Populating Occupancy Grids from LIDAR Scan Data

Course 4, Module 2, Lesson 2 – Part 1

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Learning Objectives

- Issue with the Bayesian Probability Update
- Present a solution utilizing log odds
- Bayesian log odds update derivation

Bayesian Update Of The Occupancy Grid - Summary

- Bayes' theorem is applied at each update step for each cell

Normalizer constant

$$bel_t(m^i) = np(y_t|m^i)bel_{t-1}(m^i)$$

Current measurement

Previous belief map

- There's a problem!

Issue With Standard Bayesian Update

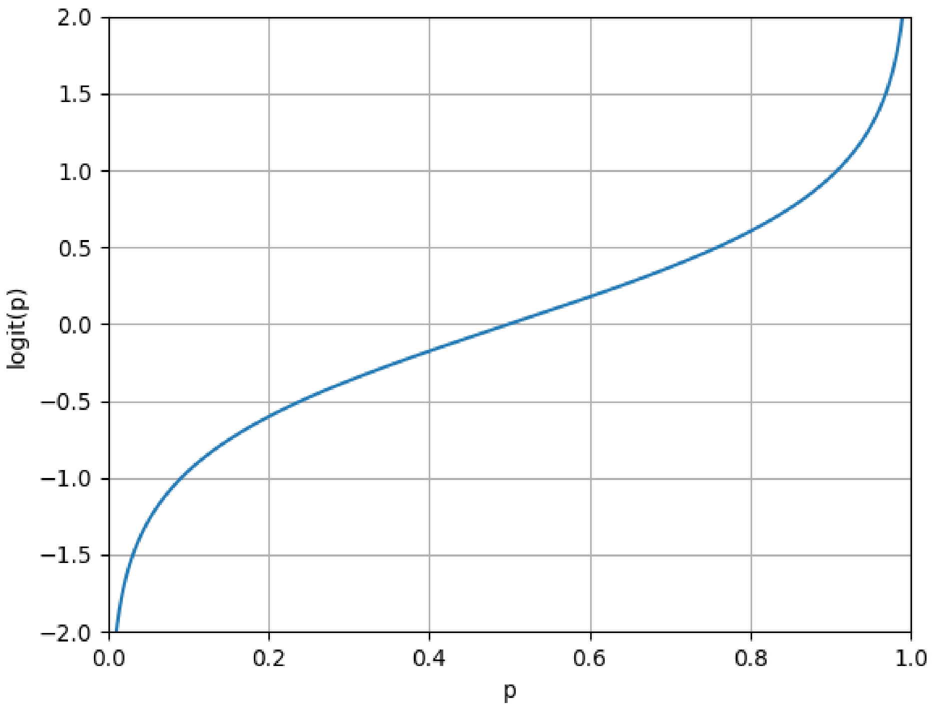
- Update a single unoccupied grid cell

$$bel_t(m) = np(y_t|m)bel_{t-1}(m)$$

0.000000008 0.000012 0.000638

- Multiplication of numbers close to zero is hard for computers
- Store the log odds ratio rather than probability

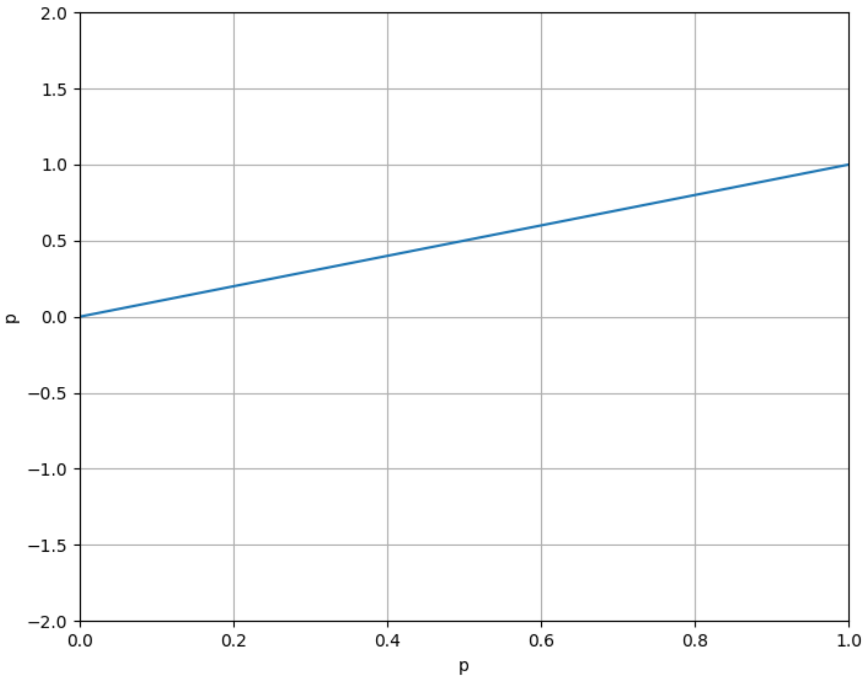
$$bel_t(m) \to (-\infty, \infty)$$

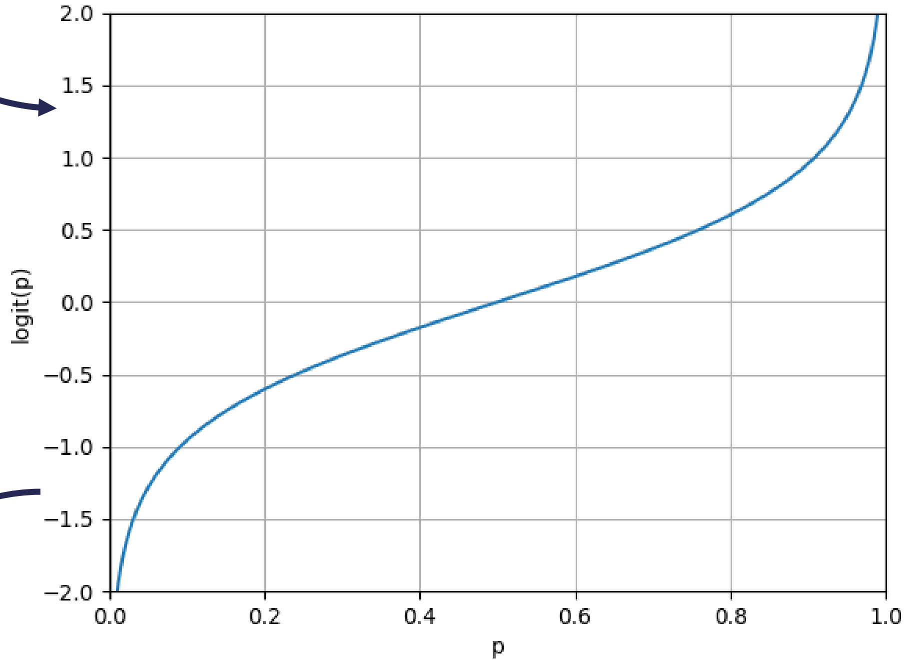


$$\log\left(\frac{p}{1-p}\right)$$

Conversion

$$logit(p) = \log\left(\frac{p}{1-p}\right)$$





$$p = \frac{e^{logit(p)}}{1 + e^{logit(p)}}$$

Bayesian Log Odds Single Cell Update Derivation

- Applying Bayes' rule:

Pulling out current measurement y_t from past measurements $y_{1:t-1}$

$$p(m^i|y_{1:t}) = \frac{p(y_t|y_{1:t-1}, m^i)p(m^i|y_{1:t-1})}{p(y_t|y_{1:t-1})}$$

Current map cell

Sensor measurement for given cell

Pulling out current measurement y_t from past measurements $y_{1:t-1}$

- Applying the Markov assumption:

$$p(m^i|y_{1:t}) = \frac{p(y_t|m^i)p(m^i|y_{1:t-1})}{p(y_t|y_{1:t-1})}$$

Bayesian Log Odds Single Cell Update Derivation

$$p(m^i|y_{1:t}) = \frac{p(y_t|m^i)p(m^i|y_{1:t-1})}{p(y_t|y_{1:t-1})}$$

- Applying Bayes' rule to measurement model:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

$$p(y_t|m^i) = \frac{p(m^i|y_t)p(y_t)}{p(m^i)}$$

- Yields:

$$p(m^i|y_{1:t}) = \frac{p(m^i|y_t)p(y_t)p(m^i|y_{1:t-1})}{p(m^i)p(y_t|y_{1:t-1})}$$

Bayesian Log Odds Single Cell Update Derivation

- Denominator: $1 - p$

$$p(\neg m^i | y_{1:t}) = 1 - p(m^i | y_{1:t}) = \frac{p(\neg m^i | y_t) p(y_t) p(m^i | y_{1:t-1})}{p(\neg m^i) p(y_t | y_{1:t-1})}$$

- Logit function

$$\operatorname{logit}(p) = \log\left(\frac{p}{1-p}\right) \qquad \frac{p(m^i | y_{1:t})}{p(\neg m^i | y_{1:t})} = \frac{\dfrac{p(m^i | y_t) \cancel{p(y_t)} p(m^i | y_{1:t-1})}{p(m^i) \cancel{p(y_t | y_{1:t-1})}}}{\dfrac{p(\neg m^i | y_t) \cancel{p(y_t)} p(m^i | y_{1:t-1})}{p(\neg m^i) \cancel{p(y_t | y_{1:t-1})}}}$$

Bayesian Log Odds Single Cell Update Derivation

- Simplifying like terms results in:

$$\frac{p(m^i|y_{1:t})}{p(\neg m^i|y_{1:t})} = \frac{p(m^i|y_t)p(\neg m^i)p(m^i|y_{1:t-1})}{p(\neg m^i|y_t)p(m^i)p(\neg m^i|y_{1:t-1})}$$

- Can rewrite by taking $\neg p$ to $1 - p$:

$$\frac{p(m^i|y_{1:t})}{p(\neg m^i|y_{1:t})} = \frac{p(m^i|y_t)(1 - p(m^i))p(m^i|y_{1:t-1})}{\left(1 - p(m^i|y_t)\right)p(m^i)\left(1 - p(m^i|y_{1:t-1})\right)}$$

- Finally, taking the log:

$$\mathrm{logit}\left(p(m^i|y_{1:t})\right) = \mathrm{logit}\left(p(m^i|y_t)\right) + \mathrm{logit}\left(p(m^i|y_{1:t-1})\right) - \mathrm{logit}\left(p(m^i)\right)$$

$$\boxed{l_{t,i} = \mathrm{logit}\left(p(m^i|y_t)\right) + l_{t-1,i} - l_{0,i}}$$

Bayesian log odds Update

Inverse Measurement Model   Previous belief   Initial belief

$$l_{t,i} = \mathrm{logit}\Big(p\big(m^i|y_t\big)\Big) + l_{t-1,i} - l_{0,i}$$

- Numerically stable
- Computationally efficient

Summary

- Identified issue with the Bayesian probability update
- Presented a solution utilizing log odds
- Bayesian log odds update derivation