INDUCTION MACHINES

Synchronous machine — stator → 3 phase (armature); rotor → DC-current (field) (or P.M.) — magnetic interlocking

one pair of poles in a multipole machine 'OR' one cycle of flux distribution = 360° (ele.) or 2π (elec. radians)

→ as there are $\left(\frac{poles}{2}\right)$ complete cycles in one revolⁿ,

$$\boxed{\theta_{ae} = \left(\frac{P}{2}\right)\theta_a}$$

electrical ← θ_{ae}; θ_a → mechanical

elec. frequency of voltage generated, $f_e = \left(\frac{P}{2}\right)\cdot\left(\frac{n}{60}\right)$ Hz (n : rpm)

INDUCTION MACHINE

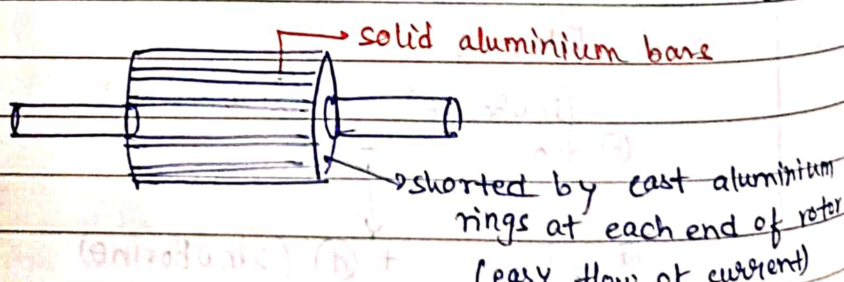
→ stator winding excited with AC current

→ Alternating currents flow in the rotor windings, due to induction i.e. $\boxed{\text{transformer action}}$

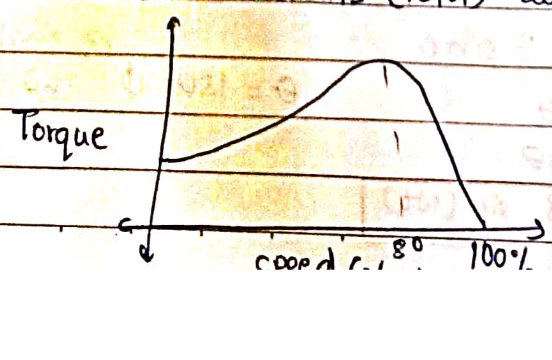
→ ind. machine ⇒ generalized transformer → elec. power transferred b/w rotor and stator with a change of frequency and a flow of mechanical power.

→ rotor windings are electrically short-circuited and frequently have no external connections → currents are induced by transformer action from the stator windings.

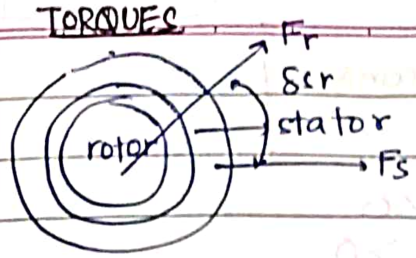
squirrel-cage

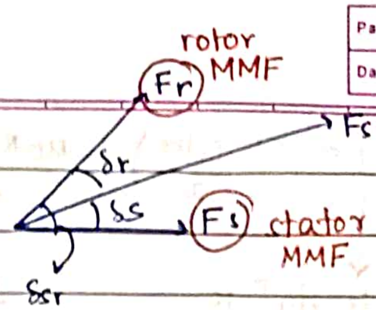


→ slipping of the rotor w.r.t. the synchronous armature flux gives rise to induced currents (rotor) and torque.



TORQUES





→ Torque is produced by the tendency of two component magnetic fields to line-up their magnetic axis.

→ Most of the flux produced by stator and rotor windings crosses the air-gap and links both windings i.e. the mutual flux, directly analogous to the magnetizing flux of a transformer.

→ some flux does not cross the air-gap → rotor-leakage and stator-leakage flux.

Torque prod^n ⇒ mutual flux.

⇒ Coenergy stored in the air-gap ⇒ Torque.

g (radial length of air-gap) $\ll R_{rotor}$ or R_{stator} ⇒ almost uniform flux radially

air-gap field → H_{ag} or B_{ag}

so, $F_{sr} = (H_{ag}) \cdot (g)$

MMF waves of stator and rotor are "spatial-sine waves" → δ_{sr} being phase angle b/w magnetic axes (electrical degrees)

$$F_{sr}^2 = F_s^2 + F_r^2 + 2F_sF_r \cos\delta_{sr}$$

$$(H_{ag})_{peak} = \frac{F_{sr}}{g}$$

$\langle \text{Energy density} \rangle = \frac{\mu_0}{2}\left(\langle H^2 \rangle\right)$ (sine wave)

$= \frac{\mu_0}{2}\left(\frac{H_{peak}^2}{2}\right)$

$$\text{avg. coenergy density} = \frac{\mu_0}{2}\left(\frac{(H_{ag})_{peak}^2}{2}\right)$$

$$= \frac{\mu_0}{4}\left(\frac{F_{sr}}{g}\right)$$

so, $\text{coenergy} = \left(\frac{\mu_0}{4}\right)\left(\frac{F_{sr}}{g}\right) \times \left(\text{Vol. of air gap}\right) = \frac{\mu_0}{4}\frac{F_{sr}}{g}\cdot(\pi D l g)$

(D – avg. diameter, l – axial length)

$$W'_{fld} = \frac{\mu_0 \pi D l}{4g}\left(F_s^2 + F_r^2 + 2F_sF_r\cos\delta_{sr}\right)$$

$$T = \left.\frac{\partial W'_{fld}}{\partial \delta_{sr}}\right|_{F_s, F_r} = -\left(\frac{\mu_0 \pi D l}{2g}\right) F_s F_r \sin\delta_{sr}$$

Page No.: Date: YOUVA

$$\boxed{T = -\left(\frac{poles}{2}\right)\left(\frac{\mu_0 \pi D l}{2}\right) F_s F_r \sin\delta_{sr}}$$

acts in dir^n to accelerate the rotor.

$\delta_{sr} > 0 \rightarrow T < 0$
$\delta_{sr} < 0 \rightarrow T > 0$

also, $F_s \sin\delta_{sr} = F_{sr} \sin\delta_r$ and $F_r \sin\delta_{sr} = F_{sr} \sin\delta_s$

POLYPHASE INDUCTION MACHINES

ROTOR
- → wound rotor → same no. of poles as stator
- → squirrel-cage rotor

⇒ rotor turning at n $\frac{rotations}{min}$ (i.e. 'n' rpm) in dir^n of rotating stator field

↳ synchronous speed of stator = n_s rpm

slip, $\boxed{s = n_s - n_r}$ fractional slip $= \left(\frac{n_s - n_r}{n_s}\right)$

rotor speed, $\boxed{n = (1-s)\, n_s}$

similarly, $\boxed{\omega_m = (1-s)\, \omega_s}$

mechanical angular velocity — synchronous angular velocity

→ The relative motion of the stator flux and the rotor conductors, induces voltages of frequencies, f_r

$\boxed{f_r = s \cdot f_e}$ — slip frequency

$$\boxed{N_s = \frac{120 f}{P}}$$

$$\boxed{f_r = \left(\frac{P}{2}\right)\left(N_s - N_r\right)}$$ → frequency of induced EMF in the rotor

speed of stator field w.r.t. stator structure $= N_s$
speed of stator field w.r.t. rotor structure $= N_s - N_r$
speed of rotor field w.r.t. rotor structure $= N_s - N_r = (s \cdot N_s)$
rotor frequency (f of current induced in rotor) $= s \times (f)$

⇒ at starting, $s = 1$ ⇒ rotor frequency = ~~slip~~ stator frequency $= f_e$

$(s=1) \rightarrow$ rotor $f = f_e$ (stator) $\rightarrow$ field of rotor currents revolves at same speed as the stator field

turn rotor in dir^n of rotation of stator-inducing field. $\Leftarrow$ $\boxed{\text{Torque}}$

↳ if the torque is sufficient to overcome the opposition to rotation due to shaft load $\rightarrow$ motor will come to its operating speed.

if $N_s = N_r$ $\rightarrow$ rotor conductors would be stationary w.r.t. stator field.

↳ No 'i' $\rightarrow$ No Torque.

→ rotor revolving in same dir^n as the stator fields $\rightarrow$ frequency of rotor currents $= \boxed{sf_e}$

↓

rotating flux which rotates at $(s \cdot n_s)$ rpm w.r.t. the rotor in FWD dir^n. $\rightarrow$ but, the rotor also revolves at n rpm.

⇒ w.r.t. stator, speed of flux wave $= sn_s + n = sn_s + (1-s)n_s$

$= \boxed{n_s}$ rpm.

↓

field due to rotor currents is in synchronism with that produced by the stator currents. $\Rightarrow$ STATIONARY w.r.t. each other ⇒ $\boxed{\text{TORQUE}}$

↓

maintain rotation of rotor.

$$T = F_s F_r \sin \delta_r \Rightarrow \boxed{T = -K I_r \sin \delta_r}$$

(proportionality) → angle by which rotor MMF leads air-gap MMF wave.

⇒ s is 2-10% → rotor frequency is very low

↳ rotor impedance is largely resistive.

↳ rotor-induced voltage ∝ slip and leads air-gap flux by 90°

so, current i_f ∝ slip and 180° out of phase with rotor voltage.

rotor MMF waves lag air-gap flux by ~90° and $\sin \delta_r = -1$

INDUCTION MOTOR EQUIVALENT

as $f_e = \left(\frac{P}{2}\right) f_m$ or $2\pi f_e = \left(\frac{P}{2}\right) \cdot 2\pi f_m$ $\quad \omega_m = \frac{N_s \cdot 2\pi}{60}$

$$2\pi f_e = \frac{P}{2} \cdot \frac{N_s (2\pi)}{60} \qquad f_e = \frac{P}{120} N_s$$

$$\boxed{N_s = \frac{120 f_e}{P}}$$

M T W T F S S

Page No.:

Date:

YOUVA

$$\boxed{\vec{B}_{res}(t) = 1.5_{\nu}\left[\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}\right]}$$

(BmR)

(max.)

STATOR

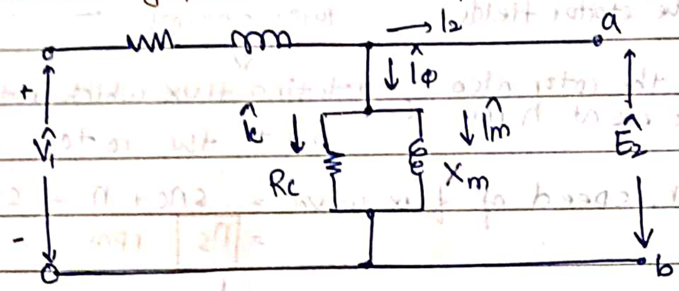
Stator leakage impedance, $Z_1 = R_1 + jX_1$

$$\hat{V}_1 = (\hat{E}_1) + (\hat{I}_1)(R_1 + jX_1)$$

Stator L-to-N terminal voltage

counter EMF in the air-gap

stator current



$\hat{I}_2 \rightarrow$ produce an MMF corresponding to the MMF of rotor current (load component)

$\hat{I}_\phi \rightarrow$ func^n of EMF $\hat{E}_2$ $\quad \hat{I}_\phi = \hat{I}_c + \hat{I}_m$

in phase with $\hat{E}_2$

lags $\hat{E}_2$ by 90°

$$Z_2 = \frac{\hat{E}_2}{\hat{I}_2} \rightarrow \text{rotor equivalent impedance}$$

→ rotor in an induction motor ⇒ equivalent rotor with a polyphase winding with same no. of phases and turns as stator but producing the same MMF and air-gap flux

→ short-circuit the rotor of an I.M. → impedance seen is the short-circuit impedance.

voltage induced in equivalent rotor

$$Z_{2s} = \frac{\hat{E}_{2s}}{\hat{I}_{2s}} = N_{eff}^2\left(\frac{\hat{E}_{rotor}}{\hat{I}_{rotor}}\right) = N_{eff}^2 (Z_{rotor})$$

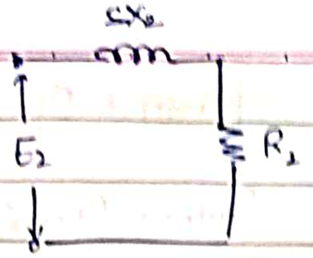
slip frequency leakage impedance of equivalent rotor

corresponding induced current

slip frequency leakage impedance of actual rotor

Slip-f leakage impedance of the referred rotor ⇒ $Z_{2s} = R_{2s} + jsX_2$

referred rotor leakage impedance at slip frequency slip-frequency equivalent circuit →



also, $\hat{I}_2 = \hat{I}_{2s}$ → current in eq. rotor with same no. of phases and same no. of turns per phase as the stator.

(speed of flux w.r.t. rotor) = $s \times$ (speed of flux w.r.t. stator)

$$E_{2s} = s\hat{E}_2$$

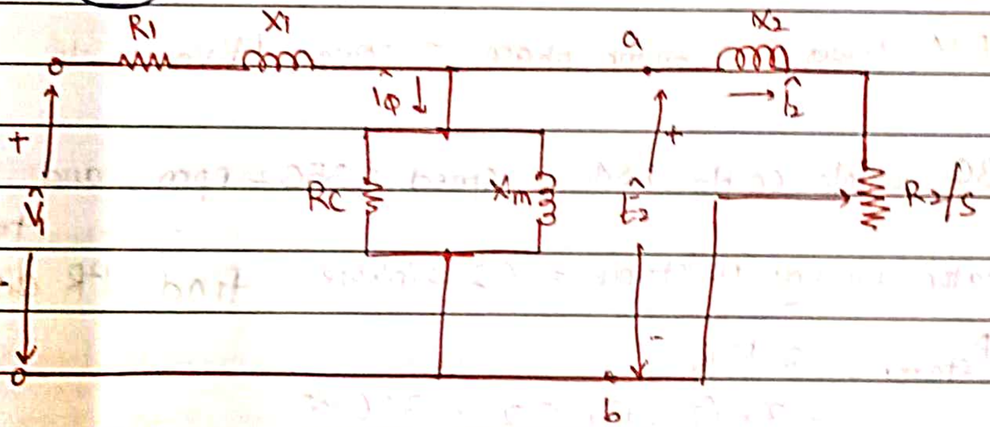
so, $$\frac{\hat{E}_{2s}}{\hat{I}_{2s}} = \frac{s\hat{E}_2}{\hat{I}_2} = Z_{2s} = R_2 + jsX_2$$

$$\boxed{Z_2 = \frac{R_2}{s} + jX_2}$$

impedance of the equivalent stationary rotor which appears across the load terminals of the stator equivalent circuit.

$\frac{R_2}{s}$ → (shaft load) + (rotor resistance) → fun^c of slip and so mechanical load.

FOR 1-phase of stator



→ current in reflected rotor impedance = load component of the stator current, $\hat{I}_2$

→ Voltage across the R.R.I. = stator voltage $\hat{E}_2$

→ frequency also changed to stator frequency.

ANALYSIS OF EQUIVALENT CIRCUIT

Total power transferred across the air-gap from the stator,

$$\boxed{P_{gap} = n_{ph} I_2^2 \left(\frac{R_2}{s}\right)}$$

(n_{ph} = no. of stator phases)

Total rotor loss, $\boxed{P_{rotor} = n_{ph}\, I_{2s}^2 R_2}$

as $I_{2s} = I_2$

$$P_{rotor} = n_{ph}\, I_2^2 R_2$$

So, $P_{mech} = P_{gap} - \underset{(lost)}{P_{rotor}} = n_{ph}\, I_2^2 \left(\frac{R_2}{s}\right) - n_{ph}\, I_2^2 R_2.$

$$\boxed{P_{mech} = n_{ph}\, I_2^2 R_2 \left(\frac{1-s}{s}\right)}$$

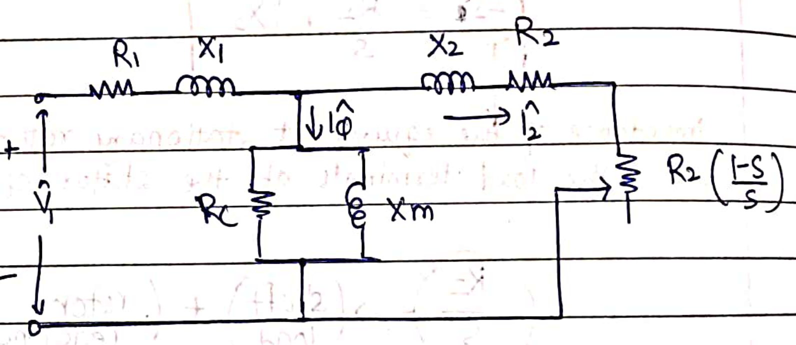
(power developed by the motor)

So,

$\boxed{P_{mech} = (1-s)\, P_{gap}}$ $\quad P_{mech} = \left(\frac{1-s}{s}\right) P_{rotor}$

$P_{rotor} = s\, P_{gap}$

redrawn eq. circuit ⇒



E.M. power per stator phase = power delivered to $R_2\left(\frac{1-s}{s}\right)$

e.g.- 3φ, 2-pole, 60 Hz I.M., speed = 3502 rpm and input P = 15.7 kW
terminal current = 22.6 A
Stator-winding resistance = 0.2 Ω/phase., find I^2R dissipated in rotor.

A) $P_{stator} = 3 \cdot I_1^2 R$

$= 3 \times (22.6)^2 \times 0.2 = 306\ W$

$N_s = \frac{120 \times 60}{2} = 3600$ rpm $\qquad P_{gap} = P_{input} - P_{stator}$

$= 15.7 - 0.3 = 15.4\ kW$

$s = \frac{98}{3600} = 0.027$

as $P_{rotor} = s \cdot P_{gap} = 0.027 \times 15.4\ kW = \boxed{419\ W}$

$P_{mech} = 15.4 - 0.4 = \boxed{15\ kW}$

rotor frequency

Now, as $P_{mech} = \omega_m T_{mech} = (1-s)\omega_s T_{mech}$

$$\boxed{T_{mech} = \frac{P_{mech}}{\omega_m} = \frac{P_{gap}}{\omega_s} = \frac{n_{ph} I_2^2 (R_2/s)}{\omega_s}}$$

(rotor)

$$\omega_s = \frac{4\pi f_e}{P} = \left(\frac{2}{P}\right)\omega_e$$

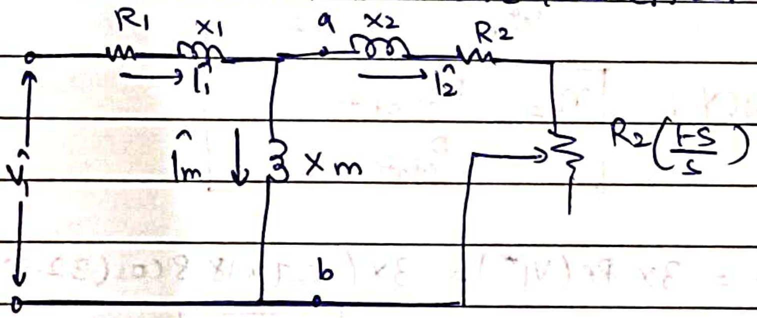
synchronous mechanical angular velocity

$$\boxed{P_{shaft} = P_{mech} - P_{rot}}$$

(losses)

$$T_{shaft} = \frac{P_{shaft}}{\omega_m} = T_{mech} - T_{rot}$$

APPROXIM^n :→ Neglect core loss resistance in eq. circuit, and subtract the core-loss effect from T_{mech}/P_{mech}.

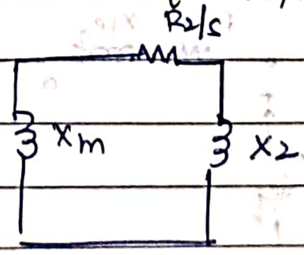


eg.- 3Φ Y-connected 220V (L-L) 7.5 KW, 60 Hz, six pole I.M. has (stator referred) $R_1 = 0.294$ $R_2 = 0.144$ Ω/phase
$X_1 = 0.503$ $X_2 = 0.209$ $X_m = 13.25$

fric^n/windage/core losses → 403 W ; slip = 2%.

operated at rated voltage and frequency.

Ans.) $V_{L\text{-to-}N} = \frac{220}{\sqrt{3}} = 127\,V$



$$Z_f = \left(\frac{R_2}{s} + jX_2\right) \| (jX_m)$$

$s = 0.02$

$$N_s = \frac{120 \times \cancel{60}^{10}}{\cancel{6}} = 1200\,rpm$$

$Z_f = 5.41 + j\,3.11$ Ω/phase

~~$7.5 \times 10^3 = \sqrt{3} \times \frac{220}{\sqrt{3}} \times |I|$~~ ~~$|I| = 19.68$~~

$V = IZ \Rightarrow I = \frac{127}{6.75\angle 32.3} = 18.8\angle -32.3$

$Z_{net} = 5.704 + j\,3.613$
$= 6.75\angle 32.3°$ Ω

Stator current: $\boxed{\hat{I}_1 = 18.8 \angle -32.3^\circ \ A}$ $\quad$ $\boxed{N_s = 1200 \text{ rpm}}$

pf = 0.845 lag $\qquad$ $\omega_s = \frac{N_s}{60} \cdot 2\pi = 125.7$ rad/sec

$$N_r = 1200(1-0.02) = 1176 \text{ rpm}$$

$$\omega_m = \left(\frac{1176}{60}\right) \times 2\pi = 123.2 \text{ rad/sec}$$

$$P_{gap} = 3 \times I_2^2 \left(\frac{R_2}{s}\right) = 3 \times I_1^2 \times R_f = 3(18.8)^2 \times 5.41$$

$$\boxed{P_{gap} = 5.704 \text{ KW}}$$

$$P_{mech} = (1-s)P_{gap} = \boxed{5.59 \text{ KW}}$$

$$P_{shaft} = 5.59 - 0.43 = \boxed{5220 \text{ W}}$$

$$T_{shaft} = \frac{P_{shaft}}{\omega_m} = \frac{5220}{123.2} = 42.4 \text{ Nm}$$

EFFKIENCY : $\boxed{\eta = \frac{P_{shaft}}{P_{stator\ input}}}$

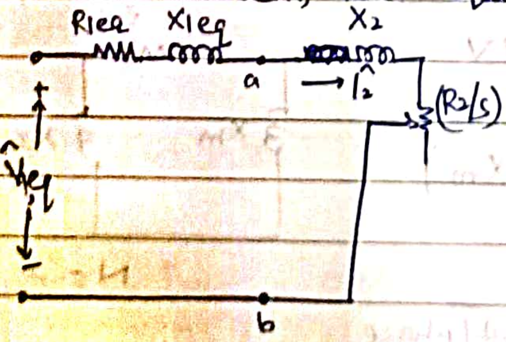
$$P_{input} = 3 \times Re(VI^*) = 3 \times (127 \times 18.8 \cos(32.3^\circ)) = 6060 \text{ W}$$

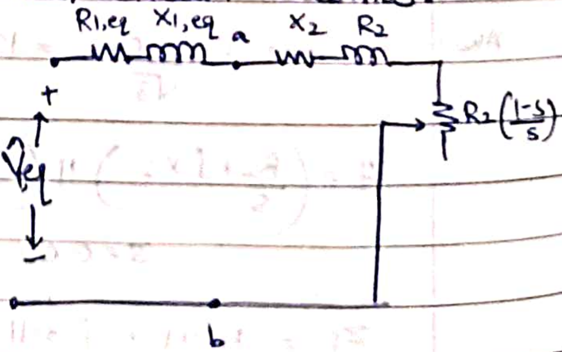
$$\eta = \frac{50220}{6060} = \boxed{86.1\%}$$

Here, (75KW → max. power the motor is designed to deliver to load under normal operating conditions)

TORQUE and POWER by THEVENIN's THEOREM

after using Thevenin's theorem, the equivalent circuit becomes :→





$$\hat{V}_{1,eq} = \hat{V}_1 \left(\frac{jX_m}{R_1 + j(X_1 + X_m)}\right) \qquad Z_{1,eq} = \frac{jX_m(R_1 + jX_1)}{j(X_m + X_1) + R_1}$$

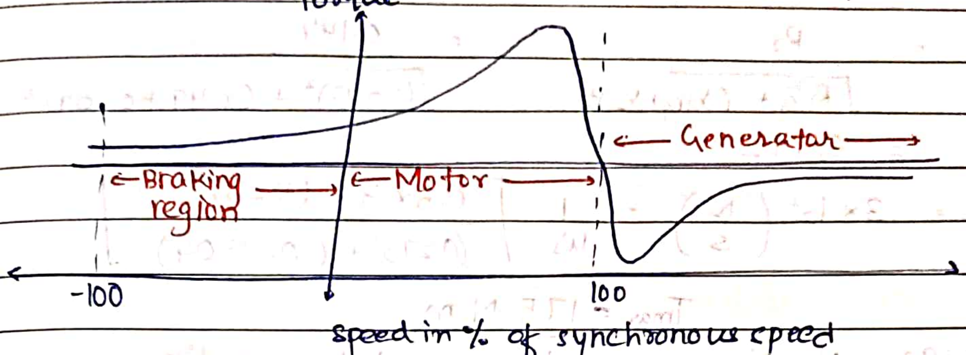
R_c → neglected

From Thevenin equivalent circuit,

$$\hat{I}_2 = \frac{\hat{V}_{1,eq}}{\hat{Z}_{1,eq} + jX_2 + (R_2/s)}$$

so,

$$T_{mech} = \frac{P_{gap}}{\omega_s} = \frac{1}{\omega_s}\left[\frac{n_{ph}\, V_{1eq}^2\,(R_2/s)}{(R_{1eq}+R_2/s)^2 + (X_{1eq}+X_2)^2}\right]$$



⇒ For $s>1$ → motor must be driven backward, against the dir^n of rot^n of its magnetic field by a source of mechanical power capable of counteracting T_{mech}.

Maximum electromagnetic torque / braking torque ⇒ T_{max} → when power delivered to (R_2/s) is maximum

↓

so, $\frac{R_2}{s} = \underbrace{\left| R_{1,eq} + j(X_{1,eq} + X_2) \right|}_{\text{magnitude of impedance b/w voltage source and resistance.}}$

(Clip for max. torque) $S_{max.T} = \dfrac{R_2}{\sqrt{R_{1eq}^2 + (X_{1eq}+X_2)^2}}$

so, the torque,

$$T_{max} = \frac{1}{\omega_s}\left[\frac{0.5\, n_{ph}\, V_{1,eq}^2}{R_{1,eq} + \sqrt{R_{1,eq}^2 + (X_{1,eq}+X_2)^2}}\right]$$

eg.3) For the previous example, find

a) load component, I_2 of stator current, T_{mech}, P_{mech} for $s = 0.03$

b) T_{max} and corresponding speed c) T_{start} and $I_{load,start}$.

Ans) $s = 0.03$ $V_{1,eq} = 122.3V$ $R_{1,eq} = 0.273\Omega$ $X_{1,eq} = j\,0.490\Omega$

$s = 0.03$ $R_2/s = 4.8\Omega$

$$|I| = \frac{V_{1,eq}}{\left| R_2/s + R_{1,eq} + j(X_{1eq} + X_2) \right|} = 23.9\text{ A}$$

Page No.: Date: YOUVA

$$\frac{P_{mech}}{\omega_m} = \frac{P_{gap}}{\omega_s}$$

$$T_{mech} = \frac{\left(n_{ph} I_2^2 \left(\frac{R_2}{s}\right)\right)}{\omega_s} = \frac{3 \times (23.9)^2 \times 4.80}{125.7} = \boxed{65.4 \text{ Nm}}$$

(using Thevenin equivalent)

$$P_{mech} = \omega_s(1-s) \times T_{mech} = \boxed{7980 \text{ W}}$$

(b) $$S_{max} = \frac{R_2}{\sqrt{R_{1,eq}^2 + (X_{1eq}+X_2)^2}} = \frac{0.144}{\sqrt{(0.273)^2 + (0.49+0.209)^2}}$$

$$= 0.192$$

$$T_{mech} = 3 \times I_2^2 \left(\frac{R_2}{s}\right) = \frac{1}{\omega_s}\left[\frac{0.5 \times 3 \times (122.3)^2}{(0.273) + (0.7504)}\right]$$

$$T_{max} = 175 \text{ N-m}$$

$$0.192 = \frac{N_s - N_r}{N_s} \Rightarrow N_r = (0.808) N_s = 969.6 \text{ rpm}$$

$$\boxed{\omega_r = 101.54 \text{ rad/sec}}$$

(c) starting torque

$S=1$ $N_r = 0$ rpm

$$I_2 = \frac{V_{1,eq}}{\sqrt{(R_{1,eq}+R_2)^2 + (X_{1,eq}+X_2)^2}} = 150 \text{ A}$$

so,

$$T_{mech} = \frac{3 \times (150)^2 R_2}{125.7} = \boxed{77.3 \text{ Nm}}$$

Braking :

For $S>1$ → drive motor backward against dir^n of rotation of its magnetic field

N_s becomes (-ve) ← $S=2$

↓

sequence changes ← interchange two stator leads in a 3-ϕ motor ← $\boxed{\text{plugging}}$

⇒ slip at max. torque $\propto R_2$

T_{max} → independent of R_2

⇒ $R_2 \uparrow\uparrow$ by inserting external resistance in a wound rotor

↓

T_{max} → same

NO-LOAD AND BLOCKED ROTOR TESTS

(#) NO LOAD TESTS

↳ exciting current and no-load losses [O.C. Test of Transformer]

⇒ rated frequency, balanced polyphase voltages at stator terminals.
↳(f_r)

V_{ln} = Line-to-Neutral Voltage I_{ln} = line-current

P_{ln} = Total polyphase electrical input power

⇒ at no load, rotor current → 0 ⇒ only produce torque to overcome the friction and windage losses.

⇒ no-load I^2R loss → 0.

⇒ no-load primary I^2R is NOT NEGLIGIBLE (like transformer)
(stator) ↓
I_ϕ is large due to air-gap flux being present.

$$\boxed{\underset{\text{(rotational losses)}}{P_{rot}} = P_{nl} - n_{ph} I_{nl}^2 R_1}$$

→ (rotational losses = CONSTT. at no-load values and load values)

⇒ IGNORE ⇒ core-losses and R_c

To determine them → operate motor at no-load and rated speed
↓
suddenly disconnect from supply.

decay in motor speed ⟷ rotational losses ⇐

$$J\left(\frac{d\omega_m}{dt}\right) = -T_{rot} = -\frac{P_{rot}}{\omega_m}$$

J (rotor inertia)

$$P_{rot} = -\omega_m J\left(\frac{d\omega_m}{dt}\right)$$

$$\boxed{P_{rot}(\omega_m) = -\omega_m J\left(\frac{d\omega_m}{dt}\right)}$$

⇕

rotational losses at any speed ω_m.

and so,

$$P_{core} = P_{nl} - P_{rot} - n_{ph} I_{nl}^2 R_1$$

P_{core}: no-load core loss → voltage of no-load test

(#) If we neglect the voltage drop across stator resistance and reactance,

Voltage across R_c = V_{ln}

$$\boxed{R_c = \frac{V_{nl}^2}{P_{core}}}$$

→ can simply keep it seperate.

$(R_2/s \to \infty)$

slip at no load, $S_{nl} \to 0$, so, $\frac{R_2}{s} \to \infty$

$\boxed{\text{load} \to 0 \quad N_s \to N_r \quad s \to 0}$

require only very little torque i.e. to overcome Prot

reactance of $\|$ comb^n $\to jX_m$

so,

apparent reactance, $X_{nl} = X_1 + X_m$

$$\boxed{\begin{array}{c} Q_{nl} = \sqrt{S_{nl}^2 - P_{nl}^2} \\ S_{nl} = n_{ph} V_{nl} I_{nl} \end{array}}$$

$$\boxed{X_{nl} = \frac{Q_{nl}}{n_{ph} I_{nl}^2}}$$

The no-load pf is usually very small, so,

$$\boxed{X_{nl} \simeq \frac{V_{nl}}{I_{nl}}}$$

BLOCKED - ROTOR TEST
(Short-circuit test)

rotor is blocked $\to$ can't rotate $\Rightarrow$ $(s=1)$

$V_{1,bl} \to$ L-N voltage

$P_{1,bl} \to$ L current

$P_{bl} \to$ Total polyphase input electrical power

$f_{bl} \to$ frequency of blocked rotor test.

⇒ it should be performed under conditions for which current & rotor frequency are approx. same as those in the machine at operating cond^n → eg., if s is supposed to be 0.01 ⇓ reduce V, f

$Q_{bl} = \sqrt{S_{bl}^2 - P_{bl}^2}$ $\quad S_{bl} = n_{ph} V_{bl} I_{bl}$

$$\boxed{X_{bl} = \left(\frac{f_{r\,\text{rated}}}{f_{bl}}\right)\left(\frac{Q_{bl}}{n_{ph} I_{bl}^2}\right)} \qquad \boxed{R_{bl} = \frac{P_{bl}}{n_{ph} I_{bl}^2}}$$

$$Z_{bl} = R_1 + jX_1 + (R_2 + jX_2) \,\|\, (jX_m) \qquad (s=1)$$

$$= R_1 + R_2\left(\frac{X_m^2}{R_2^2 + (X_m + X_2)^2}\right) + j\left(X_1 + \frac{X_m(R_2^2 + X_2(X_m + X_2))}{R_2^2 + (X_m + X_2)^2}\right)$$

ASSUMING :- $R_2 \ll X_m$

$$Z_{bl} = R_1 + R_2\left(\frac{X_m}{X_2+X_m}\right)^2 + j\left(X_1 + X_2\left(\frac{X_m}{X_2+X_m}\right)\right)$$

$$R_{bl} = R_1 + R_2\left(\frac{X_m}{X_2+X_m}\right)^2 \quad X_{bl} = X_1 + X_2\left(\frac{X_m}{X_2+X_m}\right)$$

$$X_2 = (X_{bl} - X_1)\left(\frac{X_m}{X_m + X_1 - X_{bl}}\right)$$

$$R_2 = (R_{bl} - R_1)\left(\frac{X_2+X_m}{X_m}\right)^2$$

$$\boxed{X_m = X_{nl} - X_1}$$

$$\boxed{X_2 = (X_{bl} - X_1)\left(\frac{X_{nl}-X_1}{X_{nl} - X_{bl}}\right)}$$

ASSUME :- $X_1 = X_2$

$$\boxed{X_m = X_{nl} - X_1}$$

using X_m and $X_2 \longrightarrow R_2$.

eg.- 7.5 hp, 3Φ, 220V, 19A, 60Hz, 4-pole $\boxed{X_1 = 0.429\, X_2}$ (given)

→ No-load test :- $(f = 60Hz) \Rightarrow V_{nl} = 219V$ (l-l)
$I_{1,nl} = 5.7A$ $\quad P_{1,nl} = 380W$

→ Blocked-rotor test :- (15Hz) $V_{bl} = 26.5V$ (l-l)
$I_{1,bl} = 18.57A$ $\quad P_{bl} = 675W$

→ Average dc resistance per phase : $R_1 = 0.262\Omega$ (after test 2) imm

→ Blocked-rotor test :- (60Hz) $V_{bl} = 212V$ (l-l)
$I_{1,bl} = 83.3A$ $\quad P_{bl} = 20.1KW$
$T_{start} = 74.2\ Nm$

1:) rotational losses. $P_{rot} = P_{nl} - n_{ph} I_{nl}^2 R_1$
$= (380) - (3 \times (5.7)^2 \times 0.262)$
$P_{rot} = 354\ W$

$$Q_{nl} = \sqrt{S_{nl}^2 - P_{nl}^2} = \sqrt{\left(3 \times \frac{219}{\sqrt{3}} \cdot 5.7\right)^2 - (380)^2} = 2128\ VA$$

$$X_{nl} = \frac{Q_{nl}}{3 \times (5.7)^2} = 21.8\ \Omega$$

$$Q_{1,bl} = \sqrt{S_{bl}^2 - P_{bl}^2} = \sqrt{\left(3 \times \frac{26.5}{\sqrt{3}} \times 18.75\right)^2 - (675)^2} = 520\ VA$$

Page No.:	
Date:	

$$X_{bl} = \left(\frac{f_r}{f_{bl}}\right) \frac{Q_{bl}}{(n_{ph} I_{b,l}^2)} = \left(\frac{60}{15}\right) \times \frac{520}{3(18.57)^2} = 2.01\,\Omega$$

$$X_2 = (X_{bl} - X_1)\left(\frac{X_{nl} - X_1}{X_{nl} - X_{bl}}\right) \rightarrow \text{Q.E.}$$

$X_1 = K X_2$

solving $\Rightarrow$ $X_2 = 1.48$ and $\cancel{168.1}$

$\boxed{X_2 = 1.48\,\Omega}$ $\boxed{X_1 = 0.633\,\Omega}$ $X_m = X_{nl} - X_1 = \boxed{21.2\,\Omega}$

$$R_2 = (R_{bl} - R_1)\left(\frac{X_2 + X_m}{X_m}\right)^2 \qquad R_{bl} = \frac{P_{bl}}{n_{ph} I_{bl}^2} = 0.652\,\Omega$$

$$R_2 = (0.652 - 0.262)\left(\frac{22.68}{21.2}\right)^2$$

$\boxed{R_2 = 0.447\,\Omega}$ $\boxed{R_1 = 0.262\,\Omega}$

(b) If we want T_{start}

$\hookrightarrow$ parameters change at very high slip $(s=1)$

Test 4

$P_{gap} = P_{bl} - n_{ph} I^2 R_1 = 14650\,W$

4 pole, $\omega = 188.5$ rad/s

$$T_{start} = \frac{14650}{(188.5)} = \boxed{77.7\ Nm}$$

IF $X_m \gg X_2 \Rightarrow \boxed{X_{bl} = X_1 + X_2}$

INDUCTION MOTOR - CONSTRUCTION

cage-rotors and wound rotors.

→ Y connected - 3ϕ windings similar to those on stator.

conducting bars of Al.

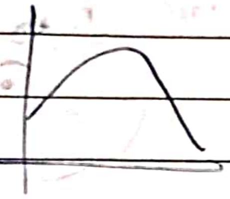
↓

rotor windings are shorted through brushes riding on the slip rings.

↓

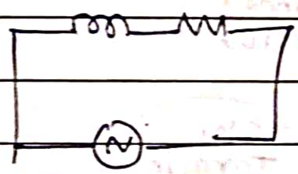
extra resistance can be added into the rotor circuit. ← wound rotors have their rotor currents accessible on ~~slip rings~~ stator brushes

squirrel cage rotors → very low starting torque



⟹ 3ϕ AC supply
↓
RMF in air-gap
↓
cuts the rotor bars
↓
EMF
↓
current (s. circuit)
↓
force

slip-ring I.M. → very high starting torque



⟹ i is not in phase with Voltage

$f\uparrow$ ⟹ $I = \dfrac{V\angle\theta}{R+j\omega L} = \dfrac{V\angle\theta}{Z\angle\tan^{-1}(\omega L/R)}$

$(\omega\uparrow)$

$\phi\uparrow$

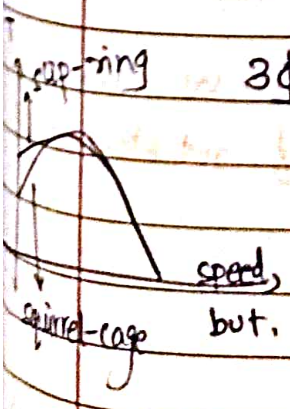
as $\boxed{f\uparrow\ \phi\uparrow}$ and as $\boxed{R\uparrow\ \phi\downarrow}$

due to the phase-lag phenomenon.

↳ max. EMF on one bar, max. current on other bar

$\boxed{\text{Max}^m \text{ torque} \Longrightarrow \text{max. current is near to the max. magnetic flux}}$

at start ⟹ (f) EMF induced $\uparrow$ ⟹ $\phi\uparrow$ ⟶ very low starting torque



3ϕ windings ⟹ $\boxed{\phi\downarrow}$

↳ EMF across terminals of windings
↓
current → lags EMF

but, we can use an external resistance $(R\uparrow\ \phi\downarrow)$.

$e_{ind} = (V[\cos\theta\,\hat{x} + \sin\theta\,\hat{y}] \times 1.5 B_m [\cos(\omega_s t)\hat{x} + \sin(\omega_s t)\hat{y}]) \cdot (l\hat{z})$

$= 1.5 B_m V l\,[\sin(\omega_s t)\cos\theta - \cos\omega_s t \sin\theta]$ $(\theta = \omega_r t)$

$= \boxed{1.5 B_m V l\,[\sin(\omega_s - \omega_r)t]}$

INDUCED TORQUE

RMF, B_s with $\boxed{N_s = \frac{120 f}{P}}$ → f: system frequency

↓

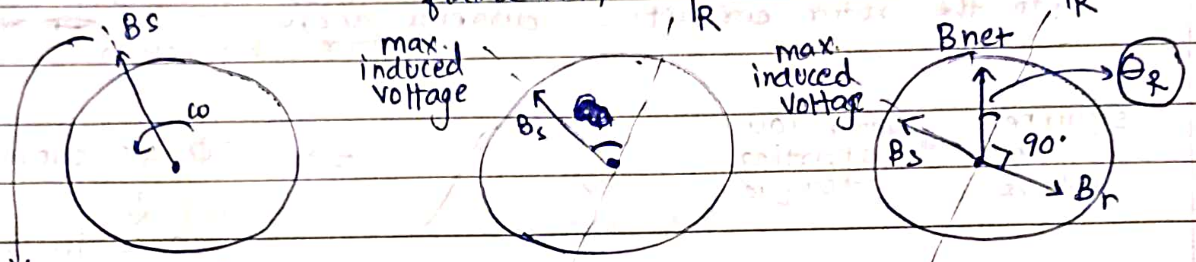
induces a current/voltage in the rotor bars

$$\boxed{e_{ind} = (\vec{V} \times \vec{B}) \cdot \vec{l}}$$

$\vec{V}$: velocity of the bar relative to M.F.

$\vec{B}$: magnetic flux density

$\vec{l}$: length of conductor in M.F.



max. induced voltage

$e = (\vec{V} \times \vec{B}) \cdot \vec{l}$

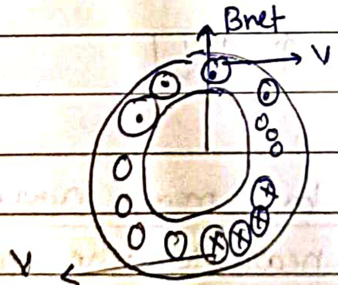
max^m induced current lags behind due to inductance of the rotor

B_r lags I_R by 90° (inductive circuit)

B_s tries to pull B_R towards it ⟹ CCW Torque

⇒ upper rotor bars → V is to the right, rel. to MF

↳ induced voltage out of page



E_{ind} in lower bars is into the page

current flow out of the upper bars and into the lower bars.

$\boxed{T_{ind} = K\,\vec{B_r} \times \vec{B_s}}$ → CCW

⇓

rotor accelerates in CCW dir^n.

⟹ B_R and B_S rotate as same N_S, but rotor itself rotates at some $N_R < N_S$.

Electrical Frequency of Rotor

→ Primary (stator) induced a voltage on secondary (rotor) but at a diff. frequency than the stator.

⇒ if the rotor is locked so that it can not move, the rotor will have the same frequency as the stator (of induced EMF).

⇒ if the rotor turns at synchronous speed, frequency of rotor = 0.

For any speed in b/w

$$\boxed{f_{re} = s \cdot f_{se}}$$

(rotor frequency) ∝ (diff. b/w n_{synch} and speed of rotor, n_r)

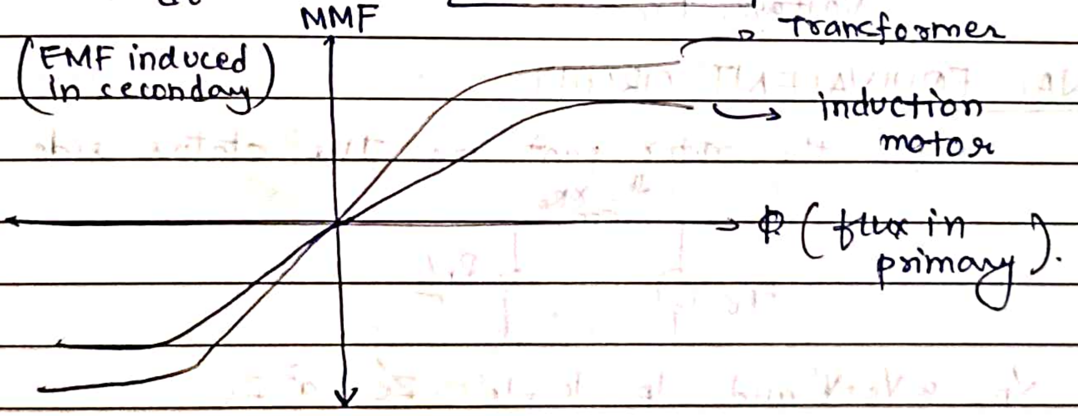
$$\boxed{f_{re} = \frac{P}{120}(n_{sync} - n_r)}$$

EQUIVALENT TRANSFORMER CIRCUIT

⇒ R_1, X_1 → stator reactance and leakage resistance.

⇒ The flux in the machine is related to the integral of the applied voltage E_1.

$$N\frac{d\phi}{dt} = E_1 \qquad \boxed{\phi = \int E_1 \, dt}$$



slope of MMF v/s Φ curve of I.M. is smaller than a transformer.
↓
due to air-gap. R↑ and so the coupling b/w the primary and secondary coils ⇓ greatly.

↳ higher reluctance ⇒ higher (IM) ⇒ X_m of an equivalent circuit will have a much smaller value.

at very low slips ⇒ $\frac{R_r}{s} \gg X_s$ ⇒ resistance predominates; varies linearly with speed.
(NO-LOAD) → steady state

ROTOR- CIRCUIT MODEL

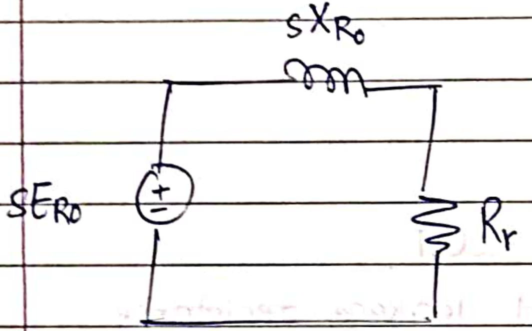
→ greater the relative motion b/w rotor and stator M.F, greater the resulting voltage and rotor frequency.

magnitude of induced voltage at any slip, $\boxed{E_R = s E_{R_0}}$ → induced voltage at blocked rotor

R_r → rotor resistance = const.

X_r → rotor reactance $= \omega_{re} L_R$
$= 2\pi f_{re} L_R$

so, $X_r = s(2\pi f_{se} L_R)$.
$\boxed{X_R = s X_{R_0}}$



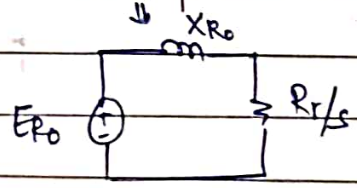
$I_R = \frac{s E_{R_0}}{j s X_{R_0} + R_r}$

so, $\boxed{I_R = \frac{E_{R_0}}{(R_r/s) + j(X_{R_0})}}$

supplied by a constant power-source voltage, E_{r_0}. ← varying impedance

⇒ FINAL EQUIVALENT CIRCUIT

refer the rotor part on the stator side.
⇓



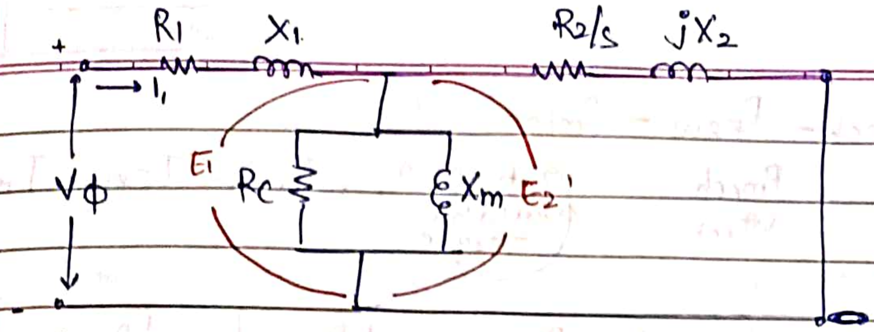
→ $V_p = a V_s = V_s'$ and $I_p = \frac{I_s}{a} = I_s'$ $\quad Z_s' = a^2 Z_s$

So, the transformations over here are →

$\boxed{E_1 = E_R' = a_{eff} . E_{R_0}}$

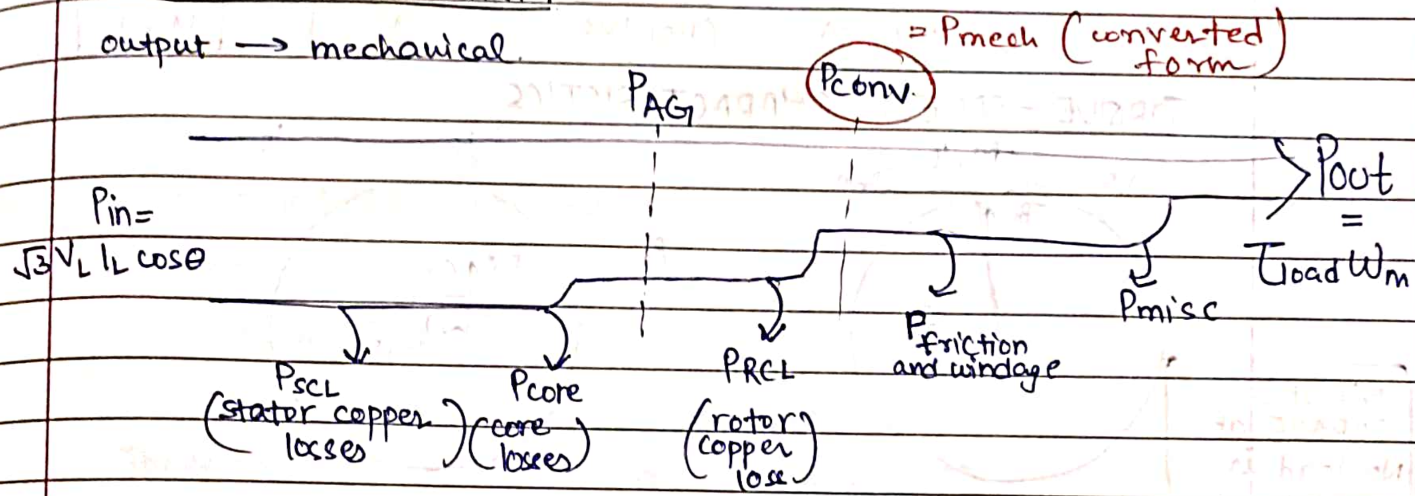
$I_2 = \frac{I_R}{a_{eff}}$ $\quad Z_2 = a_{eff}^2 \left(\frac{R_r}{s} + jX_{R_0}\right)$

$R_2 = a_{eff}^2 R_r \quad X_2 = a_{eff}^2 X_{R_0}$



POWER FLOW DIAGRAM

output → mechanical



→ because of the nature of the core losses → where they are accounted for is arbitrary.

↓ partially from rotor circuit & partially from stator circuit.

⇒ Input current to a phase of motor,

$$I_1 = \frac{V\phi}{Z_{eq}} \qquad Z_{eq} = R_1 + jX_1 + \frac{1}{(G_C - jB_m) + \frac{1}{(R_2/s + jX_2)}}$$

succeptance

stator copper losses, $P_{SCL} = 3 I_1^2 R_1$

$$\boxed{P_{core} = 3 E_1^2 G_C}$$

$P_{AG} = P_{in} - P_{SCL} - P_{core}$ → only consumed in R_2/s

$$\boxed{P_{AG} = 3 I_2^2 \left(\frac{R_2}{s}\right)}$$

$P_{RCL} = 3 I_R^2 R_R$ → as power is unchanged refeering to ideal transformer

$$\boxed{P_{RCL} = 3 I_2^2 R_2}$$

$$P_{conv} = P_{AG} - P_{RCL} = 3 I_2^2 R_2 \left(\frac{1-s}{s}\right)$$

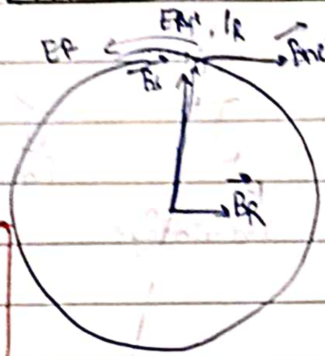
$P_{RCL} = s P_{AG}$ $\qquad P_{mech} = (1-s) P_{AG}$ $\qquad P_{mech} = \left(\frac{1-s}{s}\right) P_{RCL}$

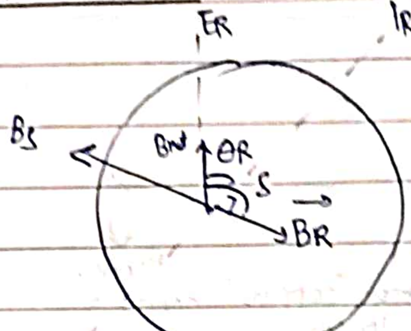
$P_{out} = P_{mech} - P_{F\&W} - P_{misc}$

Induced torque $\longrightarrow \dfrac{P_{mech}}{\omega_{rot}}$ actual available torque $\longrightarrow T_{mech} - T_{F\&W} - T_{misc}$

$$T_{ind} = \frac{P_{mech}}{\omega_s(1-s)} = \frac{(1-s)P_{AG}}{(1-s)\omega_s} = \boxed{\frac{P_{AG}}{\omega_s}} = \boxed{\frac{P_{mech}}{\omega_{rot}}}$$

TORQUE-SPEED CHARACTERISTICS





INDUCED TORQUE at No-load is just enough to overcome rotational losses.

low loads
↓
almost synchronous speed

Bnet → IM (magnetiz^n current) $\propto E_1$

if E_1 = constt. → Bnet is constt. (approx.)

No load ⇒ E_R is very small (no rel. motion)

rotor frequency ⇓ ← I_R ⇓ (small current)
↓
rotor reactance → 0 ⇒ I_R almost in phase with E_R

angle (δ+90°) ← small MF (B_r)

(stator current is quite large → must supply Bnet)

$$T_{ind} = K\, \vec{B_r} \times \vec{B}_{net}$$
$$= K|B_r|\,|B_{net}| \sin\delta$$

very small

τ ↓↓   ($\tau \to 0$) (just enough to overcome rotational losses)

high loads

load ⇑ slip ↑
↓
more rel. motion
↓
stronger E_r ⇒ produces a large rotor current
↓
angle also changes ← B_R ⇑

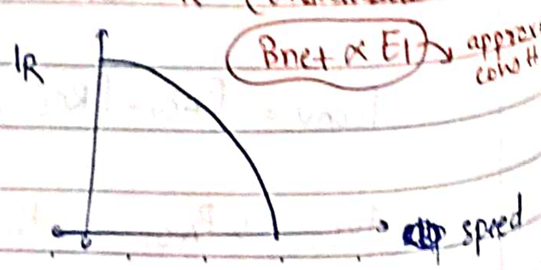
rotor frequency ↑
rotor reactance ↑ → further lags

increase in S tends to decrease the torque
↓
$\boxed{\text{MAXIMA point}}$

$$\boxed{T_{ind} = K B_r B_{net} \sin\delta}$$

$B_r \propto I_R$ (unsaturated)

$B_{net} \propto E_1$ → approx. constt.



The peak torque ... an incremental increase in load, the increase in rotor current is balanced by the decrease in the power factor.

$$I_R = \frac{E_{R_0}}{jsX_{R_0} + R_r}$$

$$\delta = 90^\circ + (PF)$$

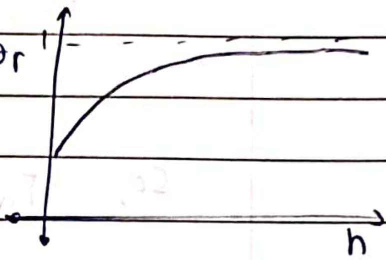
$$\boxed{\delta = \theta_R + 90^\circ}$$

$$\boxed{\sin\delta = \cos\theta_R}$$

$$\theta_R = \tan^{-1}\left(\frac{X_R}{R_R}\right) = \tan^{-1}\left(\frac{sX_{R_0}}{R_r}\right)$$

so,

$$\boxed{pf = \cos\left(\tan^{-1}\left(\frac{sX_{R_0}}{R_r}\right)\right)}$$



$$\boxed{\tau = K(I_R)(E_1)(\cos\theta_R)}$$

INDUCED TORQUE EQUATION

$$P_{AG,1\phi} = I_2^2\left(\frac{R_2}{s}\right) \qquad P_{AG} = 3I_2^2(R_2/s)$$

Thevenin's equivalent $\rightarrow$ to determine I_2

ignoring R_c,

$$V_{Th} = \left(\frac{jX_m}{R_1 + j(X_1+X_m)}\right)V_\phi$$

$$\boxed{|V_{Th}| = \frac{X_m}{\sqrt{R_1^2+(X_1+X_m)^2}}|V_\phi|}$$

$$Z_{Th} = \frac{jX_m(R_1+jX_1)}{R_1 + j(X_1+X_m)}$$

Assumptions :- $X_m \gg X_1$ and $X_m \gg R_1$

$$\boxed{V_{TH} \simeq \left(\frac{X_M}{X_1+X_M}\right)V_\phi}$$

$$\boxed{R_{TH} \simeq R_1\left(\frac{X_M}{X_1+X_M}\right)^2} \qquad \boxed{X_{TH} \simeq X_1}$$

So,

$$I_2 = \frac{V_{TH}}{Z_{TH}+Z_2} = \frac{V_{TH}}{\frac{R_2}{s} + R_{TH} + j(X_{Th}+X_2)}$$

Page No.:
Date:

$$|I_2| = \frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R_2}{s}\right)^2 + \left(X_{TH} + X_2\right)^2}}$$

So,

$$P_{Ag} = 3 I_2^2 \left(\frac{R_2}{s}\right) = \frac{3 V_{TH}^2 (R_2/s)}{\left(R_{TH} + \frac{R_2}{s}\right)^2 + \left(X_{Th} + X_2\right)^2}$$

So, $$T_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

$$\boxed{T_{ind} = \frac{3 V_{TH}^2 (R_2/s)}{\omega_{sync}\left(\left(R_{Th} + R_2/s\right)^2 + \left(X_{Th} + X_2\right)^2\right)}}$$

Characteristics

- starting torque is slightly larger than the full load torque → motor will start carrying any load.
- $\boxed{T \propto V^2}$ → speed control
- if driven faster than n_s, dirⁿ of T reverses and it becomes a generator. (mech. → ele.)

max. Torque ⇒ $$\frac{R_2}{s} = \sqrt{R_{TH}^2 + (X_2 + X_{Th})^2}$$

$$S_{max.} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_2 + X_{Th})^2}}$$

$$T_{max.} = \frac{3 V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{(X_2 + X_{Th})^2 + R_{Th}^2}\right]}$$

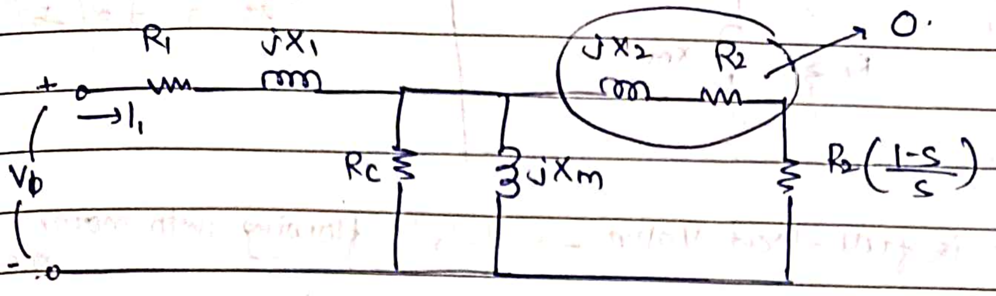
↓
Independent of R_2

LOW SLIP REGION : $\boxed{T \propto \text{slip}}$

NO-LOAD TEST

$s \to 0$ $\quad R_2\left(\frac{1-s}{s}\right) \gg R_2$ or X_2

$P_{conv} = P_{F+W} + P_{misc}$ $(s \to 0)$



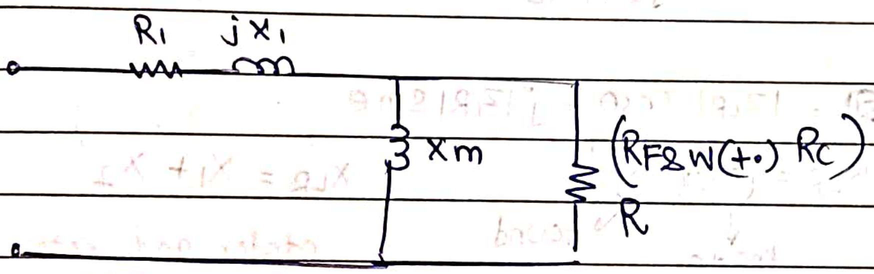
$P_{input} = P_{losses}$ $\quad$ rotor copper losses $\to 0$ $\quad$ (I_2 is ↓↓)

$P_{SCL} = 3 I_1^2 R_1$

$P_{in} = P_{SCL} + P_{core} + P_{F\&W} + P_{misc}$

$$\boxed{P_{in} = 3I_1^2 R_1 + P_{rot}}$$

$$\boxed{R_c = \frac{V_\phi^2}{P_s}}$$



$R_c \parallel R_2\left(\frac{1-s}{s}\right)$.

current needed to establish a M.F. is quite large because of the high reluctance of air-gap ⇒ X_m is ⇓ than reactances ~~in parallel~~

and so input pf → 0 $\quad$ ($\theta \to 90^\circ$, $\cos\theta \to 0$)

with the large lagging current, $\quad$ ~~large lagging~~ current.

$Re(I\angle\theta)$ $R \to 0$

$$|Z_{eq}| = \frac{V_\phi}{I_{1,nl}} = \boxed{X_1 + X_m} \quad \text{(eq. input impedance)}$$

DC Test → For stator resistance, R_1

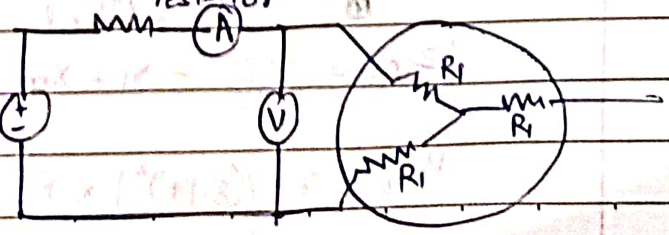
↳ DC voltage on stator windings

↓

No induced EMF in rotor circuit and no current flow.

stator resistance can now be determined. ←

Y-connected power supply $\quad \boxed{R_1 = \frac{V_{DC}}{2I_{DC}}}$



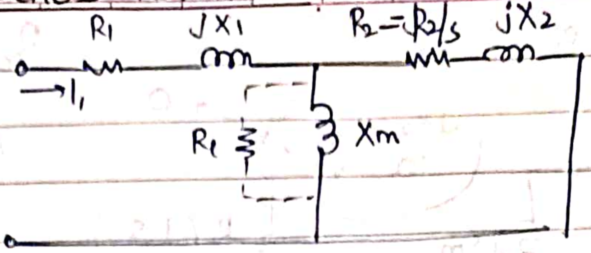
also, find $(P_{SCL})_{no\ load}$

and $P_{rot} = P_{in} - (P_{SCL})_{no\ load}$

BLOCKED ROTOR TEST (S=1)

(rotor is blocked)

S=1



at a $f \approx (25\%)$ rated frequency

current is full-load value → V, I, P flowing into motor are measured

R_2, X_2 → small → all current flows through them.

$X_m >> |R_2 + jX_2|$ $R_C >> |R_2 + jX_2|$

$\boxed{I \rightarrow \text{rated value}}$

$PF = \cos\theta = \dfrac{P_{in}}{\sqrt{3} V_T I_L}$. $|Z_{LR}| = \dfrac{V_T}{\sqrt{3} I_L} = R_{LR} + jX'_{LR}$

$Z_{LR} = |Z_{LR}|\cos\theta + j|Z_{LR}|\sin\theta$

$\boxed{R_{LR} = R_1 + R_2}$ $X'_{LR} = X'_1 + X'_2$

known found

stator and rotor reactances (AT TEST FREQUENCY)

$R_2 = R_{LR} - R_1$.

$\boxed{X_{LR} = \left(\dfrac{f_{rated}}{f_{test}}\right)\left(X_1' + X_2'\right)}$ X_1 found ⇒ X_m

e.g.) DC: $V_{DC} = 13.6\,V$ $I_{DC} = 28\,A$

7.5 hp, 4-pole, 208V, 60 Hz

$I_{rated} = 28$

No load: $V_T = 208\,V$ $f = 60\,Hz$ $I_B = 8.2\,A$

$I_A = 8.12\,A$ $P_{in} = 420\,W$ $I_C = 8.18\,A$

Locked rotor: $V_T = 25\,V$ $f = 15\,Hz$ $I_B = 28.0\,A$

$I_A = 28.1\,A$ $P_{in} = 920\,W$ $I_C = 27.6\,A$

⇒) $R_1 = \dfrac{13.6}{2(28)} = \boxed{0.243\,\Omega}$

$(I_1)_{av} = 8.17\,A$ $V_T = 208\,V$

$(V_\phi)_{nl} = 120\,V$

$|Z|_{nl} = \dfrac{120}{8.17} = X_1 + X_m = 14.7\,\Omega$

$P_{SCL} = 3 \times \left((8.17)^2\right) \times R_1 = 48.7\,W$

no-load losses, $P_{rot} = P_{in} - P_{SCL} = 420 - 48.7 = 371.3\,W$

locked rotor test, $I = 27.9\,A$

$$|Z_{LR}| = \frac{25}{27.9\sqrt{3}} = 0.517\,\Omega \qquad \cos\theta = \frac{920}{\sqrt{3}\cdot(25)(27.9)}$$

$$\theta = 40.4^\circ$$

$$R_{LR} = 0.517\cos(40.4^\circ) = R_1 + R_2$$

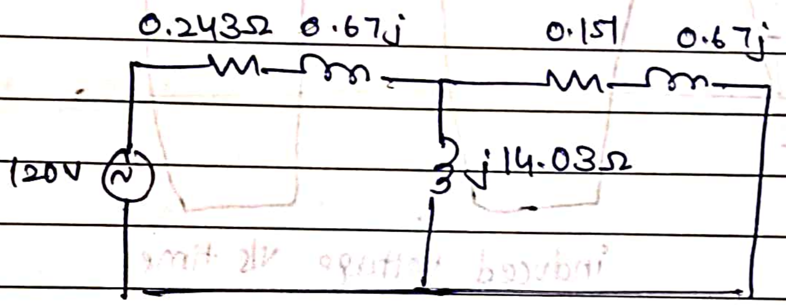
$$R_2 = 0.151\,\Omega$$

$0.517 \sin\theta$

$$X_{LR} = (4)(0.335) = 1.34\,\Omega$$

assuming $X_1 = X_2 = 0.67\,\Omega$

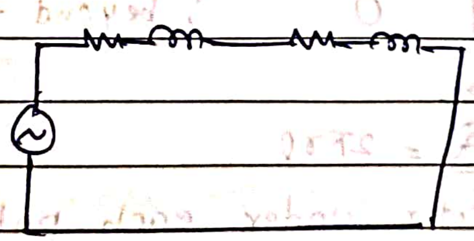
Now,



$$|V_{Th}| = |V_\phi| \cdot \frac{X_m}{\sqrt{(X_m+X_1)^2 + R_1^2}} \approx |V_\phi| \cdot \left(\frac{X_m}{X_1+X_m}\right) = 114.6V$$

$$R_{Th} = R_1\left(\frac{X_m}{X_1+X_m}\right)^2 = 0.221\,\Omega$$

$$X_{Th} = X_1 = 0.67j\,\Omega$$



$$\frac{R_2}{s} = |R_1' + j(X_1' + X_2)| \qquad s = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}}$$

$= 11.1\%$

and so, $$\tau = \frac{3}{2}\,\frac{V_{TH}^2}{\left(R_{Th} + \sqrt{R_{Th}^2 + (X + X_{Th})^2}\right)}$$

$$= \underline{66.2\ Nm}$$