EE204: Analog Circuits

Dept of Electrical Engineering, IITB

Tutorial 3

Autumn Semester 2023 Date: 30-08-2023

Q1: For the circuit in Figure 1, transistor M1 has W/L = 20um/1.8um. Use 180nm CMOS tech parameters: Vt = 0.4V, and μCox = 260μA/V^2.

(a) For *V*1 = 0.80V, calculate *ID* and *VD.*
(b) Increase V1 by 5mV, and recalculate *ID* and *VD*.
(c) Calculate $(V_D(part\ b) - V_D(part\ a))/(V_1(part\ b) - V_1(part\ a)) = \Delta V_D/\Delta V_1$. Is the absolute value of the ratio greater than 1?

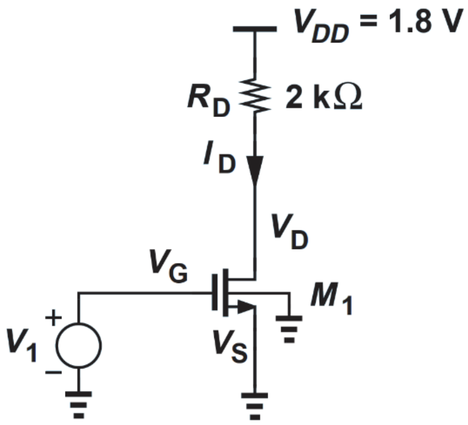


Figure 1

Q2: For the circuit in Figure 2, VDD = 5V. Photodiode sensitivity is 0.5A/W (Ampere/Watt). Op-Amp output saturates at 3.5V high and 1.5V low.

(a) If R1 = 1kΩ, what is the maximum intensity of light the circuit can sense?
(b) If Vout can be measured with a resolution of 5mV, what is the minimum change in light intensity that can be detected?

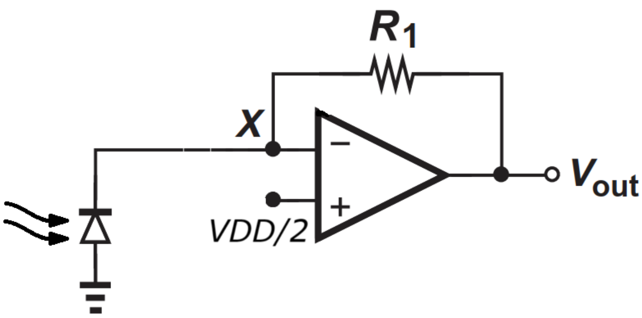


Figure 2

Q3: For the circuits in Figure 3, Op-Amps have finite open loop gain A_0. Find expressions for $A_V = V_{out}/V_{in}$ in terms of R1, R2 and open loop gain A_0. Verify that they reduce to standard inverting and non-inverting gains when $A_0 \rightarrow \infty$.

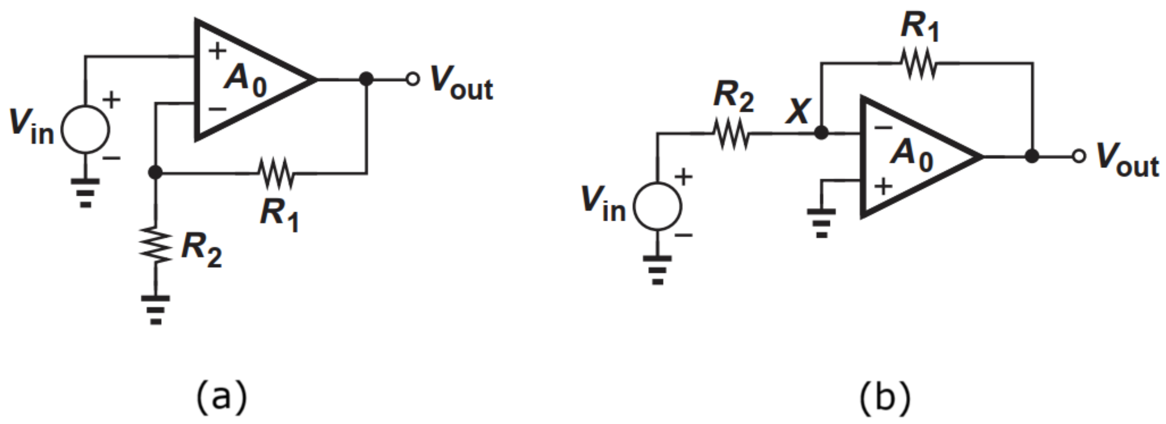


Figure 3

Q4: For the Howland Current Source in Figure 4, VDD = 5V, VSS = 0V and Vcntrl = 2V. Op-Amp output saturates at 4V.

(a) Find the values of R1, R2, R3, and R4 such that I_{OUT} = 1mA for R_L = 2kΩ.
(b) For the above design, what is the maximum value of R_L such that the Op-Amp does not saturate?

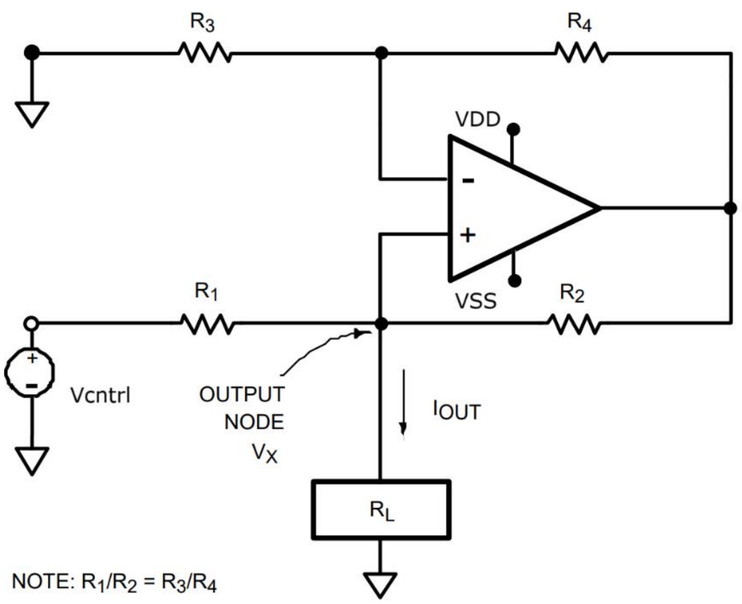


Figure 4

Q5: For the Modified Howland Current Source in Figure 5, VDD = 5V, VSS = 0V and Vcntrl = 2V. Op-Amp output saturates at 4V.

(a) Find the values of R1, R2, R3, R4, and R5 such that I_{OUT} = 1mA for R_L = 2kΩ.

(b) For the above design, what is the maximum value of R_L such that the Op-Amp does not saturate?

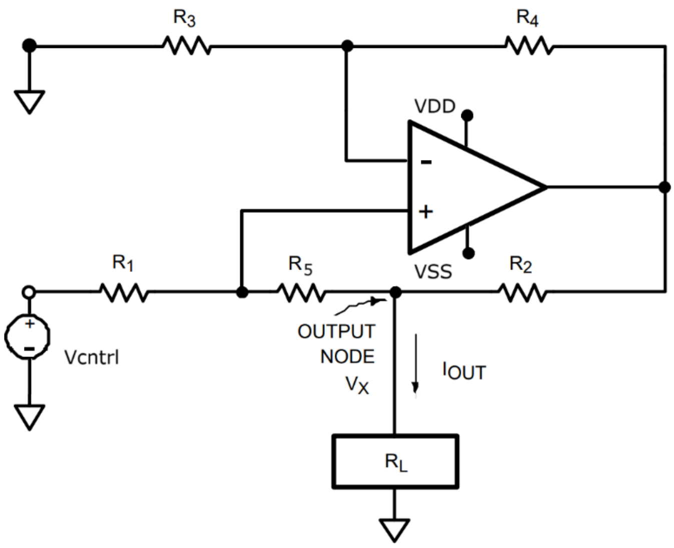
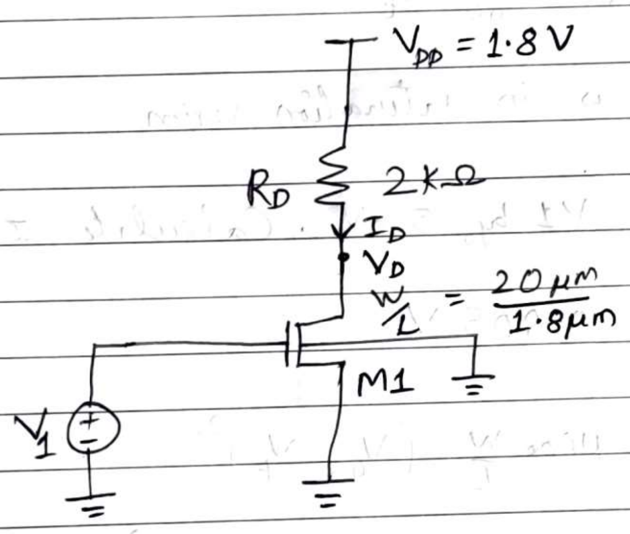


Figure 5

EE204 : Analog Circuits

Tutorial 3 Solutions

Q1 Q1: Solution



$V_t = 0.4V$, $\mu C_{ox} = 260 \mu A/V^2$.

(a) For $V1 = 0.8V$, calculate I_D & V_D.

Assuming in saturation region for M1.

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 260 \times \frac{20}{1.8} (0.8 - 0.4)^2 \ \mu A$$

$$= 231.11 \ \mu A \quad = 0.231 \ mA$$

$$V_D = V_{DD} - I_D R_D = 1.8 - 0.231 \times 2 = 1.338V$$

Let's verify our assumption of operation in saturation region for M1.

$$V_{DS} = 1.338\ V$$
$$V_{GS} - V_t = 0.8 - 0.4 = 0.4\ V$$

$\because V_{DS} > V_{GS} - V_t$

$\therefore$ M1 is in saturation region

(b) Increase V1 by 5 mV. Calculate I_D & V_D

$$V1 = 0.805\ V$$

$$I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$$

$$= \frac{1}{2} \times 260 \times \frac{20}{1.8} (0.805 - 0.4)^2\ \mu A$$

$$= 236.9\ \mu A \approx 0.237\ mA$$

$$V_D = V_{DD} - I_D R_D = 1.8 - 0.237 \times 2 = 1.326\ V$$

(c) Calculate $\frac{\Delta V_D}{\Delta V_1}$

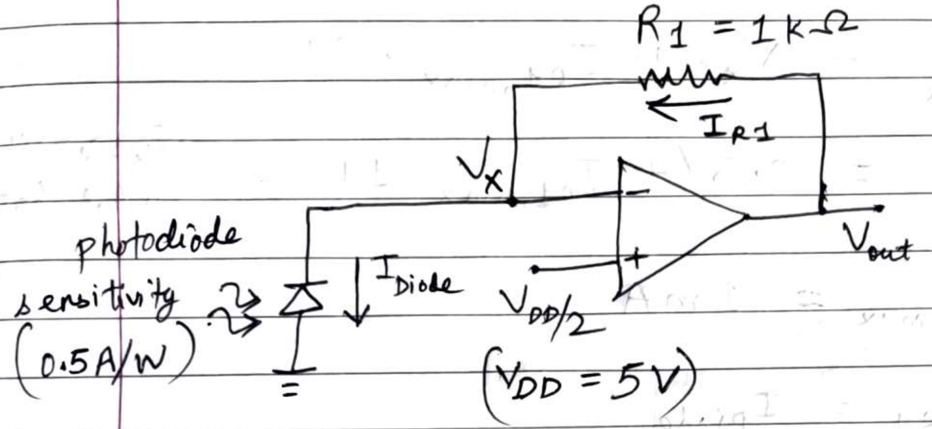
$$\frac{\Delta V_D}{\Delta V_1} = \frac{1.326 - 1.338}{0.805 - 0.8} = \frac{-0.012\ V}{0.005\ V}$$

$$= \frac{-12\ mV}{5\ mV} = -2.4$$

Absolute value $\left|\frac{\Delta V_D}{\Delta V_1}\right| = |-2.4| = 2.4 > 1$

This is a rudimentary demo of transistor as amplifier.

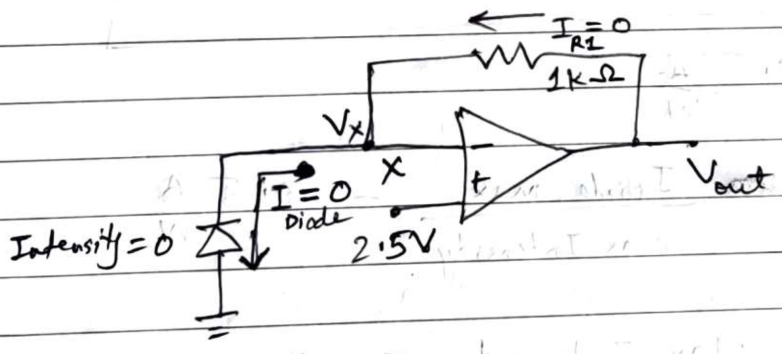
Q:2 Solution



V_{out} saturates at 3.5V high and 1.5V low.

(a) What is the max intensity of light the circuit can sense?

Solution: When intensity = 0, photodiode current = 0.



By virtual short $V_x = 2.5V$

$$V_{out} = V_x + I_{R1} \cdot R_1$$
$$= 2.5 + 0$$
$$V_{out} = 2.5V$$

For a photodiode $I_{Diode} \propto$ Intensity of light

Max intensity ~~of the cu~~ corresponds to max current without opamp saturation.

Opamp saturates at 3.5 V. Therefore

$$V_{out,max} = V_x + I_{R1,max} \cdot R1$$

$$3.5V = 2.5V + I_{R1,max} \cdot 1k\Omega$$ [$V_x = 2.5V$ by virtual short]

$$\Rightarrow I_{R1,max} = 1mA$$

$$\because \quad I_{R1} = I_{Diode}$$

$$\therefore \quad I_{Diode,max} = I_{R1,max} = 1mA$$

$$\boxed{\text{Photodiode Sensitivity} = \frac{I_{Diode}}{\text{Incident light Intensity}}}$$

$$\therefore \quad \frac{I_{Diode,max}}{\text{max Intensity}} = 0.5\,\frac{A}{W}$$

$$\text{Max Intensity} = \frac{I_{Diode\,max}}{0.5\,A/W}$$

$$\text{Max Intensity} = 2\,mW$$

Therefore, maximum intensity of light the circuit can sense befor op-amp output going to saturation is 2mW.

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~~Q3: Solution~~

(b) If V_{out} can be measured with a resolution of 5 mV, what is the minimum change in light intensity that can be detected?

Sol^n:

$$\Delta V_{out, min} = 5\,mV$$ [min. V_{out} change that can be measured]

$$V_{out} = V_x + I_{R1} \cdot R_1$$

$$V_{out} = V_x + I_{diode} \cdot R_1 \qquad [\because I_{R1} = I_{Diode}]$$

$$V_{out} = V_x + \text{Light Intensity} \times \text{Sensitivity} \cdot R_1$$

Let Light Intensity $= P_{light}$

& Sensitivity $= S$

$$V_{out} = V_x + P_{light} \cdot S \cdot R1$$

$$\Delta V_{out} = \Delta P_{light} \cdot S \cdot R1$$

($\because V_x = 2.5V$ constant; S & R1 are constant)

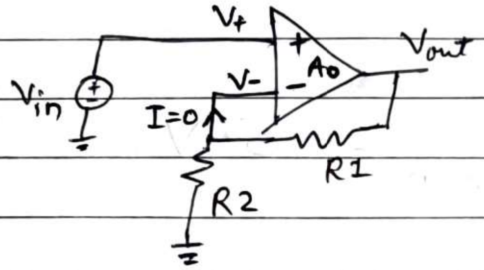
Therefore,

Minimum change in light Intensity that can be detected

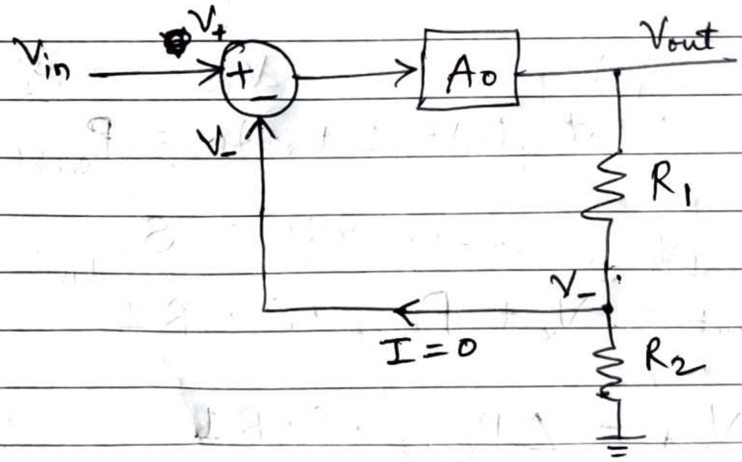
$$\boxed{\Delta P_{light, min} = \frac{\Delta V_{out, min}}{S \cdot R1} = \frac{5\,mV}{0.5\,\frac{A}{W} \cdot 1\,k\Omega} \qquad \Delta P_{light, min} = 10\,\mu W}$$

Q3: Solution

(a) Express, $A_V = \frac{V_{out}}{V_{in}}$ in terms of R_1, R_2 & A_0 for following circuit.



<u>Solution:</u> Redrawing the above circuit as below.



$$V_- = \frac{V_{out} \cdot R_2}{R_1 + R_2}$$ [Using voltage division]

$$(V_+ - V_-)\, A_0 = V_{out}$$

$$\left(V_{in} - \frac{V_{out} R_2}{R_1 + R_2}\right) A_0 = V_{out}$$

[After substituting for V_+ and V_-]

$$V_{in}A_0 - \frac{V_{out}R_2A_0}{R_1+R_2} = V_{out}$$

$$V_{out}\left[1+\frac{R_2A_0}{R_1+R_2}\right] = V_{in}A_0$$

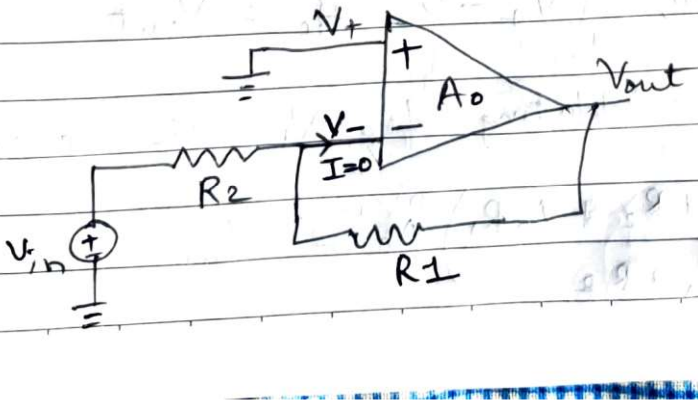
$$\boxed{\frac{V_{out}}{V_{in}} = \frac{A_0}{1+\frac{R_2A_0}{R_1+R_2}}}$$

Let $\frac{R_2}{R_1+R_2} = \beta$ (feedback factor)

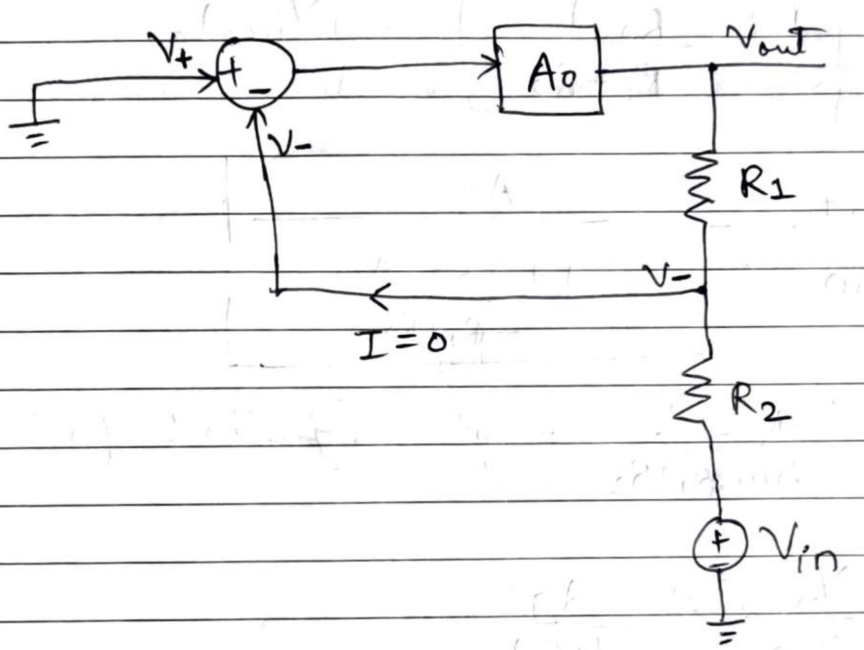
$$A_v = \frac{V_{out}}{V_{in}} = \frac{A_0}{1+\beta A_0}$$

$$\boxed{\begin{gathered}\text{if } A_0 \to \infty,\ \beta A_0 \gg 1 \\ A_v = \frac{V_{out}}{V_{in}} = \frac{A_0}{\beta A_0} = \frac{1}{\beta} = \frac{R_1+R_2}{R_2} = 1+\frac{R_1}{R_2}\end{gathered}}$$

(b) Express $A_v = V_{out}/V_{in}$, in terms of R_1, R_2 & A_0 for following circuit



Redrawing the above circuit as below



Equating Currents through R1 & R2

$$\frac{V_{out} - V_-}{R_1} = \frac{V_- - V_{in}}{R_2}$$

$$V_{out} R_2 - V_- \cdot R_2 = V_- \cdot R_1 - V_{in} R_1$$

$$V_{out} R_2 + V_{in} R_1 = V_- \cdot R_1 + V_- \cdot R_2$$

$$V_- = \frac{V_{out} R_2 + V_{in} R_1}{R_1 + R_2}$$

Now, $(V_+ - V_-) A_0 = V_{out}$

Substituting for V_+ & V_- we get

$$0 - \left(\frac{V_{out} R_2 + V_{in} R_1}{R_1 + R_2} \right) A_0 = V_{out}$$

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$$V_{out}\left(1+\frac{R_2 A_o}{R_1+R_2}\right) = -V_{in}\frac{R_1 A_o}{R_1+R_2}$$

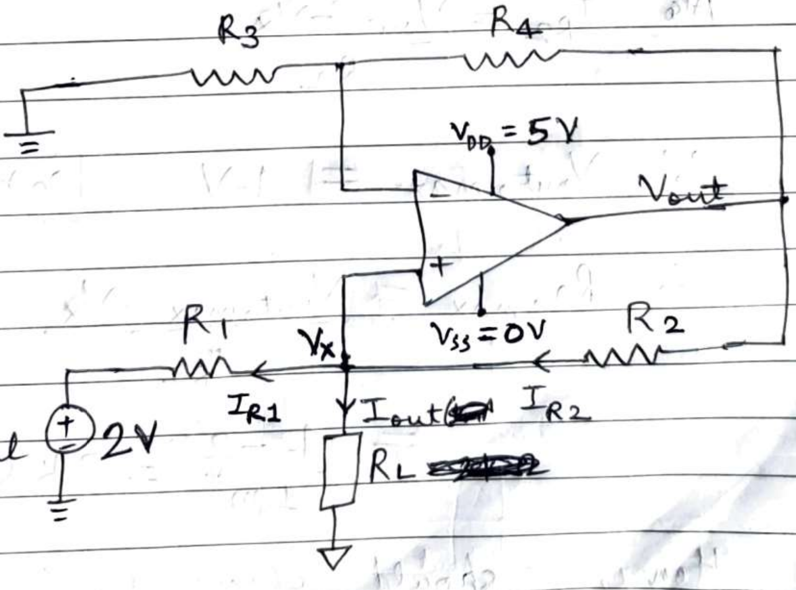
$$A_v = \frac{V_{out}}{V_{in}} = \frac{-\left(\frac{R_1 A_o}{R_1+R_2}\right)}{\left(\frac{R_1+R_2+R_2 A_o}{R_1+R_2}\right)}$$

$$\boxed{A_v = \frac{V_{out}}{V_{in}} = \frac{-R_1 A_o}{R_1+R_2+R_2 A_o}}$$

If $A_o \to \infty \Rightarrow R_2 A_o >> R_1 \,\&\, R_2$

$$\therefore \quad A_v = \frac{V_{out}}{V_{in}} = \frac{-R_1 A_o}{R_2 A_o} = \frac{-R_1}{R_2}$$

Q4: Solution



Opamp output saturates at 4V.

(a) $I_{out} = 1mA$, $R_L = 2k\Omega$.

Design the circuit and choose R1, R2, R3 and R_4 for required $I_{out} = 1mA$ and given $R_L = 2k\Omega$. Opamp should be not saturated.

Solution :-

$$I_{out} = \frac{V_{cntrl}}{R1}$$

$$\Rightarrow R_1 = \frac{V_{cntrl}}{I_{out}} = \frac{2V}{1mA} = 2k\Omega$$

$$V_x = I_{out} R_L = 1mA \cdot 2k\Omega = 2V$$

$$I_{R1} = \frac{V_x - V_{cntrl}}{R1} = \frac{2-2}{R1} = 0$$

$$\therefore I_{R2} = I_{R1} + I_{out} = 1mA$$

Also $I_{R2} = \dfrac{V_{out} - V_x}{R_2}$

$\because V_{out, max} = 4V$ [OpAmp o/p saturates at 4V]

$$\therefore R_{2, max} = \frac{V_{out, max} - V_x}{I_{R2}}$$

$$= \frac{4-2}{1m} = 2k\Omega$$

Hence, we should choose $R_2 < 2k\Omega$

Let us choose $R_2 = 1.5\,k\Omega$

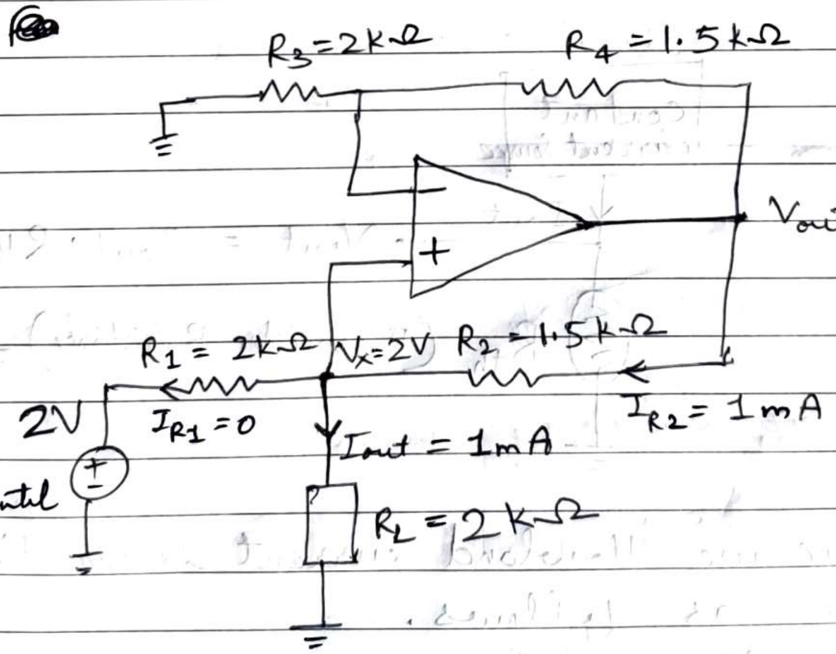
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$ [Howland current source requirement]

$$\frac{2\,k}{1.5\,K} = \frac{R_3}{R_4}$$

Let us choose, $R_3 = 2\,k\Omega$ and $R_4 = 1.5\,k\Omega$

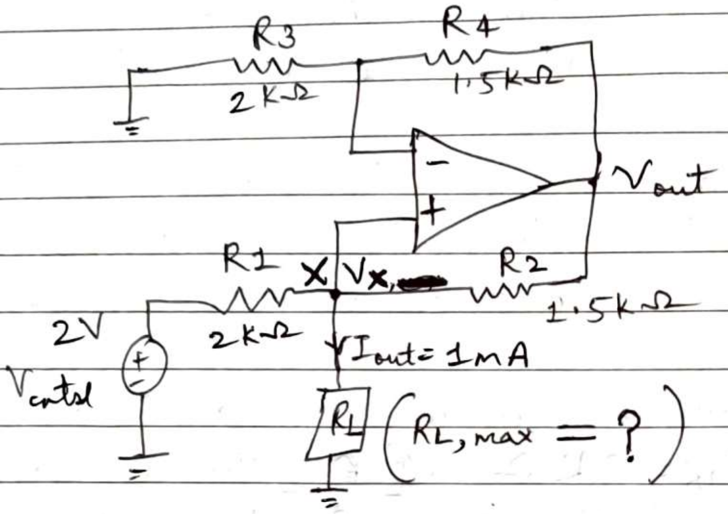
So, now our design is

$R_1 = 2\,k\Omega$, $R_2 = 1.5\,K\Omega$, $R_3 = 2\,k\Omega$, $R_4 = 1.5\,k\Omega$



For our design $V_{out} = 3.5\,V$ ($\because$ Op-Amp is not saturated and we have $0.5\,V$ margin before op-amp saturates.)

(b) For the design in part (a), what is the maximum value of R_L such that op-amp output does not saturate? [op-amp o/p is at the verge of saturation]



$(V_{out,max} = 4V)$

Writing KCL at node X

$$\frac{V_x - V_{cntrl}}{R_1} + I_{out} = \frac{V_{out} - V_x}{R_2}$$

putting $V_x = I_{out} R_L$ we get

$$\frac{I_{out} \cdot R_L - V_{cntrl}}{R_1} + I_{out} = \frac{V_{out} - I_{out} \cdot R_L}{R_2}$$

when, $V_{out} = V_{out,max} \Rightarrow R_L = R_{L,max}$

$$\frac{I_{out} \cdot R_{L,max} - V_{cntrl}}{R_1} + I_{out} = \frac{V_{out,max} - I_{out} \cdot R_{L,max}}{R_2}$$

Substituting $I_{out} = 1mA$, $V_{cntrl} = 2V$, $R1 = 2k\Omega$, $R_2 = 1.5\,k\Omega$, $V_{out,max} = 4V$, we get,

$$\frac{1mA \cdot R_{L,max} - 2}{2k\Omega} + 1mA = \frac{4 - 1mA \cdot R_{L,max}}{1.5\,k\Omega}$$

$$\frac{1m \cdot R_{L,max} - 2 + 1\cancel{m} \cdot 2\cancel{K}}{2\cancel{K}} = \frac{4 - 1m \cdot R_{L,max}}{1.5\cancel{K}}$$

$$\frac{1m \cdot R_{L,max} \cancel{-2} \cancel{+2}}{2} = \frac{4 - 1m \cdot R_{L,max}}{1.5}$$

$$1.5\,(1m \cdot R_{L,max}) = 8 - 2\,m \cdot R_{L,max}$$

$$3.5\,m \cdot R_{L,max} = 8$$

$$R_{L,max} = \frac{8}{3.5m}$$

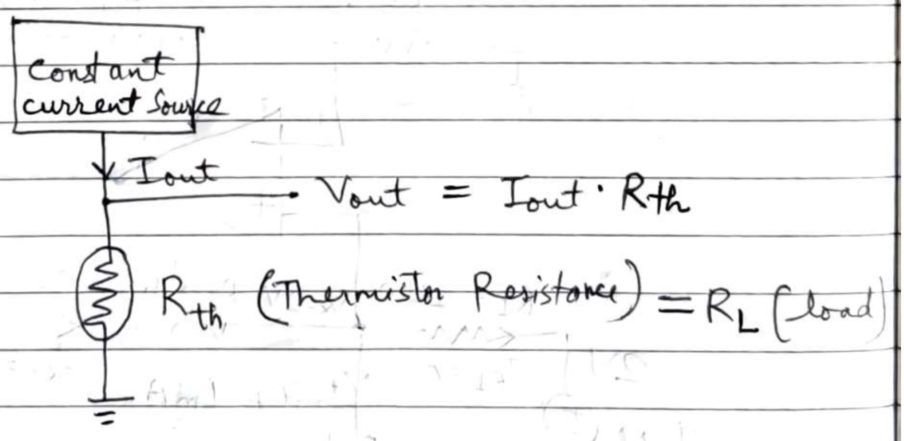
$$\boxed{R_{L,max} = 2.286\ k\Omega}$$

✗✗ Supplementary Material for Q4

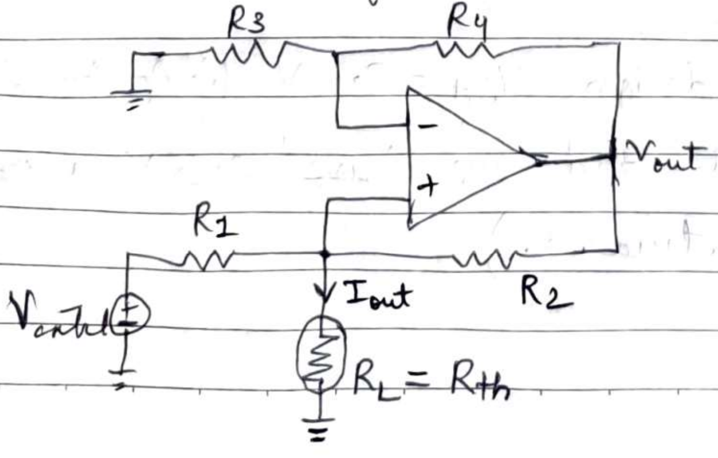
For the design in part (a) ~~if we increase R~~ if R_L (load resistance) is increased the opAmp output voltage V_{out} will increase.

R_L may increase if load is variable. One example is R_L may be sensitive to temperature and hence may change.

Let us understand through an example. We want to make a temperature sensor. ~~Using a thermistor~~. We can pass a constant current through it and sense the voltage accross thermistor.



If we use Howland current source the circuit will be as follows.



In part (a) we designed for $I_{out} = 1mA$ and $R_L = R_{th} = 2k\Omega$.

Let us say $R_L = R_{th} = 2K\Omega$ corresponds to thermistor resistance at $25°C$.

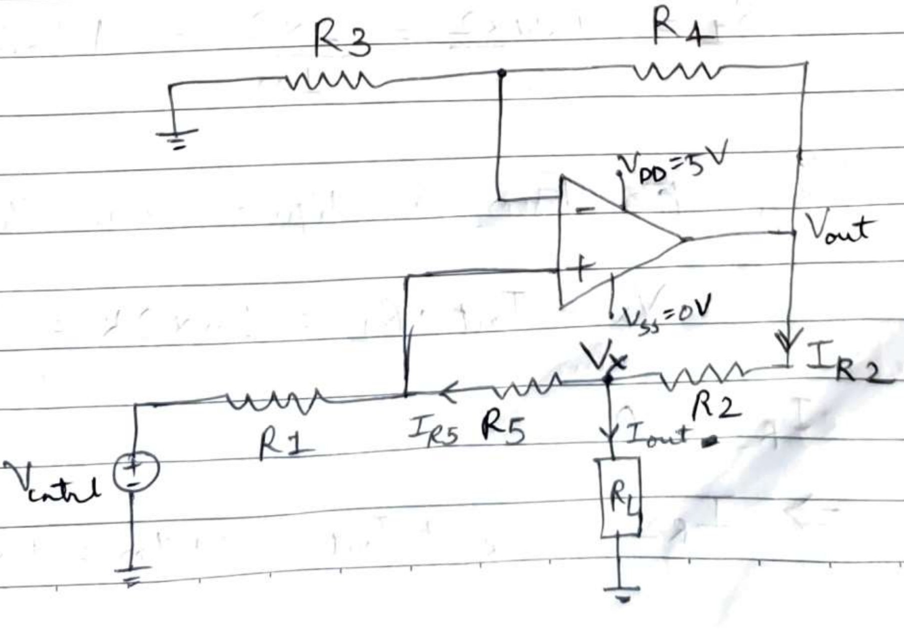
If we are using a Negative Temperature Coefficient (NTC) thermistor, the resistance, R_{th}, will increase when temperature reduces.

For our design in part (a) $V_{out} = 3.5V$ for $R_L = R_{th} = 2K\Omega$. [This is for our design choices of $R_1 = 2k\Omega$, $R_2 = 1.5k\Omega$, $R_3 = 2k\Omega$ & $R_4 = 1.5k\Omega$]

Now, in part (b) we find maximum R_L (i.e., how much R_L can be increased before Op-Amp saturates). This, $R_{L,max} = 2.286 k\Omega$.

In our thermistor example, this corresponds to the maximum R_{th} value, that is the minimum temperature value (remember we are using negative coefficient thermistor) that can be sensed before Op-Amp output saturates.

Q5: Solution



[V_{out} saturates at 4 V]

(a) $I_{out} = 1\,mA$, $R_L = 2k\Omega$.

Design the circuit and choose, R1, R2, R3, R4 and R5 appropriatly for $I_{out} = 1mA$ and $R_L = 2k\Omega$. ~~without~~ Opamp should not be saturated output.

$$\left.\begin{aligned} I_{out} &= \frac{V_{cntrl}}{R_1} \times \underbrace{\left(\frac{R_2+R_5}{R_2}\right)}_{\text{also known as Gain Factor}} \\ \frac{R_4}{R_3} &= \frac{R_2+R_5}{R_1} \end{aligned}\right\} \text{Design Equations}$$

Let us choose the following,

$$\text{Gain factor} = \frac{R_2+R_5}{R_2} = 2$$

$$\Rightarrow R_2 = R_5$$

Now,

$$\therefore \quad I_{out} = \frac{V_{cntrl}}{R_1} \times 2$$

$$R_1 = \frac{V_{cntrl} \times 2}{I_{out}} = \frac{2 \times 2}{1m} = 4\,k\Omega$$

~~V_x (see fig)~~ From the figure, using Ohm's Law.

$$V_x = I_{out} \cdot R_L = 1m \times 2k = 2V$$

$$\therefore \quad I_{R5} = 0$$

$$\Rightarrow I_{R2} = I_{R5} + I_{out} = 0 + 1m = 1mA$$

Now, $V_{out} = V_x + I_{R2} \cdot R_2$

$$\Rightarrow R_{2,max} = \frac{V_{out,max} - V_x}{I_{R2}}$$

[$V_{out,max} = 4V$ as opamp o/p saturates @ 4V]

$$= \frac{4-2}{1m} = 2\,k\Omega$$

$\therefore$ we choose $R_2 < 2\,k\Omega$.

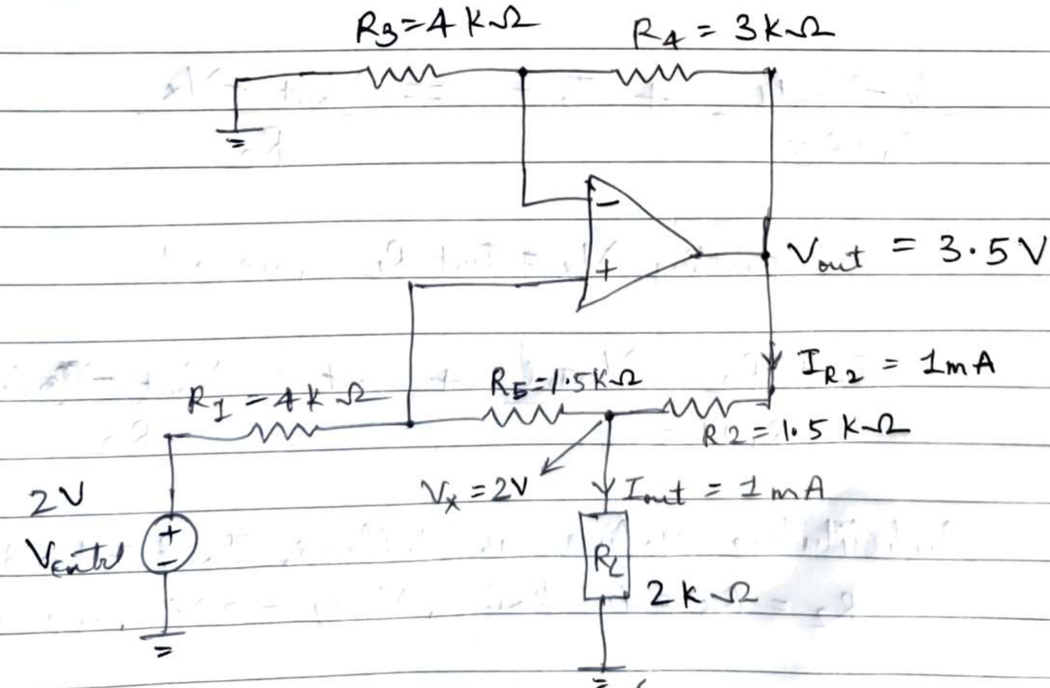
Let us choose $R_2 = 1.5\,k\Omega$.

$$\Rightarrow R_5 = R_2 = 1.5\,k\Omega$$

Now, $\frac{R_4}{R_3} = \frac{R_2 + R_5}{R_1} = \frac{1.5+1.5}{4} = \frac{3}{4}$

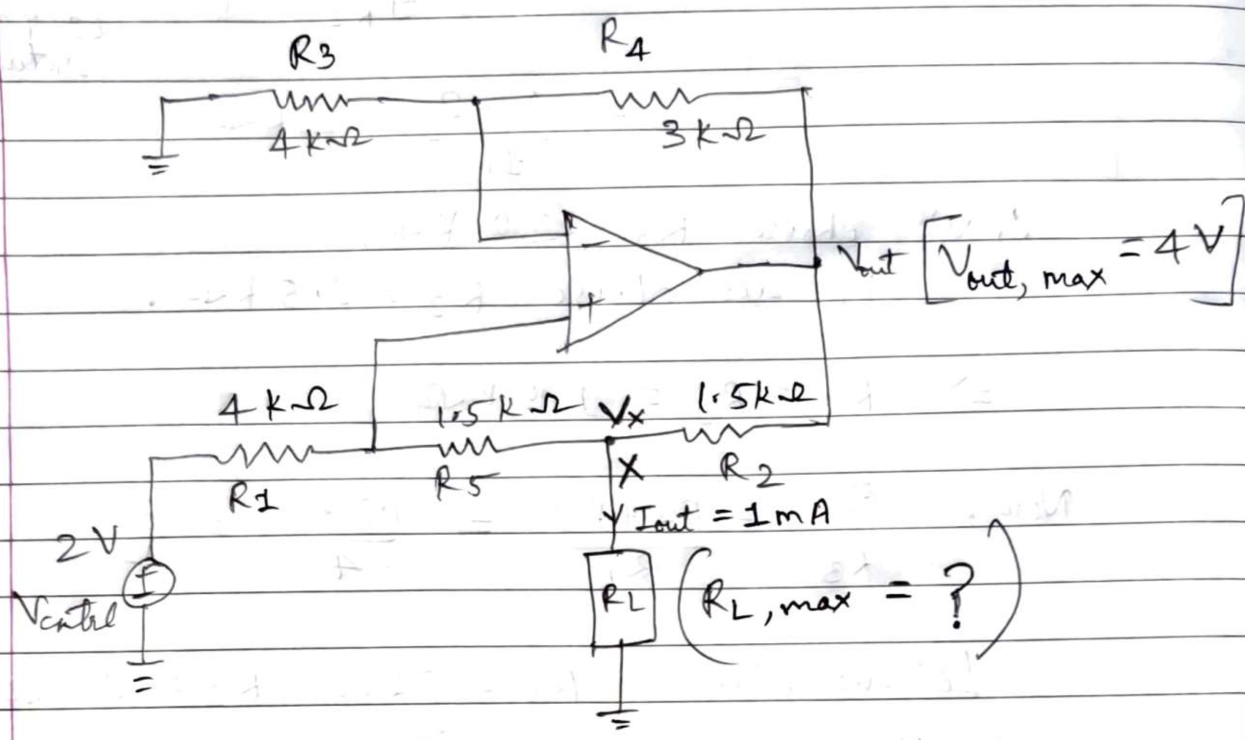
Let us choose $R_4 = 3\,k\Omega$, $R_3 = 4\,k\Omega$

So, now our designed circuit is as below.



For our design $V_{out} = 3.5\,V$ (∴ Opamp is not saturated and we have 0.5 V margin before op-Amp saturates)

(b) For the design in part (a), what is the maximum value of R_L such that op-amp is at verge of saturation?



Writing KCL at node X

$$\frac{V_x - V_{cntrl}}{R_1 + R_5} + I_{out} = \frac{V_{out} - V_x}{R_2}$$

putting $V_x = I_{out} R_L$, we get

$$\frac{I_{out} R_L - V_{cntrl}}{R_1 + R_5} + I_{out} = \frac{V_{out} - I_{out} \cdot R_L}{R_2}$$

Substituting values as follows. $I_{out} = 1\,mA$, $R_1 = 4\,k\Omega$
$R_2 = R_5 = 1.5\,k\Omega$, $V_{cntrl} = 2\,V$, we get

$$\frac{1mA \cdot R_L - 2}{4k\Omega + 1.5k\Omega} + 1mA = \frac{V_{out} - 1mA \cdot R_L}{1.5k\Omega}$$

$$\frac{1mA \cdot R_L - 2 + 5.5}{5.5} = \frac{V_{out} - 1mA \cdot R_L}{1.5}$$

when, $V_{out} = V_{out,max} \Rightarrow R_L = R_{L,max}$

$$\frac{1mA \cdot R_{L,max} - 2 + 5.5}{5.5} = \frac{V_{out,max} - 1mA \cdot R_L}{1.5}$$

put $V_{out,max} = 4V$. & simplify.

$$1.5\left(1mA \cdot R_{L,max} + 3.5\right) = 5.5\left(4 - 1mA \cdot R_{L,max}\right)$$

$$(1.5 + 5.5)\,mA \cdot R_{L,max} = 5.5 \times 4 - 1.5 \times 3.5$$

$$7\,mA \cdot R_{L,max} = 16.75$$

$$R_{L,max} = \frac{16.75}{7mA}$$

$$= \frac{16.75}{7}\,k\Omega$$

$$\boxed{R_{L,max} = 2.393\,k\Omega}$$