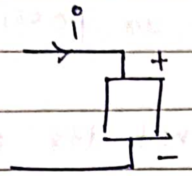
SIGN CONVENTION

→ components in which i enters through +ve terminal (V)

$P = +ve$ $\quad r = +ve$



i across voltage drop ⇒ (+ve)

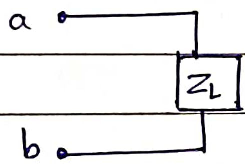
i across voltage rise ⇒ (−ve)

$P = -ve$ & $r = -ve$

Power source: $P<0 \rightarrow$ delivering

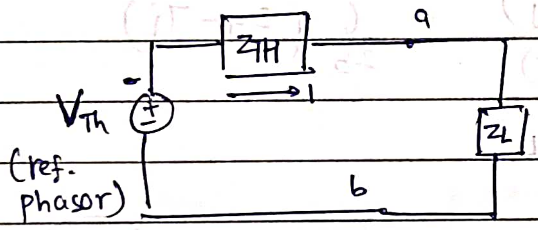
$P>0 \rightarrow$ absorbing

Maximum Power Transfer Theorem



Z_L that results in max. average power to the terminals a and b

$$\boxed{Z_L = Z_{TH}^*}$$



$$Z_{Th} = R_{TH} + jX_{Th}$$

$$Z_L = R_L + jX_L$$

$$I = \frac{V_{Th}}{(R_{TH}+R_L) + j(X_{Th}+X_L)}$$

$$(P_{avg})_{\text{to load}} = |I|^2 R_L = \frac{|V_{TH}|^2 R_L}{(R_{TH}+R_L)^2 + (X_{Th}+X_L)^2}$$

$V_{Th}, R_{Th}, X_{Th} \rightarrow$ fixed

$$\frac{\partial P}{\partial X_L} = 0 \qquad \frac{\partial P}{\partial R_L} = 0$$

$$X_L = -X_{Th} \qquad R_L = \sqrt{R_{Th}^2 + (X_L+X_{Th})^2}$$

on combining both of them, $\boxed{Z_L = Z_{Th}^*}$

when $Z_L = Z_{Th}^*$, $I = \frac{V_{Th}}{2R_L}$ $\quad (P_{max.})_{load} = \frac{|V_{Th}|^2 R_L}{4R_L^2}$

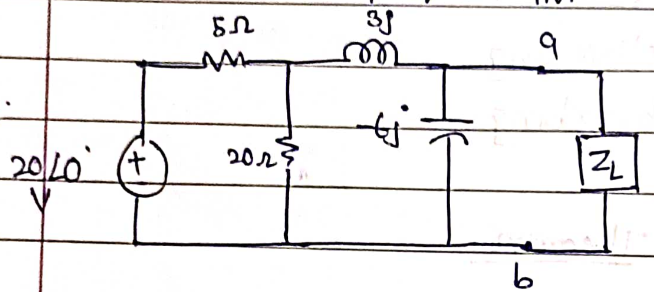
$$P_{max.} = \frac{1}{4}\frac{|V_{Th}|^2}{R_L} = \frac{1}{8}\left(\frac{V_m^2}{R_L}\right)$$

⇒ adjust X_L as close to $-X_{Th}$ as possible
↓
adjust R_L as close to $\sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$ as possible

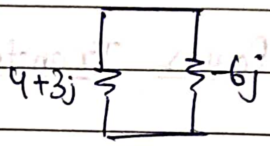
⇒ If magnitude of Z_L can be varied but not its phase angle,

$$|Z_L| = |Z_{Th}|$$

e.g.



$$Z_{Th} = \frac{(\cancel{5})(20)}{\cancel{25}} = 4$$



$$\frac{1}{Z} = \frac{4-3j}{25} + \frac{j}{6}$$

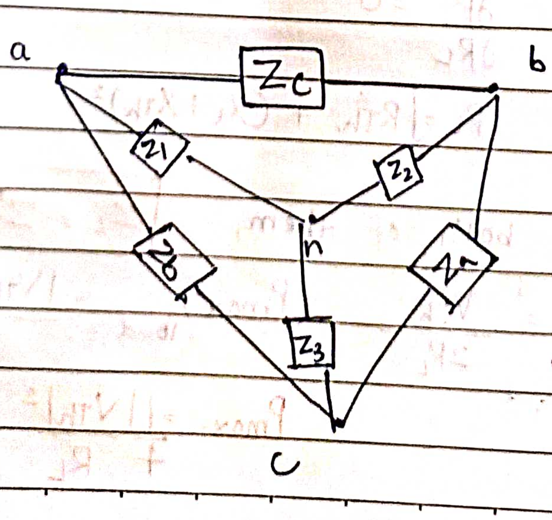
$$= \frac{24 - 18j + 25j}{150} \Rightarrow \frac{24+7j}{150} = \frac{1}{Z}$$

$$Z_{Th} = \frac{\cancel{150}\,6\,(24-7j)}{(\cancel{25})(25)} = \frac{6}{25}(24-7j)$$

$$Z_{Th}^{*} = \frac{6}{25}(24+7j) = Z_L$$

and, $(\text{max. power})_{avg.} = \frac{1}{4}\frac{|V_{Th}|^2}{R_L} = \frac{1}{4}\frac{(19.2)^2}{R_L} = 8W$ Ans.

DELTA-TO-WYE TRANSFORMATIONS



$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

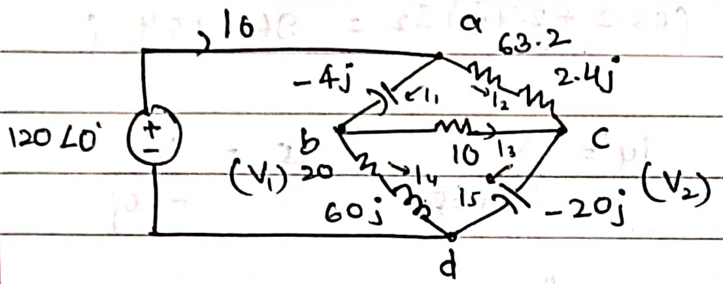
$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1} \qquad Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

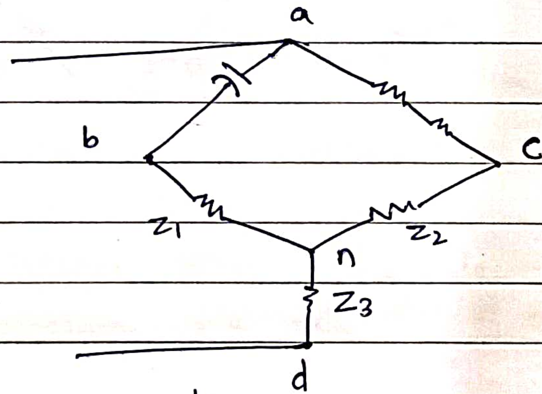
eg.



Find $I_0, \ldots, I_5, V_1, V_2$

Δ to Y transformation

upper ✗ | lower ✓ $(30+40j)$ (check denominator)



$$Z_1 = \frac{(10)(20+60j)}{(30+40j)} = \frac{(20+60j)(3-4j)}{25} = \frac{(60+240)+j(100)}{25}$$

$$Z_1 = \boxed{12+4j}$$

$$Z_2 = \frac{(1\cancel{0})(-2\overset{4}{\cancel{0}}j)(30\mp4j)}{2\cancel{5}5}$$

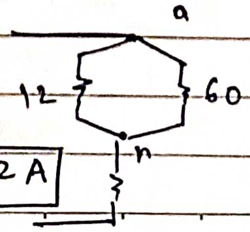
$$Z_2 = (4-3j)\times 4/5 = \boxed{-3.2-2.4j}$$

$$Z_3 = \frac{(-2\cancel{0}j)(20+60j)(3-4j)}{25} = (-2j)(12+4j) = \boxed{8-24j}$$

$$Z_{abn} = -4j + 12 + 4j = 12\,\Omega \qquad Z_{acn} = 63.2 + 2.4j - 3.2 - 2.4j = 60\,\Omega$$

$$I = \frac{120}{18-24j} = \frac{\overset{4}{\cancel{120}}}{30\angle -53^\circ}$$

$$I = 4\angle 53^\circ = \boxed{2.4 + j3.2\text{ A}}$$



$$Z_{an} = \frac{(12)(60)}{72} = 10\,\Omega$$

$$Z_{net} = 18 - 24j$$

$\cancel{12} I_1 = \overset{5}{\cancel{60}} I_2 \qquad I_1 + I_2 = I_0$

$I_2 = \frac{I_0}{6} \qquad I_1 = \frac{5I_0}{6}$

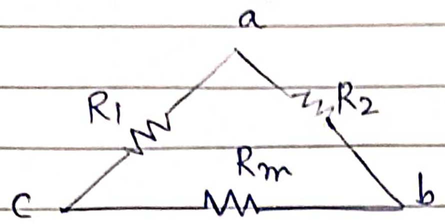
$V_1 = 120\angle 0^\circ - (4j) I_1 = \frac{328}{3} + 8j$

$V_2 = 120\angle 0 - (53.2 + 2.4j) I_2 = 96 - \frac{104}{3} j$

$I_3 = \frac{V_1 - V_2}{10} \qquad I_4 = \frac{V_1}{20+60j} \qquad I_5 = \frac{V_2}{-20j}$

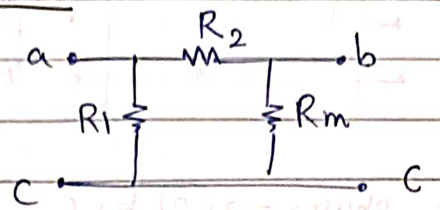
THREE PHASE CIRCUITS

Delta-To-Wye Transformations



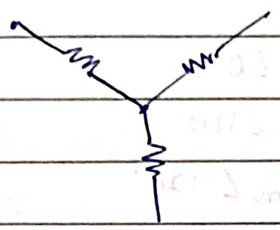
delta

≡



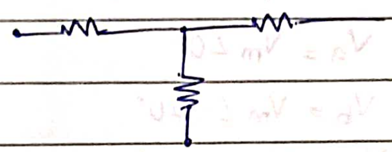
(pi-connection)

(equivalent)

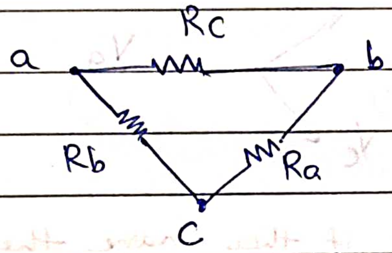


wye (Y) connection

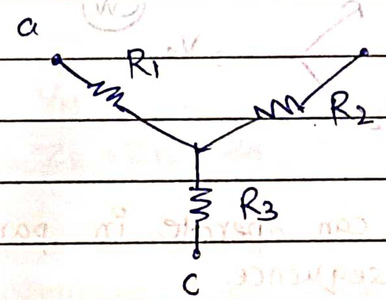
≡



T-connection



⟺



The resistance b/w terminals 'a' and 'b' must be the same whether we use Δ or Y.

$$R_{ab} = \frac{R_c(R_a+R_b)}{R_c+R_a+R_b} = R_1 + R_2$$

$$R_{bc} = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} = R_2 + R_3$$

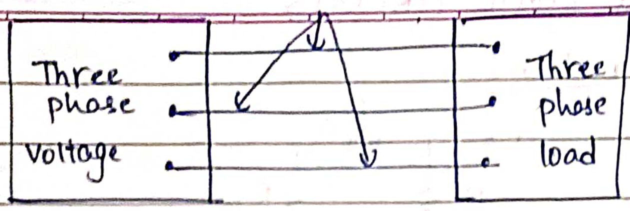
$$R_{ac} = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c} = R_1 + R_3$$

$$R_1 = \frac{R_b R_c}{R_a+R_b+R_c} \qquad R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_2 = \frac{R_a R_c}{R_a+R_b+R_c} \qquad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_3 = \frac{R_a R_b}{R_a+R_b+R_c} \qquad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Three phase line



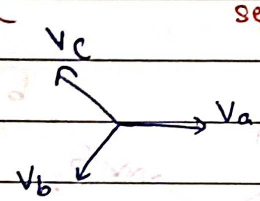
The three phases → a, b, c

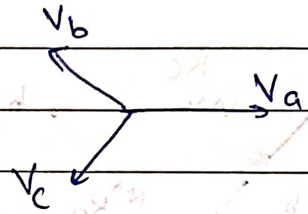
a → reference phase

b phase → 2 cases

$V_a = V_m \angle 0^\circ$
$V_b = V_m \angle -120^\circ$
$V_c = V_m \angle 120^\circ$
(abc or POSITIVE phase sequence)

$V_a = V_m \angle 0^\circ$
$V_b = V_m \angle 120^\circ$
$V_c = V_m \angle -120^\circ$
(acb or NEGATIVE phase sequence)

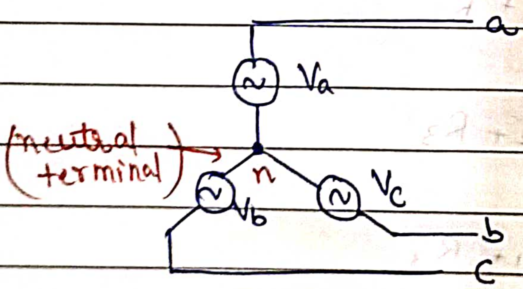


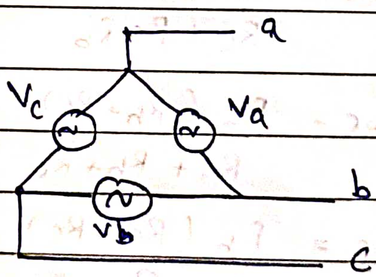


2 Circuits can operate in parallel only if they have the same phase sequence.

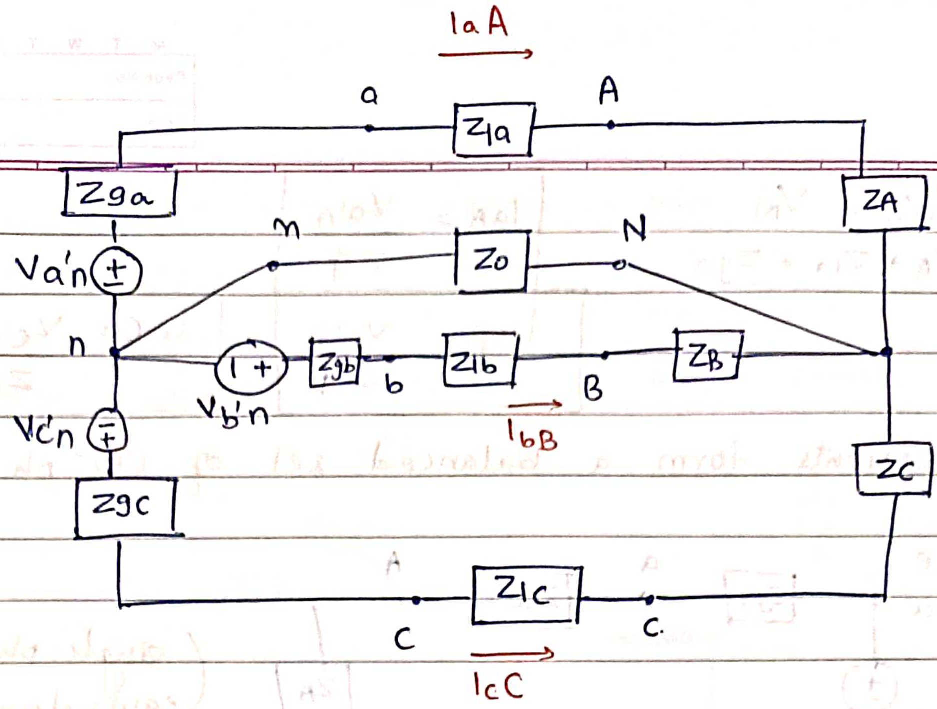
balanced set of voltages : $\boxed{\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0}$ (phasor voltages)

also, $\boxed{v_a + v_b + v_c = 0}$ (instantaneous)





WYE-WYE CIRCUIT ANALYSIS



Zga, Zgb, Zgc ⟶ internal impedances

~~Z~~ Z1a, Z1b, Z1c ⟶ line impedances

ZA, ZB, ZC ⟶ load impedances

(n ⟶ reference)

$$\frac{V_N}{Z_0} + \frac{V_N - V_{a'n}}{Z_A + Z_{1a} + Z_{ga}} + \frac{V_N - V_{b'n}}{Z_B + Z_{1b} + Z_{gb}} + \frac{V_N - V_{c'n}}{Z_C + Z_{1C} + Z_{gC}} = 0$$

balanced 3ϕ circuit ⇒

1) Voltage sources form a balanced set
2) impedance of each phase of voltage is same
3) impedance of each line is same
4) impedance of each phase load is same

So,

$$V_N\left(\frac{1}{Z_0} + \frac{3}{Z_\phi}\right) = \frac{V_{a'n} + V_{b'n} + V_{c'n}}{Z_\phi} \Rightarrow \boxed{V_N = 0}$$

(balanced 3ϕ circuit)

$$Z_\phi = Z_A + Z_{1a} + Z_{ga}$$
$$= Z_B + Z_{1b} + Z_{gb}$$
$$= Z_C + Z_{1C} + Z_{gC}$$

⇒ There is NO P.D. diff. source neutral and line neutral, and so, current in neutral conductor $(Z_0) = 0$

→ perfect short circuit and $V_N = 0$.

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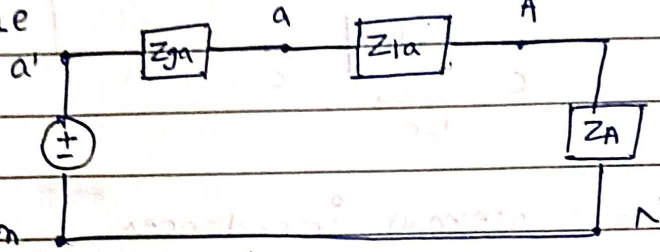
$$I_{aA} = \frac{V_{a'n} - V_{N}}{Z_A + Z_{1a} + Z_{ga}}$$

$$\boxed{I_{aA} = \frac{V_{a'n}}{Z_\phi}}$$

$$\boxed{I_{bB} = \frac{V_{b'n}}{Z_\phi}} \qquad \boxed{I_{cC} = \frac{V_{c'n}}{Z_\phi}}$$

Three line currents form a balanced set of 3-phase cu...

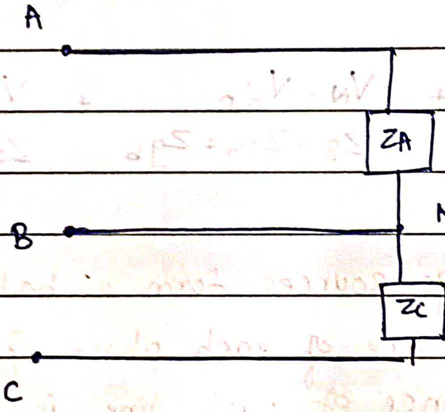
For a-phase



(single-phase equivalent circu...)

Here, current in neutral conductor is I_{aA}, not same as

$(I_0 = I_{Aa} + I_{Bb} + I_{cc})$



LOAD TERMINALS

line-to-line : V_{AB}, V_{BC}, V_{CA}

line to neutral : V_{AN}, V_{BN}, V_{CN}

$$\boxed{V_{AB} = V_{AN} - V_{BN}}$$

$$\boxed{V_{BC} = V_{BN} - V_{CN}}$$

$$\boxed{V_{CA} = V_{CN} - V_{AN}}$$

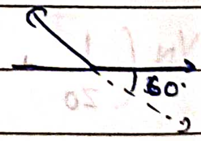
Assuming (using N)

abc (+ve seq)

$V_{AN} = V_\phi \angle 0^\circ$

$V_{BN} = V_\phi \angle -120^\circ$

$V_{CN} = V_\phi \angle 120^\circ$



−ve seq. → lag by 30°

$V_{AB} = V_\phi \angle 0^\circ - V_\phi \angle 120^\circ = \sqrt{3} V_\phi \angle 30^\circ$

$V_{BC} = V_\phi \angle -120^\circ - V_\phi \angle 120^\circ = \sqrt{3} V_\phi \angle -90^\circ$

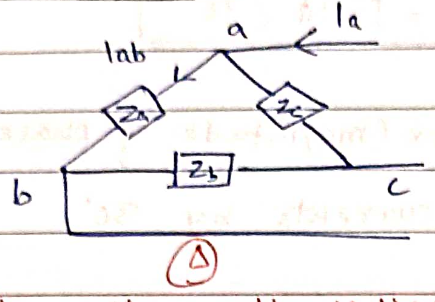
$V_{CA} = V_\phi \angle 120^\circ - V_\phi \angle 0^\circ = \sqrt{3} V_\phi \angle 150^\circ$

1) The magnitude of L-to-L voltages is $\sqrt{3} \times$ (L-to-N) voltage

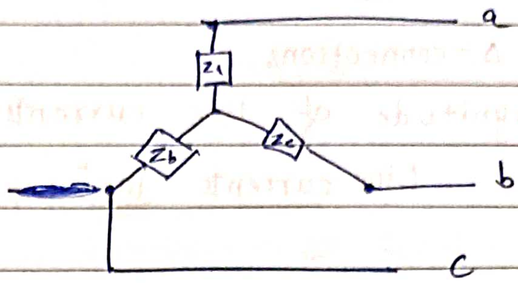
2) L-to-L voltages form a set of balanced 3-ϕ voltages.

3) L-to-L Voltages lead L-to-N voltages by 30°

Line voltage: voltage across a pair of lines
Phase voltage: voltage across a single phase
Line current: current in a single line
phase current: current in a single phase



phase voltage ≡ line voltage



phase current ≡ line current

+ve sequence (abc) ⟶ −ve sequence (acb)

Interchange (b and c) in all parameters

⇒ L-to-L voltages lag by 30° from L-to-N

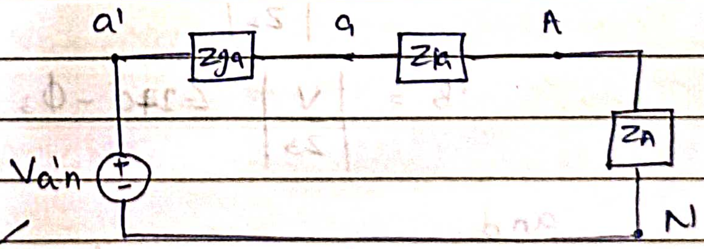
WYE-DELTA CIRCUIT

Use delta to wye transformation

(IMP.) $\boxed{Z_\Delta = 3Z_Y}$

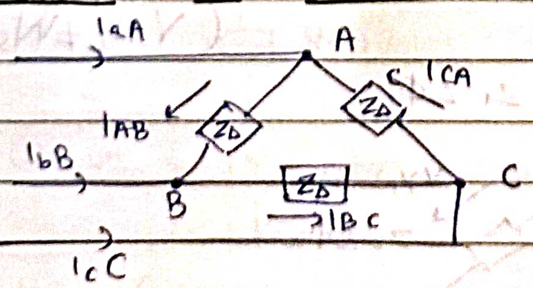
$$Z_Y = \frac{Z_\Delta}{3}$$

after the Δ-Y ⇒



use it to find line currents, I_{aA}, I_{bB}, I_{cC}

Line currents v/s Phase currents in Δ-config.



(+ve) phase sequence

$I_{AB} = I_\phi \angle 0°$

$I_{BC} = I_\phi \angle -120°$

$I_{CA} = I_\phi \angle 120°$

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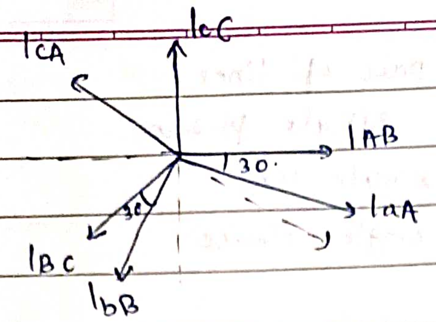


$I_{aA} = I_{AB} - I_{CA}$

$$\boxed{\begin{aligned} I_{aA} &= \sqrt{3} I_\phi \angle -30^\circ \\ I_{bB} &= \sqrt{3} I_\phi \angle -150^\circ \\ I_{cC} &= \sqrt{3} I_\phi \angle 90^\circ \end{aligned}}$$ (line currents)

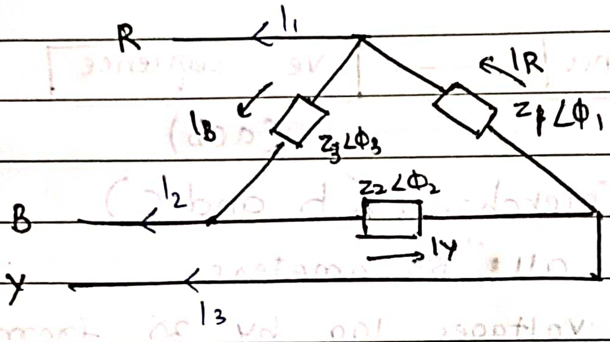
In Δ-connections

⇒ 1) magnitude of line currents $= \sqrt{3} \times$ (magnitude of phase currents)

and Line currents "lag" phase currents by 30°

UNBALANCED CIRCUITS

A)



(RYB)

$V_{RY} = V\angle 0^\circ$ $\quad V_{YB} = V\angle -120^\circ$ $\quad V_{BR} = V\angle -240^\circ$

So,

$$I_R = \frac{V_{RY}}{Z_1\angle\phi_1} = \left|\frac{V}{Z_1}\right| \angle -\phi_1$$

$$I_Y = \left|\frac{V}{Z_2}\right| \angle -120^\circ - \phi_2$$

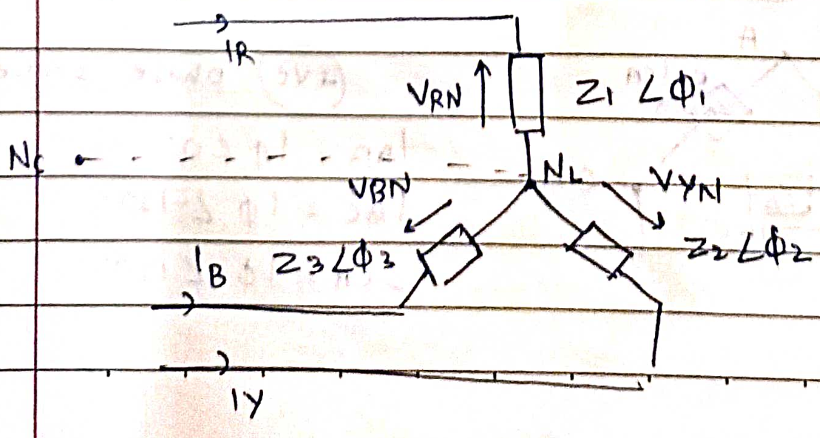
$$I_B = \left|\frac{V}{Z_3}\right| \angle -240^\circ - \phi_3$$

and

$I_1 = I_R - I_B$ (phasor difference)

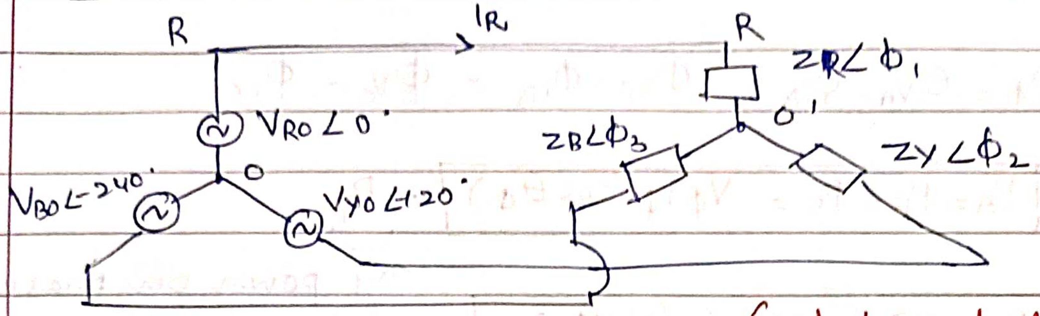
B) 4-wire star connected load

$(V_{N_L} \neq V_{N_S})$



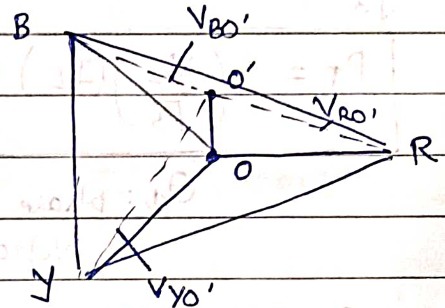
M1) Y - Δ conversion → line currents in the Δ are same as in the Y

M2) <u>Millman's method</u>



(balanced supply) (unbalanced load)

supply and load have the same line voltages, equal in magnitude and displaced by 120°



$$V_{O'O} = \frac{V_{RO} Y_R + V_{YO} Y_Y + V_{BO} Y_B}{Y_R + Y_Y + Y_B}$$

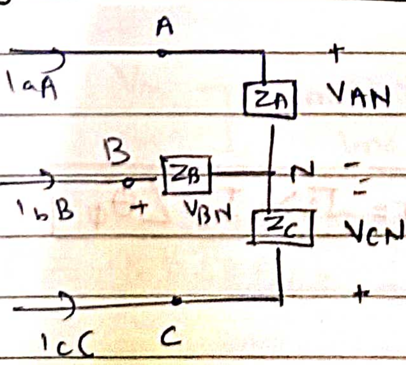
and $V_{RO} = V_{RO'} + V_{O'O}$

i.e., $V_{RO'} = V_{RO} - V_{O'O}$

POWER CALCULATIONS

~~Average~~

⇒ <u>Average Power in a Balanced Wye Load</u>



$$P_A = |V_{AN}||I_{aA}| \cos(\theta_{VA} - \theta_{iA})$$

$$P_B = |V_{BN}||I_{bB}| \cos(\theta_{VB} - \theta_{iB})$$

$$P_C = |V_{CN}||I_{cC}| \cos(\theta_{VC} - \theta_{iC})$$

as |L-to-N| voltage is same

$V_\phi = |V_{AN}| = |V_{BN}| = |V_{CN}|$

$I_\phi = |I_{aA}| = |I_{bB}| = |I_{cC}|$

and

$\theta_\phi = \theta_{VA} - \phi_{iA} = \phi_{VB} - \phi_{iB} = \phi_{Vc} - \phi_{ic}$

also,

$$\boxed{P_A = P_B = P_C = V_\phi I_\phi (\cos\theta_\phi)} = \underbrace{P_\phi}_{\text{avg. power per phase}}$$

$$\boxed{P_T = 3P_\phi = 3V_\phi I_\phi (\cos\theta_\phi)}$$

V_L and I_L are line voltages and currents,

$I_\phi = I_L \qquad \frac{V_L}{\sqrt{3}} = V_N = V_\phi$

Total real power in a balanced 3ϕ load $\Rightarrow$ $\boxed{P_T = 3\left(\frac{V_L}{\sqrt{3}}\right)(I_L)\cos\theta_\phi}$

θ_ϕ: phase angle b/w phase voltage and phase current

⇒ COMPLEX POWER IN A BALANCED WYE LOAD

$Q_\phi = V_\phi I_\phi \sin\theta_\phi$

Total complex power in a balanced 3ϕ load $\Rightarrow$ $\boxed{Q_T = (\sqrt{3}V_L I_L)\sin\theta_\phi}$

For a balanced load, complex power,

$S_\phi = V_{AN} I_{aA}^* = V_{BN} I_{bB}^* = V_{CN} I_{cC}^*$

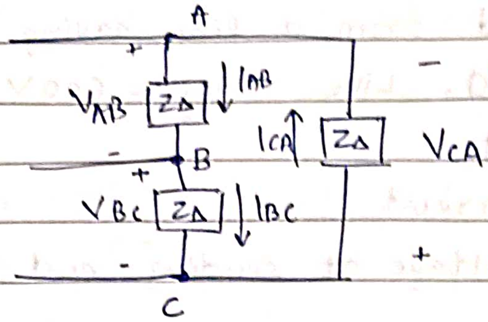
$$\boxed{S_\phi = V_\phi I_\phi^*}$$

$S_\phi = P_\phi + jQ_\phi$

$S_T = 3S_\phi \qquad \boxed{S_T = \sqrt{3} V_L I_L \angle\theta_\phi}$

BALANCED DELTA LOAD

similar to Y-connected load



$P_A = |V_{AB}||I_{AB}| \cos(\theta_{V_{AB}} - \theta_{i_{AB}})$

$P_B = |V_{BC}||I_{BC}| \cos(\theta_{V_{BC}} - \theta_{i_{BC}})$

$P_C = |V_{CA}||I_{CA}| \cos(\theta_{V_{CA}} - \theta_{i_{CA}})$

$|V_{AB}| = |V_{BC}| = |V_{CA}| = V_\phi$ and I_ϕ and θ_ϕ

$$\boxed{P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi}$$

IMP:- avg. power per phase = (RMS phase voltage) × (RMS phase current) × (cos (angle b/w phase V and I))

$$P_T = 3 V_\phi I_\phi \cos\theta_\phi$$

$$= 3 V_L \left(\frac{I_L}{\sqrt{3}}\right) \cos\theta_\phi$$

$$\boxed{P_T = \sqrt{3} V_L I_L \cos\theta_\phi}$$

and similarly,

$$\boxed{Q_T = \sqrt{3} V_L I_L \sin\theta_\phi}$$

$$\boxed{S_T = \sqrt{3} V_L I_L \angle\theta_\phi}$$

INSTANTANEOUS POWER IN 3-PHASE CIRCUITS

v_{AN} → inst. L-to-N voltage

└ reference

$P_A = v_{AN} i_{aA} = V_m I_m \cos\omega t \cos(\omega t - \theta_\phi)$

$P_B = V_m I_m \cos(\omega t - 120^\circ) \cos(\omega t - 120^\circ - \theta_\phi)$

$P_C = V_m I_m \cos(\omega t + 120^\circ) \cos(\omega t + 120^\circ - \theta_\phi)$

V_m, I_m] → max. amp. of phase voltage and line current (or phase)

$$P_T = P_A + P_B + P_C = \boxed{\frac{3}{2} V_m I_m (\cos\theta_\phi)}$$

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Eg. a balanced 3ϕ load requires 480 KW at a lagging pf of 0.8. The load is fed from a line having impedance of $0.005 + j\,0.025\ \Omega/\phi$. Line voltage = 600 V

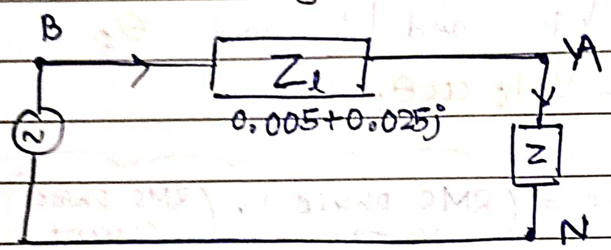
a) Single-phase equivalent

b) Magnitude of line current

c) Magnitude of line voltage at sending end

d) Pf at sending end.

A:)



~~$P_{load} = 480\angle -37° - 3P_\phi$~~

~~$P_\phi = 160\left(\frac{4}{5} - \frac{3j}{5}\right)$~~

$P_{load} =$ 160 KW at 0.8 pf $\quad = P$

$\frac{3}{4} = \frac{Q}{P} \qquad Q = 120$

$P_{load} = 160 + 120j$

$V_{Line\text{-}N} = 600\angle 0°$ (assuming A is reference node)

$P = VI^*$

$(160 + 120j) \times 1000 = \frac{6000}{\sqrt{3}} (I^*)$

$I^* = \frac{\sqrt{3}}{600}(1600 + 1200j) = 577.35\angle 37°$

$\boxed{I_L = 577.35\angle -37°}$ — (1)

$I_L Z = V_{AN}$

$Z = \frac{V_{AN}}{I_L} = \frac{600\angle 0°}{577.35\angle -37°}$

$V_B = I_L(Z_l + Z) = V_{AN} + I_L Z_L$

$= 600\angle 0° + (577.35\angle -37°)(0.005 + 0.0$

$V_B = 857.51\angle 1.57°$

$P_{sending} = V_B I_L^* = (357.57\angle 1.57)(577.35\angle 37) = 206.41\angle 38.44$

$pf = \cos(37+1.57) = \cos(38.44)$

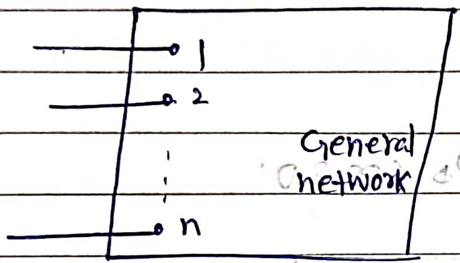
pf = angle b/w Vsource and I_{aA} (I_L)

also, V_{line} at the sending end,

$\sqrt{3}\, V_L I_L^* = 3 \times 206.41\angle 38.44$

$$\boxed{|V_L| = 619.23\ V}$$

TWO-WATTMETER METHOD



We need (n-1) currents and (n-1) voltages as we have one terminal as a reference.

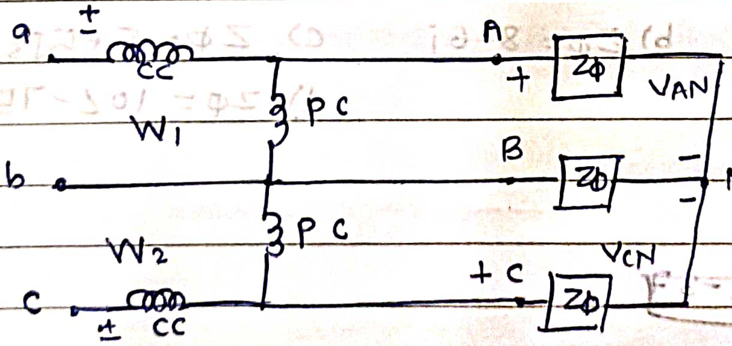
so, $P_T = v_1 i_1 + \dots + v_{n-1} i_{n-1}$

→ so, for a 3ϕ conductor-circuit, we need only two wattmeters

→ 4-conductor circuit (incl. neutral)

③ if unbalanced

② if balanced (no current in neutral line.)



$Z\phi = 12\angle\phi$

WYE

bB → reference line

CC: current coil PC: potential coil

→ I drawn by PC is negligible compared to line current measured by the CC.

→ the phase sequence

$$W_1 \rightarrow |V_{AB}||I_{aA}| \cos\theta_1 \quad \text{(angle b/w } V_{AB} \text{ and } I_{aA})$$

$$= V_L I_L \cos\theta_1$$

$$W_2 \Rightarrow V_L I_L \cos\theta_2$$

For a +ve phase sequence,

θ: phase angle b/w phase voltage and phase cu...

$$\theta_1 = \theta + 30° = \theta_\phi + 30°$$

$$\theta_2 = \theta - 30° = \theta_\phi - 30°$$

$$W_1 = V_L I_L \cos(\theta_\phi + 30°)$$

$$W_2 = V_L I_L \cos(\theta_\phi - 30°)$$

$$P_T = 2 V_L I_L \cos\theta_\phi \cos 30°$$

$$\boxed{P_T = \sqrt{3} V_L I_L \cos\theta_\phi}$$

⇒ 1) if pf > 0.5 ⟶ both W1, W2 read (+ve)

2) if pf = 0.5 ⟶ one W reads 0

3) if pf < 0.5 ⟶ one W reads negative

4) reversing phase sequence interchanges reading of the two wattmeters.

eg.- Phase voltage = 120 V and

a) $Z_\phi = 8 + 6j$ b) $Z_\phi = 8 - 6j$ c) $Z_\phi = 5 + 5\sqrt{3}\,j$

d) $Z_\phi = 10\angle -75°$

Ans. (a) $\cancel{W_1 = \frac{|V_L|^2}{Z_\phi}}$

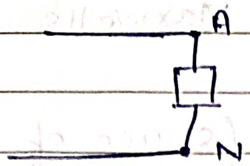
$\cancel{= \frac{120 \times 120}{10} \angle -37}$

$$W_1 = V_L \left(\frac{V_L}{Z}\right) \cos(\theta_\phi + 30°) \qquad W_2 = 2476.25\,W$$

$$= \frac{(120\sqrt{3})\,120}{10} \cos(36.87 + 30°)$$

$$= 979.75\,W$$

M2



$V_{phase} = 120\ V$

$Z_\phi = 8 + 6j$

$I_{phase} = \frac{120}{10\angle 37^\circ}$

$I_{ph} = 12 \angle -37^\circ$

$P_{net} = 3 \times (120)(12)(\angle 37^\circ)$

$= 4320 \angle 37^\circ$

$P_{real} = 3456\ W$

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$$\boxed{\begin{array}{c} W_1 = V_L I_L \cos(\theta_\phi + 30^\circ) \\ W_2 = V_L I_L \cos(\theta_\phi - 30^\circ) \\ \hline W_1 + W_2 = \sqrt{3} V_L I_L \cos\theta_\phi \end{array}}$$

(Two wattmeter method)