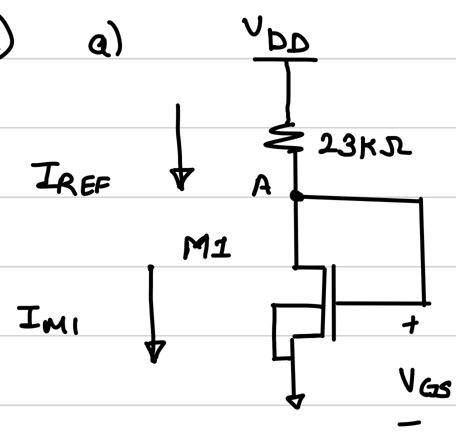
# Assignment 2 Solution

Q1) a)




At point A,

$$I_{REF} = \frac{V_{DD} - V_{GS}}{23K\Omega} \quad — ②$$ (0.5 marks)

$$I_{REF} = I_{M1} = \frac{\mu_n C_{ox} W}{2L}\left(V_{GS} - V_t\right)^2$$

$$I_{M1} = I_{REF} = \frac{260\mu}{2} \times \left(\frac{W}{L}\right)\left[V_{GS} - V_t\right]^2 \quad — ①$$ (0.5marks)

① = ②

$$\frac{1 - V_{GS}}{23K} = \frac{260\mu}{2}\left(\frac{W}{L}\right)\left(V_{GS} - 0.4\right)^2 \quad — ③$$

Solving equation ③, we get $V_{GS}$. two values of $V_{GS}$,
(0.5 marks) for discarding a value that is $\leq V_t$
For example $\frac{W}{L} = \frac{4\mu m}{0.5\mu m}$

$$\boxed{V_{GS} = 0.622V}$$

b) $\lambda = 0.025V^{-1}$ for $L_2 = 1\mu m$

$$\frac{d\lambda}{dL} = -0.05\ V^{-1}/\mu m$$

For getting $\lambda$ for $L_2 = 0.5\mu m$

$$\frac{0.025V^{-1} - \lambda}{1 - 0.5} = -0.05$$

$$\boxed{\lambda = 0.05\, V^{-1} \quad \text{for length } L_2 = 0.5\mu m}$$

0.25 marks

case 1 : $L_2 = 1\mu m$

$$\frac{W_2}{L_2} = \frac{4\mu m}{0.5\mu m} = \frac{W_2}{1\mu m}$$

$$W_2 = 8\mu m$$

(i) For $V_{DS} < V_{GS} - V_t$

$$V_{DS} < 0.622 - 0.4$$

$$V_{DS} < 0.222V$$

$M_2$ is in linear region

$$I_{out} = \frac{\mu_n C_{ox} W}{2L} \left( 2(V_{GS} - V_t)V_{DS} - V_{DS}^2 \right)$$

0.25 marks

(ii) For $V_{DS} > V_{GS} - V_t$

$V_{DS} > 0.222V$

$M2$ is in saturation region

$$I_{out} = \frac{\mu_n C_{ox} W}{2L} \left( V_{GS} - V_t \right)^2 \left( 1 + \lambda V_{DS} \right)$$

0.25 marks

For $L = 1\mu m$

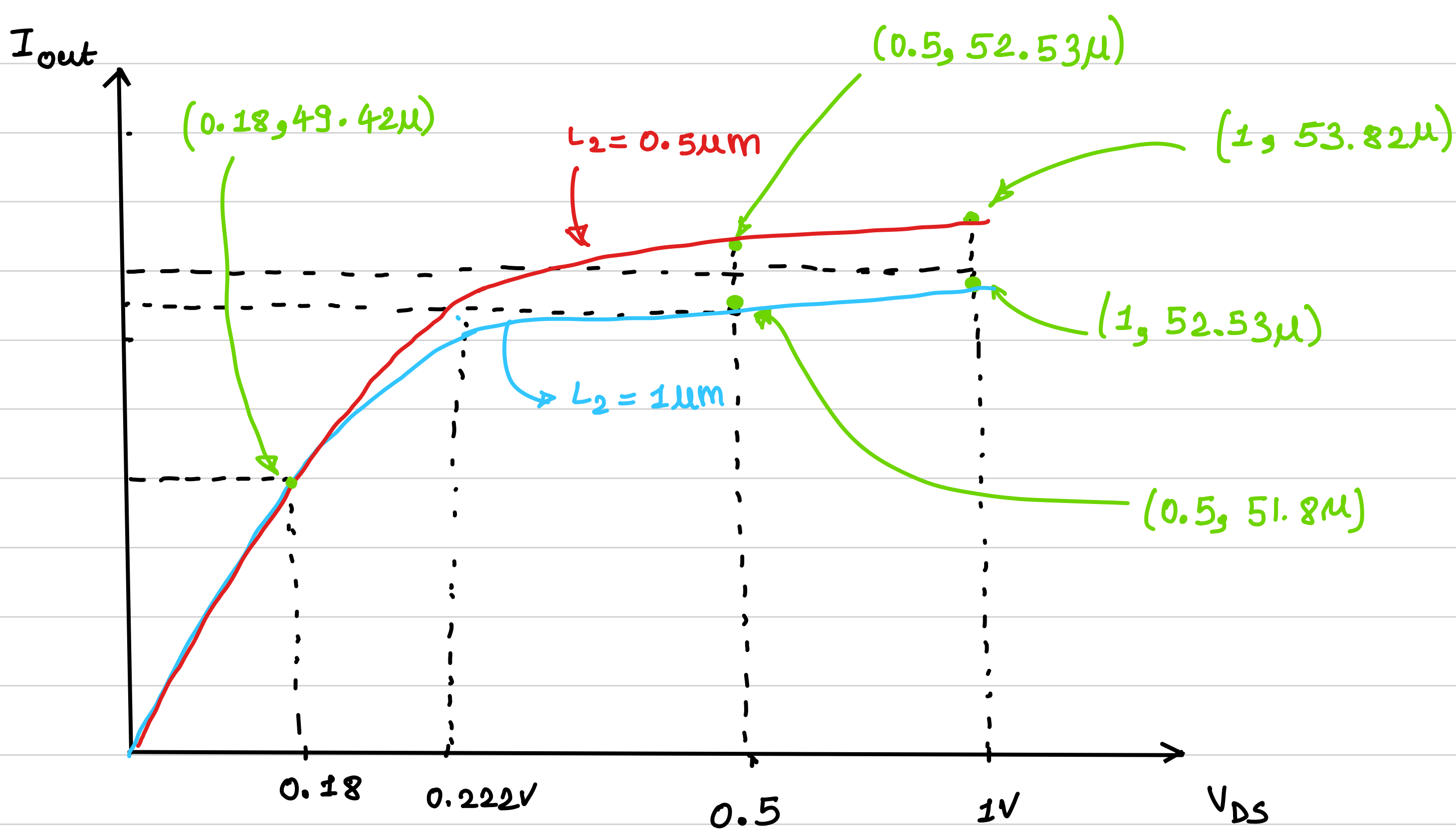
$V_{DS} = 0.18\,V$   $V_{DS} = 0.5$   $V_{DS} = 1V$

$I_{out} = 49.42\,\mu A$   $I_{out} = 51.89\,\mu A$   $I_{out} = 52.53\,\mu A$

For $L = 0.5\mu$

$V_{DS} = 0.18\,V$   $V_{DS} = 0.5$   $V_{DS} = 1V$

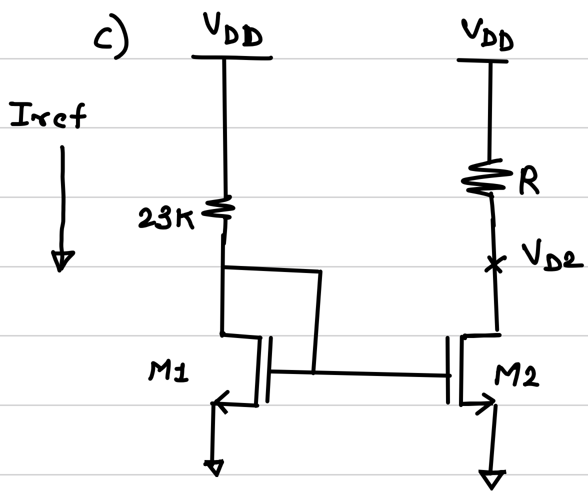
$I_{out} = 49.42\,\mu A$   $I_{out} = 52.53\,\mu A$   $I_{out} = 53.82\,\mu A$




- Take atleast 2 points in saturation and atleast 1 point in linear for both case – 0.5 marks

- 0.25 marks for plot.

c)




$$I_{ref} = \frac{1.8 - V_{GS}}{23K}$$

$$= \frac{1.8 - 0.622}{23K}$$

$$= 51.21\ \mu A$$ -0.25 marks

case 1) $R = 20K$

Assuming M2 in saturation – 0.25 marks

$V_{D2} = V_{DD} - I_{Ref}\, R$

$= 1.8 - 51.21\mu \times (20K)$

$= 0.7758$

Since $V_{D2} > V_G - V_t$ - 0.25 marks

$\boxed{\underbrace{\text{M2 is in saturation}}_{\text{0.25 marks}} \text{ and } \underbrace{I_{out} = 51.21\mu A}_{\text{0.25 marks}}}$ –

Case 2) $R = 40K$

Assuming M2 in saturation

$V_{D2} = V_{DD} - I_{Ref}\, R$

$= 1.8 - 51.21\mu \times 40K$

$= -0.248V$

so our assumption is wrong

so M2 is in linear region - 0.25 marks

$$I_{out} = \frac{V_{DD} - V_{DS}}{40K} \quad — ①$$

$$I_{out} = \frac{\mu_n C_{ox} W}{2L} \left( 2 (V_{GS} - V_t) V_{DS} - V_{DS}^2 \right) \quad — ②$$

$$① = ②$$

$$\therefore \frac{V_{DD} - V_{DS}}{40K} = \frac{\mu_n C_{ox} W}{2L} \left( 2 (V_{GS} - V_t) V_{DS} - V_{DS}^2 \right)$$ - 0.25 marks

$$\therefore 1.8 - V_{DS} = 41.6 \left( 0.444 V_{DS} - V_{DS}^2 \right)$$

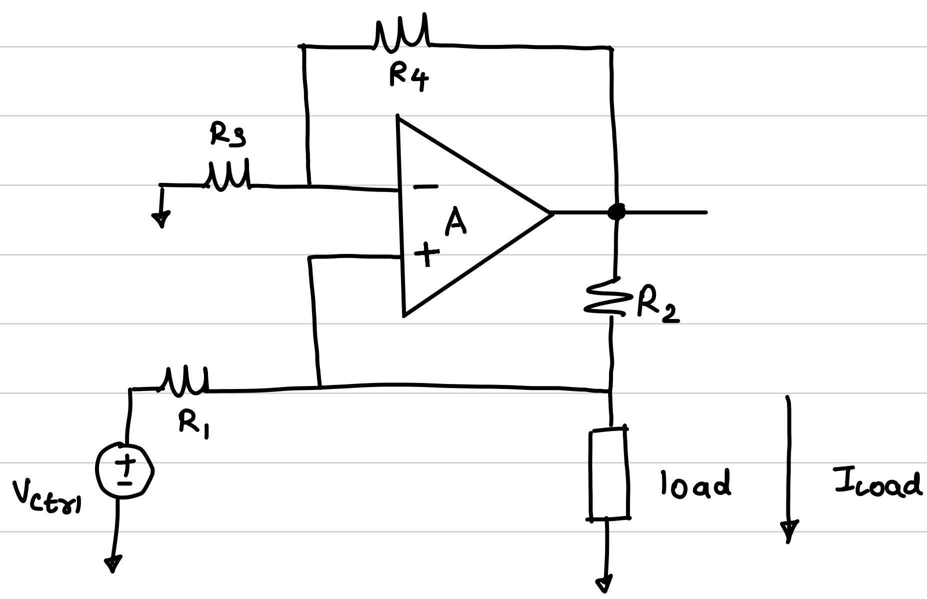
$$\therefore V_{DS} = 0.127$$

$$I_{out} = \frac{V_{DD} - V_{DS}}{40K}$$

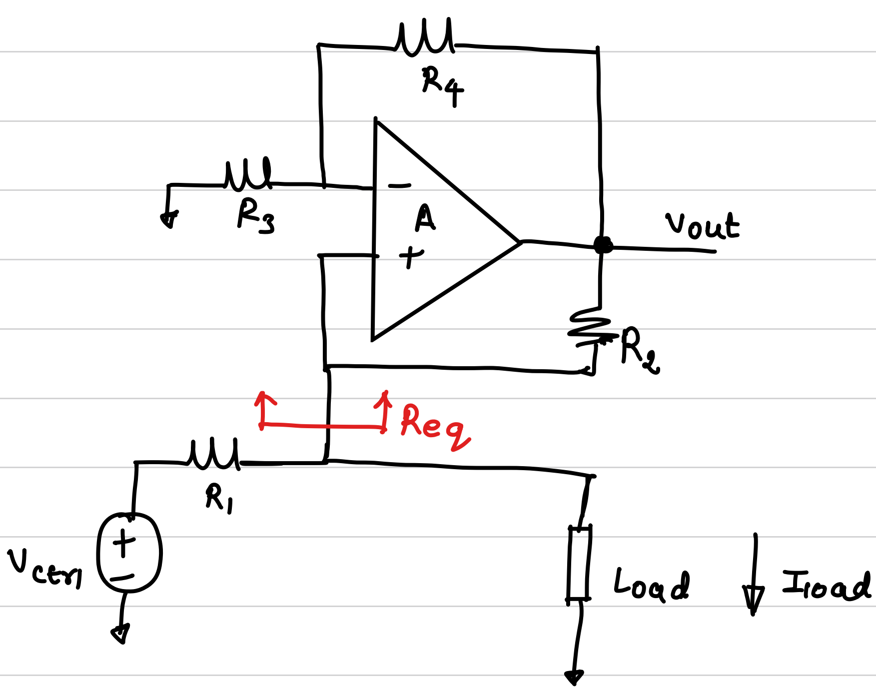
$$= \frac{1.8 - 0.127}{40K}$$

$$= 41.825 \ \mu A$$ - 0.25 marks

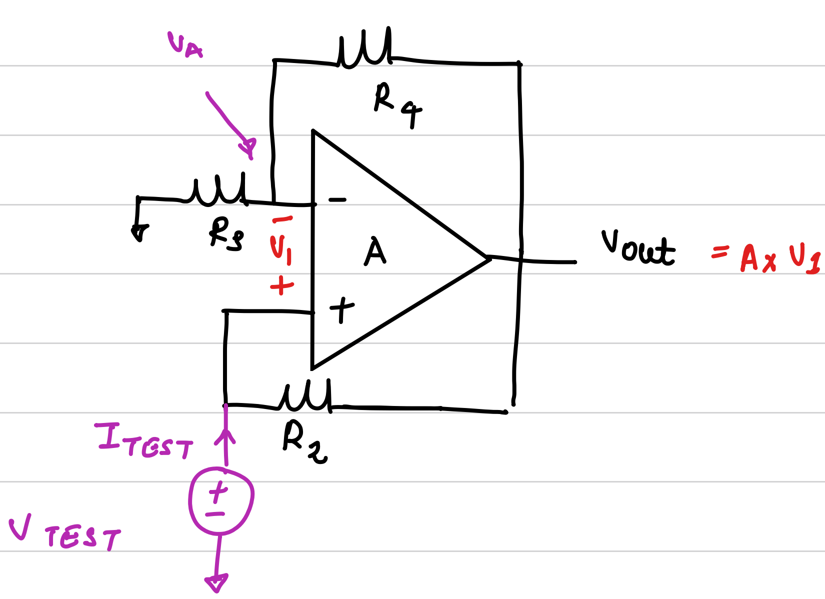
Q2)




a)




We will calculate Req




$V_{out} = A \cdot V_1$ 0.25 marks

$V_A = V_{out} \left( \frac{R_3}{R_3 + R_4} \right)$

$V_{out} = \left( 1 + \frac{R_4}{R_3} \right) V_A$

$= \left( 1 + \frac{R_4}{R_3} \right) (V_{TEST} - V_1)$ – 0.25marks

$$\therefore\ AV_1 = \left(1+\frac{R_4}{R_3}\right)\left(V_{TEST}-V_1\right)$$

$$\therefore\ \left(A+\frac{R_4}{R_3}+1\right)V_1 = \left(1+\frac{R_4}{R_3}\right)V_{TEST}$$

$$\therefore\ V_1 = \left(\frac{1+R_4/R_3}{A+1+\frac{R_4}{R_3}}\right)V_{TEST}$$

$$I_{TEST} = \frac{V_{TEST}-AV_1}{R_2}$$

$$\therefore\ I_{TEST} = \frac{V_{TEST} - A\left(\frac{1+R_4/R_3}{A+1+R_4/R_3}\right)V_{TEST}}{R_2}$$

$$\therefore\ I_{TEST} = \frac{V_{TEST}}{R_2}\left(\frac{A+1+R_4/R_3 - AR_4/R_3 - A}{A+1+\frac{R_4}{R_3}}\right)$$

$$= \frac{V_{TEST}}{R_2}\left(\frac{1+(1-A)R_4/R_3}{1+A+\frac{R_4}{R_3}}\right)$$

$$\boxed{R_{eq} = R_2\left(\frac{1+A+R_4/R_3}{1+(1-A)\frac{R_4}{R_3}}\right)}$$ - 0.5 marks

$$\boxed{R_{eq} = -R_1}$$ - 0.25 marks

0.25 marks for calculating value of $R_2$

b) $$G_{out} = \frac{-R_4}{R_3} \times \frac{1}{R_2} + \frac{1}{R_1} = \frac{1}{R_1}\left(1 - \frac{R_4/R_3}{R_2/R_1}\right)$$ − 0.5 marks

when no mismatch

$$R_4 \longrightarrow R_{40}, \quad R_3 \longrightarrow R_{30} + \Delta R_3, \quad R_2 \longrightarrow R_{20} + \Delta R_2, \quad R_1 \longrightarrow R_{10}$$

also $$\frac{R_{40}}{R_{30}} = \frac{R_{20}}{R_{10}}$$ 0.25 marks

$$G_{out} = \frac{1}{R_1}\left(1 - \frac{\frac{R_{40}}{R_{30}} \times \frac{1}{1-a}}{\frac{R_{20}}{R_{10}}(1+b)}\right)$$

$$= \frac{1}{R_1}\left(1 - \frac{1}{(1-a)(1+b)}\right)$$

$$= \frac{1}{R_1}\left(1 - (1-(b-a))\right)$$

$$G_{out} = \frac{(b-a)}{R_1}$$ 0.5 marks

$$R_{out} = \frac{R_1}{b-a}$$

$$b - a = \epsilon$$
$$0 - 0.05 < \epsilon < 0.04$$ −0.5 marks

$$R_{out} = \frac{R_1}{\epsilon}$$

$$e = \frac{R_1}{R_{out}}$$

so, $-0.05 < \frac{R_1}{R_{out}} < 0.04$

So, $-20 > \frac{R_{out}}{R_1}$ and $\boxed{R_{out} \geq 25 R_1}$ 0.25 marks

$R_{out} < -20 R_1$

$\downarrow$

This is not possible because circuit may become unstable (0.25 marks)

so, worst case error occurs when $\boxed{R_{out} = 25 R_1}$ – 0.25 marks

c) $$I_{load} = \frac{V_{ctrl}}{R_1} \left( \frac{R_{out}}{R_{out} + R_L} \right)$$

$$= \frac{V_{ctrl}}{R_1} \left( \frac{1}{1 + \frac{R_L}{R_{out}}} \right)$$ - 0.5 marks

$V_{ctrl}$ must be between $V_{DD}$ and $V_{SS}$ - 0.25 marks

0.25 marks for finding numerical value.