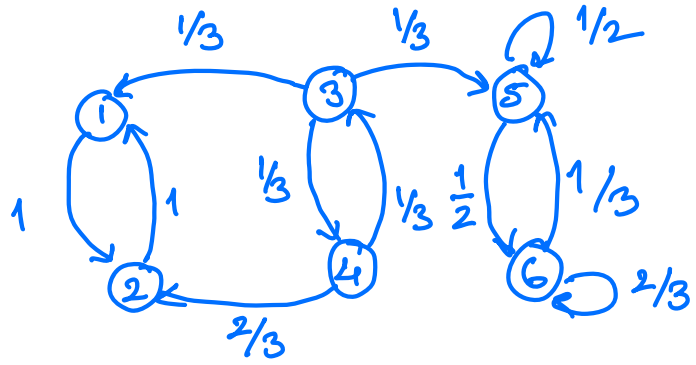
Q.1 Consider a DTMC shown below:



(a) Find communicating classes.

(b) Find prob that the DTMC gets absorbed in communicating class $\{1,2\}$, assuming that initial state is uniformly chosen.

(c) Assuming, $X_0 = 5$ w.p.1. Find fraction of time the DTMC spends in each of the states.

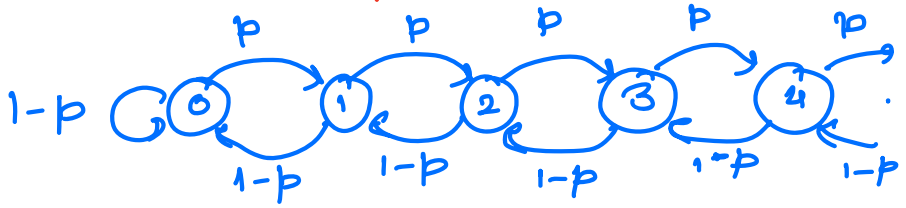
(d) Solve (c) assuming $X_0 = 3$ w.p.1.

Q.2 Consider a DTMC on finite $\mathcal{S}$ with TPM P.

(a) show that $\max_i p_{ij}^{(n+1)} \leq \max_i p_{ij}^{(n)}$

(b) Show that $\min_i p_{ij}^{(n+1)} \geq \min_i p_{ij}^{(n)}$. $\forall\, n \geq 1$.

Q.3 Consider a reflected random walk as shown:

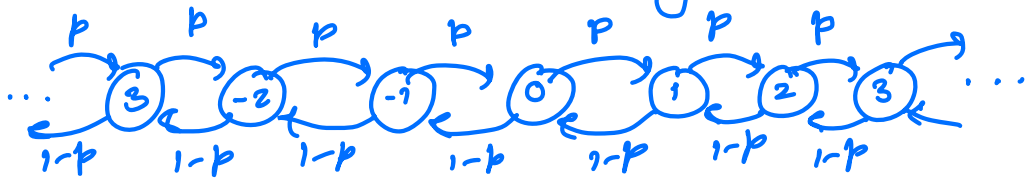


with $p \in (0, 1/2)$.

(a) Find steady state distribution (if exists).

(b) Find the expected time to return to state 0 (ν_{00}).

Q.4 Consider a following random walk:



(a) Use your intuition to find value of p for which the DTMC is recurrent.

(b) Solve $\bar{y} = Q\bar{y}$, to formally prove (a)

(c) Let β_k denote the prob. that the DTMC ever visits state k. Find β_k (Assume $X_0 = 0$).

Q.5 Consider an irreducible DTMC on $\mathcal{S}$ with TPM P. For any $i \in \mathcal{S}$ define

$$D_i := \text{g.c.d.}\{n : f_{ii}^{(n)} > 0\}.$$

Show that $D_i = D_j \quad \forall i, j$.