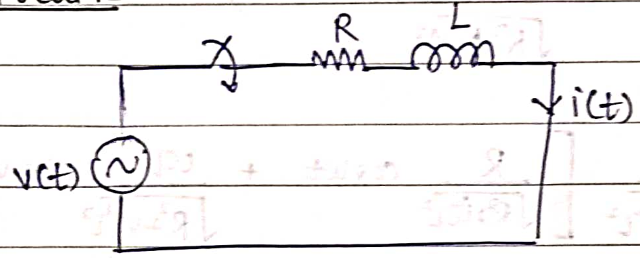
PHASOR ANALYSIS

① R-L circuits



Using KVL,

$$v(t) = iR + L\frac{di}{dt}$$

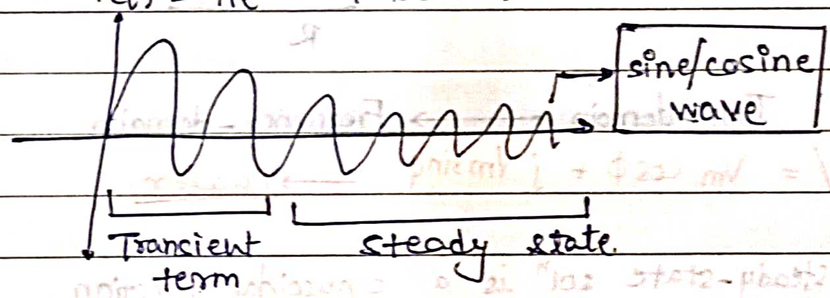
$$V_m \cos\omega t = iR + L\frac{di}{dt}$$

~~Assuming~~ For the homogenous system, $L\frac{di}{dt} + iR = 0$

$$i = Ae^{-Rt/L}$$

Assuming the full solution as

$$i(t) = Ae^{-Rt/L} + B\cos\omega t + C\sin\omega t$$



sudden opening/closing of a switch

$$V_m \cos\omega t = ARe^{-Rt/L} + BR\cos\omega t + CR\sin\omega t + L\left(\frac{-AR}{L}e^{-Rt/L} - \omega B\sin\omega t + \omega C\cos\omega t\right)$$

comparing co-efficients of cos and sine,

$V_m = BR + \omega LC \qquad CR - BL\omega = 0$

$V_m = \frac{CR^2}{\omega L} + C(\omega L) \qquad B\omega L = CR$

$$C = \left(\frac{\omega L}{R^2 + (\omega L)^2}\right)V_m \qquad B = \left(\frac{R}{R^2 + (\omega L)^2}\right)V_m$$

A turns out to be $\dfrac{-V_m}{\sqrt{R^2+(\omega L)^2}} \cos(\phi-\theta)$

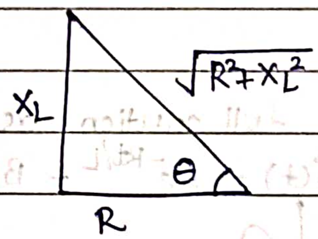
and so,

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left[\frac{R}{\sqrt{R^2+\omega^2L^2}}\cos\omega t + \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}\sin\omega t\right]$$
$$- \frac{V_m}{\sqrt{R^2+\omega L^2}}\cos(\phi-\theta)e^{-Rt/L} + I_0 e^{-Rt/L}$$

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\Big[\underbrace{\cos(\omega t+\phi-\theta)}_{\text{Steady state response}} - \underbrace{\cos(\phi-\theta)e^{-Rt/L}\Big]}_{\text{Transient response}} + I_0 e^{-Rt/L}$$

$\boxed{\tan\theta = \left(\frac{X_L}{R}\right)}$

$$\cos\theta = \frac{R}{\sqrt{R^2+\omega^2L^2}}$$



PHASORS : Time-domain $\longrightarrow$ Frequency-domain

$V = V_m\cos\phi + jV_m\sin\phi \longrightarrow$ phasor

1) steady-state solⁿ is a sinusoidal function
2) frequency of response signal is identical to frequency of source signal
3) max. amplitude of steady-state response is $\dfrac{V_m}{\sqrt{R^2+X_L^2}}$ and max. amplitude of signal source is V_m.
4) Phase angle of voltage source : ϕ ; response signal : $\phi-\theta$

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

PHASORS

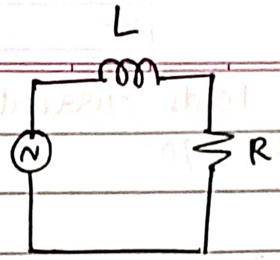
$v = V_m\cos(\omega t+\phi)$

$= \mathrm{Re}\left(V_m e^{j\phi} \cdot e^{j\omega t}\right)$

$\underbrace{V_m}_{\text{magnitude}}\underbrace{e^{j\phi}}_{\text{phase}}$

$\Rightarrow 1\angle\phi = e^{j\phi}$

Steady-state solution : $\mathrm{Re}(Ae^{j\beta}e^{j\omega t})$



$$i_{steady\text{-}state} = Re\{ I_m e^{j\beta} e^{j\omega t} \}$$

$$L\frac{di}{dt} + iR = V$$

$$Re\{ j\omega L I_m e^{j\beta} e^{j\omega t} \} + Re\{ R I_m e^{j\beta} e^{j\omega t} \} = Re\{ V_m e^{j\phi} e^{j\omega t} \}$$

$$Re\{ (j\omega L + R) I_m e^{j\beta} e^{j\omega t} \} = Re\{ V_m e^{j\phi} e^{j\omega t} \} \quad \text{—— (1)}$$

if instead we would have used sine in the analysis
i.e. $V = V_m \sin(\omega t + \phi)$, we have

$$Im\{ (j\omega L + R) I_m e^{j\beta} e^{j\omega t} \} = Im\{ V_m e^{j\phi} e^{j\omega t} \} \quad \text{—— (2)}$$

From (1) and (2)

$$(j\omega L + R) I_m e^{j\beta} = V_m e^{j\phi}$$

$$\boxed{I_m e^{j\beta} = \frac{V_m e^{j\phi}}{(R + j\omega L)}}$$

(A) RESISTOR

$$v = iR$$

$$v = R(I_m \cos(\omega t + \theta_i))$$

↓ Phasor Transformation

in-phase

$$\mathbf{V} = R I_m e^{j\theta_i} = R I_m \angle \theta_i$$

$$\boxed{\mathbf{V} = R\mathbf{I}}$$

(phasor forms)

no phase-shift b/w the current and voltage across a resistor

(B) INDUCTOR

$$i = I_m \cos(\omega t + \theta_i)$$

$$v = L\frac{di}{dt} = L I_m \omega (-\sin(\omega t + \theta_i))$$

$$v = -\omega L I_m \cos(\omega t + \theta_i - 90^\circ)$$

$$\mathbf{V} = -\omega L I_m e^{j(\theta_i - 90^\circ)} = -\omega L I_m e^{j\theta_i} e^{-j\pi/2} = (j\omega L)(e^{j\theta_i} I_m)$$

$$\boxed{\mathbf{V} = (j\omega L)\mathbf{I}}$$

(Phasor voltage) = $(j\omega L)$ x (phasor current)

$V = (\omega L \angle 90^\circ)(I_m \angle \theta_i)$

$V = \omega L I_m \angle(\theta_i + 90^\circ)$

$\boxed{\text{Voltage leads current by } 90^\circ}$

phase shift of 90° $\longrightarrow$ Time shift of $(T/4)$.

Ⓒ **CAPACITOR**

$v = V_m \cos(\omega t + \theta_i)$

$v = \frac{1}{C}\int i \cdot dt$

$i = C\frac{dv}{dt} = C V_m \omega \cdot (-\sin(\omega t + \theta_i))$

$i = C\omega V_m \cos(\omega t + \theta_i + 90^\circ)$

$\boxed{\text{current leads the voltage by } 90^\circ}$

$\downarrow$ Phasor Transform

$I = \omega C\, e^{j(90^\circ)}(e^{j\theta_i} V_m)$

$\boxed{V = \left(\frac{1}{j\omega C}\right) I} \longrightarrow V = \frac{1}{\omega C} I_m \angle\theta_i \angle -90^\circ$

i.e. $\cancel{V = I \cdot (1/\omega C)\, e^{-j(90^\circ)}}$

$\boxed{V = \frac{I_m}{\omega C} \angle \theta_i - 90^\circ}$

$\cancel{V = \frac{I_m}{\omega C} \angle -90^\circ}$

IMPEDANCE & REACTANCE

$V = Z I$

$\downarrow$

Impedance = ratio of phasor voltage to its current phasor.

$\Longrightarrow$ Kirchoff's Laws in Frequency Domain

$v_1, \ldots, v_n$ represent voltages around a closed path in a circuit.

if the circuit is in a sinusoid steady state:

$v_1 + v_2 + \ldots + v_n = 0$

i.e. $V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2) + \ldots + V_{mn}\cos(\omega t + \theta_n) = 0$

$Re\{(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2} + \ldots + V_{mn}e^{j\theta_n})e^{j\omega t}\} = 0$

$Re\{(V_1 + V_2 + \ldots + V_n)e^{j\omega t}\} = 0$ as $e^{j\omega t} \neq 0$

$V_1 + V_2 + \ldots + V_n = 0$ ← KVL in frequency domain

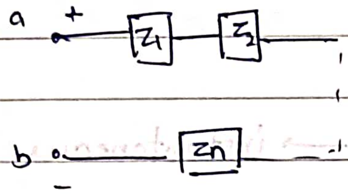
$I_1 + I_2 + \ldots + I_n = 0$ ← KCL in frequency domain

COMBINING impedances in SERIES and PARALLEL

⇒ **Series:-**

same phasor current I is carried

and,



$V_{ab} = Z_1 I + Z_2 I + \ldots + Z_n I$

$ZI = (Z_1 + Z_2 + \ldots + Z_n) I$

$$\boxed{Z = Z_1 + \ldots + Z_n}$$

imp. identities

$Z = a + jb = \left(\sqrt{a^2+b^2}\right) \angle \tan^{-1}(b/a)$ $\qquad \dfrac{1}{a\angle b} = \boxed{\dfrac{1}{a} \angle -b^\circ}$

$\dfrac{a\angle b^\circ}{K\angle m^\circ} = \left(\dfrac{a}{K}\right) \angle (b-m)^\circ$ $\qquad (a\angle b^\circ)(K\angle m^\circ) = (aK)\angle (b+m)^\circ$

⇒ **Parallel:-**

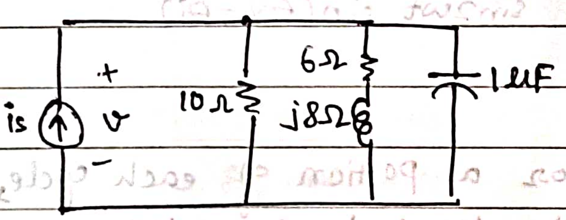
$$\boxed{\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \ldots + \frac{1}{Z_n}}$$

conductance: $Y = \dfrac{1}{Z}$

using $I = I_1 + \ldots + I_n$

$\dfrac{V}{Z_{eq}} = \dfrac{V}{Z_1} + \ldots + \dfrac{V}{Z_n}$

Q:)



$i_s = 8\cos(200{,}000 t)$ A

A:)

$\dfrac{1}{Z} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}$

$Z_1 = 10 \quad Z_2 = 6 + 8j \quad Z_3 = -5j$

$\dfrac{1}{Z} = \dfrac{10}{100} + \dfrac{6-8j}{100} + \dfrac{20j}{100} = \dfrac{16+12j}{100}$

$Z = \dfrac{100}{400}(16 - 12j) = 4 - 3j$ $\qquad \boxed{Z = 5\angle -37^\circ}$

$\boxed{I = 8\angle 0^\circ}$

$V = ZI = 40\angle -37^\circ$

$i_1 = \dfrac{40\angle -37^\circ}{10\angle 0^\circ} = 4\angle -37^\circ$

$i_2 = \dfrac{40\angle -37^\circ}{10\angle 53^\circ} = 4\angle -90^\circ$

$i_3 = \dfrac{40\angle -37^\circ}{5\angle -90^\circ} = 8\angle 53^\circ$

Adding cosines with Phasors

$y_1 = 20\cos(\omega t - 30^\circ)$ $\qquad$ $y_2 = 40\cos(\omega t + 60^\circ)$

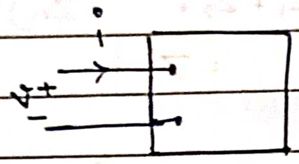
$= 20\angle -30$ $\qquad$ $= 40\angle 60^\circ$

$17.32 - 10j + 20 + 34.64j$

$y = 37.32 + 24.64j$

$$\boxed{y = 44.72\angle 33.43^\circ}$$

POWER



$p = vi \longrightarrow$ instantaneous power

(#) dir^n of current is dir^n of voltage drop $\longrightarrow$ (+)

$v = V_m\cos(\omega t + \theta_v)$ $\qquad$ $i = I_m\cos(\omega t + \theta_i)$

$\Rightarrow v = V_m\cos(\omega t + \theta_v - \theta_i)$ $\qquad$ $i = I_m\cos(\omega t)$

$$p = V_m I_m \cos(\omega t + \theta_v - \theta_i)\cos(\omega t)$$

$$= \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v - \theta_i)\right]$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos 2\omega t \cos(\theta_v - \theta_i)$$

$$- \frac{V_m I_m}{2}\sin 2\omega t \sin(\theta_v - \theta_i)$$

$\rightarrow f = 2 \times f_{\text{voltage/current}}$

$P_{inst.}$ may be $-$ve for a portion of each cycle, even if the network b/w the terminals is passive.

$\downarrow$

energy stored in inductors/capacitors $\longrightarrow$ extracted.

AVERAGE and REACTIVE POWER

$$p = P + P\cos 2\omega t - Q\sin 2\omega t$$

(Average power) $P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$ $\qquad$ $Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$ (Reactive power)

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p \cdot dt = \frac{1}{T}\int_0^T p \cdot dt$$

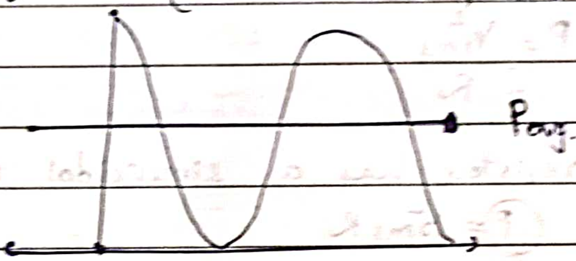
$\langle \sin 2\omega t \rangle = 0$ $\quad \langle \cos 2\omega t \rangle = 0$ (over one 'T')

so. $P_{avg.} = \frac{V_m I_m}{2}(\cos\phi)$

Purely Resistive circuits

$p = P + P\cos 2\omega t$ $\quad$ ($\cos$ $\theta_v - \theta_i = 0$; $\sin 0° = 0$; $\cos 0° = 1$)

instantaneous real power



p is never -ve ⟶ power can't be extracted but is only dissipated as heat.

Purely inductive circuits

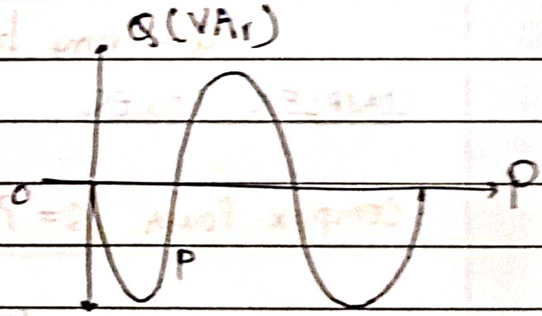
$\theta_v - \theta_i = 90°$ $\quad \cos 90 = 0$ $\quad \sin 90° = 1$

(reactive power)

$p = -Q \sin 2\omega t$

$P \Rightarrow$ avg. power $= 0$

power is continually exchanged by inductor $(f = 2\omega)$

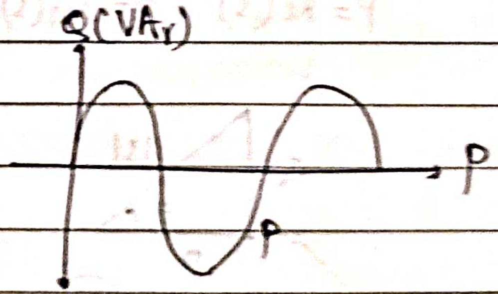


Purely capacitive circuits

$\theta_v - \theta_i = -90°$

$p = Q \sin 2\omega t$

$P = 0$ ⟶ No transform^n of energy



POWER FACTOR

pf angle ⟶ $\quad pf = \cos(\theta_v - \theta_i)$

$rpf = \sin(\theta_v - \theta_i)$

lagging pf / leading pf ⟶ (current leads voltage) $\theta < 0$

$\theta > 0$ ↓ lags (current voltage)

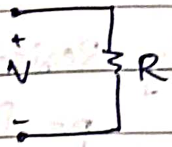
capacitive load

inductive load

$Q > 0$: inductors $\longrightarrow$ demand/absorb magnetizing VArs
$Q < 0$: capacitors $\longrightarrow$ furnish/deliver magnetizing VArs.

RMS POWER

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_m^2 \cos^2(\omega t+\phi_v)}{R} \cdot dt$$



$$P = \frac{V_{rms}^2}{R}$$

if resistor has a sinusoidal current $I = I_m \cos(\omega t + \phi_i)$,

$$P = I_{rms}^2 R$$

$\hookrightarrow$ effective value

100V DC delivers same ~~power~~ energy in T seconds as a 100 V_{rms} AC

$$\boxed{P = (V_{rms} I_{rms}) \cos(\theta_v - \theta_i)}$$

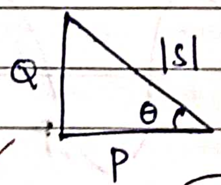
$$Q = (V_{rms}\ I_{rms}) \sin(\theta_v - \theta_i)$$

COMPLEX POWER

Complex Power, $\boxed{S = P + jQ}$ Unit $\Rightarrow$ VA

$\rightarrow$ can be found from V and I phasors directly.

$P = Re(S)$ $Q = Im(S)$



θ is the pf angle
as $\frac{Q}{P} = \tan(\theta_v - \theta_i) = \tan\theta$

θ : pf

if any two of
$P, Q, |S|, \theta \rightarrow$ known $\Rightarrow$ all 4 known.

apparent power $\Rightarrow$ $|S| = \sqrt{P^2 + Q^2}$ VA

$\downarrow$

VA capacity required to supply the average power unless $\theta(pf) = 0^\circ$, $|S| > P \longrightarrow$ (VA capacity required) $>$ (avg. power used)

Q1:) 240 Vrms, $P_{avg.} = 8$ KW, lagging p.f. of 0.8
a) $S = ?$ b) $z = ?$

A1:) a) Q is +ve (lagging pf)

$\cos\phi = \frac{4}{5}$ $\tan\phi = \frac{3}{4} = \frac{Q}{8}$

$Q = +6KW$ $S = (8+6j)$ KW

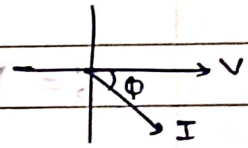
b) $\cancel{P = V_{rms} I_{rms} \Rightarrow 8000 = 240\, I_{rms}}$

$\cancel{I_{rms} = \frac{100}{3}}$

$\cancel{\frac{3}{4} = \frac{Z_1}{R} \quad Z_1 = \frac{3R}{4} \quad Z = \frac{5}{4}R = \frac{5}{4}\left(\frac{240 \cdot 3}{100}\right) = 9}$

$P = V_{rms} I_{rms} \cos\phi \Rightarrow I_{rms} = 41.67$ Amp

$|Z| = \frac{240}{41.67} = 5.76$ $\phi = +37^\circ$ (lagging)



$Z = 5.76 \angle -37^\circ$

POWER CALCULATIONS

$$S = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + j\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$$

$$= \frac{V_m I_m}{2}\left[e^{j(\theta_v - \theta_i)}\right] = \frac{1}{2} V_m I_m \angle(\theta_v - \theta_i)$$

$$\boxed{S = V_{rms} I_{rms} \angle(\theta_v - \theta_i)}$$

complex power $\longrightarrow$ product of rms phasor voltage and conjugate of rms phasor current.

$$S = V_{rms} I_{rms}\, e^{j\theta_v} e^{-j\theta_i} = V_{eff} I_{eff}^*$$

$$\boxed{S = V_{eff} I_{eff}^*}$$ (V, I → phasor)

$$\boxed{S = \frac{1}{2} V I^*}$$

or

$V_{eff} = Z I_{eff}$ (V_{eff} → phasor, RMS)

$$S = Z\, I_{eff} \cdot I_{eff}^* = Z\, |I_{eff}|^2$$
$$= |I_{eff}|^2 (R + jX)$$
$$= \underbrace{|I_{eff}|^2 R}_{P} + \underbrace{j\, |I_{eff}|^2 X}_{Q}$$

$$P = |I_{eff}|^2 R = \frac{1}{2} I_m^2 R \qquad S = Z\, |I_{eff}|^2$$

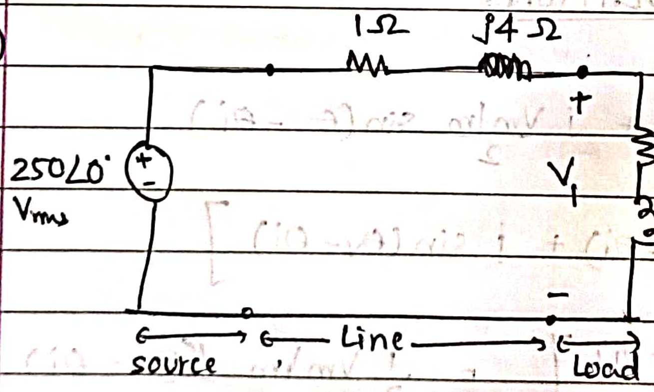
$$Q = |I_{eff}|^2 X = \frac{1}{2} I_m^2 X$$

$$S = V_{eff} \left(\frac{V_{eff}}{Z} \right)^* = \frac{|V_{eff}|^2}{Z^*} = P + jQ$$

if Z is purely resistive / Z is purely reactive

$P = \frac{|V_{eff}|^2}{R}$ $\qquad$ $P = \frac{|V_{eff}|^2}{X}$

Q1:)



$Z_{load} = (39 + 26j)\,\Omega$
$Z_{line} = (1 + 4j)\,\Omega$
$(V_{rms})_{source} = 250$

A1) I_L and V_L

$$Z_{net} = Z_{line} + Z_{load}$$
$$Z_{net} = (40 + 30j)$$
$$= 50 \angle 37^\circ$$

$$I_L = \frac{V_1}{Z} = \frac{250 \angle 0^\circ}{50 \angle 37^\circ} = 5 \angle -37^\circ$$
$\quad \llcorner (I_{rms})$

$$V_L = I_L Z = (5 \angle -37^\circ)\left(13\sqrt{13} \angle \tan^{-1}(2/3)\right)$$
$\sqrt{13}\,65 \angle \ldots)$

$$\boxed{V_L = 234.3 \angle -3.18}$$

b) $S_{load} = |I_{eff}|^2 Z = |I_L|^2 Z$

$= (25)(39+26j) = 975 + 650j$ VA

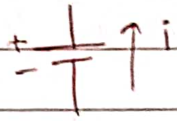
c) $S_{line} = |I_{line}|^2 z = 25(1+4j) = 25 + 100j$ VA

$Q = 100$ VAr $\quad P = 25$ W

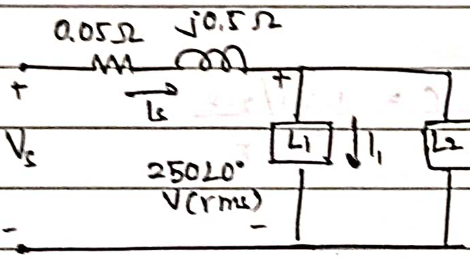
d) $S_{source} = |I_L|^2 Z_{net} = 25(40+30j)$ VA

$P = 1000$ W $\quad Q = 750$ VAr

$S_s = -V_s I_L^*$ → current reference is in the dir^n of voltage rise



Q2:)



L1 absorbs $P_{avg} = 8$ KW lead pf = 0.8

L2 absorbs 20 KVA lag pf = 0.6

A2) $0.8 + j0.6 \qquad 0.6 + j0.8$

$S = (250\angle 0^\circ)(I_s^*) = 250 I_1^* + 250 I_2^* = S_1 + S_2$

$\boxed{S = S_1 + S_2}$ sum the complex powers geometrically

$\frac{3}{4} = \frac{Q}{P}$

$S_1 = 8000 - j6000 = (8-6j)$ KW

$S_2 = (12 + j16)$ KW

$P^2 + \frac{16P^2}{9} \Rightarrow P\cdot\frac{5}{3} = 20$ KVA

$\frac{S}{P} = \frac{4}{3}$

$Q = \frac{4P}{3}$

$S = 20 + 10j = 250(I_s^*)$

$I_s^* = \frac{20+10j}{250} \qquad I_s = \frac{(20-10j)}{250} \times 1000$

$I_s = 80 - 40j \qquad V_s = 250$

$Z = \frac{250\angle 0^\circ}{89.44\angle -26.56^\circ} = 2.8\angle 26.56^\circ$

pf $= 26.56^\circ \longrightarrow \cos(26.56^\circ) = 0.894$ lagging

$S = 22.3\angle 26.56^\circ \qquad |I_s| = 89.44$ A $\qquad P = 20000$ VA

$(P_{loss})_{line} = |I_L|^2 R = (89.44)^2 \times 0.05 = 400$ W

$\cancel{Z = 2.8\angle 26.56^\circ = (2.5) + 1.25j}$ $(Z_L = 2.45 + 0.75j)$

$\cancel{1.25j - 1.25j}$ $\cancel{\frac{1}{\omega C} = 1.25}$

$\cancel{Z_{net} = (0.05 + 0.5j) + Z_L}$ $\cancel{C = \frac{1}{(120\pi)\times 1.25} = 2.1\ mF}$

$\cancel{\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_{cap}}}$ $\cancel{\frac{1}{Z_{eq}} = \frac{1}{Z_L} + \frac{1}{Z_C}}$

$\cancel{= \frac{1}{2.5 + 1.25j} + \frac{1}{(1/j\omega C)}}$ $\cancel{= \frac{2.45 - 0.75j}{2.56} + \frac{1}{(1/j\omega C)}}$

$\cancel{\frac{1}{Z_{eq}} = \frac{2.5 - 1.25j}{2.8} + j\omega C}$ $\cancel{0.75 = 2.5\,\omega C}$

$\cancel{1.25 = 2.8\,\omega C}$ $\cancel{C = \frac{1.25/2.8}{120\pi}}$

$S = S_1 + S_2 + S_3$ due to capacitor

so, $Q_{cap} = -10\ KVAr = Z_c |V_L|^2$

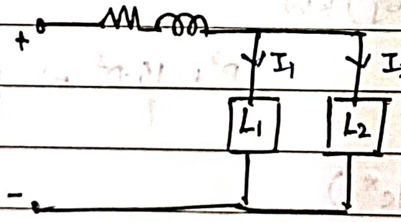
$10 \times 10^3 = Z (250)^2$

$Z_c = -6.25\ \Omega$

$\frac{1}{\omega C} = 6.25$ $\boxed{C = 424.41\ \mu F}$

It will make the pf = 1

Power (Apparent) in series



$S = V_{rms} I_{rms}^* = V_{rms} (I_1 + I_2)^*$

$= V_{rms} I_1^* + V_{rms} I_2^*$

$\boxed{S = S_1 + S_2}$ (L_1, L_2 are in parallel)

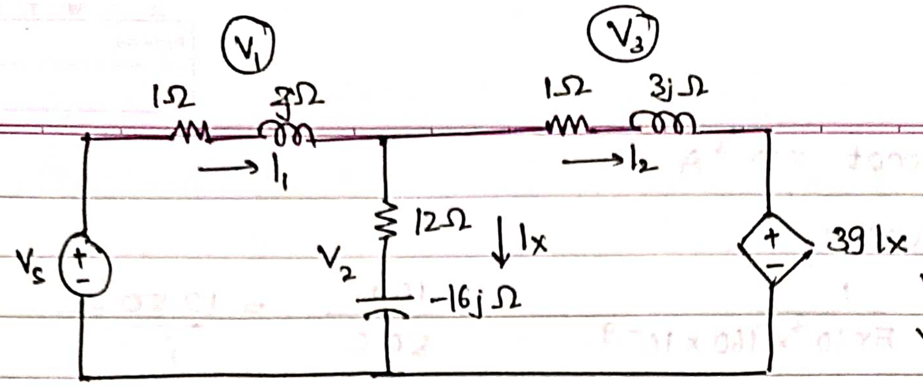
combined pf $\rightarrow$ Find S and use $\tan\phi = \frac{Q}{P}$

Power supplied = Re(S) [avg. P] + [Pavg. lost in the transmission line]

$= P + [|I_L|^2 R]$

$\boxed{Q = |V_{eff}|^2 / X}$

Q3:)



$V_s = 150\angle 0$

$V_1 = 78 - 104j$ $\quad$ $I_1 = -26 - 52j$ $\quad$ $V_2 = 72 + 104j$ $\quad$ $I_2 = -2 + 6j$

$V_3 = 150 - 130j$ $\quad$ $I_3 = -24 - 58j$

a) ① $Z = 1 + 2j$ $\qquad$ $\left(I = \frac{\sqrt{26^2 + 52^2}}{\sqrt{2}}\right)$ RMS

~~116.27 VAr~~

$S_1 = \frac{1}{2} V_1 I_1^* = \frac{1}{2} (-26 + 52j)(78 - 104j)$

$= \underbrace{1690}_{P_1} + \underbrace{3380j}_{Q_1}$ VA

$\boxed{S = \frac{1}{2} V I^*}$
if V, I given in complex forms

$S_2 = \frac{1}{2}(72 + 104j)(-2 - 6j) = \underbrace{240}_{P_2} - \underbrace{320j}_{Q_2}$

$S_3 = \frac{1}{2}(150 - 130j)(-24 + 58j) = 1970 + 5910j$

② V_s $\quad$ V_2

$V_s - V_2 = V_1$

$V_s = V_1 + V_2 = 150$

$I_s = I_1 = -26 - 52j$

$S_{s_1} = \frac{1}{2}(150)(-26 + 52j) = (1950 + 3900j)$

V_{s_2}

$V_2 - 39 I_x = V_3$ $\qquad$ $V_{s_2} = V_2 - V_3 = 72 + 104j - 150 + 130j$

$V_{s_2} = -78 + 234j$

$S_{s_2} = \frac{1}{2} V_{s_2} I_2^* = (-5850 - 5070j)$ VAr

c) $(P)_{delivered} = 1690 + 1970 + 240 + 1950$

$= 5850 = (P)_{absorbed}$

$(Q)_{delivered} = 3380 + 5910$. $\qquad$ $(Q)_{abs} = 5070 + 3900 + 320$

$= 9290$ VAr $\qquad = \qquad = 9290$ VAr