$\boxed{Q.1}$ Let X be a random variable s.t. $P(X<\infty)=1$, then show that $\lim_{t\uparrow\infty} \frac{X}{t} = 0$ w.p. 1.

$\boxed{Q.2}$ Prove ERT, when $A \neq F$. Write all the steps carefully, and assume $0 < EX_2 < \infty$.

$\boxed{Q.3}$ Prove ERT, when $EX_2 = \infty$. Note that SLLN requires that the random variables are absolutely integrable.

$\boxed{Q.4}$ A critical component of a machine lasts for time $X \sim F$. Two options exists: (a) Replace after failure or (b) Replace after $\min\{X,T\}$. If replacement occurs after failure, cost incurred is C_F, otherwise cost is C_T, where $C_F > C_T$. Replacement is instantaneous. Find T s.t. avg. replacement cost is minimized.

$\boxed{Q.5}$ Define, $X(t) = Z_{m(t)+1} - Z_{m(t)}$. Find

$$\lim_{t\uparrow\infty} \frac{1}{t}\int_0^t X(u)\,du .$$

$\boxed{Q.6}$ Consider the discrete time queue as described in the class, except that the system can hold at most B customers. Arrivals in the full system are turned away. Find the fraction of arrivals that are turned away.