

Q.1 If CTMC $\{X(t)\}_{t \geq 0}$ is positive recurrent with prob. mass function π on \mathcal{S} satisfying $\pi Q = 0$ and $\pi_j > 0, \forall j \in \mathcal{S}$. Show that

$$\pi_j = \frac{1/a_j}{E[S_j]}, \text{ where}$$

S_j is the life-time of the renewal process defined with respect to visits to state j .

Hint: You can use expression for $E[S_j]$ defined in the class.

Q.2 For a positive recurrent CTMC with steady state prob. π . Show that $\forall \tilde{\mathcal{S}} \subseteq \mathcal{S}$,

$$\sum_{i \in \tilde{\mathcal{S}}} \sum_{j \in \tilde{\mathcal{S}}^c} \pi_i q_{ij} = \sum_{i \in \tilde{\mathcal{S}}^c} \sum_{j \in \tilde{\mathcal{S}}} \pi_i q_{ij}$$

steady state rate from $\tilde{\mathcal{S}}$ to $\tilde{\mathcal{S}}^c$
steady state rate from $\tilde{\mathcal{S}}^c$ to $\tilde{\mathcal{S}}$.

Q.3 For M/M/1 queue with arrival rate $\lambda > 0$ and service rate $\mu > 0$, consider a RP with respect to successive visits to state j . Define, $C(t) = \#$ of transitions from j to $j+1$ until time t .

(a) Find $\lim_{t \rightarrow \infty} C(t)/t$.

(b) Find $\lim_{t \rightarrow \infty} C(t)/\Lambda(t)$, where $\Lambda(t) = \#$ of arrivals until t .

(c) Note that $C(t)/\lambda(t)$ denotes number of arrivals that see the system in state j . Does your answer confirm PASTA?

Q.4 Consider the departure process of $M/M/1$ queue, i.e. Let $D(t)$ denote # of departures until time t .

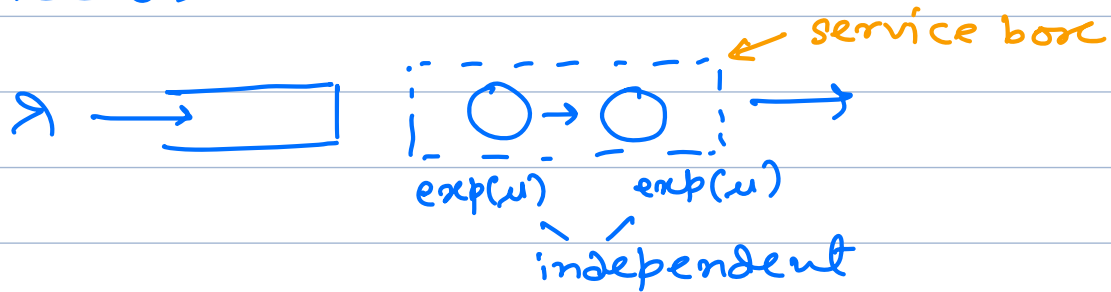
Q.5 Consider two $M/M/1$ queues in tandem as shown below.



Once a customer finishes service in Queue 1, it queues up in Queue 2, and after finishing service there leaves the system.

- Show that the system can be modeled as a CTMC with appropriate state space.
- Find condition under which the CTMC is positive recurrent. Assume this to be true.
- Find avg waiting time for the customers in this system.
- Find waiting time in individual queues.

Q.6 Consider a queueing system with Poisson(λ) arrival process and multistage service as shown below.



Here, one customer enters service box at a time. When in the service box, it finishes service stages one at a time. Upon finishing the second stage, it departs.

(a) Show that the system can be modeled as a CTMC with appropriate state space.

(b) Find avg waiting time in the system.