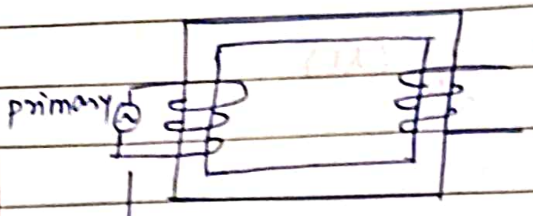
TRANSFORMERS

⇒ two or more windings coupled by mutual magnetic flux.



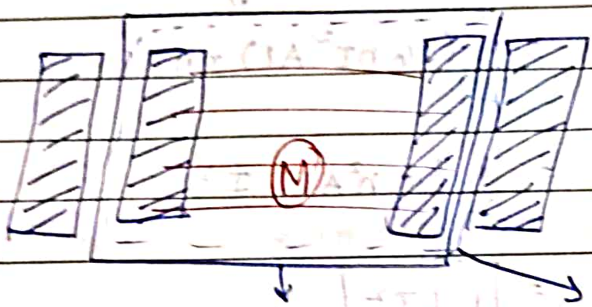
secondary ← induces a voltage

↑

mutual flux links it to the other winding

iron-core transformer

alternating flux → mag. depends on primary voltage



core

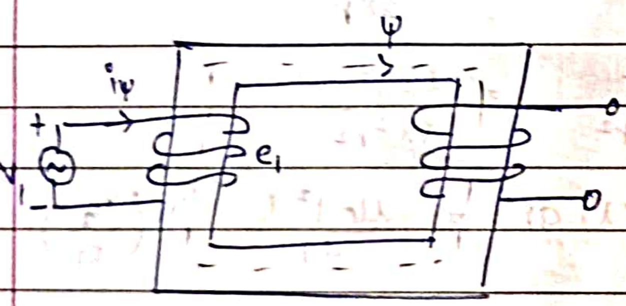
ϕ links both the windings

windings also produce an additional flux: leakage flux

↓

links one winding without linking the other.

NO-LOAD CONDITIONS



(Windings are actually linked)

a small steady state current i_φ → exciting current flows in primary and establishes an alternating flux in the magnetic circuit

The flux induces an eMF in primary. $e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi}{dt}$

$\boxed{e_1 = N_1 \frac{d\phi}{dt}}$ $\boxed{V_1 = R_1 i_\phi + e_1}$

(leakage flux is ignored)

λ: flux linkage

ϕ: flux in core linking winding

N_1: turns

$\varepsilon = \frac{d\phi}{dt}$ $\quad (\varepsilon = \omega N_1 \phi_{max} . \sin(\omega t + 90^\circ))$

IF, $\phi = \phi_{max} \sin \omega t$ (assumed)

$e_1 = N_1 \phi_{max} \omega \cos \omega t$ → leads ϕ by 90°

rms value, $E_1 = \frac{2\pi}{\sqrt{2}} f N_1 \phi_{max}$ $\quad \boxed{E_1 = \sqrt{2}\pi f N_1 \phi_{max}.}$

if $V_1 = E_1$ $(R \to 0)$

$\boxed{\phi_{max.} = \frac{V_1}{\sqrt{2}\pi f N_1}}$ → core flux depends SOLELY on the applied voltage.

If resistance and leakage-inductance voltage drops are negligible ⇒ core flux depends on applied voltage.

⇒ Because of non-linear magnetic properties of iron ⇒ waveform of exciting current is diff. from that of flux.

↓

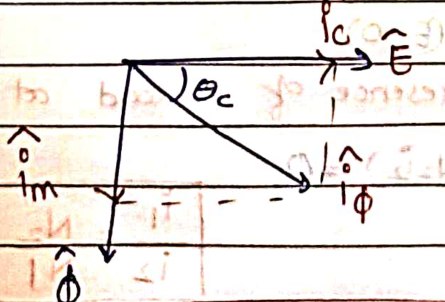
i_ϕ analyzed by Fourier series → Fundamental + Odd Harmonics

- in phase with EMF → core-loss component (I_{he})
  - ↓ supplies the power absorbed by hysteresis & eddy-current losses
- lagging the EMF by 90° → magnetizing current

(core-loss component of exciting current.)

(exciting current) − (core-loss component) = magnetizing current → fundamental lag component + odd harmonics

⇒

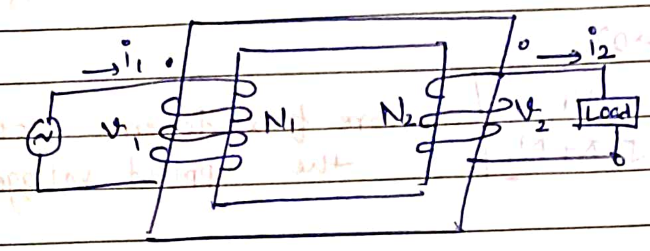


exciting current = sinusoidal current with same ω, RMS value as the exciting current & produces same avg. power

core-loss, $\boxed{P_c = E_1 i_\phi \cos\theta_c}$

$\hat{I}_m \rightarrow$ equivalent sine wave current, in-phase with the flux, having the same RMS value as magnetizing current

SECONDARY CURRENT, IDEAL TRANSFORMER



positive secondary current $\rightarrow$ out of the winding
$\downarrow$
MMF opp. to created by the positive primary current.

IDEAL
- 1) Winding $R \rightarrow 0$
- 2) ϕ confined to core
- 3) all ϕ links both winding (leakage flux $\rightarrow 0$)
- 4) μ is so high $\rightarrow$ negligible exciting MMF to establish the flux.

v_1 (applied) $= e_1 = N_1 \dfrac{d\phi}{dt}$ $\qquad v_2 = e_2 = N_2 \dfrac{d\phi}{dt}$

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}} \longrightarrow \text{Transforms voltage directly in ratio of no. of turns}$$

For a load in secondary, current i_2, MMF $N_2 i_2$

core flux $\longrightarrow$ primary voltage ONLY $\therefore$ independent of load
$\hookrightarrow$ won't change as $\mu \rightarrow \infty$ $(R \rightarrow 0)$ $(E \rightarrow 0)$

core flux is unchanged by presence of a load at secondary
$\hookrightarrow$ exciting MMF $= (N_1 i_1 - N_2 i_2) = 0$

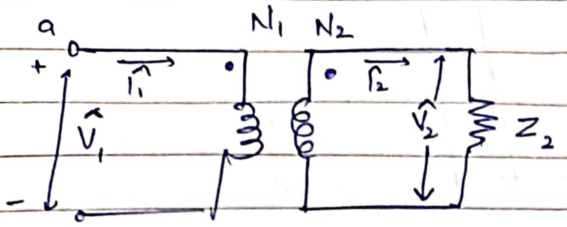
$\Rightarrow \boxed{N_1 i_1 = N_2 i_2}$ $\qquad \boxed{\dfrac{i_1}{i_2} = \dfrac{N_2}{N_1}}$

$\downarrow$

any change in secondary MMF as a result of a load $\Rightarrow$ accompanied by a change in the primary MMF.

$V = \phi R \rightarrow 0$ $(\phi \rightarrow 0)$

Also, $\boxed{V_1 i_1 = V_2 i_2}$ (net power balance ; loss = 0)



voltage across a dot-marked to an unmarked terminal will be of the same instantaneous polarity for primary and secondary.

$\hat{I}_1$ and $\hat{I}_2$ are in-phase $\qquad$ $\hat{V}_1$ and $\hat{V}_2$ are in-phase

$(N_1 i_1 = N_2 i_2)$

into dotted terminal $\qquad$ out of the dotted terminal ⇐ IMP.

$$\hat{V}_1 = \left(\frac{N_1}{N_2}\right)\hat{V}_2 \qquad \hat{V}_2 = \left(\frac{N_2}{N_1}\right)\hat{V}_1$$

$$\hat{I}_1 = \left(\frac{N_2}{N_1}\right)\hat{I}_2 \qquad \hat{I}_2 = \left(\frac{N_1}{N_2}\right)\hat{I}_1$$

$$\boxed{\frac{\hat{V}_1}{\hat{I}_1} = \left(\frac{N_1}{N_2}\right)^2 \frac{\hat{V}_2}{\hat{I}_2}} \Rightarrow \boxed{Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2}$$

⇓

an impedance Z_2 in secondary circuit can be replaced by equivalent Z_1 in primary circuit

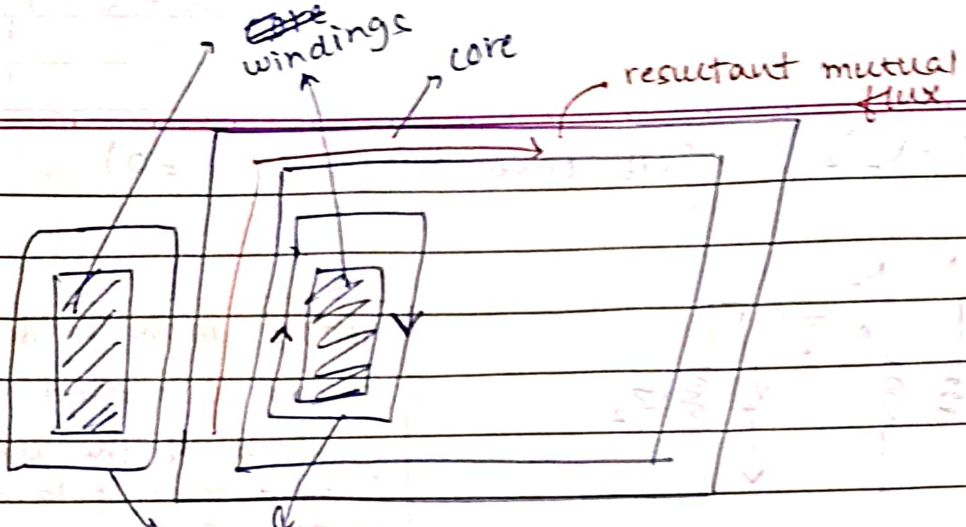
⇒ Voltages → direct ratio ; currents → inverse ratio
impedances → direct ratio squared ; power (V.A.) → unchanged.

ACTUAL TRANSFORMER

winding resistances ; leakage fluxes ; finite exciting currents (finite permeability)

- → equivalent-circuit method
- → theory of magnetically-coupled coils

Primary → Total flux linking the primary winding → resultant mutual flux (iron core)
→ primary leakage flux



primary leakage flux
↳ induces voltage in the ~~primary winding~~ → adds to that produced by the primary flux.

→ As large part of leakage flux is in AIR
↳ FLUX and voltage induced vary linearly with i

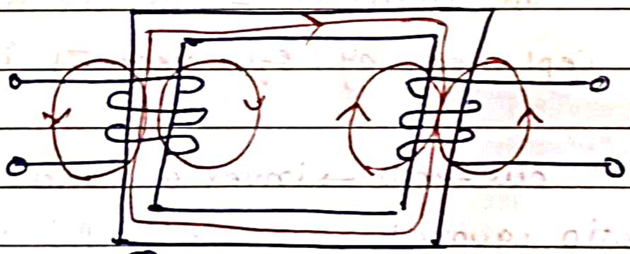
⇓

represented as primary leakage inductance L_1

$$\boxed{L_1 = \frac{\lambda_1}{i_1}}$$

λ_1 → leakage-flux linkage
i_1 → primary current

primary leakage reactance, $\boxed{X_1 = 2\pi f L_1}$ + voltage drop across resistance R_1



(leakage flux adds to primary flux)

CONSIDERING ① COIL ONLY

$$\phi_1 = \phi_{1K-1} + \phi_{12}$$

$$\lambda_1 = N_1\phi_1 = N_1\phi_{1K-1} + N_1\phi_{12} = \lambda_{1K-1} + \lambda_{12}$$

So, $$\frac{d\lambda_1}{dt} = \frac{d\lambda_{1K-1}}{dt} + \frac{d\lambda_{12}}{dt}$$

$$\boxed{L_{self} = \frac{d\lambda_{1K}}{dt} + \frac{d\lambda_{magnetizing}}{dt}}$$

(self inductance) = (leakage inductance) + (magnetizing inductance) considering both the coils :-

$$(\phi_{net})_1 = \phi_{1K-1} + \phi_{12} + \phi_{21}$$
$$(\phi_{net})_2 = \phi_{1K-2} + \phi_{21} + \phi_{12}$$

self inductance

mutual inductance

$$\phi_{1K-1} = \frac{N_1 i_1}{R_{1K-1}} \qquad \phi_{core} = \frac{N_1 i_1}{R_{core}} = \frac{N_1 i_1 \mu A_c}{l_{core}}$$

$\cancel{\phi_{mutual}} =$

$$\boxed{(\phi_{net})_1 = \phi_{magnetizing} + (\phi)_{leakage}}$$

depends on i_1 (primary) current ONLY

$$(\lambda_{net})_1 = \lambda_{1K-1} + \lambda_{11} + \lambda_{12}$$
$$= i_1 L_{self} + i_2 M$$

$$\boxed{(\lambda_{net})_1 = i_1 \left(\frac{\mu_0 N_1^2 A_{air}}{l_{1K-1}} + \frac{\mu_0 N_1^2 A_c}{l_{core}} \right) + \left(\frac{\mu_0 N_1 N_2 A_c}{l_{core}} \right) i_2}$$

and similarly,

$$(\lambda_{net})_2 = i_1 \left(\frac{\mu N_1 N_2 A_c}{l_{core}} \right) + i_2 \left(\frac{\mu_0 N_2^2 A_{air}}{l_{1K-2}} + \frac{\mu_0 N_1 N_2 A_C}{l_{core}} \right)$$

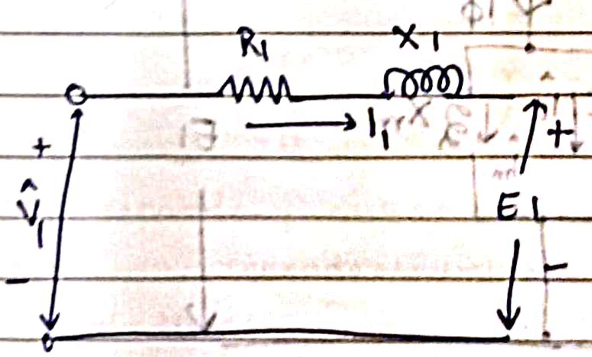
$$Hl = Ni \qquad \lambda = N\phi = NAB \qquad V = \frac{d\lambda}{dt} = \frac{d}{dt}(NAB)$$

$$V = NA\left(\frac{dB}{dt}\right) \qquad B(t) = \frac{1}{NA} \int V(t) \cdot dt$$

---

primary leakage reactance, $X_L = (2\pi f) L_1$ $\qquad L_1 = \frac{\lambda_1}{i_1}$ → leakage flux linkage; i_1 → primary current

EQUIVALENT PRIMARY CIRCUIT :-



$$\hat{V}_1 = \hat{I}_1 R_1 + \hat{I}_1 X_1 + \hat{E}_1$$

resultant mutual flux → links both primary & secondary windings & is created by combined MMF

primary current
→ produce MMF to produce resultant mutual flux
→ counteract effect of secondary to demagnetize the core
→ (supply current to the load of the secondary)

Iprimary, $\hat{I_1} = \hat{I_\phi} + \hat{I_L}$

$\hat{I_\phi}$ → exciting component → produce the core magnetic flux (NON-SINUSOIDAL)

$\hat{I_L}$ → load component $(\hat{I_2'})$ → component current in primary to counteract the MMF of the secondary current $\hat{I_2}$.

$$N_1 \hat{I_\phi} = N_1 \hat{I_1} - N_2 \hat{I_2}$$
$$= N_1 (\hat{I_\phi} + \hat{I_2'}) - (N_2 \hat{I_2})$$

So, $\boxed{\frac{\hat{I_2'}}{\hat{I_2}} = \frac{N_2}{N_1}}$

load component of primary = secondary current referred to primary of an ideal transformer

$\hat{I_\phi}$ → resolve into $\hat{I_m} + \hat{I_c}$

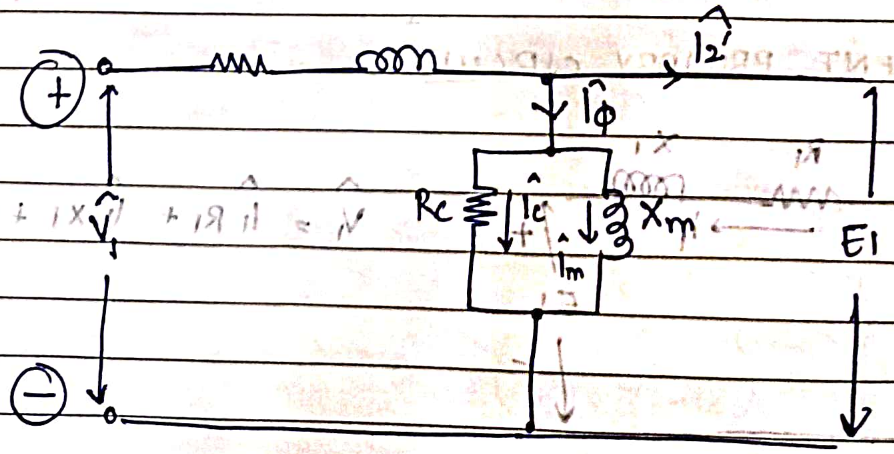
$\hat{I_m}$: lag by 90°

$\hat{I_c}$: in phase with $\hat{E_1}$

magnetizing inductance

$\boxed{X_m = 2\pi f L_m}$

(R_c: core-loss resistance)



The power, $\frac{E_1^2}{R_c}$ → core-loss due to the resultant mutual flux.

R_c → magnetizing resistance/ core-loss resistance

|| impedance of R_c and X_m ⇒ exciting impedance (Z_ϕ)

⇒ If R_c = constt,

loss $\propto E_1^2 \propto \phi_{max}^2 f^2$ (sinusoidal waves)

ϕ_{max} ↓ (max value of the resulting mutual flux.)

X_m → varies with saturation of iron

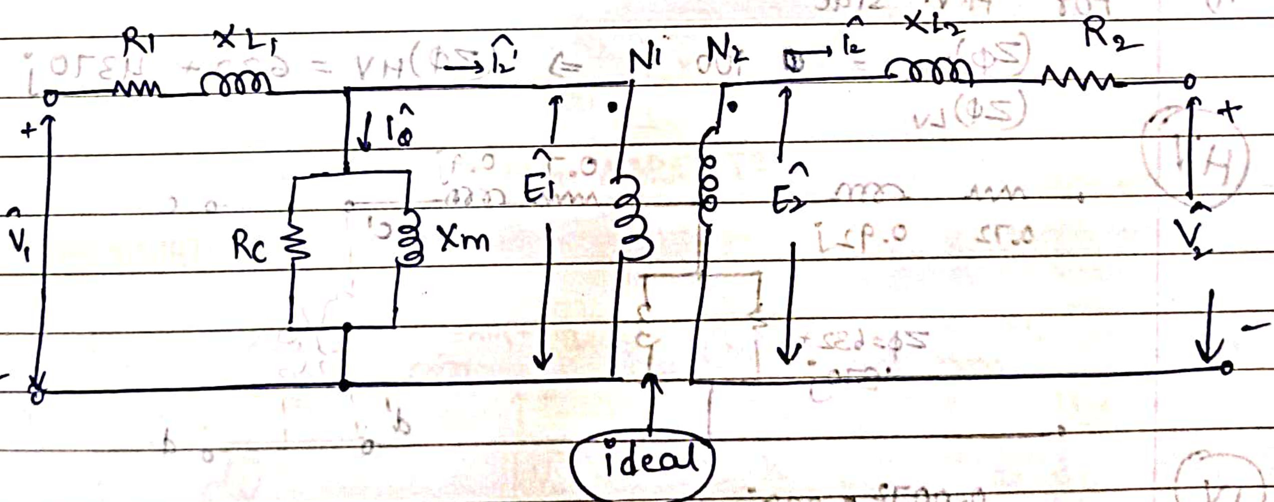
if X_m = constt. ⟹ $\hat{I}_m$ is independent of 'f' and ($\hat{I}_m \propto$ resultant mutual flux)

SECONDARY

resultant mutual flux, ϕ_0 induces an EMF, $\hat{E}_2$ in the secondary and since this flux links both windings,

$$\boxed{\frac{\hat{E}_1}{\hat{E}_2} = \frac{N_1}{N_2}}$$

(ϕ: links both windings) → just as an ideal transformer



$\left(\frac{\hat{I}_2'}{\hat{I}_2} = \frac{N_2}{N_1}\right)$ and $\left(\frac{\hat{E}_1}{\hat{E}_2} = \frac{N_1}{N_2}\right)$

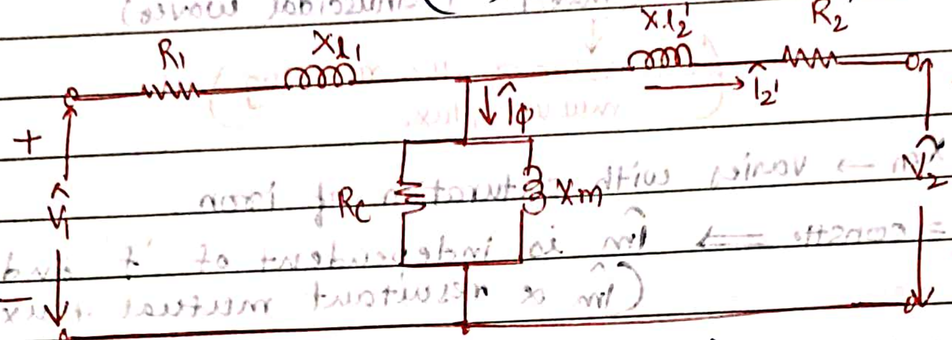
⇒ The secondary current $\hat{I}_2$ also forms a secondary leakage flux → secondary terminal voltage $\hat{V}_2$ differs from the induced voltage $\hat{E}_2$ by voltage drops due to

secondary resistance, R_2 ← → secondary reactance (leakage), $X_{L2} = 2\pi f L_2$ (lkg) so, (Actual transformer) = (Ideal transformer) + (External impedances)

$$Xl_2' = \left(\frac{N_1}{N_2}\right)^2 Xl_2 \qquad R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2$$

(impedances shifted)

$$V_2' = \left(\frac{N_1}{N_2}\right) V_2$$



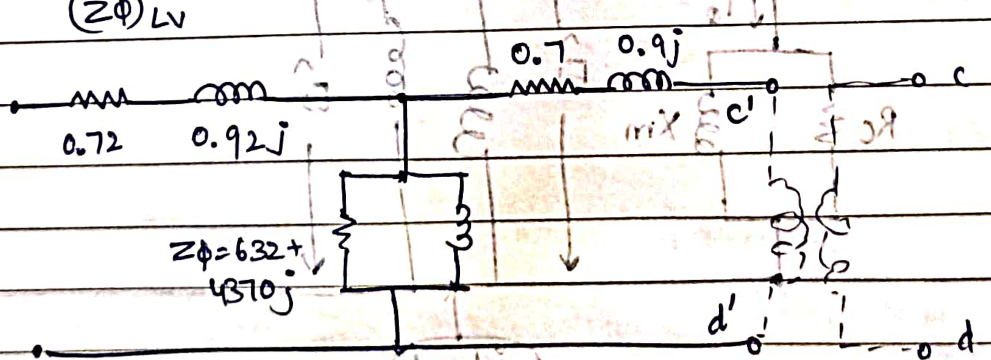
(T-equivalent circuit)

e.g.- 50 KVA, 2400 : 240 V, 60Hz transformer,
leakage impedance = $0.72+0.92j\ \Omega$ in H.V. winding, $0.007+0.009j$ in LV
impedance of exciting branch, $Z\phi = 6.32+43.7j\ \Omega$ viewed from L.V. side

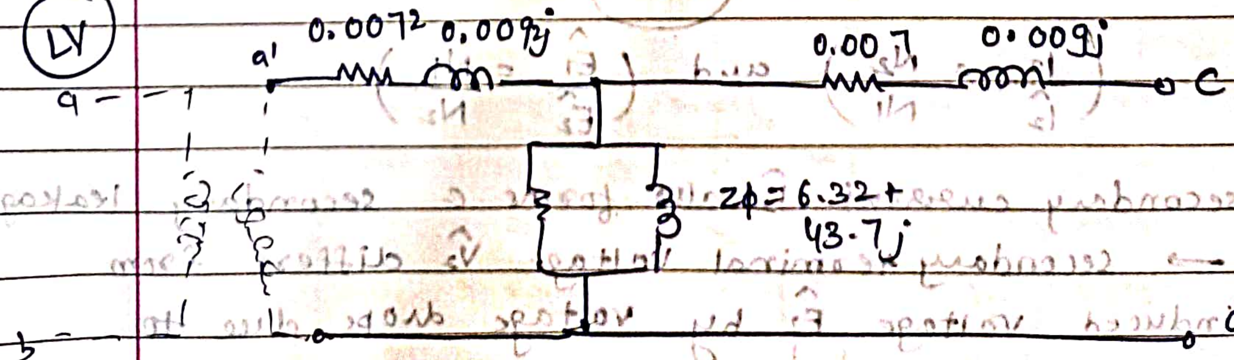
A) For H.V. side

$$\frac{(Z\phi)_{HV}}{(Z\phi)_{LV}} = 100 \Rightarrow (Z\phi)_{HV} = 632 + 4370j$$

HV



LV



Let $2400\angle 0^\circ$ be applied on HV side

so, we should get $2400\angle 0^\circ$ on LV side referred to HV side

so,

$$\cancel{\frac{V-2400}{0.72+0.92j}} + \frac{\cancel{V-2400}}{0.7+0.9j} + \frac{\cancel{V}}{\cancel{632+4370j}} \xrightarrow{(\to 0)} = 0$$

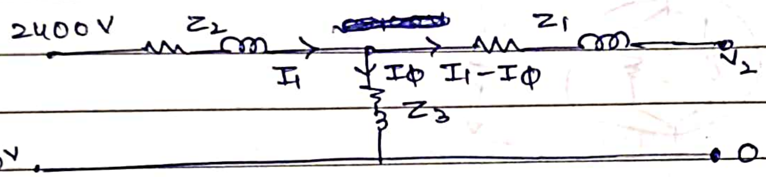
(neglect)

$$\cancel{V-2400\left(\frac{\angle -51.95^\circ}{1.16} + \frac{\angle -65.32^\circ}{1.14}\right) = 0}$$

$$\frac{2400}{(632+4370j)} = \frac{2400}{4415\angle 81.7^\circ} = 0.543\angle -81.7^\circ$$

$$\boxed{I_\phi = 0.543\angle -81.7^\circ \text{ Amp.}}$$

I (LV as ref.) $= 10 \times 0.543 = \boxed{5.43 \text{ A rms}}$



L.V

$$2400 - I_1 Z - I_\phi Z_3 = 0$$
$$V_2 + (I_1 - I_\phi) Z_1 - I_\phi Z_3 = 0$$
$$2400 - I_1 Z - (I_1 - I_\phi) Z_1 = 0$$

variables $\Rightarrow V_2, I_1, I_\phi$

3 eq^n, 3 variables

$\Downarrow$

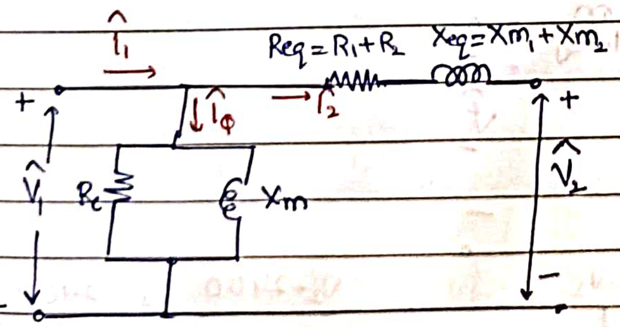
solve

$$\text{regulation} = \left(\frac{2400 - V_2}{2400}\right) \times 100\ \%$$

ENGINEERING ASPECTS

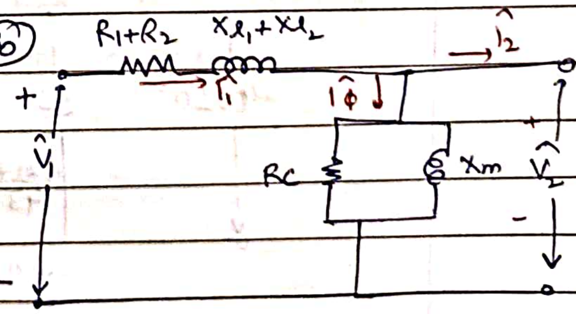
APPROXIMATIONS

(a)

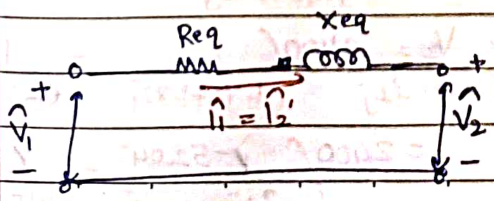


or

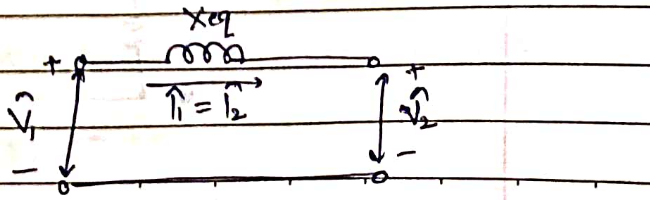
(b)



(c)



(d)

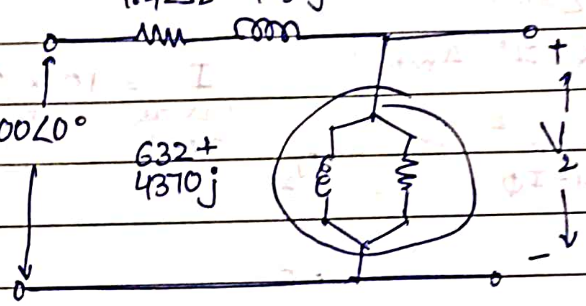


(a) and (b) → move exciting current out of middle to either the primary or secondary side

↳ series resistance is the combined leakage reactance and resistance, referred on same side.

eg.– Use earlier data, Using LV terminal O.C. and 2400 V at the HV terminal, find voltage at LV using (a) and (b) equivalent circuits.

Ans.) (b) ⇒ referred on HV side



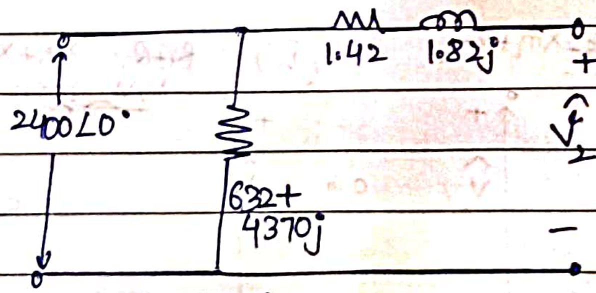
$$V_2 = \frac{2400}{(1.42+1.82j)+(632+4370j)} \times (632+4370j)$$

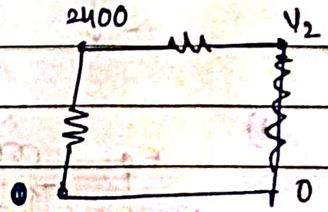
$$= \frac{2400}{(633.42+4371.82j)} \times (632+4370j)$$

$$= \frac{2400 \times 4415\angle 81.7^\circ}{4417.46\angle 81.75^\circ}$$

$$= \boxed{2398.66\ \angle -0.05^\circ}$$

(a) ⇒ referred on HV side





$$\frac{V_2-2400}{1.42+1.82j} + \frac{2400}{632+4370j} = 0$$

$$\frac{V_2}{1.42+1.82j} = 2400\left(\frac{1}{1.42+1.82j} - \frac{1}{632+4370j}\right)$$

$$= 2400\left(\frac{\angle -52.04^\circ}{2.3} - \frac{\angle -81.7^\circ}{4415.46}\right)$$

$$V_2 = 2400\left(1 - \frac{1.42+1.82j}{632+4370j}\right) = 2400\left(1 - \frac{2.3\angle 52^\circ}{4415.46\angle 81.7^\circ}\right)$$

$$= 2400\left(1 - 5.2\times10^{-4}\angle -30^\circ\right)$$

$$= 2400\left(0.9995 + j2.6\times10^{-4}\right)$$

$$= \boxed{2398.8\angle 0.014^\circ}$$

⇒ Some more approximations

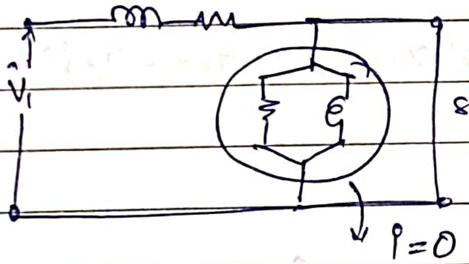
↳ Neglect the exciting current (several hundred KVA or more)

↓

also R_{eq} can be neglected w.r.t. X_{eq}.

→ in ideal cases → neglect everything.

⇒ Using ⓐ and ⓑ → we can determine R_{eq}, X_{eq} using one single test ONLY → SHORT-CIRCUIT ONE TERMINAL



short circuit

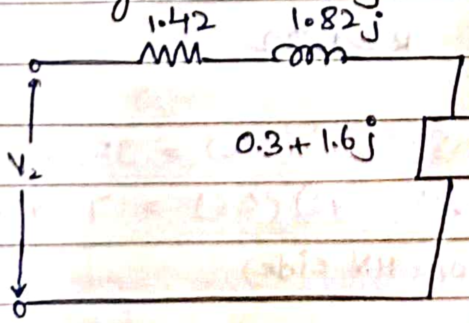
so, $\boxed{\dfrac{\hat{V}_1}{R+jX} = \hat{I}_1}$

eg. 2.5) For same values as earlier eg., 50 KVA, 2400 – 240 V transformer is used to step-down voltage at load end of a feeder with $Z = 0.3+j1.6$. V_{SRC} (sending end) $= 2400$ V

↳ Find voltage at secondary when load draws rated current from transformer at pf = 0.8 lag.

(neglect exciting current voltage drops).

⇒



$Z = (0.3+1.6j)$

pf = 0.8 lag

rated current $= \dfrac{50000}{2400} = 20.8$ A

pf = 0.8 lag

$\boxed{I_{load} = 20.8\angle -37^\circ}$

~~I = 20.8∠-37 Z = 0.3+1.6j~~

~~V = IZ = (20.8∠-37°)(1.63∠79.38°)~~

~~V = 33.904∠42.38°~~

~~(Zload) on HV side = (0.3+1.6j) × 100~~

~~= 30 + 160j~~

~~I = 20.8∠-37°~~

~~Z = 162.79∠79.38°~~

~~so, V =~~

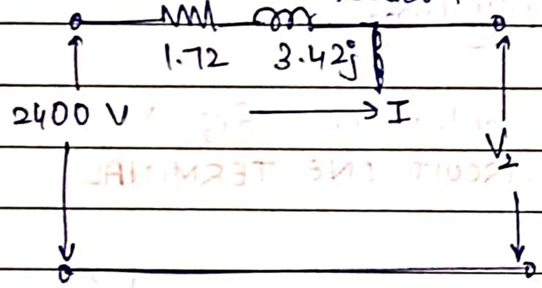
~~$P = \frac{|V|^2}{Z}$~~

~~$|V|^2 = (50000\angle\ )(1.62\angle\ )$~~

~~V = 285.29 V~~

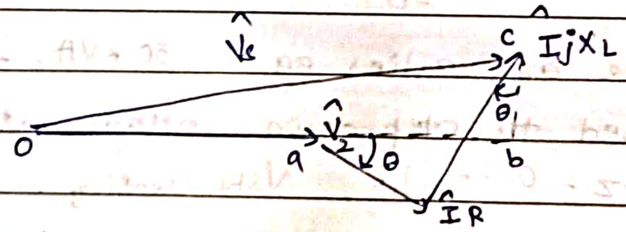
load current on HV side = $\frac{50000}{2400}$ = 20.8 A rms
(rated)

feeder + leakage = 0.3 + 1.6j
+ (1.42 + 1.82j)
= 1.72 + 3.42j



M1) $I = 20.8\angle -37°$ $V_2 = 2400 - I(1.72 + 3.42j)$

M2) $\boxed{\text{PHASORS}}$



$ob = \sqrt{V_s^2 - (bc)^2}$ and $V_2 = ob - ab$

$bc = IX\cos\theta - IR\sin\theta$ $ab = IR\cos\theta + IX\sin\theta$

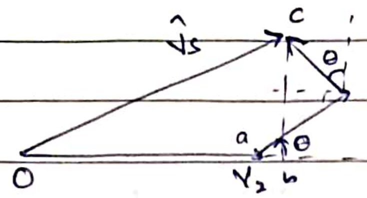
$\theta = +36.87°$ $X = 3.42$ $R = 1.72$

$bc = (20.8)(3.42)(0.8) - (20.8)(1.72)(0.6) = 35.5V$

$ab = (20.8)(1.72)(0.8) + (20.8)(3.42)(0.6) = 71.4V$

and so, $V_2 = 2330V$ (on HV side)

$\boxed{V_{load} = 233\,V}$ (referred on LV side) if pf = 0.8 leading



$ab = IR\cos\theta - IX\sin\theta$
$bc = IX\cos\theta + IR\sin\theta$ } (exchange signs)

$\boxed{Ob = \sqrt{V_s^2 - (bc)^2}}$ $Ob = V_2 + ab$

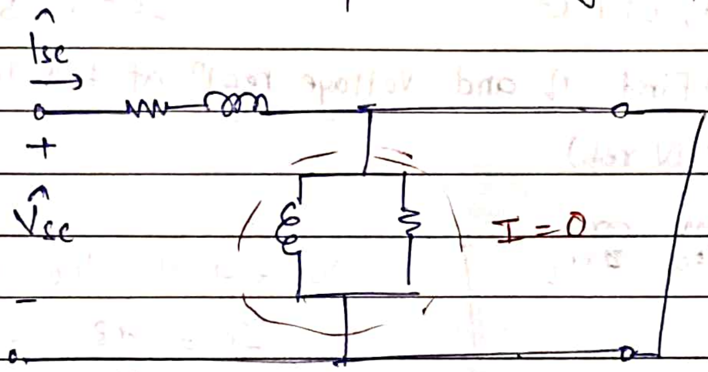
$\boxed{V_2 = Ob - ab}$

SHORT-CIRCUIT TEST

→ Find $R_{eq} + jX_{eq}$

→ short-circuit on the secondary and voltage applied on primary.

(referred on primary side)



$$Z_{sc} = (R_1 + R_2) + j(X_1 + X_2) = R_{eq} + jX_{eq}.$$

instruments → measure V_{sc}, I_{sc}, P_{sc}

$$|Z_{eq}| = \boxed{|Z_{sc}| = \frac{|V_{sc}|}{|I_{sc}|}} \qquad R_{eq} = \boxed{R_{sc} = \frac{P_{sc}}{I_{sc}^2}}$$

$$\boxed{X_{sc} = \sqrt{|Z_{sc}|^2 - R_{sc}^2}}$$

and assume $R_1 = R_2 = \frac{R_{eq}}{2}$ and $X_{l_1} = X_{l_2} = \frac{X_{eq}}{2}$

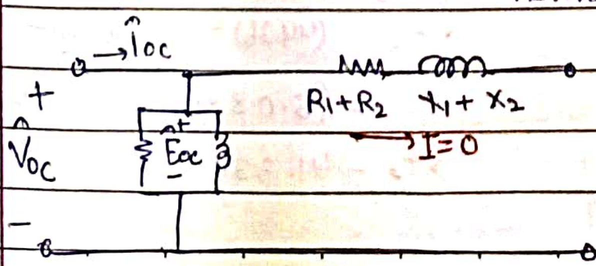
(for all impedances referred on one side)

OPEN CIRCUIT TEST

⇒ secondary open-circuited and rated voltage on primary.

LV SIDE (taken as primary)

magnetizing reactance will be operating at a ~~flux level close to which will exist under~~ normal operating conditions.



$\hat{V}_{oc} \approx \hat{E}_{oc}$ (EMF induced by core flux)

$I_{oc}^2 R_1 \to 0$ (negligible loss due to exciting current)

$$\boxed{P_{oc} = \frac{E_{oc}^2}{R_c}} \qquad \boxed{Z_{oc} = \frac{R_c(jX_m)}{(R_c + jX_m)}}$$

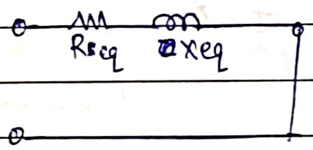
$$R_c = \frac{V_{oc}^2}{P_{oc}} \qquad |Z_\phi| = \frac{|V_{oc}|}{|I_{oc}|} \qquad X_m = \frac{1}{\sqrt{\left(\frac{1}{|Z_\phi|}\right)^2 - \left(\frac{1}{R_c}\right)^2}}$$

eg.- LV side short-circuited, instruments on the HV side
48V, 20.8A, 617W

open-circuit test, LV side energized, readings on LV side
240 V, 5.41 A, 186 W

⇒ Find η and voltage reg^n at full load, 0.8 pf lag.

Ans.) S.C. test (HV ref.)



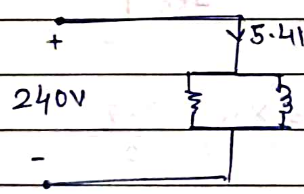
$V_{sc} = 48V \quad I_{sc} = 20.8A$

$Z_{sc} = \frac{48}{20.8} = 2.31\,\Omega$

$P = |I|^2 R_{eq} \qquad R_{eq} = \frac{617}{(20.8)^2} = \boxed{1.43\,\Omega}$

$\boxed{X_{eq} = 1.81\,\Omega}$

O.C. test (LV ref.)



$V_{oc} = 240 \quad I_{oc} = 5.41\,A \quad P = 186\,W$

$|Z| = \frac{240}{5.41} = 44.36$

$P = I_{oc}^2 R_{oc}$

$R_c = \frac{186}{(5.41)^2} = \boxed{6.36\,\Omega}$ (LV)

$\boxed{X_m = 6.42j\,\Omega}$ (LV)

$\frac{1}{X^2} = \cancel{\frac{1}{|Z|^2 - R^2}}$

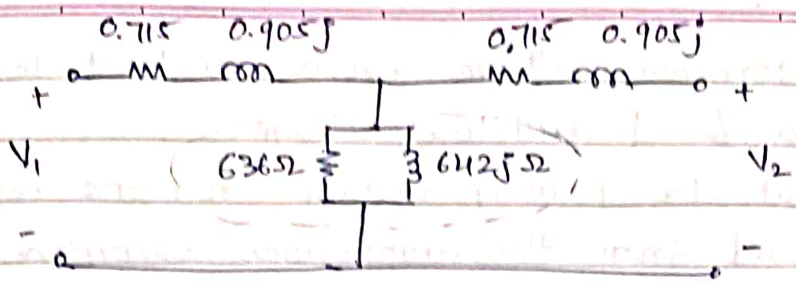
$\frac{1}{|Z|^2} = \frac{1}{R^2} + \frac{1}{X^2}$

$\frac{1}{X^2} = \cancel{\frac{1}{5.08\times10^{-4}}}$

$\frac{1}{X^2} = \frac{1}{(44.36)^2} - \frac{1}{(6.36)^2}$

$= (5.08\times10^{-4}) - (0.0247)$

$X^2 = -41.336$



$\frac{N_1}{N_2} = 10$

load at full-rated power = 50 KVA at 0.8 pf lag

$V_2 = 2400\angle 0°$ (scaled up)

$10^3 \times 50\angle +37° = 2400\angle 0° \cdot I^*$

$\boxed{I_{rated} = 20.8\angle -37°}$

$V_1 - (1.43 + j1.81)(20.8\angle -37°) = 2400$

$V_1 = 2400 + (2.3 \times 20.8)\angle 51.69 - 37°$

$= 2400 + 47.84\angle 14.69°$

$= 2400 + 46.27 + j12.13$

$V_1 = 2446.27 + 12.13j = 2446.3\angle 4.9 \times 10^{-3}°$

$\boxed{V_1 = 2446.3V}$

Voltage regulation $= \left(\frac{2446.3 - 2400}{2400}\right) \times 100 = 1.93\%$

efficiency $= \left(\frac{2400}{2446.3}\right) \times 100 = 98.11\%$

M2> "Shorter"

From S.C. test $\Rightarrow$ $|Z_{eq,H}| = \frac{48}{20.8} = 2.31\Omega$ $R_{eq,H} = \frac{617}{20.8^2} = 1.42\Omega$

$X_{eq,H} = 1.82\Omega$

at full-load operation,

$I_H = \frac{50000}{2400} = 20.8A$

$P_{output} = P_{load} = (0.8)\,50000 = 40,000W$ $\boxed{P_{output} = (\cos\theta)\,P_{full}}$

$P_{winding} = I_H^2 R_{eq,H} = 617W$ $\boxed{P_{winding} = P_{sc}}$

core loss, from O-C test $\Rightarrow$ $P_{core} = 186W$ $\boxed{P_{core\text{-}loss} = P_{oc}}$

$P_{loss} = P_{sc} + P_{oc} = 803W \Rightarrow P_{input} = P_{output} + P_{loss}$

$$\boxed{\text{efficiency} = \left(\frac{P_{out}}{P_{out} + P_{sc} + P_{oc}}\right) \times 100\ \%}$$

Regulation

→ change in secondary terminal voltage from no-load to full-load and expressed as a % of full-load voltage.

→ under assumption that the primary voltage remains constant as the load is removed from the transformer secondary

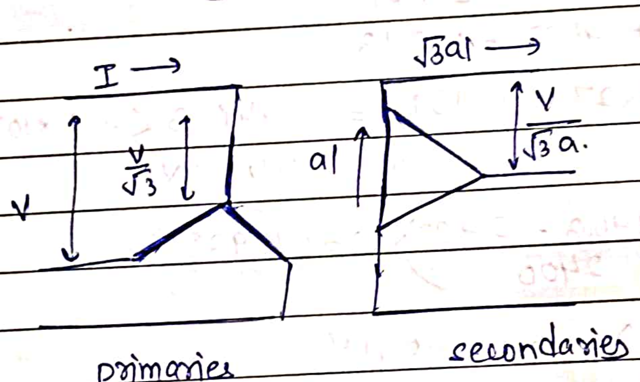
$$\boxed{\text{Regulation} = \left(\frac{|V|_{FL} - |V|_{NL}}{|V|_{NL}}\right) \times 100}$$

(same as rated)

(rated + IZ)

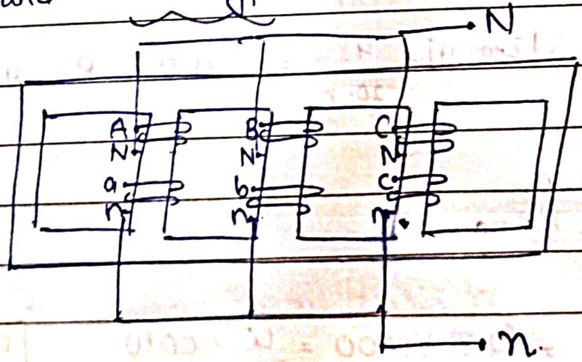
THREE-PHASE TRANSFORMERS

→ connect 3 single-phase transformers
(Y-Δ Δ-Δ Y-Y Δ-Y)

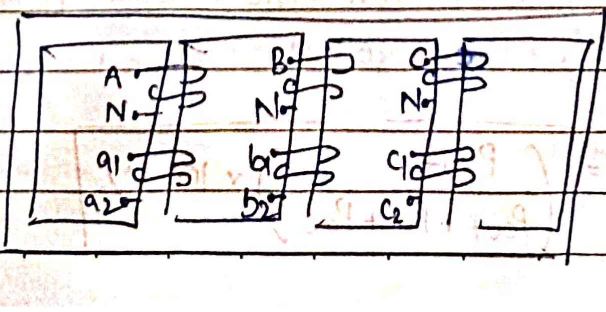


(parallel lines) → (Pri. Sec.)

⇒ core-type and shell-type

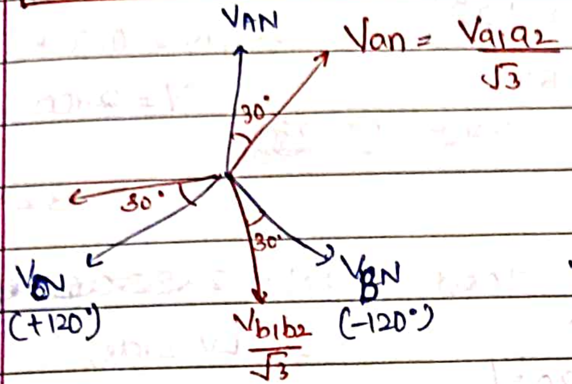


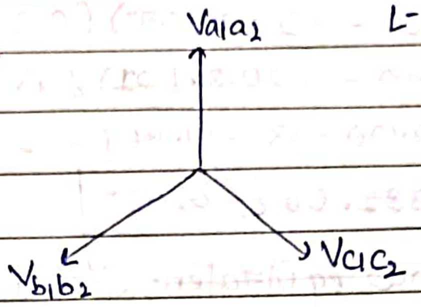
windings in the same side of the core are in the same phase



$V_{AN} \rightarrow V_{a_1a_2}$ $V_{BN} \rightarrow V_{b_1b_2}$ $V_{CN} \rightarrow V_{c_1c_2}$

$$\boxed{\begin{array}{ll} \text{in } Y: \; V_{LL} = V_{LN}\sqrt{3}\,\angle 30^\circ & ; \; I_{LL} = I_\phi \\ \text{in } \Delta: \; V_{LL} = V_\phi & ; \; I_{LL} = I_\phi\sqrt{3}\,\angle -30^\circ \end{array}}$$



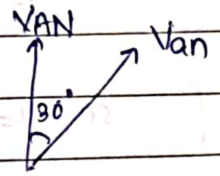


L-L Voltages

$V_{a_1a_2} = V_{AN}$

$\Delta \to Y$ shift : L-L same

$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \angle -30^\circ$



$$\boxed{V_{an} = \frac{V_{AN}}{\sqrt{3}} \angle -30^\circ}$$

if $N_{a_1a_2} = N_{AN} \Rightarrow V_{AN} = V_{a_1a_2}$ i.e. $\boxed{V_{AN} = \sqrt{3}\, V_{an} \angle 30^\circ}$

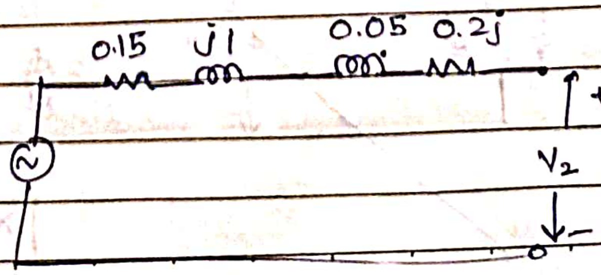
Yd $\boxed{11}$

primary secondary shift as seen by a clock.

e.g.- 3 single-phase 50 KVA 2400-240 V transformers, Y-Δ connected ; 3 phase 150 KVA step-down bank to step down voltage at load end of a feeder whose $Z = 0.15 + j1.00\,\Omega$/phase. Voltage at sending end is 4160 L-L. on secondary side, the transformers supply a balanced load through a feeder with $Z = 0.0005 + j\,0.0020\,\Omega$/phase. Find L-L voltage at load end when load draws rated current from transformers at pf = 0.8 lag.

Ans.) Single-phase ; HV side. $V_{LL} = 4160 \angle 30^\circ$ $V_{LN} = \frac{4160}{\sqrt{3}} \angle 0^\circ$ assume

$V_{LN} = 2400$ V



~~at load end → full-load, pf = 0.8 (50KVA)~~

~~rated current = 50000 / 2400~~ = 20.8 ∠−37°

rated current on HV side $= \frac{50 \times 10^3}{2400} = 20.8$ A/phase

LV feeder impedance on HV side $= \left(\frac{2400}{240/\sqrt{3}}\right)^2 (0.0005 + j0.0020)$

$= 0.15 + j0.60\,\Omega$

~~$V = 2400 - (20.8\angle -37°)(0.2 + 1.2j)$~~

~~$= 2400 - (20.8 \times 1.22)\angle 43.5°$~~

~~$= 2400 - 18 - 17.46j = 2382 - 17.46j$~~

~~$\boxed{V = 2382.06 \angle -0.42°}$~~

$(Z_{net})_{HV} = 0.3 + j1.6\,\Omega$

$V = 2400 - (20.8\angle -37°)(0.3 + 1.6j)$

$= 2329\text{ V}$

~~so, for the equivalent Y on load, $V_{LN} = 2382.06\angle 0°$~~

~~so $V_{LL} = V_{LN}\sqrt{3}\angle 30°$~~

~~$\boxed{V_{LL} = 4125.84 \angle 30°}$~~

~~on the LV side,~~

~~$\boxed{V_{LL} = 412.58 \angle 30°}$~~

on LV side,

$(2329) \times \left(\frac{240}{2400\sqrt{3}}\right) = 134$ V (L-N)

so, $V = 134\sqrt{3} = \boxed{233\text{V}}$ line-to-line

---

eg. (oc and sc Test)

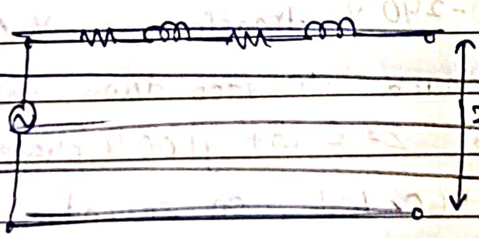
20 KVA, 8000/240 V, 60 Hz

$V_{oc} = 240$ V $\qquad V_{sc} = 489$ V

$I_{oc} = 7.133$ A $\qquad I_{sc} = 2.5$ A

$P_{oc} = 400$ W $\qquad P_{sc} = 240$ W

A) Short ~~Open~~-circuit Test (LV side).



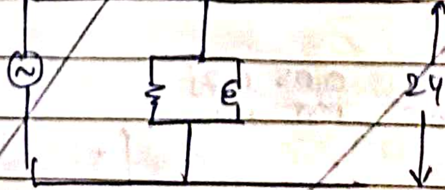
~~$|Z| = \frac{V_{oc}}{I_{oc}} = 33.65\,\Omega$~~

~~$\frac{400}{(7.13)^2} = R_{eq} = \boxed{7.87\,\Omega}$~~

~~$(33.65)^2 = (7.87)^2 + (X_L)^2$~~ $\boxed{(X_L) = 32.71\,\Omega}$

(HV) side

B Open-circuit Test



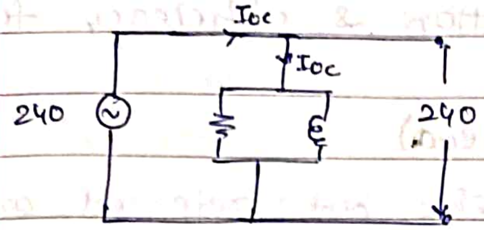
$Z_{oc} = 33.65\,\Omega$

$R_{oc} = \frac{400}{(7.133)^2} = \boxed{7.87\,\Omega}$

$X = \frac{1}{\sqrt{\frac{1}{(33.65)^2} - \frac{1}{(7.87)^2}}}$

$\boxed{X_m = 8.09j\,\Omega}$

Open-circuit test



$|Z| = \cancel{\frac{V_{oc}}{I_{oc}}} =$

$P_{oc} = \frac{V_{oc}^2}{R_c}$ $\quad R_c = \frac{(240)^2}{400} = 144\Omega$

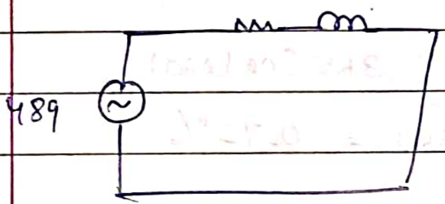
$Z_{oc} = \frac{(R)(jX)}{(R+jX)} = \frac{V_{oc}}{I_{oc}}$

$|Z_{oc}| = \frac{|V_{oc}|}{|I_{oc}|} = 33.65\Omega$

$\frac{1}{Z} = \frac{1}{R} - \frac{j}{X}$ $\quad \left|\frac{1}{Z}\right|^2 = \sqrt{\frac{1}{R^2} + \frac{1}{X^2}}$

$\frac{1}{(33.65)^2} = \frac{1}{(144)^2} + \frac{1}{(X)^2} \Rightarrow \boxed{X = 34.63\Omega}$

Short-circuit Test



$I_{sc} = 2.5\ A$ $\quad P_{sc} = 240W$ $\quad V_{sc} = 489\ V$

$Z_{sc} = \frac{489}{2.5} = 195.6\Omega$

$\boxed{X = 191.79\Omega}$ $\quad R_c = \frac{240}{(2.5)^2} = \boxed{38.4\Omega}$

eg.- The nameplate on a 50 MVA, 60 Hz single-phase transformer indicates that it has a voltage rating 8KV : 78 KV.
Open-circuit test → 8KV, 62.1 A, 206 KW (LV side)
short-circuit test → 674V, 6.25KA, 187 KW (LV side)

Ans.)

S.C

$|Z_{eq,LV}| = \frac{|V_{sc,L}|}{|I_{sc,L}|} = 107.8\ m\Omega$ $\quad X_{eq,LV} = \sqrt{Z_{LV}^2 - R_{LV}^2} = 107.7\ m\Omega$

$|R_{eq,LV}| = \frac{P_{sc}}{|I_{sc}|^2} = 4.78\ m\Omega$ $\quad \boxed{(Z_{eq})_{LV} = (4.8 + 108j)\ m\Omega}$

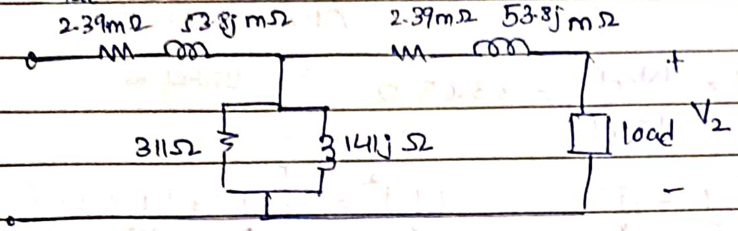
O.C

$R_{c,LV} = \frac{V_{oc}^2}{P_{oc}} = 311\Omega$ $\quad Z = \frac{V_{oc}}{I_{oc}} = 128.82\ \Omega$

$X_m = \frac{1}{\sqrt{\left(\frac{1}{Z}\right)^2 - \left(\frac{1}{R}\right)^2}} = 141\Omega$

⇒ If a load is connected, find regulation & efficiency for rated voltage and load i) (unity pf)
ii) (0.9 pf lead)

Ans) Here, load is connected on 78 KV side but referred on the 8 KV side



rated Low-voltage current, $I_{rated} = \frac{50\times10^6}{8\times10^3} = 6.25\ KA$

and so, $V_L = |V_{load} + Z_{eq,L} \cdot I_{rated}|$
$= 8KV + (\quad)(I\angle 0°)$

$V_L = 8.058\ KV$ (full load) 8 KV (no load)

and so, regulation $= \left(\frac{8.058 - 8}{8}\right) \times 100 = 0.72\%$

$$\eta_{eff} = \frac{P_{out}}{P_{in}} = \frac{50\times10^6}{50\times10^6 + (187+206)\times10^3} = \boxed{99.2\%}$$

⇒ (ii) $I_L = 6.25\angle 25.8°$

3-PHASE TRANSFORMERS

⇒ carry out computations on a single-phase (per-phase Y, L-N) basis

$$Z_Y = \frac{1}{3} Z_\Delta$$

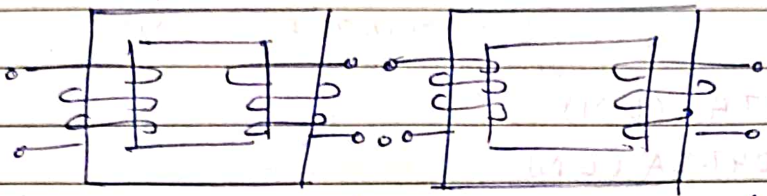
e.g.) The 3 phase-transformer of earlier example are Δ-Δ connected and supplied with 2400 V (line to line) 3-phase feeder with reactance 0.80 Ω/phase. At its sending end, it is connected to the secondary terminals of a 3-phase Y-Δ transformer with rating 500 KVA, 24 KV : 2400 V (line-to-line). The equivalent impedance of the sending-end transformer is 0.17 + j0.92 Ω/phase referred on 2400 V side. The voltage at primary terminals of sending-end transformer is 24 KV (line-to-line).

A 3-phase short circuit occurs at the terminals of 240V receiving-end transformers. Find

1) steady-state Isc in 2400V feeder phase wires, primary and secondary windings of receiver-end transformers and at the 240-V terminals.

$\boxed{Y-\Delta-\Delta}$

24KV : 2400 V   2400V : 240V



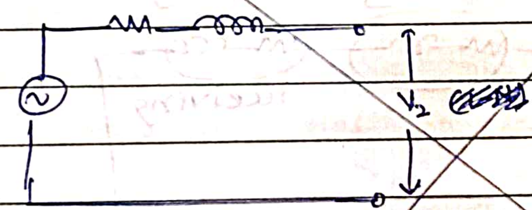
Y   Δ   Δ

i) Y-Δ transformer

on the 24 KV (L-L) side,

$$Z = \left(\frac{24\times10^3}{2400}\right)^2 (0.17+0.92j)\ \Omega/\text{phase}$$

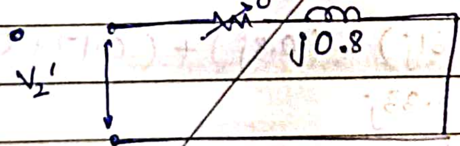
$$Z = (17+92j)\ \Omega$$



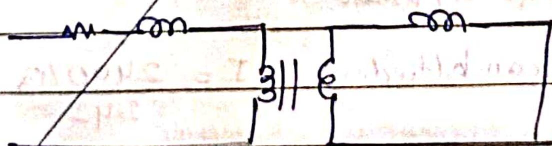
$I_{rated\,(HV)} = \frac{500\times10^3}{24\times10^3}$

$= 20.8\text{ A}$

$I = \frac{V_2'}{0.8j}$

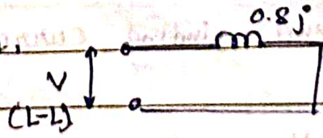


short circuit on 240V side



short circuit

For the Δ-Δ transformer,



$I = \frac{V}{0.8j}$

current has a phase of 90°

So, $V'_{(L-L)} = 10\text{ V}$ $I'_{(L-L)} = \frac{V}{8j}$

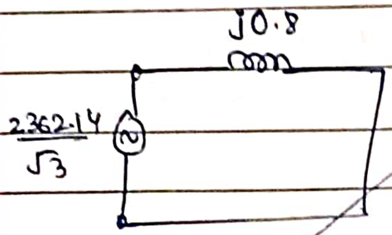
$V'_{(L-N)} = \frac{10V}{\sqrt{3}}$ $I'_{(L-N)} = \frac{V}{8j}$

$\frac{10^3 \times 24}{\sqrt{3}} - (20.8\angle)(17+92j) = 13502.81 - j1913.6$

$= 13637.73\angle 8.02^\circ$

$V_2(L-L) = 13637\sqrt{3}$

$\boxed{V_{2\,(L-L)} = 2362.14\ V}$ → input to the second transformer



$I = \frac{2362.14/\sqrt{3}}{0.8j}$

$I = 1704.72\ A$ (L-N)
(2400V)

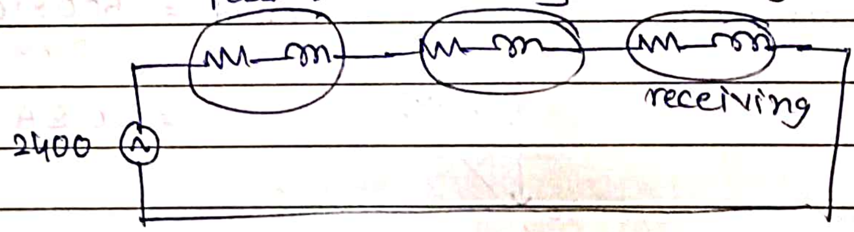
$I_{secondary} = 170.47\ A$ (L-N)

$I_{feeder} = 17047.2\ A$ (L-N)

all references to 2400V side

source voltage $= \frac{2400}{\sqrt{3}}$ (LN) = 1385 V

single-phase equivalent Δ-Δ has $Z = \frac{1.42 + j1.82}{3} = 0.47 + j0.61\ \Omega/\text{phase}$



so, $Z = (0.47 + 0.61j) + (0.8j) + (0.17 + 0.92j)$

$= 0.64 + 2.33j$

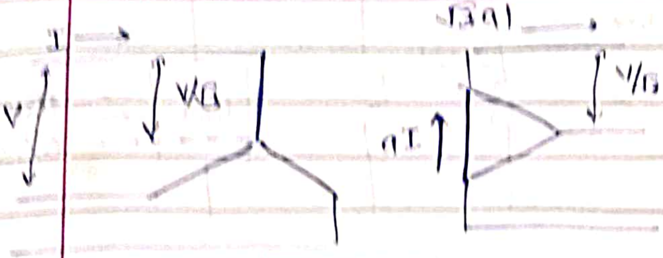
$|Z| = 2.42\ \Omega/\text{phase}$

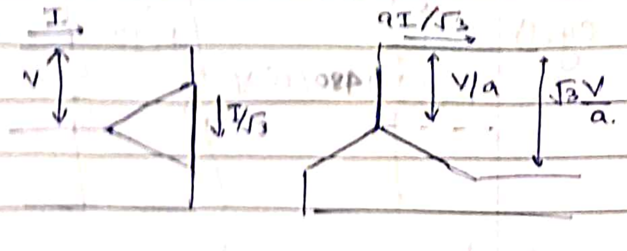
in 2400V branch (feeder) $I = \frac{2400/\sqrt{3}}{2.42} = 572\ A$

so, receiving-end winding current $= \frac{572}{\sqrt{3}} = 330A$ Ans

and in 240 V winding = 3300A

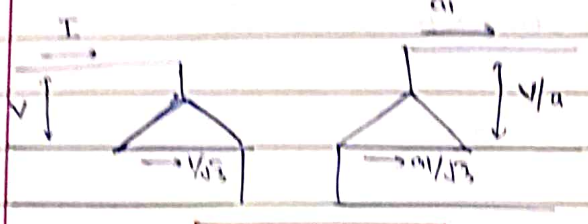
phase current $= 3300\sqrt{3} = 5720\ A$

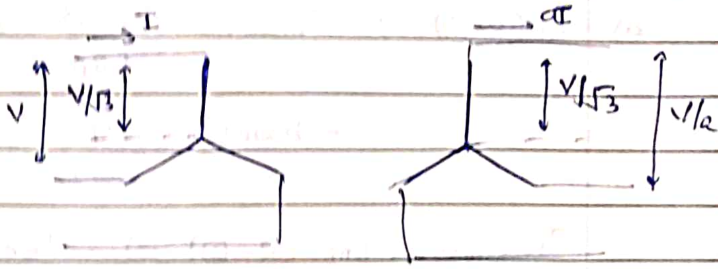




Y-Δ connection

Δ-Y connection





Δ-Δ connection

Y-Y connection

PER-UNIT SYSTEM

→ No need for explicit voltage-level conversions at every transformer in the system.

→ per unit (pu) system of measurements.

Ω/phase

⇒ $\text{Quantity per unit} = \dfrac{\text{Actual value}}{\text{Base value of quantity}}$ ⟶ value in V, Amp. etc.

→ select ② base quantities to define a given p.u. system

↙ Voltage ↘ power (or apparent power)

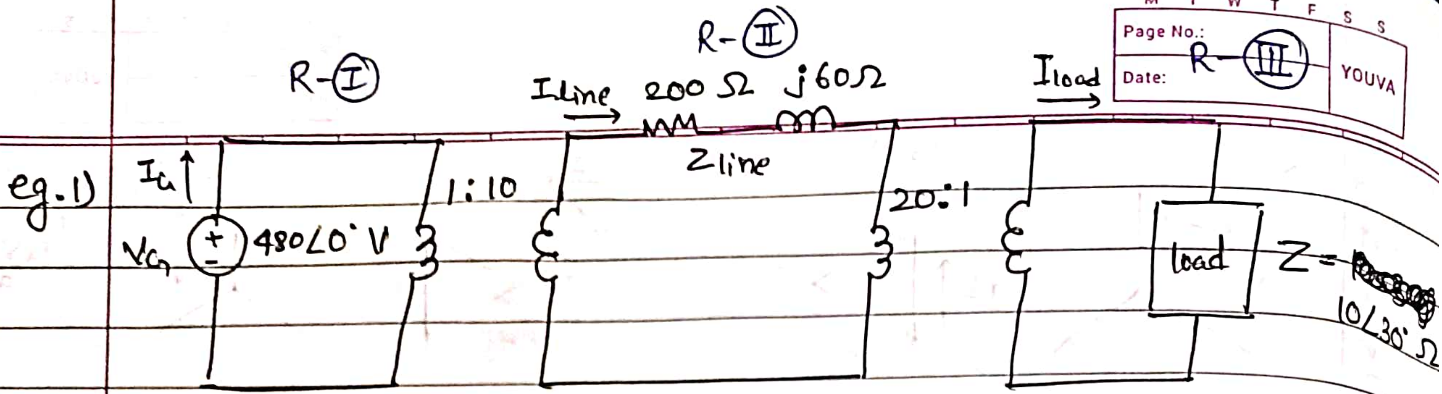
single phase ⇒ $S_{base} = V_{base}\, I_{base}$ (P_{base}, Q_{base} or S_{base})

$$Z_{base} = \frac{V_{base}}{I_{base}} \quad (R_{base}, X_{base} \text{ or } Z_{base})$$

$$Y_{base} = \frac{I_{base}}{V_{base}} \qquad Z_{base} = \frac{(V_{base})^2}{S_{base}}$$

⇒ A base apparent power and voltage are selected at a specific point in the system.

→ Transformer has no effect on the base apparent power of the system since $\boxed{S_{in} = S_{out}}$, V_{base} changes acc. to turns ratio eg. 1)



⇒ base values →

$V_{base} = 480\,V$ $S_{base} = 10\,KVA$ at the generator (G)

a) G (R-①) ⇒ $I_{base1} = \dfrac{10\times10^3}{480} = 20.83\,A$

$Z_{base1} = \dfrac{480}{20.83} = 23.04\,\Omega$

Transmission line (R-②) ⇒ $V_{base_2} = \dfrac{480}{0.1} = 4800\,V$

$S_{base_2} = 10\,KVA$ (same)

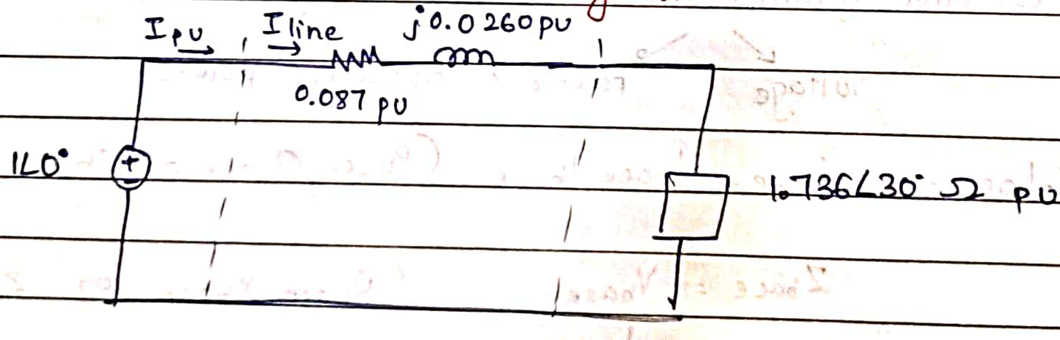
$I_{base\,2} = 2.083\,A$ $Z_{base_2} = 2304\,\Omega$

Load (R-③) ⇒ $V_{base_3} = \dfrac{4800}{20} = 240\,V$ $S_{base\,3} = 10\,KVA$

$I_{base\,3} = 41.67\,A$ $Z_{base\,3} = 5.76\,\Omega$

CONVERTING TO PER-UNIT EQUIVALENT SYSTEM

divide each component by its base value in that region.



$\boxed{I_{G,pu} = I_{line,pu} = I_{load,pu} = I_{pu}}$

⇒ Now, $I_{pu} = \dfrac{1\angle 0^\circ}{(0.087 + 0.0260j) + (1.503 + j\,0.868)}$ 1.736∠30° Ω

$I_{pu} = 0.569\angle -30.6^\circ$ pu

so, $I_{load} = I_{load,pu} \times I_{base\text{-}3} = (0.569 \times 41.67)\angle -30^\circ$

$P_{pu,load} = |I_{pu}|^2 R = (0.569)^2 \times (1.503) = \boxed{0.487}$

so, $P_{load} = P_{pu} \times S_{base3} = \boxed{4870\ W}$

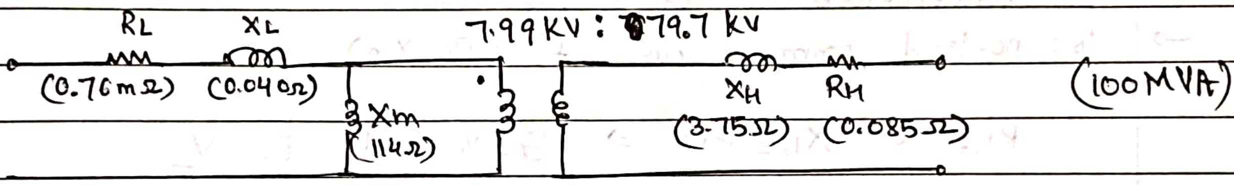
d) per unit power lost in transmission line $= (0.569)^2 \times 0.087$

so, $P = 0.00282 \times 10 \times 10^3 = \boxed{28.2 W}$

⇒ If more than one machine and one transformer are included in a single power system, the base can be chosen arbitrarily BUT the entire system must have the same base.

$$(P,Q,S)_{pu\ on\ base\text{-}2} = (P,Q,S)_{pu\ on\ base 1} \times \frac{S_{base 1}}{S_{base 2}}$$

eg.2)



LV side

$S_{base} = 100\ MVA$

$V_{base} = 7.99\ KV$

$I_{base} = \frac{100\ MVA}{7.99\ KV} = 12515.65\ A$

$Z_{base} = \frac{(7.99)^2}{100\ MVA} = 0.635\ \Omega$

HV side

$S_{base} = 100\ MVA$

$V_{base} = 79.7\ KV$

$Z_{base} = \frac{(79.7)^2}{100} = 63.5\ \Omega$

$R_L \rightarrow 0.0012\ pu$ $X_L \rightarrow 0.063\ pu$ $X_m = 179.53\ pu$

$X_H \rightarrow 0.06\ pu$ / $R_H \rightarrow 0.0013\ pu$

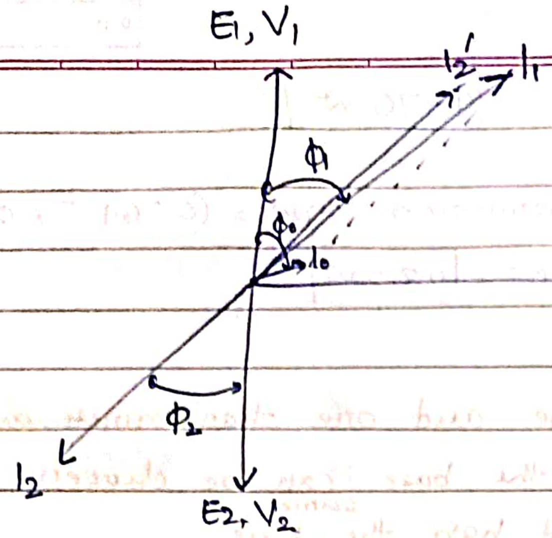
per-unit turns ratio $= \left(\frac{7.97}{7.97}\right) : \left(\frac{79.9}{79.9}\right) = 1:1$

V_1

V base

(no need of transformer in b/w)

PHASORS



→ mutual flux ϕ taken as reference phasor

→ EMF E_1 induced in primary leads ϕ by 90°, E_2 drawn opp for convenience

→ I_0 : no-load primary current (R_c, X_m)

$R_{1,2} \sim 0$ $\quad X_{1,2} \sim 0$ $\Rightarrow$ $E_1 = V_1$ and $E_2 = V_2$

→ load impedance on secondary is lagging by $\cos(\phi_2)$
i.e $Z_L = |Z_L| \angle \phi_2$

I_2' → secondary current on primary side

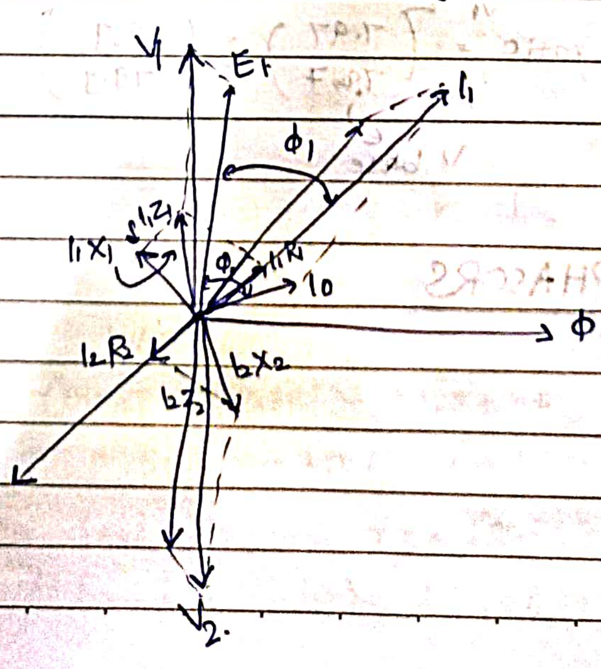
$I_1 = I_0 + I_2'$ → Total current → lags V_1 by ϕ_1

⇒ including R_{12} and X_{12}

$$V_2 = E_2 - I_2 Z_2 = E_2 - I_2 R_2 - I_2 j X_2$$

$$\Rightarrow E_2 = V_2 + I_2 R_2 + I_2 j X_2$$

$$V_1 = E_1 + I_1 R_1 + I_1 j X_1.$$



also, voltage regulation = $\frac{I_1 (R_{eq} \cos\phi_2 + X_{eq} \sin\phi_2)}{V_1}$

(load with lagging pf $\cos\phi_2$)

$= \frac{I_1 (R_e \cos\phi_2 - X_e \sin\phi_2)}{V_1}$ (leading pf $\cos\phi_2$)

MAXIMUM CORE EFFICIENCY

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_2 I_2 \times pf}{V_2 I_2 \times pf + P_c + I_2^2 R_{eq}}$$

$$= \frac{V_2 \cos\phi}{V_2 \cos\phi + \frac{P_c}{I_2} + I_2 R_{eq}}$$

$$\frac{d}{dI_2}\left(V_2 \times pf + \frac{P_c}{I_2} + I_2 R_{eq}\right) = 0$$

$$\Rightarrow -\frac{P_c}{I_2^2} + R_{eq} = 0 \quad \text{i.e.} \quad P_c = I_2^2 R_{eq}$$

max. efficiency when variable power loss (due to I^2R) = constant core loss (H + E.C.)