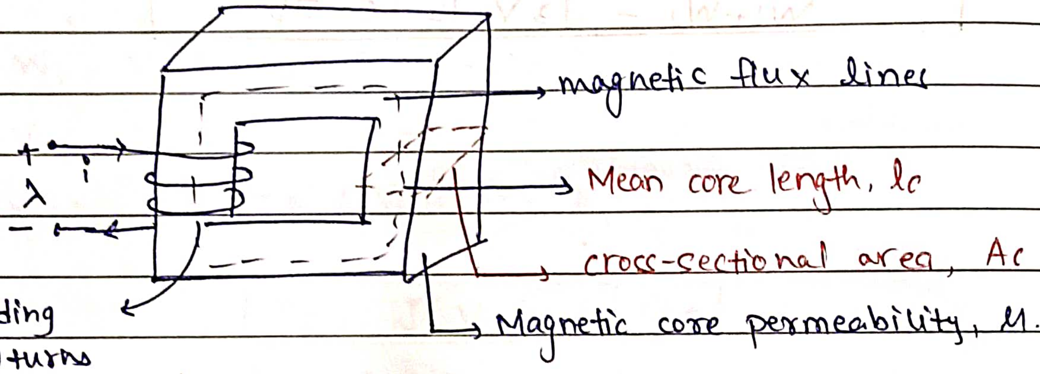
MAGNETIC CIRCUITS

→ neglect displacement - current term in Maxwell's equations.

Maxwell's equations :→ $\boxed{\oint_C H \cdot dl = \int_S J \cdot da}$ (source of H is J)

$\boxed{\int_S B \cdot dl = 0}$ (no net flux enters or exits any loop.)

→ high-permeability material is used causing magnetic flux to be defined in paths determined by these structures.



$\mu >> \mu_{air}$

winding → M.F. in the core

→ Because of the high permeability
- → ϕ entirely confined in core
- → field lines follow path of core
- → flux density is uniform over a cross-section as $A_{c.s.}$ is uniform.

→ Source of M.F. in the core is the ampere-turns product, Ni → Magnetomotive Force (MMF).

$$(\mathcal{F})\ MMF = \sum_{K=1}^{n} N_K I_K \quad (n: \text{no. of coils})$$

Magnetic flux, $\boxed{\phi = \int_C B \cdot da}$ Weber (Wb)

Net ϕ entering or leaving a closed surface is ZERO ⟹ all flux which enters a surface enclosing a volume MUST leave that volume over some other portion of that surface as the cross-section area is the same over all surfaces,

we have, $\phi = \int B \cdot (dA)$

$$\boxed{\phi_c = B_c A_c}$$

ϕ_c: Flux in core; B_c: Flux density in core; A_c: area of core

⇒ The relation b/w MMF acting on a circuit and magnetic field intensity in a circuit is :

$$\boxed{\oint H \cdot dl = Ni = MMF}$$ (Ampere's law)

→ The core dimensions are such that the path length of any flux line is close to mean core length, l_c.

so,

$$\boxed{F = Ni = H_c \cdot l_c}$$

$H_c \cdot l_c$: scalar product of the magnitude of H and mean path length l_c.

⇝ H_c : average magnitude of $\vec{H}$ in the loop.

↳ direc^n : right-hand thumb rule.

→ The rel^n b/w the magnetic field intensity $\vec{H}$ and magnetic flux density B is a property of the material and so,

$$\boxed{\vec{B} = \mu \vec{H}}$$

B: $\left(\frac{Wb}{m^2}\right)$ = Tesla; μ: magnetic permeability; H: A/m

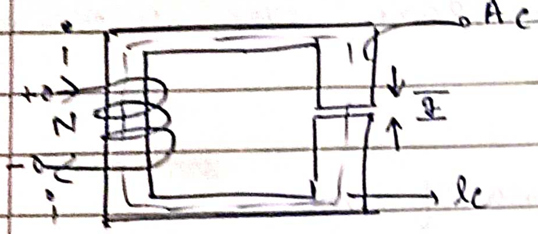
μ: $\frac{Wb}{A\text{-turn-}m}$ = $\frac{Henry}{meters}$

$$\boxed{\mu_0 = 4\pi \times 10^{-7} \text{ henrys/m}}$$

$$\boxed{\mu = \mu_0 \mu_r}$$

⇒ MMF drop across any segment of a magnetic circuit = $\int H \cdot dl$ over that portion of magnetic circuit.

AIR-GAP



→ energy-conversion devices must have air-gaps

if $g \ll l_c$ → NO 'FLUX LEAKAGE'

$$B_c = \frac{\phi_c}{A_c} \qquad B_g = \frac{\phi_c}{A_g}$$

also, $\mathcal{F} = H_c l_c + H_g l_g$

$$= \frac{B_c}{\mu_c} l_c + \frac{B_g}{\mu_a} l_g$$

produce MMF in core

produce MMF in air-gap

⇒ B_c is often a non-linear fun^c of H_c.

$$F = \phi\left(\frac{l_c}{\mu_c A_c} + \frac{g}{\mu_0 A_g}\right)$$

$$\boxed{Ni = \phi\left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}\right)}$$

MMF ← current ← reluctance

Reluctance, $\boxed{R = \frac{l}{\mu A}}$

$$R_c = \frac{l_c}{\mu A_c} \qquad R_g = \frac{l_g}{\mu A_g}$$

$$\mathcal{F} = \phi(R_c + R_g)$$

$$\boxed{\phi = \frac{\mathcal{F}}{R_{total}}}$$

Permeance, $P_{tot} = \frac{1}{R_{tot}}$

→ Fraction of ~~flux~~ MMF to drive flux through each portion of the magnetic circuit, called the MMF drop, is proportional to its reluctance.

$\frac{\mu A_c}{l_c} \gg \frac{\mu_0 A_g}{g}$ High μ and low g

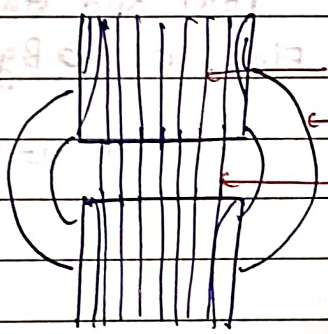
$\boxed{R_c \ll R_g}$

so, $\phi = \frac{F}{R_g} = \frac{F\mu_0 A_g}{g}$

$$\boxed{\phi = \frac{Ni\mu_0 A_g}{g}}$$

→ as long as μ is sufficiently large, its variatⁿ does not affect the performance of the magnetic circuit.

FRINGING FIELDS :-



Fringing fields increase the effective area of air-gap, A_g

→ neglecting :- $A_g = A_c$

⇒ For general magnetic circuits,

$$\boxed{F = \int_S J \cdot dA = \sum_K F_K = \oint H \cdot dl} \quad \boxed{F_K = H_K l_K}$$

analogy to Kirchoff's law,

$\boxed{\sum_n \phi_n = 0}$ → sum of flux into a node is ZERO.

e.g.- $A_c = A_g = 9\,cm^2$, $g = 0.05\,cm = 5\times10^{-4}\,m$ $l_c = 3\times10^{-1}\,m$ $N = 500$

$\mu_r = 7\times10^4$

a) $R_c = \frac{l_c}{\mu A_c} = \frac{3\times10^{-1}}{7\times10^{4}\mu_0\times9\times10^{-4}} = \frac{0.3}{63\mu_0} = 3.79\times10^3$

$R_g = \frac{g}{\mu A_g} = 4.42\times10^5$

b) $B_c = 0.1\,T$ $Ni = \phi(R)$ $\phi = \frac{500\times i}{R} = B_c A_c$

$0.1 = \mu_c(H_c)$

LIST OF ALL FORMULAES :-

1) $MMF = Ni$

2) $\phi = B_c A_c$

3) $Ni = \sum H_k l_k$

$Ni = H_c l_c + H_g l_g$

$\left(\begin{array}{l} MMF = \oint H \cdot dl \\ \oint H \cdot dl = \int_s J \cdot da \end{array} \right)$

4) $B = \mu H$; $B = \dfrac{\phi}{A}$

$B_c = \mu_c H_c$ $\quad B_g = \mu_0 H_g$

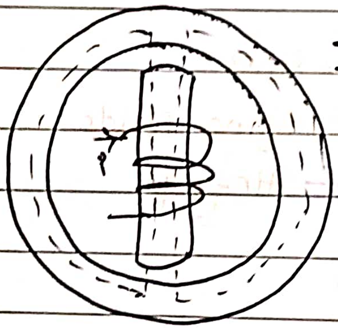
5) $R = \dfrac{l}{\mu A}$ $\quad \phi = \dfrac{Ni}{R_{total}}$

$\phi = \dfrac{Ni \mu A}{l}$

6) $(\phi_{net})_{node} = 0$ (KCL) $\quad$ 7) $Ni = \sum N_k i_k$

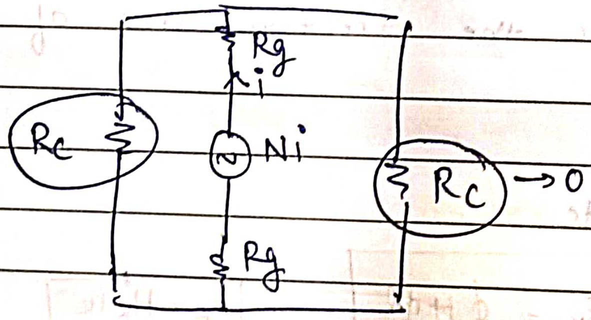
eg.-

rotor and stator have $\mu \to \infty$



Ig Find ϕ_g, B_g.

$I = 10\,A$, $N = 1000$, $g = 1cm$, $A_g = 2000$



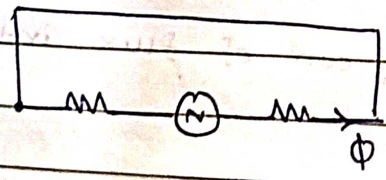
$\phi = \dfrac{Ni}{2R_g}$

$= \dfrac{1000 \times 10}{\dfrac{2 \times 10^{-2}}{\mu_0 \times \cancel{2} \times 10^3 \times 10^{-4}}}$

$= \mu_0 \times 10^4 \times 10^{-1} \times 10^2$

$= \mu_0 \times 10^5$

$\Rightarrow$



$\phi_g = 4\pi \times 10^{-2} = 0.13\ Wb$

$B_g = \dfrac{\phi_g}{A_g} = \dfrac{4\pi \times 10^{-2}}{2 \times 10^{-1}} = 2\pi \times 10^{-1} = 0.2\pi\ \dfrac{wb}{m^2}$

$= \underline{0.65\,T}$

FLUX LINKAGES

→ M.F. varying with time, an Electric field is produced in space as determined by the Faraday's law imp. $\boxed{\text{FLUX-LINKAGE} = \left(\begin{array}{c}\text{No. of turns of} \\ \text{the winding}\end{array}\right) \times \left(\begin{array}{c}\text{flux flowing through} \\ \text{the coil.}\end{array}\right)}$

$$\underbrace{\oint_C E \cdot ds}_{\text{line-integral of } \vec{E}} = -\underbrace{\frac{d}{dt}\int_S B \cdot da}$$

rate of change of magnetic flux ~~density~~ through that contour.

→ if E_{wire} is extremely small and so can be neglected.

as winding is done 'N' times,

induced EMF $\boxed{e = N\frac{d\phi}{dt}} = \frac{d\lambda}{dt}$ λ: Flux linkage of winding, $\boxed{\lambda = N\phi}$ (instt. value of time-varying flux).

Flux-linkage of a coil $= \int_S B \, dA$

B: normal component of magnetic flux density over any area spanned.

⇒ direcⁿ of induced EMF :- if the winding terminals were short-circuited, a current would flow in a dirⁿ to oppose the flux linkage.

→ for a circuit of magnetic material with constt. μ or a dominating air-gap, inductance, $\boxed{L = \frac{N\phi}{i}}$ (linear) $\left(L = \frac{\lambda}{i}\right)$

i.e. $L = \frac{N\phi}{i} = \frac{N}{i} \cdot \frac{Ni}{R}$ $\boxed{L = \frac{N^2}{R_{tot}}}$

assuming $R_c \ll R_g$

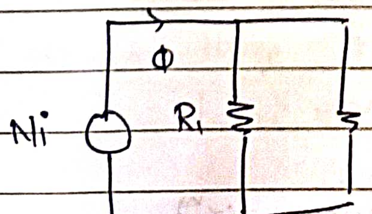
$$L = \frac{N^2}{(g/\mu_0 A g)} = \frac{\mu_0 N^2 A}{g} \qquad \boxed{L = \frac{\mu_0 N^2 A}{l}}$$

$L \propto \mu$ $\quad L \propto N^2$ $\quad L \propto A$ $\quad L \propto \frac{1}{l}$

requires linear relⁿ b/w Ni and ϕ

$\boxed{\text{(MMF) and (flux)}}$

e.g.



$\phi_1 = \frac{Ni}{R_1}$ $\quad \phi_2 = \frac{Ni}{R_2}$ $\quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$$L = \frac{N^2(R_1+R_2)}{R_1 R_2} = \mu_0 N^2\left(\frac{A_1}{g_1} + \frac{A_2}{g_2}\right)$$

⇒ linearalizing effect of a dominating air-gap in a magnetic circuit

If μ_{core} say changes from 70000 to 3000

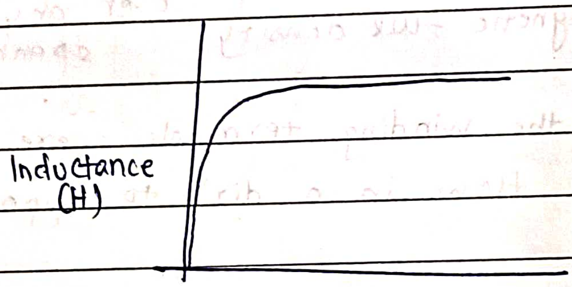
$$R_c = \frac{l}{\mu_r \mu_0 A_c} \qquad R_c' = \frac{l}{\mu_r' \mu_0 A_c}$$

but $R_c + R_g \approx R_c' + R_g$ if μ_r' is comparatively higher

$$R_T \approx R_T'$$

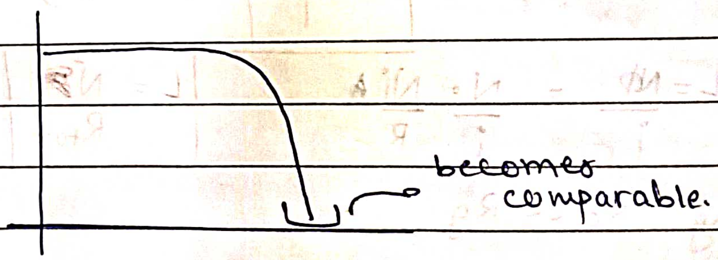
$$L = \frac{N^2}{R_{Tot}} \quad \text{so} \quad \boxed{L_T \approx L_T'}$$

i.e. on changing μ_c by 25 times, L changes by 1.16 times

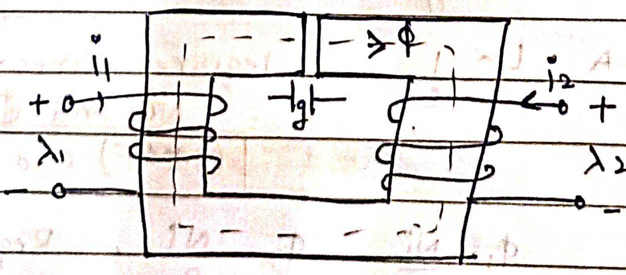


⇒ Effect of air-gap length

as l_g changes, R_g changes.



TWO WINDING CIRCUITS



$F = N_1 i_1 + N_2 i_2$ (produce ϕ in same dirn)

$$\phi = \boxed{(N_1 i_1 + N_2 i_2) \frac{\mu_0 A_c}{g}} \rightarrow \text{resultant core flux}$$

M	T	W	T	F	S	S

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$$\lambda_1 = N_1\phi = N_1^2\left(\frac{\mu_0 A_c}{g}\right) i_1 + N_1 N_2 \left(\frac{\mu_0 A_c}{g}\right) i_2$$

$$\boxed{\lambda_1 = \underbrace{L_{11} i_1}_{\text{self}} + \underbrace{L_{12} i_2}_{\text{mutual}}} \qquad \boxed{\lambda_2 = L_{21} i_1 + L_{22} i_2}$$

$$L_{11} = \frac{N_1^2 \mu_0 A_c}{g} \qquad L_{12} = \frac{N_1 N_2 \mu_0 A_c}{g} \qquad \boxed{L_{12} = L_{21}}$$

→ The resol^n of resultant flux linkages into components produced by i_1 and i_2 is due to the superpos^n of individual effects and implies a linear (FLUX – MMF) relationship.

$$e = \frac{d(Li)}{dt} \qquad \boxed{\mathcal{E} = L\frac{di}{dt}}$$

in general, $e = L\frac{di}{dt} + i\frac{dL}{dt}$ (use flux-linkage of both windings)

→ The power, $P = ie$ $\boxed{P = i\frac{d\lambda}{dt}}$ $\boxed{\Delta W = \int_{\lambda_1}^{\lambda_2} i \cdot d\lambda}$

$$\Delta W = \int_{\lambda_1}^{\lambda_2} i\,d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{\lambda}{L}\,dL = \frac{1}{2L}(\lambda_2^2 - \lambda_1^2)$$

$$\Delta W = \frac{1}{2L}(\lambda_2^2 - \lambda_1^2)$$

Putting $\lambda_1 = 0$ (reference)

Total magnetic energy stored $= \frac{1}{2L}\lambda^2 = \boxed{\frac{1}{2}Li^2}$

$$\boxed{EMF = NA\left(\frac{dB}{dt}\right)}$$

PROPERTIES OF MAGNETIC MATERIALS

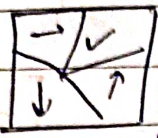
obtain large B with less MMF

↳ as M.Force and Energy density ↑ with B

Ferromagnetic materials

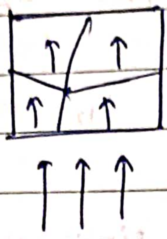
$\mu_r >> 1$ → iron & alloys of iron

↳ constrain and direct M.F. in well-defined paths.



absence of B_{ext}

(random)



(align with B_{ext})

remove B_{ext} → tend to retain initial orientation

remanant magnetic field.

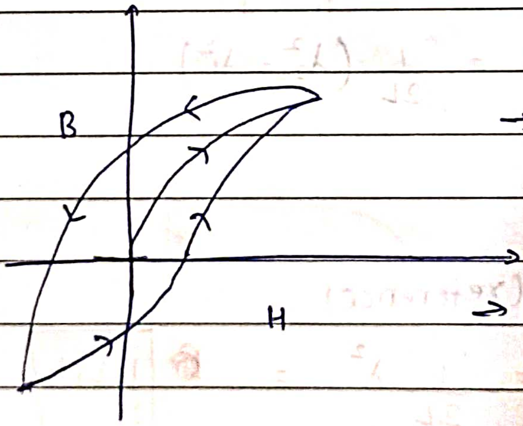
$\mu = \frac{B}{H}$ → very high

(domain moments add to the applied field)

~~until~~ all are oriented – saturation

→ when $B = 0$ → try to come to initial orientation, but NOT completely random → net magnetization exists, along the dir^n of applied field.

↳ Magnetic Hysteresis



→ after several cycles, forms closed loops

→ as H↑, curves flatten out as they tend to saturation.

when $H = 0 \Rightarrow B \neq 0$ → remanant magnetization when $H = 0$.

eg- $H_c = 11$ A/m ($B_c = 0.1T$ and see hysteresis curve), find i, $B_g g \to 5 \times 10^{-4}$

μ_0

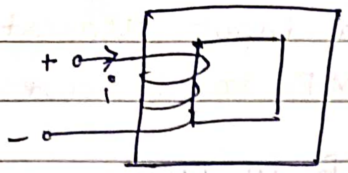
↓

$F_c = 11 \times \overset{l_c}{(0.3)} = 3.3$ A turns

$F_g = H_g \times (g) = \left(\frac{B_g}{\mu_0}\right) g = 396$

so, $Ni = 396 + 3.3 = 399.3$ $\quad i = \frac{399.3}{500} = 0.8A$

AC EXCITATION



V, ϕ approximate sinusoidal func of time.

Assume, $\varphi(t) = \phi_{max} \sin\omega t = A_c B_{max} \sin\omega t$

ϕ_{max}: amplitude of core flux; B_{max}: amplitude of flux density

and so, induced voltage,

$$e(t) = N\frac{d\varphi(t)}{dt} = \omega N A_c B_{max} \cos\omega t = E_{max}\cos\omega t$$

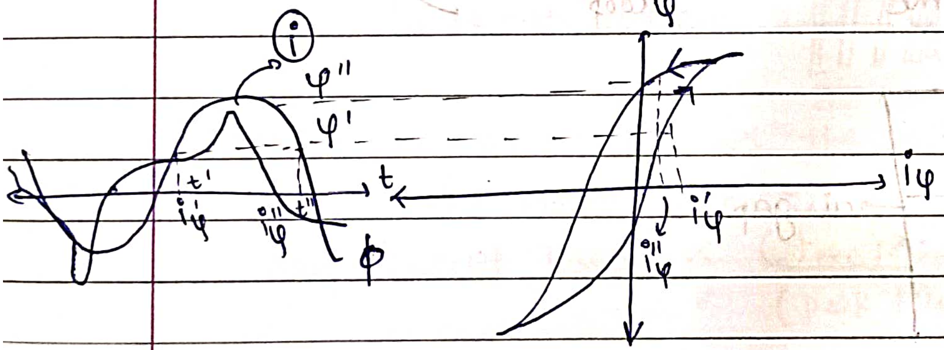
$$\boxed{E_{max} = \omega N \phi_{max}} = 2\pi f N \phi_{max}$$

$$\boxed{E_{rms} = \sqrt{2}\pi f N \phi_{max}}$$

→ We need an exciting current, i_φ in the winding.

non-linear core → waveform differs from sinusoidal. (so it follows the B-H curve)

$\varphi = B_c A_c$ and $i_\varphi = \frac{H_c l_c}{N}$



as hysteresis loop 'flattens out' due to saturation effect, the waveform of exciting currents is $\boxed{\text{sharply - peaked}}$

rising flux values from rising part of hysteresis loop.

$$\boxed{(I_\varphi)_{rms} = \frac{l_c (H_c)_{rms}}{N}}$$

$$(E_{rms})(I_{\varphi,rms}) = \sqrt{2} f \cancel{N} A_c \pi B_{max} \frac{l_c H_{rms}}{\cancel{N}}$$

$$= \sqrt{2}\pi f B_{max} H_{rms} (A_c l_c)$$

$A_c l_c$ → volume

$$P = \frac{E_{rms}\cdot(I_{\varphi,rms})}{mass} = \left(\frac{\sqrt{2}\pi f}{\rho_c}\right) B_{max}\cdot H_{rms}$$

exciting RMS voltamperes per unit mass,

$$\boxed{P_a = \left(\frac{\sqrt{2}\pi f}{\rho_c}\right) B_{max}\cdot H_{rms}}$$

→ exciting current → core flux i.e. power input associated with the energy of the M.F. in the core

↓

rest → reactive power associated with B in core

↓

NOT DISSIPATED ; cyclically absorbed and supplied

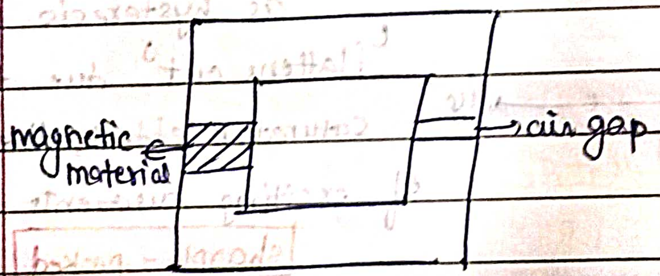
Losses → I^2R (I: induced currents) (eddy currents)

$$W = \oint i_\varphi \, d\lambda = \oint \left(\frac{H_c l_c}{N}\right) A_c N \, dB_c = A_c l_c \oint H \, dB_c$$

$$\boxed{W = A_c l_c \oint H \cdot dB_c}$$

hysteresis loop ∝ frequency of AC excitation (as loss per cycle)

$$\left(\frac{\text{Energy lost}}{\text{unit volume}} = \text{area of hysteresis loop}\right)$$



$$F = 0 = H_g g + H_m l_m$$

$$H_g = -H_m\left(\frac{l_m}{g}\right)$$

and $B_g A_g = B_m A_m$ $\quad B_g = \left(\frac{A_m}{A_g}\right) B_m$

$$B_g^2 = \mu_g H_g \cdot \left(\frac{A_m}{A_g} B_m\right)$$

$$= \mu_0 \left(\frac{-H_m l_m}{g}\right)\left(\frac{A_m}{A_g}\right) B_m$$

$$B_g^2 = \mu_0 \left(\frac{(\text{Vol.})_m}{(\text{Vol.})_g}\right)(-H_m B_m)$$

$$\boxed{(\text{Vol.})_{mag} = \frac{(\text{Vol.})_g \, B_g^2}{\mu_0 (-H_m B_m)}}$$

P_a = exciting RMS voltamperes per unit mass $= \dfrac{E_{rms} I_{\varphi,rms}}{m} = \dfrac{\left(\dfrac{l_c (H_c)_{rms}}{\cancel{N}}\right)\left(\sqrt{2}\pi f \cancel{N} \cancel{A_c} B_{max}\right)}{\cancel{A_c l_c} S_c}$

$$\boxed{P_a = \left(\frac{\sqrt{2}\pi f}{S_c}\right) B_{max} (H_c)_{rms}}$$

function of $(H_c)_{rms}$ alone or of B_{max} only as B_{max} depends on H_{rms}

Losses → I^2R (ohmic)

↳ Time varying MF → EF → eddy currents

↓

oppose changes in flux density of material

To counter the demagnetizing effect, current in excited winding must increase, making the B-H loop "fatter".

↳ ↓↓ eddy currents → thin sheets of laminations.

layers of insulation → interupt the path → ↓↓ eddy currents

↳ Thinner laminations ⇒ lower losses.

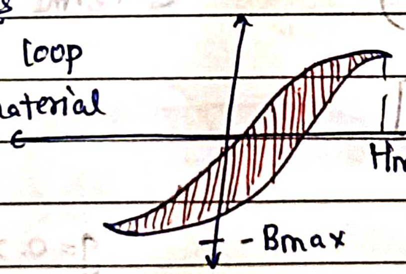
eddy-current losses $\propto (f_{exc})^2$

$\propto$ (peak flux density)^2

Hysteric losses

$\propto$ area of loop

$\propto$ Vol. of material



energy input to magnetic core : $W = \oint i_\varphi \, d\lambda = \oint \left(\frac{H_c l_c}{\cancel{N}}\right)(\cancel{N} A_c \, dB_c)$

$= A_c l_c \oint H_c \cdot dB_c$

$$\boxed{W = (\text{Volume}) \times \oint H_c \cdot dB_c}$$

area of hysteresis loop

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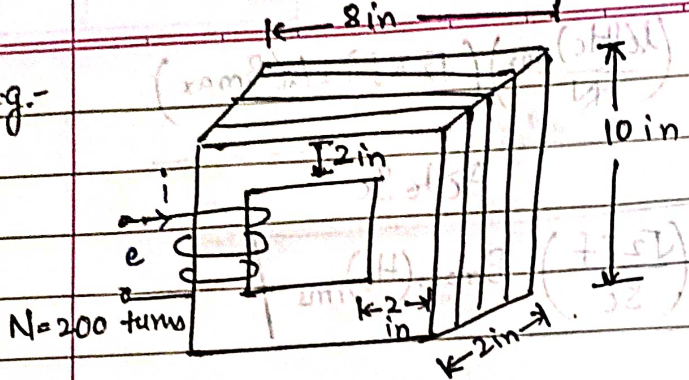
eg.-



$f = 60Hz$ excited

$B = 1.5 \sin \omega t$ $\quad \omega = 2\pi 60 \approx 377$

steel : 0.94 % of core C.S. Area

$\delta_{steel} = 7.65$

$B_{max.} = 1.5T \xrightarrow{corr.} H_{max} = 36$ A-turns/m

$P_a = 1.5$ VA/kg

A:) a) Applied Voltage, $e = N\frac{d\phi}{dt} = NA_c(1.5\ \omega \cos\omega t)$

$$e = (200) \times \left(\frac{A_{coil\ is}}{wound}\right) \frac{dB}{dt}$$

$$= (200 \times 4\, in^2 \times 1.5\omega)\cos\omega t$$

$$\boxed{e = 274 \cos \omega t \text{ Volts}}$$

b) $B_{max.} = 1.5T \longrightarrow H = 36A$ turns/m

$$\mu_r = \frac{B_{max}}{\mu_0 H_{max}} = 33000$$

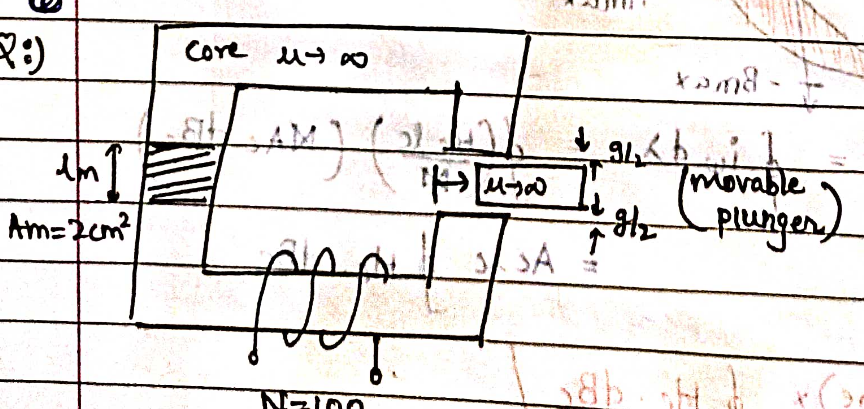
$$l_c = (6+6+8+8) \times \frac{1\ m}{39.04\ in} = 0.71\ m$$

$$I_{peak} = \frac{H_c l_c}{N} = \boxed{0.13A}$$

c) $(P_a)_{\times} W = I_{\varphi,rms}\left(\frac{274}{\sqrt{2}}\right)$ $\quad (I_\varphi)_{rms} = \frac{P_a}{(E_{\varphi,rms})}$

$$\Rightarrow I_{\varphi,rms} = \boxed{0.10\ A}$$

Q:)



$g = 0.2cm$

$2cm^2 \leq A_g \leq 4\ cm^2$

a) If $B_{max.} = 1T$, $H_{max.} = 40$ kA/m, find l_m.

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$$\frac{B_m}{-H_m} = \frac{B_g}{H_g}\,\frac{A_g}{A_m}\,\frac{l_m}{g}$$

$$l_m = g\left(\frac{A_m}{A_g}\right)\left(\frac{B_m}{\mu_0 H_{m}}\right)$$

(as $\mu \to \infty$ of core, $H_c \to$ negligible
Use flux continuity and MMF relations)

$$\left(\begin{aligned} B_m A_m &= B_g A_g \\ B_g &= \frac{B_m}{A}\left(\frac{A_m}{A_g}\right)\end{aligned}\right) \quad \text{and} \quad \left(0 = H_g g + H_m l_m\right)$$

and so, when there is no external Ni (MMF),

$l_m = 3.98$ cm

b) $Ni = H_m l_m + H_g g$ $\qquad B_m A_m = B_g A_g = \mu_0 H_g A_g$

$$B_m = -2.5\times10^{-5} H_m + 6.28\times10^{-2}\, i$$

$i \to \max$

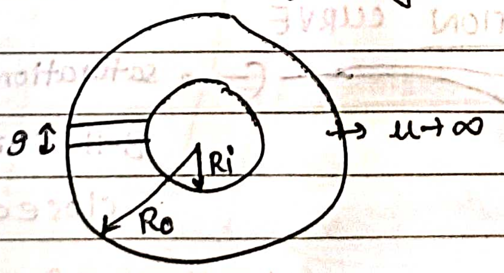
$$i_{max.} = \frac{B_{max.} + 2.5\times10^{-5} H_{max}}{6.28\times10^{-2}}$$

INDUCTANCE OF A TOROID

e.g.- rings in a stack of height 'h'.

$R_i = 3.4$ cm $\quad R_o = 4.0$ cm

$h = 2$ cm $\quad g = 0.2$ cm



$$l_c = 2\pi\left(\frac{R_i+R_o}{2}\right)-g = 2\pi\left(\frac{4+3.4}{2}\right)-g = \frac{3.7\times2\pi}{2}-g = 3.7\pi - g$$

$l_{core} = 23.05$ cm

$A_c = (R_o - R_i)h = 1.2\ \text{cm}^2$

~~$R_c = \frac{l_c}{\mu A_c}$~~ $\quad R_c = 0 \quad R_g = \frac{g}{\mu_0 A_g} = 1.327\times10^{7}$

as $N = 65$ turns,

$$L = \frac{N^2}{R_{total}} = 0.318 \text{ mH}$$

$i = ?$ if $B_g = 1.35T$

$\phi = BA$ $\quad L = \frac{N(BA)}{i}$ $\quad i = B\left(\frac{NA}{L}\right)$

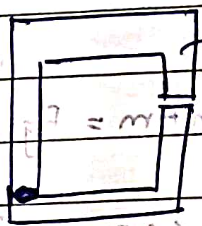
$$i = B\left(\frac{\cancel{N}\cancel{A} \cdot g}{N^{\cancel{2}} \mu_0 \cancel{A}}\right) = \frac{B \cdot g}{\mu_0 N} = \underline{33.05A}$$

Flux linkage of coil, $\lambda = N\phi$ $\quad B = \frac{\phi}{A}$

$= NBA$

$= \underline{10.53 \text{ mWb}}$

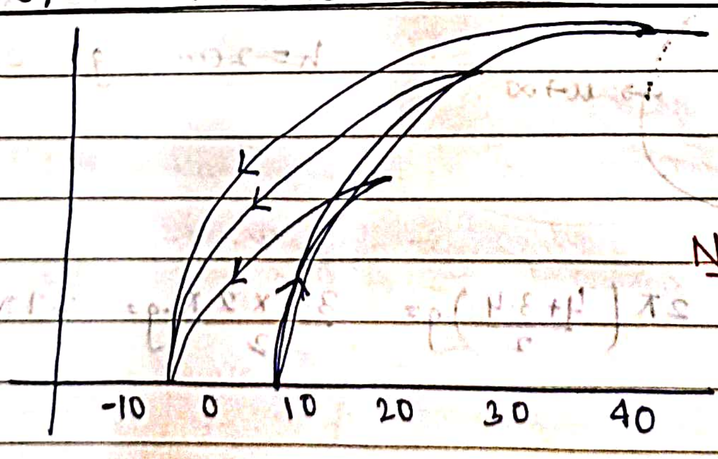
⇒ **INDUCTANCE :-**



$$L = \frac{N^2}{R_{total}} = \frac{N^2}{\frac{l_c}{\mu_r \mu_0 A_c} + \frac{g}{\mu_0 A_c}}$$

$$= \frac{N^2 \mu_0 A_c}{(g + l_c/\mu_r)}$$

⇒ **DC / NORMAL MAGNETIZATION CURVE**



(→ saturation)

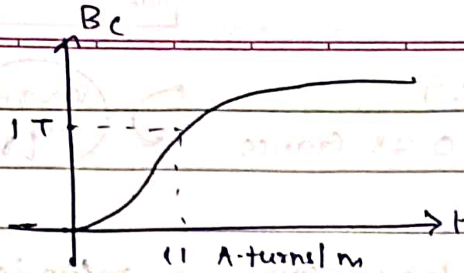
B-H curves form closed loops

Non-linear & multivalued

→ form closed loops after several cycles

⇒ remanant magnetization when $H = 0$

→ curve by joining locus of tips of hysteresis curve
→ DC/normal magnetizⁿ curve



eg. at $B_c = 1T$ $H_c = 11$ A-turns/m $l_c = 0.3$ m

$\frac{Ni}{l} = H \Rightarrow \cancel{Ni = H_c l_c = 3.3 A}$

$$Ni = H_c l_c + H_g l_g = (3.3 + \underbrace{396}_{\text{given}})$$

$$i = \frac{3.3 + 396}{500} = 0.8\,A$$ current to produce $B_c = 1T$

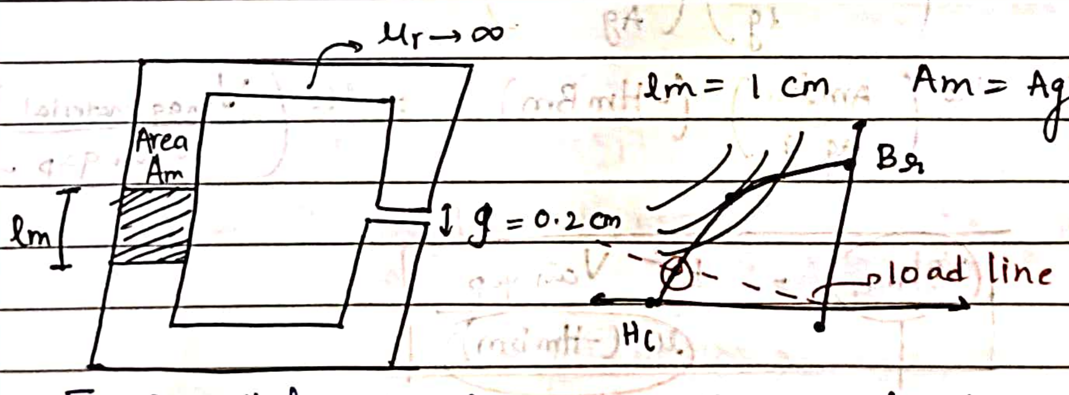
PERMANENT MAGNETS

remanent magnetization, $B_r \rightarrow$ B in a closed structure that would remain if applied MMF = 0 (or H = 0)

co-ercivity, $H_c \rightarrow$ H (magnetic field intensity ($\propto$ MMF)) when magnetic flux density, B = 0.

$B_r \rightarrow$ produce magnetic flux in a magnetic circuit, in the absence of external excitation

eg.-



$l_m = 1$ cm $A_m = A_g = 4\,cm^2$

$\Rightarrow F = 0 = H_g l_g + H_m l_m$ $H_g = -\left(\frac{H_m l_m}{l_g}\right)$

ϕ = const. $A_g B_g = A_m B_m$

$$B_g = \left(\frac{A_m}{A_g}\right) B_m$$

$$B_m = -\mu_0 \left(\frac{A_g}{A_m}\right)\left(\frac{l_m}{g}\right) H_m = -5\mu_0 H_m$$

$\boxed{B_m = -6.28 \times 10^{-6} H_m} \rightarrow$ linear reln : LOAD LINE $B_g = B_m = 0.3T$

Date:

if we used { AlNiCo-5 → $B_g = 0.3T$
M-5 ele. steel → $B_g = 0.38$ Gauss } large diff.

Co-ercivity (H_c)
↳ magnitude of MMF to demagnetize a material.
↳ measure of capacity to magnetize (i.e. produce flux) in a magnetic circuit which includes an air-gap.

Permanent Magnets → large H_c.
↳ Maximum-Energy product → largest B-H product $(B\text{-}H)_{max}$.

(energy density dimensions) 2nd Quad. of hysteresis loop

⇒ operatⁿ on MEP-point will need smallest volume of that material to produce the same flux across an air-gap

$$B_g = \left(\frac{A_m}{A_g}\right) B_m \qquad \frac{H_m l_m}{H_g l_g} = -1$$

$$H_g = -\left(H_m\left(\frac{l_m}{l_g}\right)\right)$$

$$B_g = \mu_0 H_g$$

$$B_g^2 = \mu_0 \left(-H_m \frac{l_m}{l_g}\right)\left(\frac{A_m}{A_g} B_m\right)$$

$$B_g^2 = \mu_0 \left(\frac{A_m l_m}{A_g \cdot g}\right)(-H_m B_m) = \mu_0 \left(\frac{V_{mag.\ material}}{V_{air\ gap}}\right)(-B_m H_m)$$

$$\boxed{Vol_{mag.} = \frac{B_g^2\, V_{air\text{-}gap}}{\mu_0\,(-H_m B_m)}}$$

MIN. ⇐ MAX.

→ larger the value of B-H product across diff. materials smaller the size of magnet to produce the same flux

NOTE :- $V_{rms} = \frac{\omega N A_c B_{max}}{\sqrt{2}}$ $\qquad i_{rms} = \frac{V_{rms}}{\omega L}$

$$W_{peak} = \frac{1}{2} L (i_{rms}\sqrt{2})^2$$

ENERGY DENSITY

$$\boxed{u_m = \frac{B^2}{2\mu}}$$

Energy stored $= u_m \times (\text{Vol.}) = \frac{B^2}{2\mu}(Al)$

Across an inductor (solenoid)

$L = \left(\frac{BA}{i}\right) \times n = \mu_0 n^2 AL$ $\quad B = \mu_0 n I$

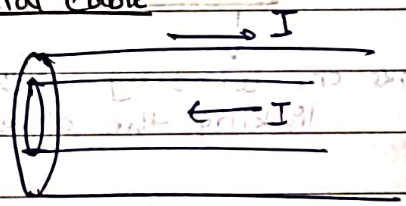
$u_m = \frac{(In\mu_0)^2}{2\mu_0}$ $\quad U = \int u_m \, dV = \int \frac{n^2 I^2 \mu_0^{\cancel{2}}}{2\cancel{\mu_0}} (A\,dl)$

$$= \frac{1}{2}(\mu_0 n I^2 Al) \times n$$

$$= \frac{1}{2}(\mu_0 n^2 Al) I^2$$

$$= \boxed{\frac{1}{2} L I^2}$$

e.g.- Coaxial cable



$B(2\pi r) = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi r}$

$$u_m = \frac{B^2}{2\mu_0} = \frac{\mu_0^{\cancel{2}} I^2}{\cancel{\mu_0}\, 8\pi^2 r^2} = \frac{\mu_0 I^2}{8\pi^2 r^2}$$

$$U = \int_{R_1}^{R_2} \frac{\mu_0 I^2}{8\pi^2 r^2}(2\pi r l)\,dr = \frac{\mu_0 I^2 l}{4\pi} \ln\left(\frac{R_2}{R_1}\right)$$

$$\frac{1}{2} L \cancel{I^2} = \frac{\mu_0 \cancel{I^2} l}{4\pi} \ln\left(\frac{R_2}{R_1}\right)$$

$$L = \frac{\mu_0 l}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

$$\frac{L}{l} = K = \boxed{\frac{\mu_0}{2\pi} \ln\left(\frac{R_2}{R_1}\right)}$$ inductance per unit length