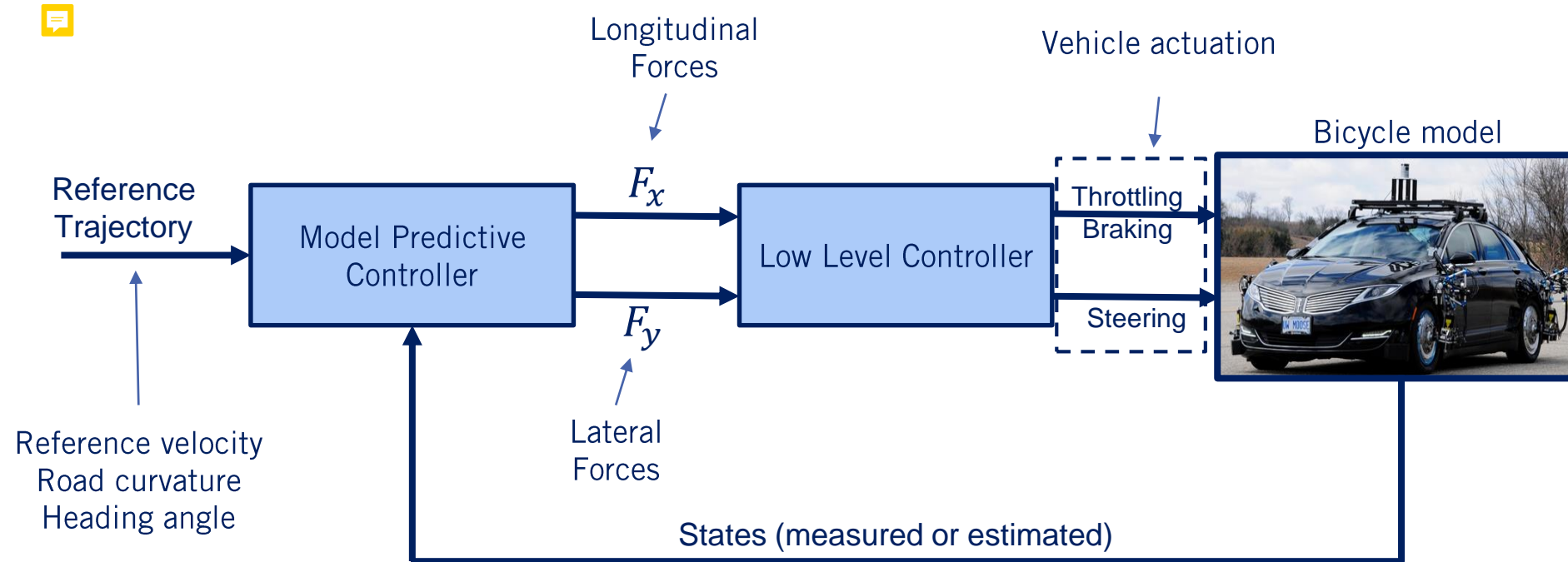
(Non)Linear MPC formulation

- Constrained (non)linear finite horizon discrete time case

$$\begin{aligned} \min_{U \triangleq \{u_{t|t}, u_{t+1|t}, \dots\}} J(x(t), U) &= \sum_{j=t}^{t+T} C(x_{j|t}, u_{j|t}) \\ \text{s.t.} \quad x_{j+1|t} &= f(x_{j|t}, u_{j|t}), & t \leq j \leq t+T-1 \\ x_{\min} &\leq x_{j+1|t} \leq x_{\max}, & t \leq j \leq t+T-1 \\ u_{\min} &\leq u_{j|t} \leq u_{\max}, & t \leq j \leq t+T-1 \\ g(x_{j|t}, u_{j|t}) &\leq 0, & t \leq j \leq t+T-1 \\ h(x_{j|t}, u_{j|t}) &= 0, & t \leq j \leq t+T-1 \end{aligned}$$

- No closed form solution, must be solved numerically

Vehicle Longitudinal & Lateral Control



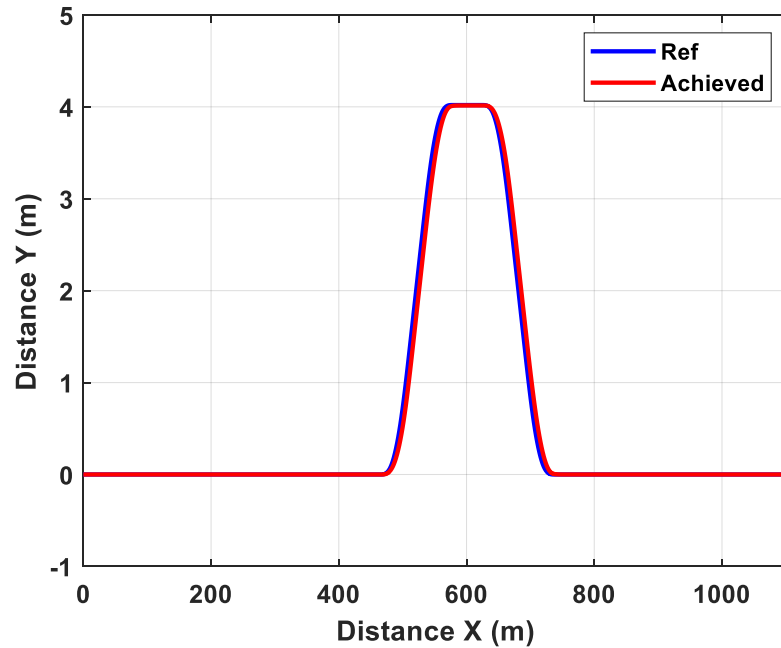
Model Predictive Controller

- Cost Function - Minimize
 - Deviation from desired trajectory
 - Minimization of control command magnitude
- Constraints - Subject to
 - Longitudinal and lateral dynamic models
 - Tire force limits
- Can incorporate low level controller, adding constraints for:
 - Engine map
 - Full dynamic vehicle model
 - Actuator models
 - Tire force models

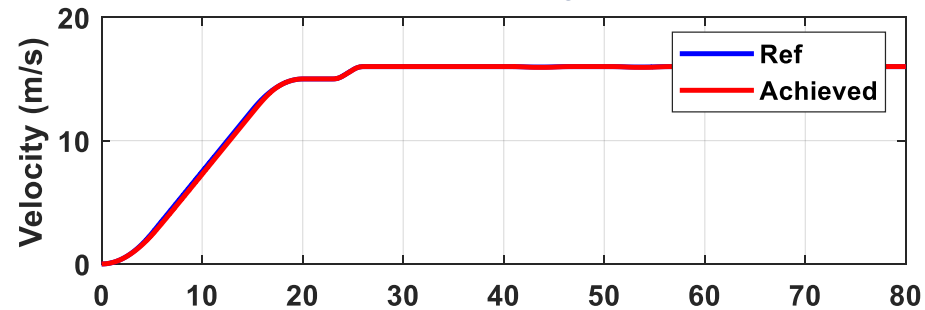
Vehicle Lateral Control

- Vehicle trajectory (double lane change)

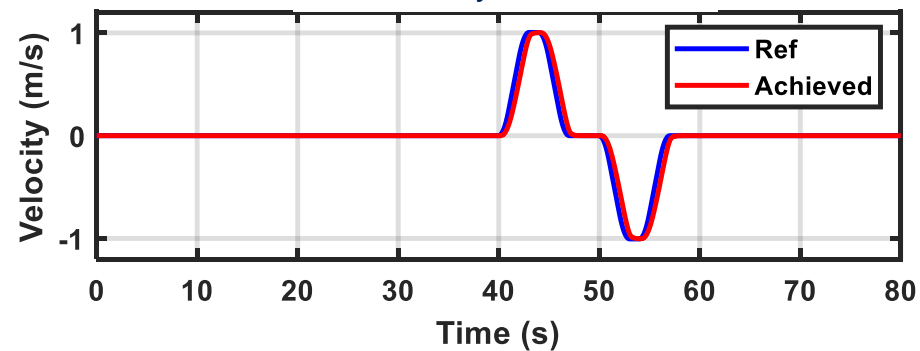
Lateral Motion



Velocity – Longitudinal

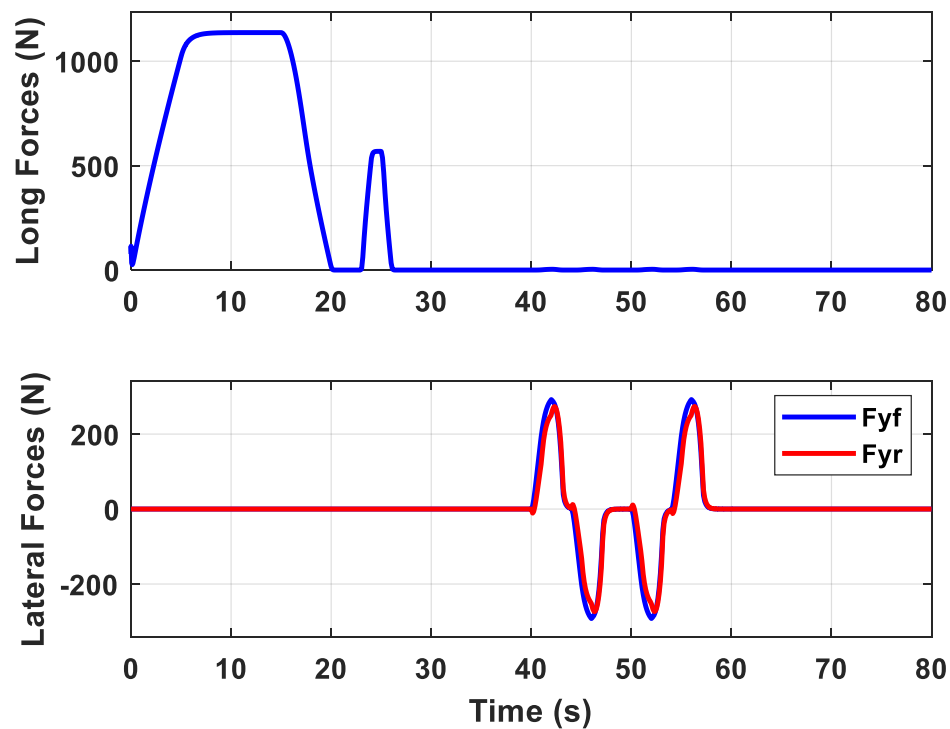


Velocity – Lateral

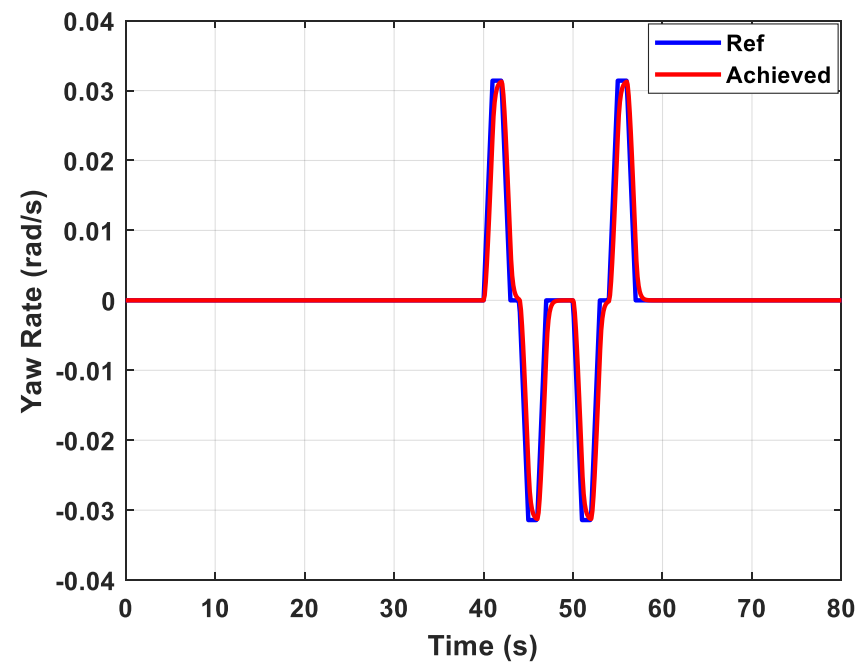


Vehicle Lateral Control

Tire Forces



Yaw Rate



Summary

What we have learned from this lesson:

- The concepts of Model Predictive Control (MPC)
- MPC costs and constraints
- Applied MPC to a lane change maneuver

Summary

In this module, you ...

- Defined the lateral path tracking problem
- Applied two geometric path tracking controllers, the pure pursuit and Stanley controllers, to the path tracking problem.
- And define a model predictive controller for joint lateral and longitudinal control.

What is next?

- Apply lateral and longitudinal control in the Carla simulator