$\boxed{Q.1}$ In a sequence of coin tosses: find the expected number of tosses until we see 10 heads in a row for the first time.

Hint: Find approach that does not require you to maintain 2^{10} states.

How to generalize of length "n" instead of 10?

$\boxed{Q.2}$ Consider a DTMC $\{X_n\}_{n\geq 0}$ with parameters (α, P) on state space $\mathcal{S}$. We say that state j is reachable from state i if $\exists\, n \geq 0$ such that $p_{ij}^{(n)} > 0$ (recall that $p_{ii}^{(0)} = 1 \;\forall\, i \in \mathcal{S}$).

We say that i and j communicate if i is reachable from j and j is reachable from i.

Show that:

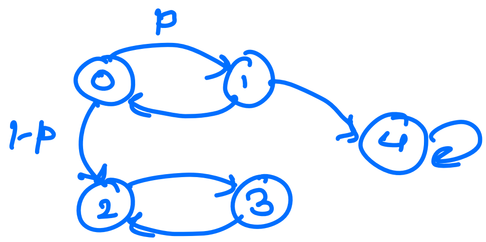
(i) i communicates with i.

(ii) if i communicates with j, then j communicates with i.

(iii) If i communicates with k and k communicates with j, then i communicates with j.

(iv) $\tilde{\mathcal{S}} \subseteq \mathcal{S}$ is called a communicating class if $\forall\, i, j \in \tilde{\mathcal{S}}$, i communicates with j. Show that $\mathcal{S}$ can be partitioned in communicating classes.

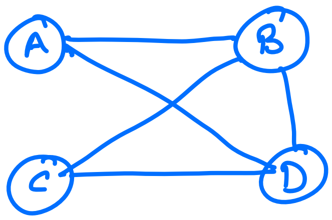
(v) Consider a DTMC with following state transition diagram:



Find the communicating classes.

(vi) Find $f_{00}^{(n)}$ and $f_{22}^{(n)}$. Classify these states.

$\boxed{\text{Q.3}}$ Consider an undirected graph shown below:



Assume that nodes indicate cities. Each day when you wake up, you are equally likely to travel to one of the neighboring city.

Model this as DTMC.

Find ν_{AA}.