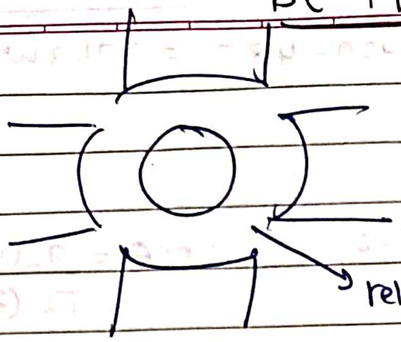
DC MACHINES



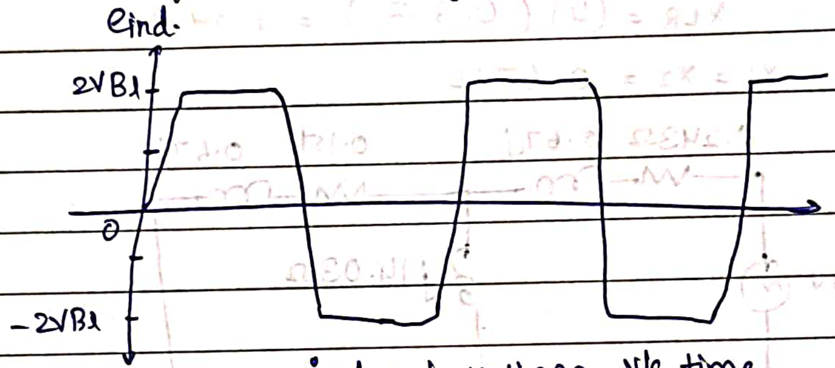
width of air gap = constt.

↓

shortest path ⇒ ($\perp^{ar}$) to the rotor surface everywhere

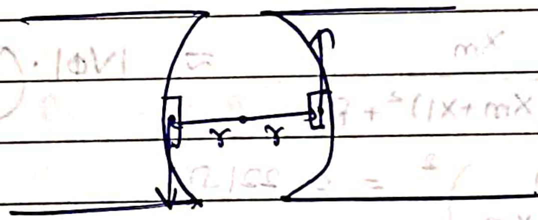
reluctance same everywhere ⇒ $\boxed{B = \text{constt.}}$

$$e_{ind} = \begin{cases} 2VBl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases}$$



induced voltage v/s time.

AC



$V_T = r\omega$

$$e_{ind} = \begin{cases} 2rlB\omega_m & ; \text{under pole faces} \\ 0 & ; \text{beyond the pole edges} \end{cases}$$

rotor ⇒ cylinder

└→ area = (A) = $2\pi r l$

area of rotor under each pole = $\pi r l$

$$e_{ind} = \begin{cases} \frac{2}{\pi} A_p B \omega_m \\ 0 \end{cases}$$

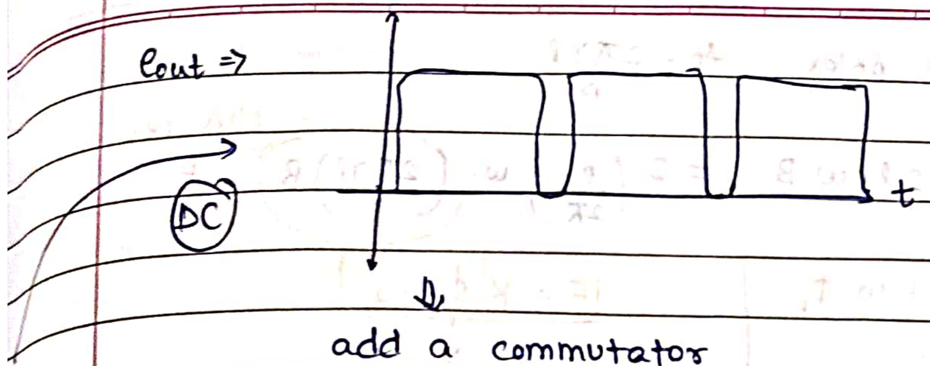
∴ $e_{ind} \propto$ (flux) × (speed of rotn) × (a constt.)

uniform

$\phi = B A_p$

$$e_{ind} = \begin{cases} \frac{2}{\pi} \omega \phi_m \\ 0 \end{cases}$$

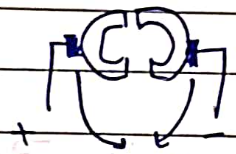
$e_{out} \Rightarrow$



$e_{ind} = 0$ when coil is at 90°

$\Downarrow$

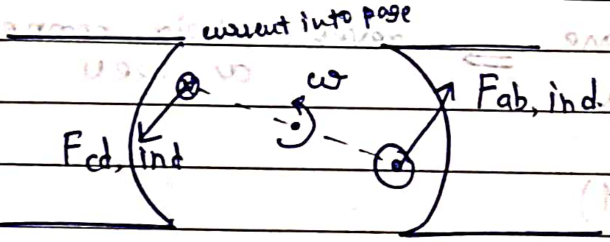
add a commutator

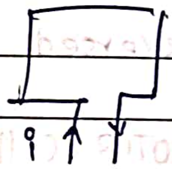


→ rotate → interchange (+) and (−) after 180°.

(#) when voltage in the loop = 0, the contacts S.C. the two segments ← fixed contacts (carbon brushes)

INDUCED TORQUE IN ROTATING LOOP





$F = i(\vec{l} \times \vec{B})$ $\quad \tau = rF\sin\theta$

$\tau_{ab} = ilBr$ ccw $\quad \tau_{bc} = 0$

$\tau_{cd} = ilBr$ ccw

$\tau_{ind} = \begin{cases} 2ilBr \\ 0 \end{cases}$ $\quad \tau_{ind.} = \begin{cases} \frac{2}{\pi}\phi i & \text{under pole faces} \\ 0 & \text{beyond pole edge} \end{cases}$

if we connect a battery to the coil

↓

coil at rest, $e_{ind.} = 0$

↓

$i = \frac{V_B}{R}$ ⟶ $\tau = \frac{2}{\pi}\phi i$ ccw → rotate

current gets induced ↑ , $e_{ind} = \frac{2}{\pi}\phi\omega_m$

↓

$i \Downarrow$

$\tau \downarrow$

steady state, $\boxed{\tau_{ind} = 0}$, $\boxed{V_B = e_{ind.}}$

at steady state,

$V_B = e_{ind.} = \frac{2}{\pi}\phi\omega_m$ $\quad \boxed{\omega_{steady\ state} = \frac{V_B}{2rlB}}$

If there are 'p' poles, $A_p = \frac{2\pi r l}{p}$

$$E = 2Blv = 2lr\omega B = 2\left(\frac{p}{2\pi}\right)\omega \cdot \left(\frac{2\pi r l}{p}\right)B$$

flux per pole

$$\boxed{E = 2\left(\frac{p}{2\pi}\right)\omega \cdot \phi_p}$$

$$\boxed{E = K'\phi_p \omega}$$

ϕ_p → flux per pole

$$\boxed{\tau = K\phi_p I}$$

force density $= \vec{J} \times \vec{B}$

$$F = \left(\frac{I}{A_c} \times B\right) \cdot A_c l = I\vec{l} \times \vec{B}$$

volume

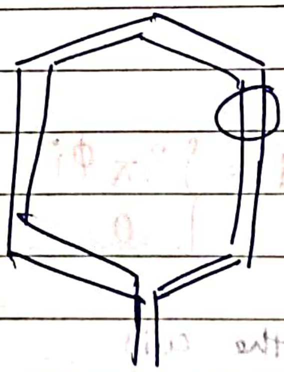
COMMUTATION

whenever voltages are reversed in a loop ⇒ reverse their connections as well.

ROTOR COILS (DC-M)

each coil → no. of turns (loops) of wire

↳ each side of a turn is ⇒ CONDUCTOR



2 turns

$Z = 2N$

No. of conductors, $\boxed{Z = 2CN_c}$

C → no. of coils on rotor

N_c → no. of turns on each coil

coil → spans 180 electrical degrees.

$$\boxed{\theta_e = \frac{P}{2}\theta_m}$$

two-layer windings ⇒ sides from two diff. coils are inserted in each slot → one at bottom and one at top.

simplex rotor w. → single, complete, closed winding wound on a rotor
duplex rotor w. → two complete and independent sets of rotor windings

Lap → coils containing one or more turns of wire → two ends of each coil coming on adjacent commutator segments

progressive → $(Y_c = 1)$ end connected to segment after beginning
retrogressive → vice-versa $(Y_c = -1)$

⇒ As many parallel current paths as the poles on the machine

duplex ⇒ $\boxed{Y_c = \pm 2}$

m-plex lap winding ⇒ $\boxed{Y_c = \pm m}$

no. of current paths ⇒ $\boxed{a = mP}$ (P: no. of poles)

lap: $a=P$

simplex wave winding: $Y_c = \frac{2(C \pm 1)}{P}$ C: no. of coils on rotor, P: poles

wave: $a=2$

$\boxed{a = 2m}$

Problems with commutations → armature rxn, $L\, di/dt$ voltages

connect a load across armature windings → current will flow → M.F. → distort MF from machine's poles → armature rxn

current carrying conductor produces a M.F.

"neutral plane shift" → where V is ∥ to B, $\boxed{e_{ind} = 0}$

motor → opp. to dirⁿ of rotⁿ
generator → in dirⁿ of rotⁿ

brushes will no longer be calibrated.

$L\,di/dt$ Voltages

commutator is shorted $\longrightarrow$ dir^n of current reverses $\rightarrow$ very small time $\left(\frac{di}{dt}\uparrow\uparrow\right)$

No. of commutator segments = No. of coils
No. of brushes = No. of poles
No. of parallel paths = No. of poles
No. of coils in series = $\left(\frac{\text{No. of coils}}{\text{No. of poles}}\right)$

TORQUE & VOLTAGE IN REAL DC MACHINES

Voltage in a single conductor under pole faces = $e_{ind} = e = vBl$

voltage out of armature = $\boxed{\frac{ZvBl}{a}}$

Z: total no. of conductors
a: no. of current paths

$$E_A = \frac{Zr\omega_m Bl}{a} \qquad BA_p = \phi \qquad A_p = \frac{A}{P} = \frac{2\pi rl}{P}$$

$$\phi\text{(flux per pole)} = BA_p = \frac{2\pi r l B}{P}$$

$$E_A = \left(\frac{ZP}{2\pi a}\right)\left(\frac{2\pi r l B}{P}\right)\omega_m$$

$$\boxed{E_A = \left(\frac{ZP}{2\pi a}\right)\phi_p\,\omega_m} \qquad \boxed{E_A = K\phi\omega}$$

$\tau = Z \times \tau_{each\ conductor} \rightarrow r\, I_{cond}\, lB$

a current path, $I_A \rightarrow$ split among 'a' paths $= \left(\frac{I_A}{a}\right) = I_{cond.}$

$$\tau_{cond.} = \frac{rI_A lB}{a}$$

$$\boxed{\tau_{ind} = \frac{ZrlBI_A}{a}} \qquad \phi_p = BA_p = \frac{2\pi r l B}{P}$$

$$\boxed{\tau_{ind} = K\phi I_A} \qquad \boxed{K = \frac{ZP}{2\pi a}}$$

eg- duplex, lap-wound, six-pole, six brush sets, each spanning 2 commutator segments, 72 coils on armature, each has 12 turns, $\phi_p = 0.039$ Wb, speed = 400 rpm.

$\Rightarrow$ $Z = 2 \times C \times N_c = 2 \times 72 \times 12$

$= 1728$ conductors

no. of current paths, $a = mP = 2 \times 6 = 12$

$$E = \left(\frac{ZP}{2\pi a}\right) \phi_p \cdot \frac{\overset{20}{\cancel{400}} \cdot \cancel{2\pi}}{3\cancel{60}} = \left(\frac{20ZP}{3a}\right)\phi_p$$

$$E = \frac{20 \times 1728 \times 6}{3 \times (12)} \times \phi_p \Rightarrow \boxed{E_A = 224.6\ V}$$

effective armature resistance $\Rightarrow$ 'a' parallel paths with $\frac{Z}{a}$ conductors in each

so, Resistance/path $= (Z/a)\, r_1$

and net resistance $= \boxed{Zr_1/a^2}$

POWER

$$\eta = \frac{P_{out}}{P_{in}} \times 100\ \%$$

Losses → 1) electrical/copper losses → armature and field winding

$P_A = I_A^2 R_A$

$P_F = I_F^2 R_F$

2) Brush losses → power lost across contact potential,

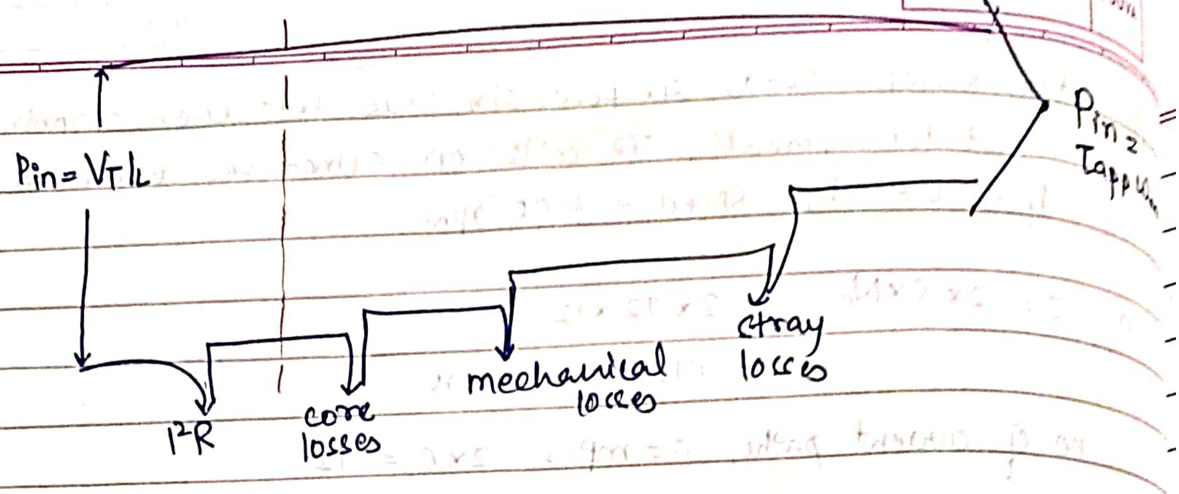
$\boxed{P_{BD} = V_{BD} I_A}$

3) Core losses → (H) + (E)

4) Mechanical losses

frict^n ↙ ↘ windage

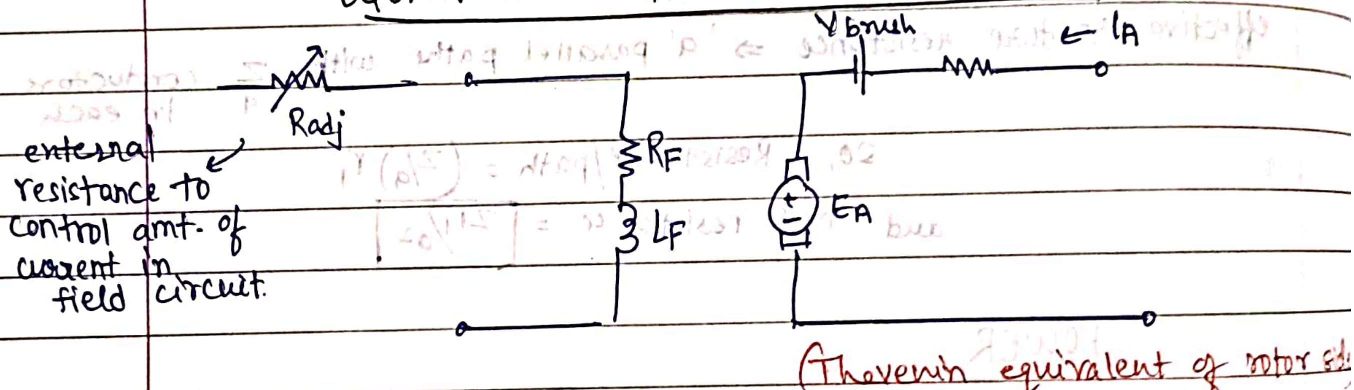
$P_{conv} = E_A I_A = \tau_{ind}\,\omega_m$



$$\text{Speed regulation (SR)} = \left| \frac{\omega_{m,nl} - \omega_{m,fl}}{\omega_{m,fl}} \right| \times 100\%$$

(+ve) regl^n ⇒ speed ↓ as load ↑

EQUIVALENT CIRCUIT OF DC MOTOR



→ V_{brush}: in dir^n of current drop

$\boxed{E_A = K\phi\omega_m}$ / $\boxed{\tau_{ind} = K\phi I_a}$

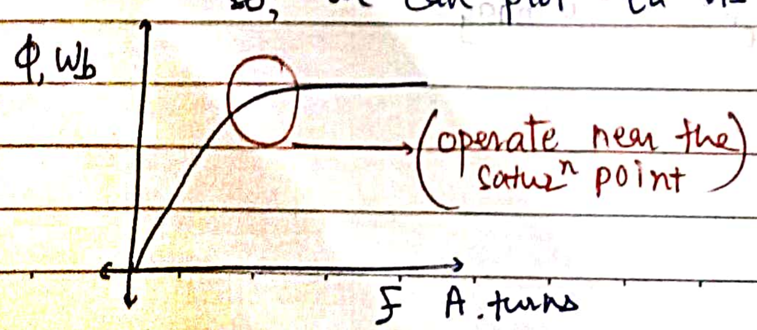
MAGNETIZATION CURVE

E_A v/s field current ⇒ ??

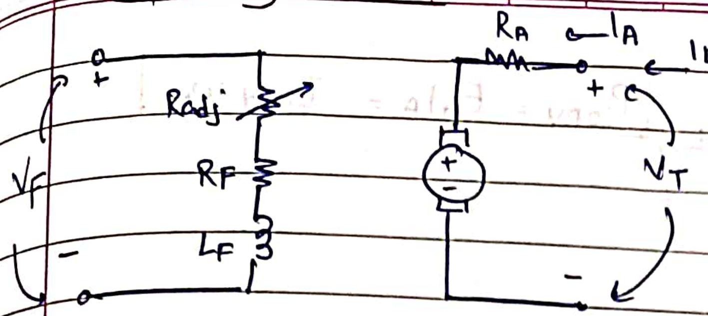
Field current ⇒ MMF $F_F = N_F I_F$ ⇓ ϕ

as $I_F \propto$ MMF and MMF $\propto \phi$

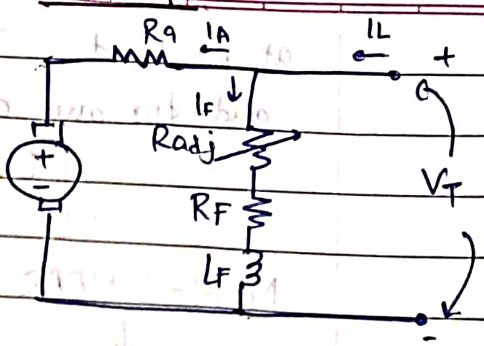
so, we can plot E_a v/s I_F for a given ωb



$I_F \to$ ↑↑ ↓ $E_a \to$ ↑ (near full load) seperately Excited



Shunt



$I_F = \frac{V_F}{R_F}$ $V_T = E_a + I_a R_a$

$I_L = I_A$

$I_F = \frac{V_T}{R_F}$ $V_T = E_a + I_A R_A$

$I_L = I_A + I_F$

→ Terminal characteristics of shunt DC motors

output torque v/s speed

load on shaft ↑ → $\tau_{load} > \tau_{ind.}$

↓

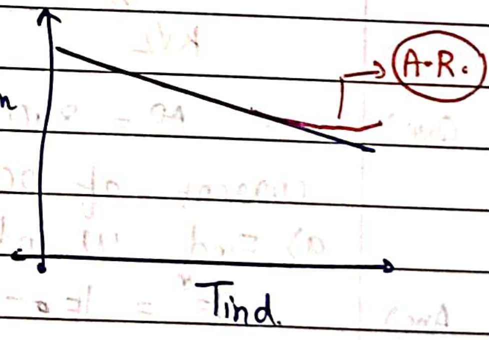
machine starts to slow down $\Rightarrow E_A \Downarrow \Rightarrow I_A = \left(\frac{V_T - E_a}{R_a}\right) \Uparrow \Rightarrow \tau_{ind.} = K\phi I_a \Uparrow$

$\tau_{ind.} = \tau_L$ at a lower speed ω_m.

$V_T = E_a + I_a R_a$

$V_T = K\phi\omega_m + \left(\frac{\tau_{ind}}{K\phi}\right) R_a$

$\omega_m = \frac{V_T}{K\phi} - \left(\frac{R_a}{(K\phi)^2}\right)\tau_{ind.}$



eg. 50 hp, 250 V, 1200 rpm, $R_a = 0.06\,\Omega$, $R_{adj} + R_F = 50\,\Omega$, no load speed = 1200 rpm. 1200 turns per pole on shunt field winding

a) Find ω when $I = 100$ A, $I = 200$ A, $I = 300$ A.
input

Ans) as $\frac{E_2}{E_1} = \frac{n_{m2}}{n_{m1}}$ so, $n_{m2} = \left(\frac{E_2}{E_1}\right) n_{m1}$

at no load, $I_a = 0$ $E_1 = \frac{V_T}{R_a} = 250$

when $I_L = 100$ A ~~$E_2 = 250 - 100(0.06) = 250 - 6 = 244\,\Omega$~~

$I_L = I_a + I_F$ $I_F = \frac{V_T}{50} = 5$ A $I_a = I_L - 5$

at no load, $\tau_{ind} = 0$

and for any other load, $\boxed{P_{conv} = E_A I_A = \tau_{ind}\, \omega_m}$

$$\boxed{\tau_{ind} = \frac{E_A I_A}{\omega_m}}$$

NON-LINEAR ANALYSIS.

↓

magnetization curve → plot of E_A v/s I_F for a given ω_0

eg. $\mathcal{F}_{net} = N_F I_F - \mathcal{F}_{AR}$

equivalent field current,

$$\boxed{I_F^* = I_F - \frac{\mathcal{F}_{AR}}{N_F}}$$

also, $E_A = K\phi_p \omega_m = K\phi' n_m$

For a given effective $I_F^* \Rightarrow \phi$ is fixed

so, $\boxed{\frac{E_A}{E_{A_0}} = \frac{n_m}{n_0}}$

KVL

Que.) Let AR = 840 Amp. turns in earlier example, at a load current of 200 A.

a) Find ω when $I = 200A$

Ans.) $I_F^* = I_F - \frac{840}{N_F} = \left(\frac{250}{50}\right) - \left(\frac{840}{1200}\right) = 4.3\,A$

$I_L = I_F + I_A \Rightarrow I_A = 200 - 5 = \boxed{195\,A}$

and $E_A = 238.3V$

~~no load ⇒ ω = 1200 rpm~~

~~$I_F = 4.3A$ $E_a = V_T$ $\omega = 1200$ rpm~~ ~~$E_a = V_T - I_a R_a$ $\omega = ?$~~

↓ ~~$\frac{250 - (195.7)(0.06)}{250}$~~ = ~~$\frac{\omega\, rpm}{1200\, rpm}$~~

~~$E_{a_0} = 250 - (195.7)(0.06)$~~ = 238.3V $250 - 195(0.06) = 238.3V$

↓

~~when~~ $V_T = 250V$

(Voltage when no load is applied.)

$\frac{\omega}{1200} = \frac{238.8}{233}$ ⇒ $\boxed{\omega = 1227\ rpm}$

when we $I_F^* = I_F = \frac{MMF}{N_F} = 4.3V_T \xrightarrow{\text{magnetiz. curve}} 233V$

no-load speed = 1200 rpm $\longrightarrow$ $\boxed{E_0 = 233V}$

$233V \rightarrow 1200$ rpm
$228.3V \rightarrow \omega$

$\omega = 1227$ rpm

SPEED CONTROL

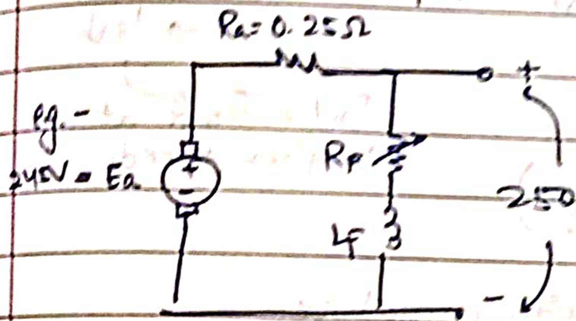
1) Adjust field resistance, R_F
↳ adjust the field flux

$R_F \uparrow \rightarrow I_F = \frac{V_T}{R_F \uparrow} \Rightarrow I_F \downarrow \Rightarrow \phi \downarrow$ (MMF↓) $\Rightarrow E_A = K\phi\omega \downarrow$

$I_a = \frac{V_T - E_a \downarrow}{R_a} \Rightarrow I_a \uparrow\uparrow$

$T = K I_a \uparrow\uparrow \phi \downarrow$ (T_{ind} ?)

eg.-



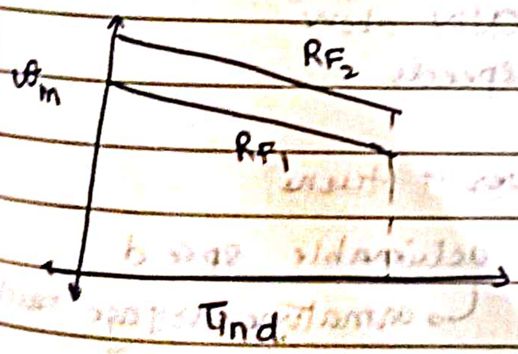
$E_a = 250 - I_a R_a$ $\quad I_a = 20$ A
$= 245$

→ 1% decrease in flux → E_A decreases by 1%; $E_A' = 242.55V$

$I_A' = 29.8A$ increase in current = 49% >> 1%.

so, $T_{ind} \uparrow$; and as $T_{ind} > T_{load} \Rightarrow$ speeds up

$T_{ind} = T_{load}$ at a higher steady state speed. ⇐ T_{ind} falls ← I_A falls ← E_A rises



(over the operating range of motor)

$R_{F_2} > R_{F_1}$

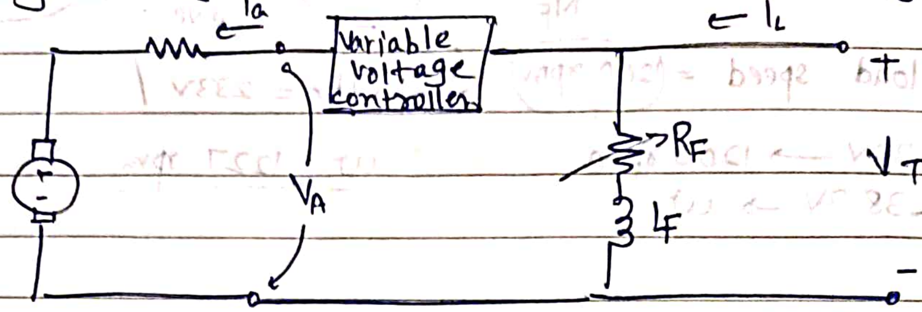
as $\phi \downarrow$

$(\propto 1/\phi)$ no-load speed ↑

$(\propto \frac{1}{\phi^2})$ slope of $T-\omega$ curve becomes steeper.

⇒ Changing Armature Voltage

Change voltage to armature without changing to fi



In effect, motor must be $\boxed{\text{seperately encited}}$

$V_a \uparrow \Rightarrow I_a = \frac{V_a \uparrow - E_a}{R_a} \Rightarrow I_a \uparrow \quad T_{ind} \uparrow \quad T_{ind} > T_{load}$

$\Downarrow$

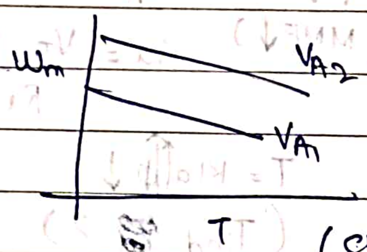
$\omega \uparrow$

$\Downarrow$

$E \uparrow \rightarrow I_a \downarrow$

$T_{ind} = T_{load}$ at higher speeds

(S.E.)



$V_{A2} > V_{A1}$

(slope remains CONSTT.)

⇒ Insert a Resistance in series with armature

drastically ↑↑ slope of motor-torque characteristics.

$I_a = \frac{V_T - E_a}{R_a + R_a'} \downarrow \quad \downarrow T_{ind} = K\phi I_a \downarrow$

Operation Ranges ⇒ Field resistance control ⇒ lower the field current

↓

faster it turns

$I_F \uparrow \Rightarrow$ speed ↓ ⇒ minimum achievable speed

minimum speed occurs when max. permissible current flows th

base speed (rated) ⇒ F. control ONLY above base speeds.

$\boxed{\text{A. Vol. control}}$ ⇒ AV ↓↓ ⇒ slower it turns

↑ Va ⇒ ↑ speed ⇒ max. achievable speed

↳ armature voltage reaches max. rated

⇓

below base speeds ONLY.

armature voltage control

ϕ = constt. $\boxed{T_{max} = K\phi I_{a,max}}$ → constt. irr. of ω_m.

$P = T\omega = T_{max} \cdot \omega_m$

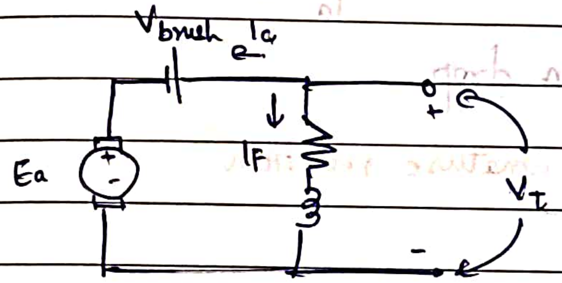
$\boxed{P_{max} = T\omega}$ → constt. $\boxed{T_{max} \propto \frac{1}{\omega}}$ (Field control)

EFFICIENCY

- 50 hp, 250 V, 1200 rpm shunt DC motor, rated I_A = 170 A, rated I_F = 5A. When rotor is blocked, V_t = 10.2 produces rated I_a and a field voltage of 250V produces I_F = 5A. Brush voltage drop = 2V. At no load with V_t = 240 V, the I_a = 13.2A and I_F = 4.8 A and ω = 1150 rpm

a) Find P_{out} b) η = ?

⇒ $R_a = \frac{V_t}{I_a} = \frac{10.2}{170} = 0.06\Omega$ $R_F = \frac{250}{5} = 50\Omega$



~~I_a = 13.2A~~

~~$I_a^2 R_a$ = 10.45W~~

~~$I_F^2 R_F$ = 1152 W~~

~~P_{brush} = 26.4 W~~

~~$E_a = V_t - I_a R_a$ = 239.2 V~~

~~$P_{out} = E_a I_a$~~

at no-load, P_{in} = Prot. losses (assuming armature & Cu drops are negligible)
$= (240)(13.2)$
$= 3168$ W

at rated load, $P_{in} = (250)(170) = 43750$W

$P_F = I_F^2 R_F = (5)^2(50) = 1250$ $P_a = (170)^2(0.06) = 1734$W

$P_{brush} = V_b I_a = (170)(2) = 340$ W

So,

$$P_{out} = P_{in} - P_{brush} - P_F - P_a - \underbrace{(P_{core} + P_{mech} + P_{stray})}_{\approx \text{ Prot. losses at no load}}$$

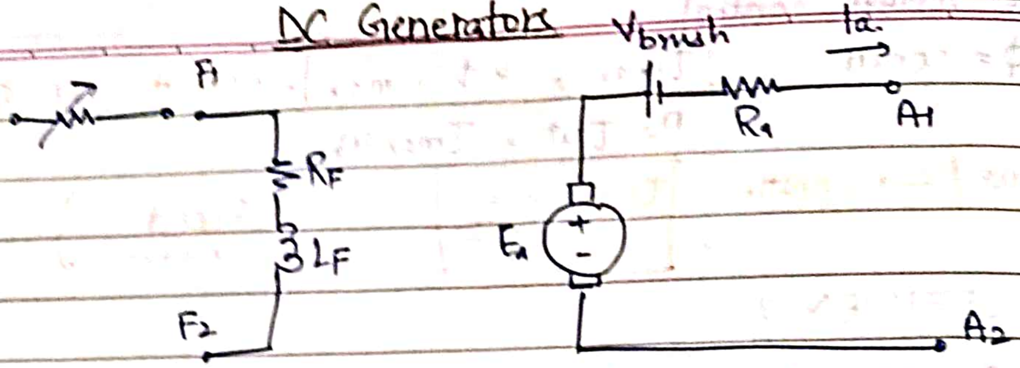
$= 43750 - (340 + 1734 + 1250 + 3168)$

P_{out} = 37258 W

$\eta = 0.85 = \boxed{85.17\%}$

Ans.

DC Generators

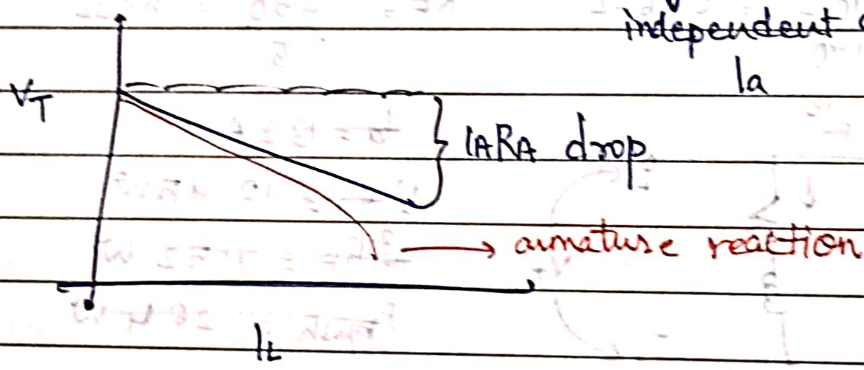


(seperately excited DC generator) $\Rightarrow$ field current supplied by diff. cnt. voltage source

$I_L = I_A$ $\quad V_T = E_A - I_A R_A$ $\quad I_F = \frac{V_F}{R_F}$

Output characteristics $\Rightarrow$ $\boxed{V_T = E_a - I_a R_A}$

~~independent of~~ Ia



load $\Uparrow$ $\rightarrow$ $I_L (= I_a) \Uparrow$ $\rightarrow$ $I_A R_A$ drop $\uparrow$ $\rightarrow$ V_T falls.

control V_T
1) increase/decrease E_a i.e. ω_m
2) change I_F (i.e. ϕ and hence E_a)

NON-LINEAR

$$\mathcal{F}_{net} = N_F I_F - \mathcal{F}_{AR}$$

equivalent field current $\rightarrow$ $I_F^* = I_F - \frac{\mathcal{F}_{AR}}{N_F}$

locate E_{A_0} on the magnetization curve (at a particular 'ω_0')

$$\frac{E_A}{E_{A_0}} = \frac{\omega}{\omega_0}$$

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SERIES DC MOTORS

$V_T = E_a + I_A(R_a + R_s)$

Here, $\phi \propto I_a$ (armature current)

$\tau_{ind} = K\phi I_a$

as load ↑ ⇒ I_a ↑ ⇒ ϕ ↑

$\phi = cI_A$

$\boxed{\tau_{ind} = Kc I_A^2}$

$I_A = \sqrt{\frac{\tau_{ind}}{Kc}}$ $V_T = K\phi\omega_m + \sqrt{\frac{\tau_{ind}}{Kc}}(R_a + R_s)$

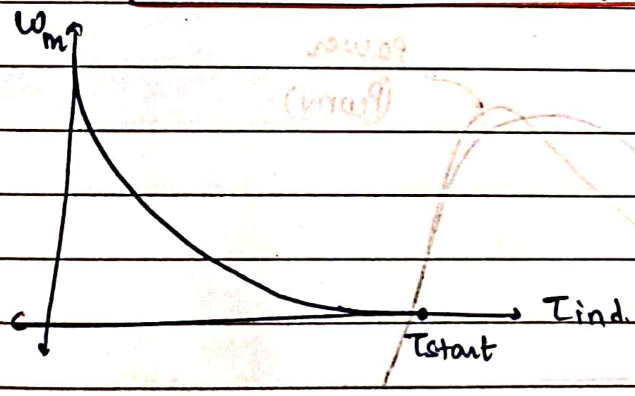
$\phi = \frac{\tau_{ind}}{K I_a} \Rightarrow \tau_{ind} = \frac{K}{c}\phi^2$ $\phi = \sqrt{\frac{c}{K}}\sqrt{\tau_{ind}}$

$$V_T = K\sqrt{\frac{c}{K}}\sqrt{\tau_{ind}}\,\omega_m + \sqrt{\frac{\tau_{ind}}{Kc}}(R_a + R_s)$$

$$\sqrt{Kc}\sqrt{\tau_{ind}}\,\omega_m = V_T - \left(\frac{R_a + R_s}{\sqrt{Kc}}\right)\sqrt{\tau_{ind}}$$

$$\omega_m = \frac{V_T}{\sqrt{Kc}\sqrt{\tau_{ind}}} - \left(\frac{R_a + R_s}{Kc}\right)$$

$$\boxed{\omega_m = \frac{V_T}{\sqrt{Kc}}\left(\frac{1}{\sqrt{\tau_{ind}}}\right) - \left(\frac{R_A + R_s}{Kc}\right)}$$

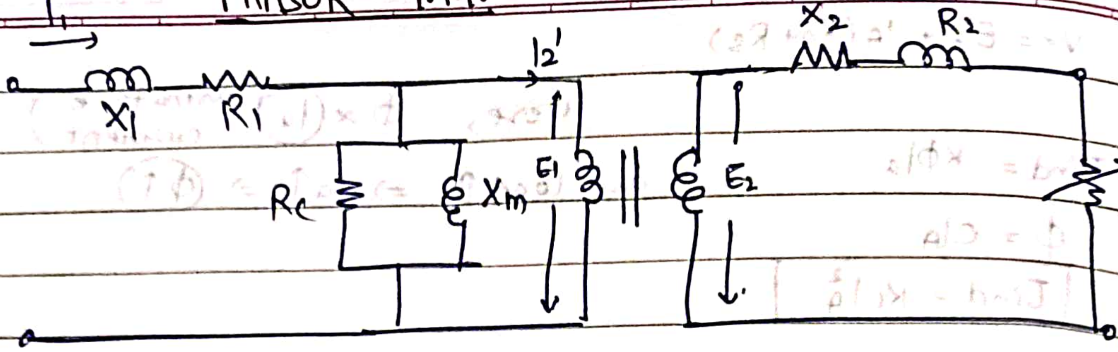


LAP and WAVE

- as many ∥ current paths through the machine as there are poles on the machine in a simplex lap winding.

c coils and P is no. of poles ⇒ $\left(\frac{c}{p}\right)$ coils in each of the P parallel current paths

PHASOR- I.M.



PHASOR