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22B1232.

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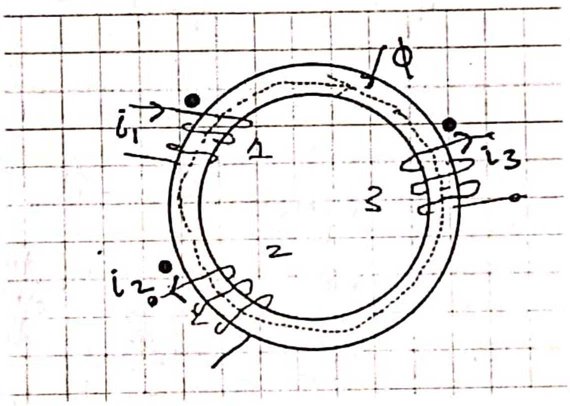
25

Time:30 minutes | *Quiz 03* | Maximum Marks: 25

1. Three coils are wound closely together on a common magnetic circuit. Assume that the permeability is constant and high. Therefore, resultant core flux links all the turns of all windings, and that magnetic leakage between windings is negligible. The winding turns are $N_1$=100 $N_2$=500 $N_3$=100 The winding resistances in ohms are negligible. The self-inductance of winding-1 is 200 H.

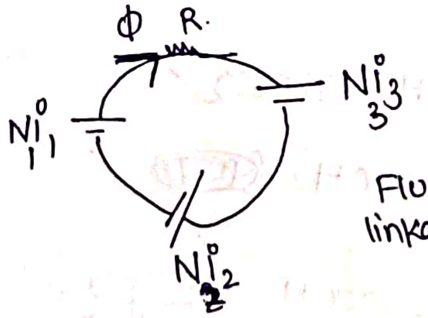



1. Determine self inductance of coil-2 and coil-3.
2. Determine the voltage equation of the circuit write in matrix form.
3. Determine all mutual inductances

[15]

(a). Let R be the reluctance of the core

so, $\phi = \dfrac{N_1 i_1 \pm N_2 i_2 \pm N_3 i_3}{R}$ wb




For coil-1,

Flux linkage, $\lambda = N_1\phi = \left(\frac{N_1^2}{R}\right) i_1 \pm \left(\frac{N_1N_2}{R}\right) i_2 \pm \left(\frac{N_1N_3}{R}\right) i_3$

as $\lambda = L_{11} i_1 + M_{12} i_2 + M_{13} i_3$

$L_{11} = N_1^2/R = 200 \Rightarrow \frac{(100)^2}{R} = 200 \Rightarrow \boxed{R_c = 50 \ H^{-1}}$

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Question 1 continues on the next page...

$v = \frac{d\lambda}{dt}$



Similarly, $L_{22} = \frac{N_2^2}{R} = \frac{(500)^2}{50} = \boxed{5000\ H}$ $\quad L_{33} = \frac{N_3^2}{R} = \frac{(100)^2}{R} = \frac{(100)^2}{50} = \boxed{200\ H}$

(b) $v = \frac{d\phi}{dt}$ $\quad \phi = \frac{N_1 i_1 - N_2 i_2 - N_3 i_3}{R}$ wb.

$$\cancel{v = \frac{d\phi}{dt} = \left(\frac{N_1}{R}\right)\frac{di_1}{dt} + \left(\frac{-N_2}{R}\right)\frac{di_2}{dt} + \left(\frac{-N_3}{R}\right)\frac{di_3}{dt}}$$

Voltage across coil-1, $\cancel{v_1 = \frac{N_1}{R}\left(\frac{di_1}{dt}\right)}$ $\quad \cancel{v_2 = \left(\frac{-N_2}{R}\right)\frac{di_2}{dt}} \quad \cancel{v_3 = \left(\frac{-N_3}{R}\right)\frac{di_3}{dt}}$

$$v_1 = \frac{d\lambda_1}{dt} = \frac{d}{dt}(N_1\phi) = \frac{d}{dt}\left(\frac{N_1^2}{R}i_1 - \frac{N_2^2}{R}i_2 - \frac{N_3^2}{R}i_3\right)$$

$$v_1 = \frac{N_1^2}{R}\left(\frac{di_1}{dt}\right) + \left(\frac{-N_2 N_1}{R}\right)\frac{di_2}{dt} + \left(\frac{-N_3 N_1}{R}\right)\frac{di_3}{dt} = (200)\frac{di_1}{dt} + (-1000)\frac{di_2}{dt} + (-200)\frac{di_3}{dt}$$

Similarly,

$$v_2 = \frac{d\lambda_2}{dt} = -\left(\frac{N_1 N_2}{R}\right)\frac{di_1}{dt} + \left(\frac{N_2^2}{R}\right)\frac{di_2}{dt} + \left(\frac{N_2 N_3}{R}\right)\frac{di_3}{dt} = \left(1000\cdot\frac{di_1}{dt} - 5000\frac{di_2}{dt} - 1000\frac{di_3}{dt}\right.$$

$$v_3 = \frac{d\lambda_3}{dt} = -\left(\frac{N_1 N_3}{R}\right)\frac{di_1}{dt} + \left(\frac{N_2 N_3}{R}\right)\frac{di_2}{dt} + \left(\frac{N_3^2}{R}\right)\frac{di_3}{dt} = \left(200\frac{di_1}{dt} - 1000\frac{di_2}{dt} - 200\frac{di_3}{dt}\right.$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} N_1^2/R & -\frac{N_1 N_2}{R} & -\frac{N_1 N_3}{R} \\ -\frac{N_1 N_2}{R} & +\frac{N_2^2}{R} & +\frac{N_2 N_3}{R} \\ -\frac{N_1 N_3}{R} & +\frac{N_2 N_3}{R} & +\frac{N_3^2}{R} \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ di_3/dt \end{bmatrix}$$

(The dir$^n$ of $\phi$ and EMF are drawn)

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 200 & -1000 & -200 \\ -1000 & +5000 & +1000 \\ -200 & +1000 & +200 \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \\ di_3/dt \end{bmatrix}$$

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(c) $M_{12} = M_{21} = -\frac{N_1 N_2}{R}$ (co-eff. of $i_2$ in $\lambda_1$ or $i_1$ in $\lambda_2$ (magnitude))

$$= -\frac{100 \times 500}{50} = -1000\ H \text{ (mag)}$$

$$M_{23} = M_{32} = \frac{N_2 N_3}{R} = \frac{500 \times 100}{50} = 1000\ H \text{ (mag)}$$

$$M_{13} = M_{31} = -\frac{N_1 N_3}{R} = -\frac{100 \cdot 100}{50} = -200\ H \text{ (mag)}$$

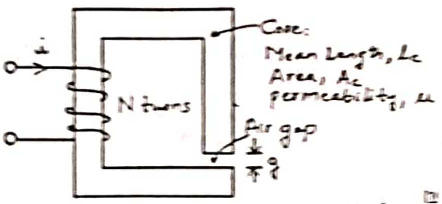
$B = \mu H$

$\frac{Wb}{m^2} = \mu \left( \frac{A\,turns}{m} \right)$ $\quad \frac{m}{\mu A}$

$\left( \mu = \frac{Wb}{m\,A\,turns} \right).$



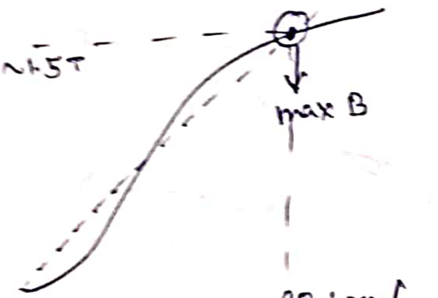
2. From the dc magnetization curve given in figure 1, it is possible to calculate the relative permeability $\mu_r = B_c/(\mu_0 H_c)$ for M-5 electrical steel as a function of the flux level. Assuming the core of the figure ?? to be made of M-5 electrical steel with the dimensions as follow: $A_c = A_g = 9\text{cm}^2$ $g$=0.05 cm $l_c$=30 cm $N$=500. Calculate the maximum flux density such that the reluctance of the core never exceeds 5 percent of the reluctance of the total magnetic circuit. [10]




$B_c/H_c \geq 0.014$

$B_c/H_c = 0.014$

→ approx. graph of $\left( \frac{B_c}{H_c} = 0.014 \right)$

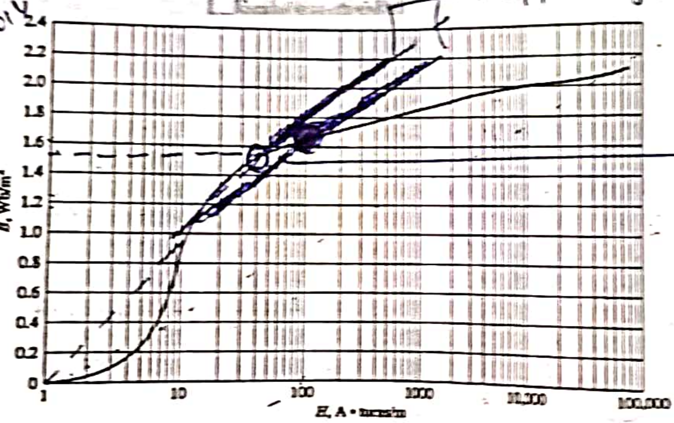


$\frac{B_c}{H_c} \geq 0.014$

~1.5T

max B

~90 Aturns/m.




P.O.I. → gives the max possible flux density (B)

**Figure** (*Armco Inc.*) Dc magnetization curve for M-5 grain-oriented electrical steel 0.012 in thick.

Figure 1: Magnetization Curve M5 material

$A_c = A_g = 9\times10^{-4}\,m^2 \qquad g = 5\times10^{-4}\,m \qquad l_c = 0.3\,m \qquad N = 500$

$R_c \leq 5\%$ of $R_{net}$ $\qquad R_{net} = R_c + R_g$

$R_c \leq 0.05(R_c + R_g)$

$\cancel{0.95 R_c \leq R_g}$

$\cancel{0.95 \times \frac{l_c}{\mu_0 \mu_r A_c} \leq \frac{l_g}{\mu_0 A_g}}$ $\quad \cancel{(A_c = A_g)}$

$0.95 \frac{l_c}{l_g} \leq \mu_r \Rightarrow \cancel{\mu_r \geq 0.95 \left(\frac{l_c}{l_g}\right) = \frac{0.95 \times 0.3}{5\times10^{-4}}}$

$\cancel{\mu_r \geq \frac{0.19}{\cancel{0.95 \times 0.3 \times 10^4}}} = 570$

$\cancel{(\mu_r)_{min} = 570}$



~~For B → max ⇒ $\mu_r$ → min = 570~~

~~(relative)~~

$\frac{wb}{m^2} = \mu \cdot \frac{A\text{-turns}}{m}$



Contd.. $R_c \le 0.05(R_c + R_g) \Rightarrow 0.95R_c \le 0.05 R_g$

$19 R_c \le R_g$ i.e. $19 \cdot \frac{l_c}{(\mu_r \mu_0) A_c} \le \frac{l_g}{\mu_0 A_g}$

$(A_c = A_g)$

so, $\mu_r \geq 19 \cdot \left(\frac{l_c}{l_g}\right) = \frac{19 \times 0.3}{5 \times 10^{-4}} = 11400$

$\boxed{\mu_r \geq 11400}$

so, core permeability, $\mu \geq 11400 \times 4\pi \times 10^{-7}$

$\boxed{\mu_c \geq 0.014}$ (wb/m. A-turns).

i.e. $\frac{B_{core}}{H_{core}} \geq 0.014$

Now, $H_c l_c + H_g l_g = Ni$ and $B_c A_c = B_g A_g$

as $A_c = A_g \Rightarrow B_{core} = B_{gap}$

so, $Ni = \left(\frac{l_c}{\mu_r} H_g + l_g H_g\right)$. i.e. $(\mu_r \cancel{\mu_0} H_{core}) = (H_{gap}) \cancel{\mu_0}$

$\left(H_{core} = \frac{H_{gap}}{\mu_r}\right)$

$H_g = \frac{Ni}{(l_g + l_c/\mu_r)}$

$\phi = \frac{Ni}{(R_c + R_g)} \Rightarrow Ni = \phi(R_c + R_g)$

$B_c = B_g = \mu_0 H_g = \frac{\mu_0 Ni}{(l_g + l_c/\mu_r)}$

$\boxed{\frac{B_c}{H_c} = 0.014.}$ (min.)

$\boxed{B_{core} = 0.014 H_{core}}$

wherever this line intersects the DC magnetization curve at the highest B

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(approx.) $B \approx 1.5\,T$ $H \approx 900$ A-turns/m

$\boxed{\text{highest } B}$

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