EE114: Power Engineering-I

Lecture Notes: Single and Three Phase AC Systems

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Contents

Objectives

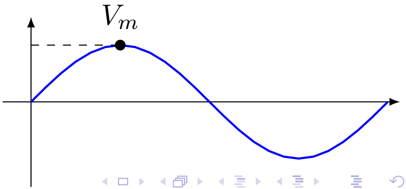
- Understand and Use concept of Phasors and inverse transform
- Transform sinusoidal source to frequency domain using phasor
- Apply know analysis techniques in frequency (phasor) domain
- Analyze circuit with linear transformers using phasors
- Understand constraints of ideal transformer concepts and analyze circuits with ideal transformer using phasors
- Phasor Diagrams

Sinusoidal Source

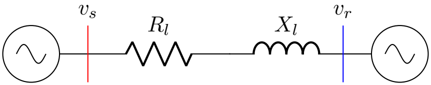
- Source produces sinusoidally varying output with respect to time

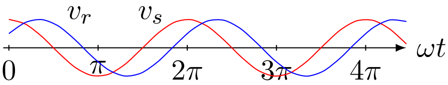
$$v(t) = V_m \cos{(\omega t + \phi)} \qquad (1)$$

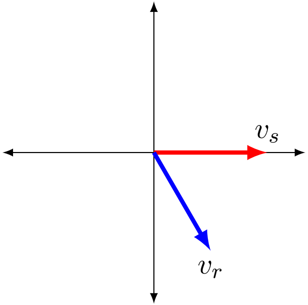
- Can be expressed as a function of sin or cos
- Repeats at regular intervals i.e. periodic. The duration of each period is Tseconds.
- Number of repetitions per cycle is frequency measured in Hz
- Angular Frequency is given $\omega = 2\pi f = 2\pi/T$ measured as $radians/sec$
- ϕ is phase angle usually measured in degrees (convert to radians)
    - Indicates value at $t = 0$
    - Changing ϕ shifts waveform
        - $\phi \rightarrow +ve$ shifts left
        - $\phi \rightarrow -ve$ shifts right



Phasor and Time Domain Representations

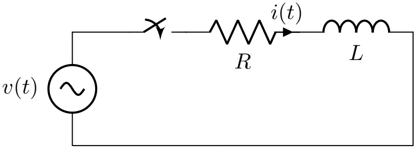






- $v = |V| \cos(\omega t - \theta)$
- Assumption *Pure* sinusoid and *constant* frequency
- cos wave is taken as the reference
- Euler's identity
  - $e^{j\theta} = \cos(\theta) + j \sin(\theta)$

Response to Sinusoidal Excitation



Response

- All circuit analysis rules are applicable
- Use KVL to analyze the circuit

$$i(t) = \underbrace{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}}_{\text{Peak Value}} \left[\overbrace{\cos\left(\omega t + \phi - \theta\right)}^{\text{Steady State Response}} - \underbrace{\cos\left(\phi - \theta\right) e^{-Rt/L}}_{\text{Transient Response}} \right] \tag{2}$$

The aim is to avoid solving differential equations: Solution is to use phasors

Phasors

- Carries the information of phase angle and amplitude of sinusoidal signals
- Concept is based in Eulers Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta \tag{3}$$

- We can think of cos as $\mathbf{Re}(e^{j\theta})$ and sin as $\mathbf{Im}(e^{j\theta})$
- $v(t) = V_m \cos(\omega t + \phi) = \mathbf{Re}(V_m e^{j(\omega t + \phi)})$
- Carries Amplitude and Phase Information, No frequency Information. Complex number is Polar Form in (4).

$$v(t) = \underbrace{V_m e^{(j\theta)}}_{\text{Complex Number}} \quad e^{(j\omega t)} \tag{4}$$

- Phasor Transforms time domain to frequency domain
- Use bold face letter to depict phasors
- Rectangular Form: $\mathbf{V} = V_m \cos\phi + jV_m \sin\phi$

Phasors

- Phasor domain transformation is useful since it reduces the task to calculation of amplitude and phase.
- We are only interested in steady state solution
- We solve the (5) using to get phasor domain results. Assume $i_{ss}(t) = \mathbf{Re}(I_m e^{(j\beta)} e^{(j\omega t)})$

$$v(t) = L\frac{di}{dt} + Ri \tag{5}$$

$$\mathbf{Re}(V_m e^{j\phi} e^{j\omega t}) = \mathbf{Re}(I_m(R + j\omega L) e^{j\beta} e^{j\omega t}) \tag{6}$$

$$I_m e^{j\beta} = \frac{V_m e^{j\phi}}{R + j\omega L} \tag{7}$$

Passive Elements in Frequency Domain

Resistor

- A current source $i(t) = I_m \cos(\omega t + \theta_i)$ is applied across resistor
- Voltage is in phase with the current

$$v = RI_m \cos(\omega t + \theta_i) \tag{8}$$

$$\mathbf{Re}(v) = \mathbf{Re}(RI_m e^{j\theta_i} e^{j\omega t}) \tag{9}$$

$$\mathbf{V} = RI_m e^{j\theta_i} \tag{10}$$

Passive Elements in Frequency Domain

Inductor

- A current source $i(t) = I_m \cos(\omega t + \theta_i)$ is applied across inductor
- Voltage is 90^o out of phase with the current. In this case voltage leads current

$$v = L\frac{dI_m \cos(\omega t + \theta_i)}{dt} \tag{11}$$

$$= -\omega L I_m \cos(\omega t + \theta_i - 90^o) \tag{12}$$

$$\mathbf{V} = -\omega L I_m e^{j(\theta_i - 90^o)} \text{use } e^{-j90^o} = -1 \tag{13}$$

$$\mathbf{V} = j\omega L \mathbf{I} \tag{14}$$

Passive Elements in Frequency Domain

Capacitor

- A voltage source $v(t) = V_m \cos(\omega t + \theta_v)$ is applied across capacitor
- Voltage is 90^o out of phase with the current. In this case voltage lags current

$$i = C\frac{dV_m \cos(\omega t + \theta_v)}{dt} \tag{15}$$

$$= \omega C V_m \cos(\omega t + \theta_i + 90^o) \tag{16}$$

$$\mathbf{V} = \omega C V_m e^{j(\theta_v + 90^o)} \text{use } e^{j90^o} = 1 \tag{17}$$

$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I} \tag{18}$$

Impedance in Series and Parallel

Series Impedances

Acts as Voltage Divider

$$Z_{eq} = Z_1 + Z_2 + Z_3 + Z_4 + Z_5 = \sum_{m=1}^{k} Z_m \tag{19}$$

Parallel Impedances

Acts as Current Divider

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4} = \sum_{m=1}^{k} \frac{1}{Z_m} = Y_{eq} = \sum_{m=1}^{k} Y_m$$

$$Y = \frac{1}{Z} = G + jB \tag{20}$$

$$= \text{Conductance} + j\text{Reactance} \tag{21}$$

Kirchoff's Voltage Law

Kirchoff's Voltage Law

The sum of all voltages in a closed loop of a circuit is zero

$$0 = v_1 + v_2 + ... + v_n \tag{22}$$

$$0 = V_{1m}\cos\left(\omega t + \theta_1\right) + V_{2m}\cos\left(\omega t + \theta_2\right) + ... + V_{nm}\cos\left(\omega t + \theta_n\right) \tag{23}$$

$$0 = \mathbf{Re}\left[\left(V_{1m}e^{(j\theta_1)} + V_{2m}e^{(j\theta_2)} + ... + V_{nm}e^{(j\theta_n)}\right)\right] \tag{24}$$

$$0 = \sum_{k=1}^{l} \mathbf{V}_{km} \tag{25}$$

Kirchhoff's Current Law

Kirchhoff's Current Law

The sum of all currents at a node of a circuit is zero

$$0 = i_1 + i_2 + i_3 + i_4 + i_5 \tag{26}$$

$$0 = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 + \mathbf{I}_5 \tag{27}$$

$$0 = \sum_{k=1}^{l} I_k \tag{28}$$

Adding Cosines: Example

Use of Phasors

Using two methods: 1. Expand and Add 2. Phasor

$$y_1 = 20\cos(\omega t - 30^o) \tag{29}$$

$$y_2 = 40\cos(\omega t + 60^o) \tag{30}$$

Expand and Add

$$\begin{aligned} y &= 20\cos\omega t\cos 30^o + 20\sin\omega t \\ &\quad \sin 30^o + 40\cos\omega t\cos 60^o \\ &\quad - 40\sin\omega t\sin 60^o \\ &= 44.72(\frac{37.32}{44.72})\cos\omega t \\ &\quad - 44.72(\frac{24.64}{44.72})\sin\omega t \\ &= 44.72\cos(\omega t + 33.43^o) \end{aligned}$$

Phasor Addition

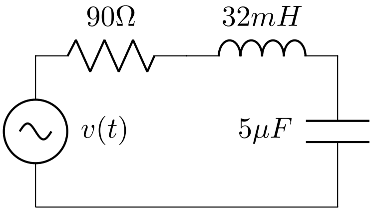
$$y_1 = 20\angle - 30^o$$

$$y_2 = 40\angle 60^o$$

Add as Phasors

$$\begin{aligned} y &= 17.32 - j10 \\ &\quad + 20 + j34.64 \\ y &= 44.72\angle 33.43^o \end{aligned}$$

Combining Impedances: Series Circuit



$$v(t) = 750\cos(\omega t + 30^o) = 750\angle 30^o$$

$$Z_L = j\omega L = j160\Omega$$

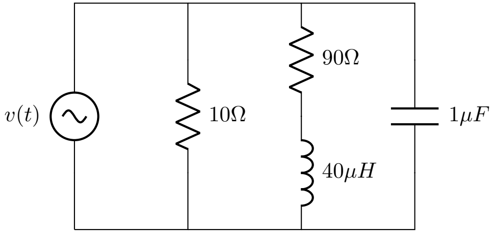
$$Z_C = \frac{1}{j\omega C} = -j40\Omega$$

$$Z_{series} = Z_L + Z_C + Z_R$$

$$= 90 + j160 - j40$$

$$I = \frac{V}{Z_{series}} = \frac{750\angle 30^o}{150\angle 53.13^o}$$

Combining Impedances: Complex Circuits



$$v(t) = 8\cos(2\times 10^5 t)$$

$$Z_1 = 100\Omega$$

$$Y_1 = \frac{1}{Z_1} = 0.01\text{S}$$

$$Y_2 = \frac{1}{6 + j2\times 10^5 \times 40 \times 10^{-6}}$$

$$Y_3 = \frac{1}{\frac{1}{j\omega C}} = j2\times 10^5 \times 1 \times 10^{-6}$$

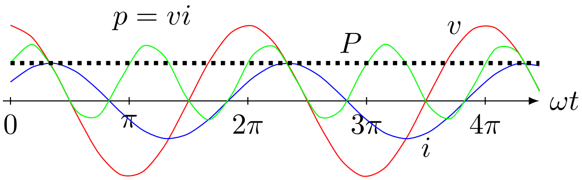
$$Y = Y_1 + Y_2 + Y_3$$

$$V = \frac{I}{Y}$$

Power in ac circuits

Instantaneous Power

$p = \frac{dE}{dt}$ $p = v \times i$
$p = \frac{V_{max}I_{max}}{2}\cos\theta\,(1+\cos 2\omega t) + \frac{V_{max}I_{max}}{2}\sin\theta\sin 2\omega t$
Phasor Form : $p = VI^*$



- Positive Power: Current in direction of Voltage drop → Power to Load
- Negative Power: Current in direction of Voltage rise → Power from Load

Power in ac Circuits

Apparent Power

$$S = \text{Apparent Power}$$

$$P = \frac{V_{max} I_{max}}{2} \cos(\theta_v - \theta_i)$$

$$Q = \frac{V_{max} I_{max}}{2} \sin(\theta_v - \theta_i)$$

$$S = P + jQ$$

$$\begin{aligned} S &= V_{rms} \angle \theta_v (I_{rms} \angle \theta_i)^* \\ &= V_{rms} \angle \theta_v (I_{rms} \angle - \theta_i) \end{aligned}$$

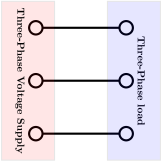
Root Mean Square (RMS) Value

$$V_{rms} = \sqrt{\frac{1}{T} \int_{t-T}^{t} V_{max}^2 \cos^2 (\omega t + \theta_v) \, dt}$$

$$V_{rms} = \text{Sine Waveform} = \frac{1}{\sqrt{2}}$$

Hence, current and voltage phasors are defined with rms values and angle instead of peak because the apparent power can be directly calculated.

Three Phase ac Circuits



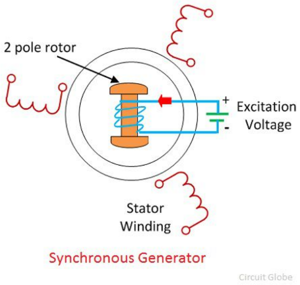
What is three phase supply?

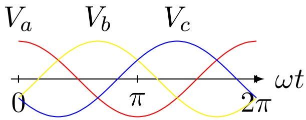
- Three identical frequency ac sources
- Phase shifted with respect to each other
- Subset of polyphase power
- Polyphase power was independently invented by: Gallileo Ferraris, Mikhail Dolivo-Doborovolsky,Jonas Wenstrom, John Hopkinson, Nikola Tesla

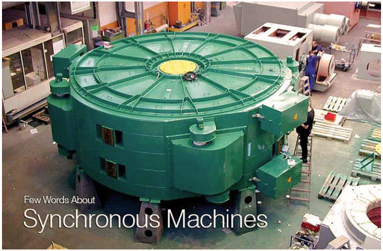
Important Aspects

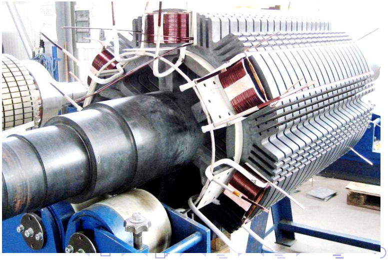
- Balanced and Unbalanced Systems
- Positive and Negative Sequence
- Wye or Delta Connected
- Synchronous machine: typical source

Three phase ac sources



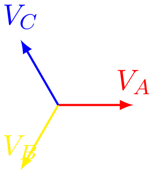




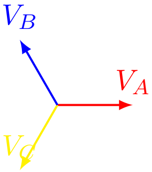


Three phase ac Systems

Positive Sequence



Negative Sequence



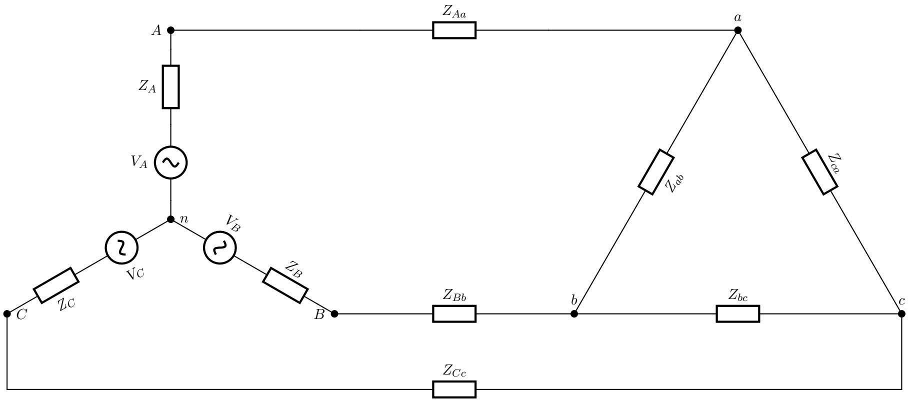
Balanced System

- Three identical frequency ac sources
- Phase shifted with respect to each other by 120^o
- Equal in magnitude
- Line impedances are equal
- Load impedances are equal
- Sum of voltages is zero

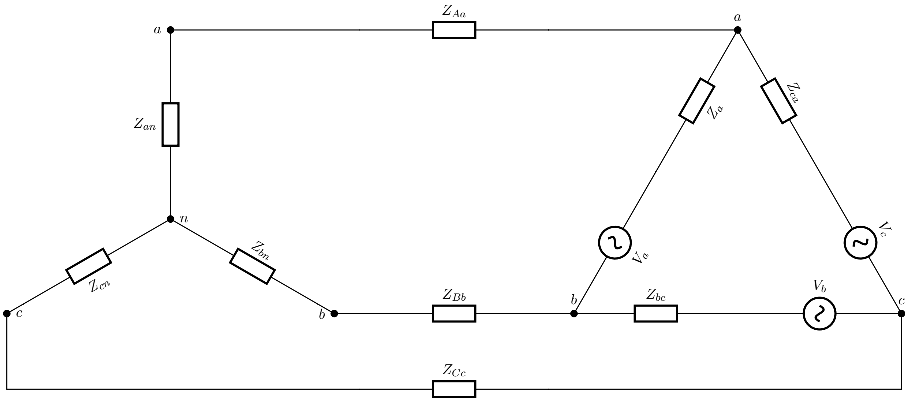
Unbalanced System

- Any one of the conditions is not met
- Neutral

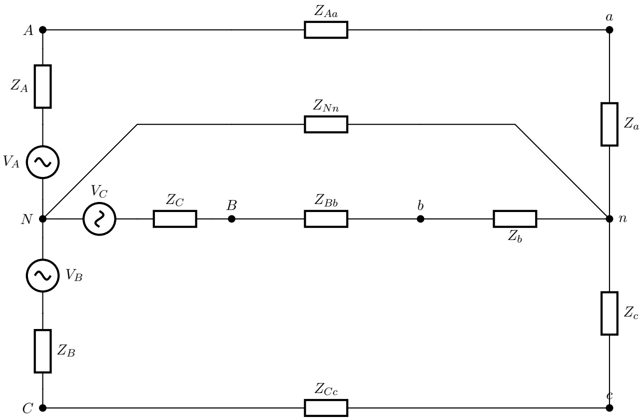
Source Star (Wye) and Load Delta (Pi) Connection



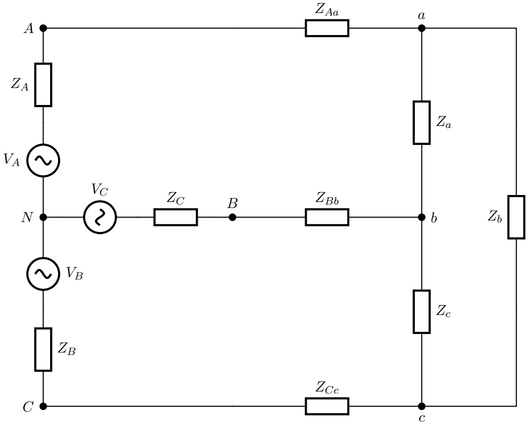
Load Star (Wye) and Source Delta (Pi) Connection



Source Star (Wye) and Load Wye (Star) Connection



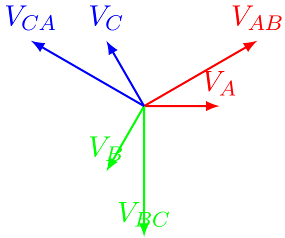
Source Star (Wye) and Load Delta Connection



Voltages and Currents in Balanced Wye System

Relationship between Voltage and Current

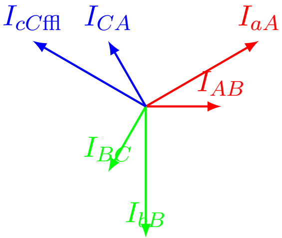
$$
\begin{aligned}
I_{phase} &= I_{Line-Line} \\
V^{AB}_{Line-Line} &= V_{AN} - V_{BN} \\
&= V_m\angle 0^o - V_m\angle - 120^o \\
&= V_m(1 + 0.5 + j0.866) \\
&= \sqrt(3)V_m(0.866 + j0.5) \\
&= \sqrt(3)V_m\angle 30^o
\end{aligned}
$$



Voltages and Currents in Balanced Delta System

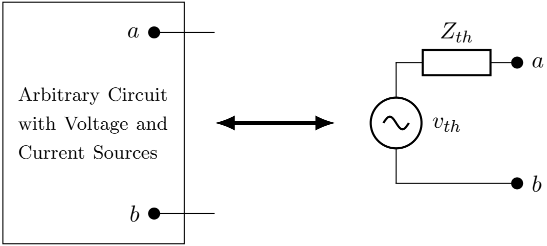
Relationship between Voltage and Current

$$
\begin{aligned}
V_{phase} &= V_{Line-Line} \\
I^{aA}_{Line-Line} &= I_{AB} - I_{CA} \\
&= I_m\angle 0^o - I_m\angle -120^o \\
&= I_m(1 + 0.5 + j0.866) \\
&= \sqrt(3) I_m(0.866 + j0.5) \\
&= \sqrt(3) I_m\angle 30^o
\end{aligned}
$$



Thevenin and Norton Equivalent

- We wish to focus on what happens on the pair of terminals or a sub-circuit
- Thevenin and Norton Equivalents are circuit simplifications that focus on terminal behavior
- Used to represent circuits made of linear elements
- The equivalent source and resistance replaces the large network but produces same voltage and current in the load



How to obtain v_{th} and R_{th}?

Thevenin Equivalent

- Open circuit terminals $a - b$ and calculate open circuit voltage v_{ab}
- Place a short circuit across terminals $a - b$ and calculate the short circuit current i_{sc}

Procedure to Calculate v_{th}

- Open circuit terminals $a - b$
- Calculate voltage across the open terminals using all known techniques like KVL, KCL, source transformations etc..

Procedure to Calculate R_{th}

- Short circuit terminals $a - b$
- Calculate current in the shorted terminals using all known techniques like KVL, KCL, source transformations etc..
- $Z_{th} = \frac{V_{th}}{i_{sc}}$

Alternate way to obtain Z_{th}?

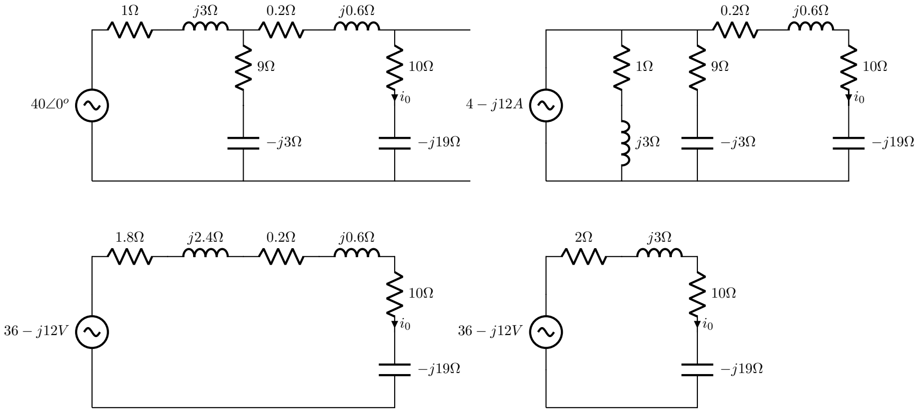
Procedure to Calculate R_{th}

- Short circuit all voltage sources and open circuit all current sources
- Use all known simplifications methods to calculate resistance /impedance looking back in to the circuit from terminals of interest

Norton Equivalent

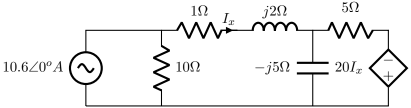
Norton Equivalent can be obtained by source transformation of the Thevenin Equivalent Circuit.

Source Transformation and Thevenin Equivalent Example



Node Voltage and Mesh Current Analysis:Example

Node Voltage Method

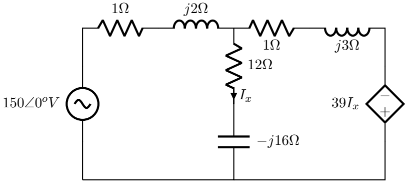


$$0 = \frac{V_1}{10} + \frac{V_1 - V_2}{1 + j2} - 10.6\angle 0^o$$

$$0 = \frac{V_2}{-j5} + \frac{V_2 - V_1}{1 + j2} + + \frac{V_2 - 20I_x}{5}$$

$$I_x = \frac{V_1 - V_2}{1 + j2}$$

Mesh Current Analysis



$$0 = 150\angle 0^o - I_1(1 + j2) + (I_1 - I_2)(12 - j6)$$

$$0 = I_2(1 + j3) + 39I_x + (I_2 - I_1)(12 - j16)$$

$$I_x = I_1 - I_2$$

Superposition

Superposition

Superposition states that whenever a linear circuit is excited/energized by multiple source then the total response can be obtained by sum of responses due to individual sources

Linear Equation or System

System is said to be linear if it satisfies the following:

- Additive Property: $f(x_1 + x_2) = f(x_1) + f(x_2)$
- Homogeneity: $f(\alpha x) = \alpha f(x)$ [a]

[a] Note: Terms of linear equations cannot contain products of distinct or equal variables, nor any power (other than 1) or other function of a variable, equations involving terms such as xy, x2, y1/3, and sin(x) are nonlinear.

Introduction to Magnetic Circuits

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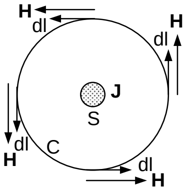
- Current carrying conductor produces magnetic field

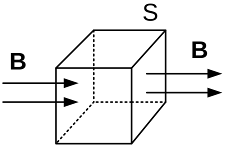
$$\oint_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{da} \qquad (1)$$

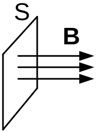
$$\oint_S \mathbf{B} \cdot \mathbf{da} = 0 \qquad (2)$$

- Magnetic field lines form closed loops
- Magnetic monopole does not exist
- Flux, ϕ crossing a surface S with field density B

$$\phi = \int_S \mathbf{B} \cdot \mathbf{da} \qquad (3)$$







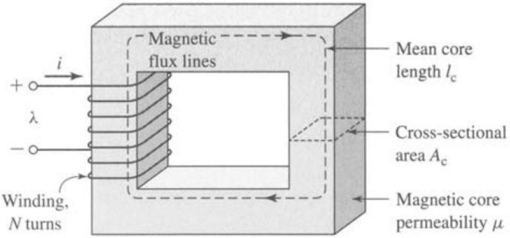


Figure 1: Simple magnetic circuit (Image taken from reference[1])

Direction of H_c in core- Right hand rule

- Fingers of right hand- Current
- Thumb of right hand - Magnetic field in the core

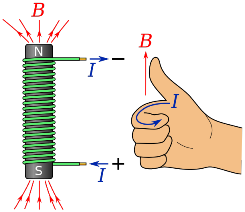


Figure 2: Right hand rule (Image available online[2])

[1]'Electric Machinery' by Fitzgerald, Kingsley and Umans
[2]https://www.wikiwand.com/en/Right-hand_rule

- Flux in the core, ϕ_c

$$\phi_c = B_c A_c \qquad (4)$$

where B_c = flux density in core

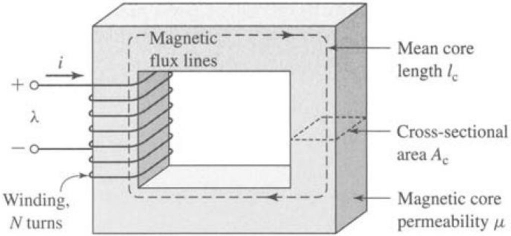
- MMF acting on a magnetic circuit, $\mathcal{F}$

$$\mathcal{F} = Ni = \oint \mathbf{H} \cdot \mathbf{dl} \qquad (5)$$

- Path length of flux = l_c

$$\mathcal{F} = Ni = H_c l_c \qquad (6)$$

where H_c - average magnitude of H in the core



- Relation between magnetic field density in the material **B** and magnetic field intensity **H**

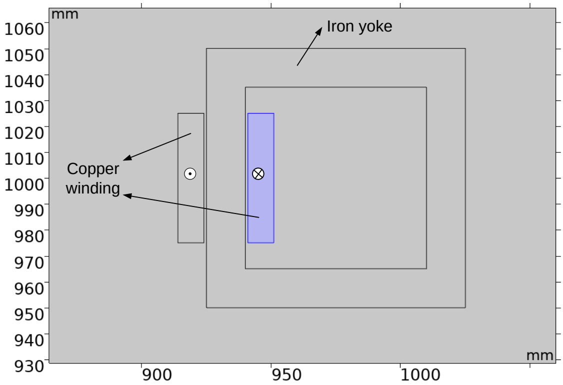
$$\mathbf{B} = \mu \mathbf{H} \tag{7}$$

where μ- magnetic permeability of the material

$$\mu = \mu_r \mu_0 \tag{8}$$

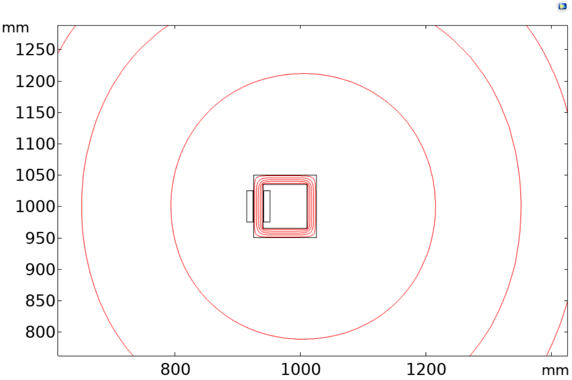
where μ_0- permeability of free space $= 4\pi \times 10^{-7}$ H/m

- μ_r = 2000 to 80000 for transformers and rotating machines



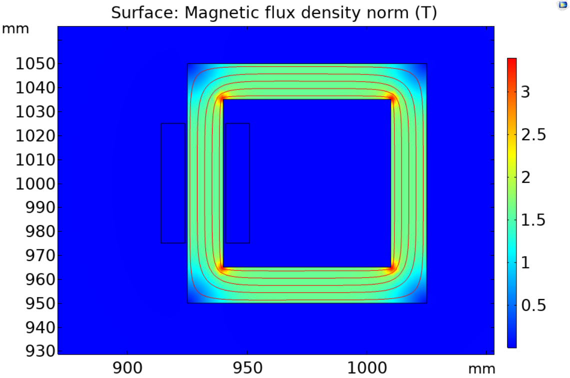
- Copper winding around an iron core
- DC current density $= 0.2\ \text{A}/\text{mm}^2$
- μ_r of iron $= 4000$

Field Lines in a magnetic core



- Field density is higher in the core

Field Density in a magnetic core



- Field density is higher in the core
- Field is directed through the iron core
- Negligible field outside the core

Magnetic Circuit with Air Gap

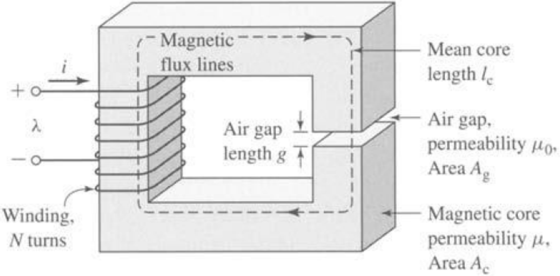


Figure 3: Magnetic circuit with air gap (Image taken from reference[3])

Two magnetic circuits in series

- A magnetic core of permeability μ, cross-sectional area A_c, mean length l_c
- Air gap of permeability μ_0, cross-sectional area A_g, length g

[3]'Electric Machinery' by Fitzgerald, Kinsley and Umans

- Flux density in core B_c and in the air gap B_g

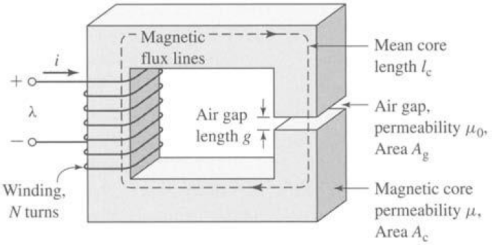
$$B_c = \frac{\phi}{A_c} \quad (9)$$

$$B_g = \frac{\phi}{A_g} \quad (10)$$

$$\mathcal{F} = H_c l_c + H_g g \quad (11)$$

$$\mathcal{F} = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g \quad (12)$$

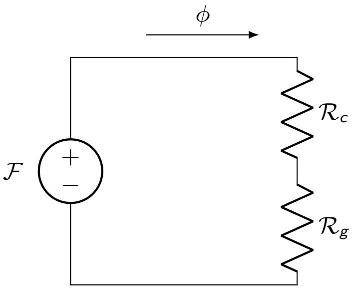
- $\mathcal{F} = Ni$ - mmf applied to the magnetic circuit



- $\mathcal{F}_c = H_c l_c$ - produces magnetic field in the core
- $\mathcal{F}_g = H_g g$ - produces magnetic field in the air gap
- With electrical- magnetic circuit analogy, we can write

$$\mathcal{F} = \phi(\mathcal{R}_c + \mathcal{R}_g) \quad (13)$$

Equivalent magnetic circuit



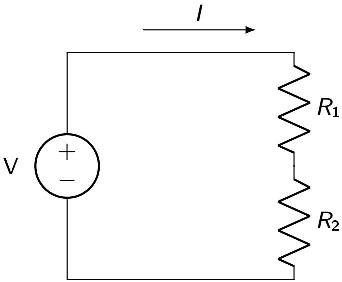
$$\mathcal{F} = \phi(\mathcal{R}_c + \mathcal{R}_g) \tag{14}$$

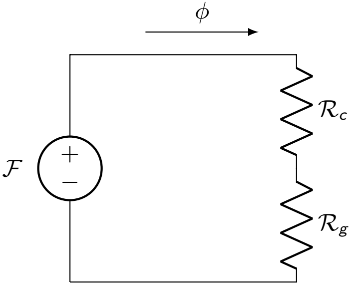
where $\mathcal{R}_c$ - reluctance of core
$\mathcal{R}_g$ - reluctance of air gap

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} \text{ (A.turns/Wb)} \tag{15}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} \text{ (A.turns/Wb)} \tag{16}$$

Electric- Magnetic Analogy





$$I = \frac{V}{R_1 + R_2} \tag{17}$$

$$\phi = \frac{\mathcal{F}}{\mathcal{R}_c + \mathcal{R}_g} \tag{18}$$

$$\phi = \frac{\mathcal{F}}{\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}} \tag{19}$$

$$\phi = \frac{\mathcal{F}}{\mathcal{R}_{tot}} \tag{20}$$

Permeance of magnetic circuit- Conductance of electric circuit

$$P_{tot} = \frac{1}{\mathcal{R}_{tot}} \tag{21}$$

Fringing Effect

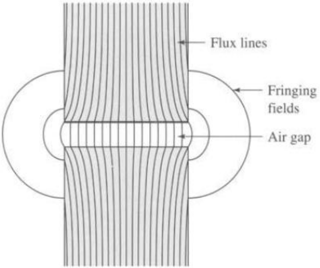


Figure 4: Fringing fields (Image taken from reference[4])

- Assumption of uniform field lines in air gap
- Practically, magnetic field "fringe" outward while crossing the air gap
- Increase in the effective cross-sectional area A_g
- Neglecting fringing, $A_g = A_c$

[4] 'Electric Machinery' by Fitzgerald, Kinsley and Umans

- For magnetic circuits with multiple elements in series and parallel

$$\mathcal{F} = \oint \mathbf{H} \cdot \mathbf{dl} = \sum_k \mathcal{F}_k = \sum_k H_k l_k \qquad (22)$$

The magnetic circuit in Figure 3 has dimensions $A_c = A_g = 9\ \text{cm}^2$, $g = 0.050$ cm, $l_c = 30$ cm, and $N = 500$ turns. Assume the value of $\mu_r = 70{,}000$ for core material.

(a) Find the reluctances $\mathcal{R}_c$ and $\mathcal{R}_g$.

For the condition that the magnetic circuit is operating with $B_c = 1.0$ T, find (b) the flux ϕ and (c) the current i.

Ans: (a) $\mathcal{R}_c = 3.79 \times 10^3\ \frac{\text{A}\cdot\text{turns}}{\text{Wb}}$, $\mathcal{R}_g = 4.42 \times 10^5\ \frac{\text{A}\cdot\text{turns}}{\text{Wb}}$

(b) $\phi = 9 \times 10^{-4}$ Wb

(c) $i = 0.80$ A

- When magnetic field varies with time, an electric field is produced in space

$$\oint_C \mathbf{E}\cdot\mathbf{ds} = -\frac{d}{dt}\int_S \mathbf{B}\cdot\mathbf{da}$$

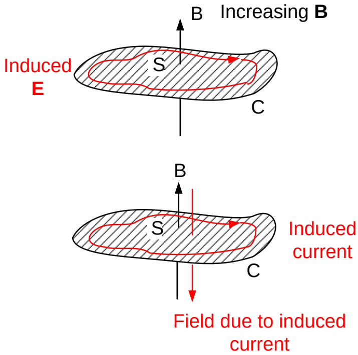
- If contour C has N turns, induced voltage e in C

$$e = N\frac{d\varphi}{dt} = \frac{d\lambda}{dt} \qquad (23)$$

where λ - flux linkage of the winding (Wb.turns)

$$\lambda = N\varphi \qquad (24)$$

φ - instantaneous value of time varying flux



- For a magnetic circuit composed of magnetic material of constant magnetic permeability or including dominating air gap,

$$L = \frac{\lambda}{i} \tag{25}$$

$$L = \frac{N.\mathrm{mmf}}{Ri} \tag{26}$$

$$L = \frac{N.Ni}{\frac{g}{\mu_0 A_g} i} = \frac{N^2 \mu_0 A_g}{g} \tag{27}$$

- Unit of inductance is Henrys (H)

Important- Above calculation is valid only for linear relationship between mmf and flux

- Total mmf $= \mathcal{F} = N_1 i_1 + N_2 i_2$
- As μ_r of core $\gg \mu_0$, the reluctance of core is neglected
- Flux linkage of coil 1, $\lambda_1 = N_1 \phi = {N_1}^2 \left(\frac{\mu_0 A_c}{g}\right) i_1 + N_1 N_2 \left(\frac{\mu_0 A_c}{g}\right) i_2$
- $\lambda_1 = L_{11} i_1 + L_{12} i_2$
- L_{11} - self inductance of coil 1, $L_{11} i_1$ - flux linkage of coil 1 due to i_1
- $L_{11} = {N_1}^2 \frac{\mu_0 A_c}{g}$
- $L_{12} = N_1 N_2 \frac{\mu_0 A_c}{g}$

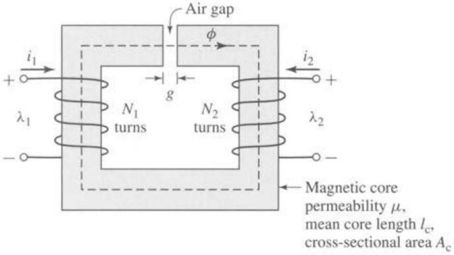
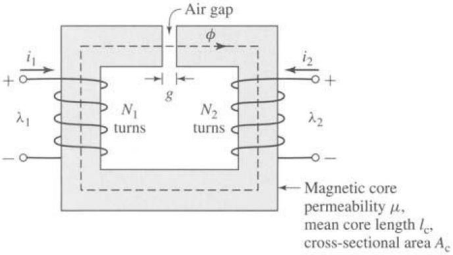


Figure 5: Magnetic circuit with two windings (Image taken from reference [5])

[5]'Electric Machinery' by Fitzgerald, Kinsley and Umans

Magnetic Circuit with Two Windings

- Total mmf $= \mathcal{F} = N_1 i_1 + N_2 i_2$
- As μ_r of core $\gg \mu_0$, the reluctance of core is neglected
- Flux linkage of coil 2, $\lambda_2 = N_2 \phi = N_1 N_2 \left(\frac{\mu_0 A_c}{g}\right) i_1 + {N_2}^2 \left(\frac{\mu_0 A_c}{g}\right) i_2$
- $\lambda_2 = L_{21} i_1 + L_{22} i_2$
- L_{22} - self inductance of coil 2, $L_{21} i_1$ - flux linkage of coil 2 due to i_1
- $L_{22} = {N_2}^2 \frac{\mu_0 A_c}{g}$
- $L_{21} = N_1 N_2 \frac{\mu_0 A_c}{g}$



Induced EMF under Time Varying Condition

- Induced emf in the winding, e

$$e = \frac{d}{dt}(Li) \tag{28}$$

- When inductance is fixed

$$e = L\frac{di}{dt} \tag{29}$$

- Electromechanical energy conversion devices- time varying inductances

$$e = L\frac{di}{dt} + i\frac{dL}{dt} \tag{30}$$

- Power p at the terminals of winding (W)

$$p = ie = i\frac{d\lambda}{dt} \tag{31}$$

- The change in stored magnetic energy ΔW in time interval t_1 to t_2

$$\Delta W = \int_{t_1}^{t_2} p\ dt = \int_{\lambda_1}^{\lambda_2} i\ d\lambda \tag{32}$$

- When inductance is constant

$$\Delta W = \int_{\lambda_1}^{\lambda_2} i\ d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{\lambda}{L}\ d\lambda = \frac{1}{2L}({\lambda_2}^2 - {\lambda_1}^2)\ \mathrm{J} \tag{33}$$

- Total magnetic energy stored at given λ with zero initial condition (i.e. $\lambda_1 = 0$)

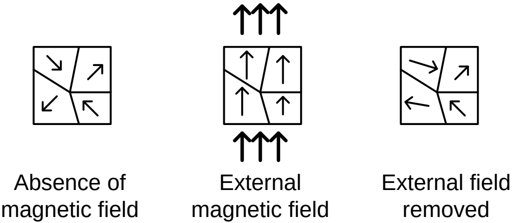
$$W = \frac{1}{2L}\lambda^2 = \frac{L}{2}i^2 \tag{34}$$

- Is it possible to obtain large flux density with small mmf? (Hint: Think of electric-magnetic circuits analogy)

Magnetic Materials

- Is it possible to obtain large flux density with small mmf? (Hint: Think of electric-magnetic circuits analogy)
- Yes, if material has high magnetic permeability
- Ferromagnetic materials - $\mu_r >> 1$
- Most commonly used ferromagnetic materials - Iron and alloys of iron
- These can also constrain and direct magnetic fields in well-defined paths

Ferromagnetism



- Absence of magnetic fields- random alignment of domain magnetic moments
- External magnetic field- Magnetic moments align along field direction
- Net field inside material- Domain magnetic moments + Applied field
- Removal of external field- Domain moments try to retain initial orientation
- Remanant magnetic field- Non-zero magnetic field in the material after external field is zero

Hysteresis

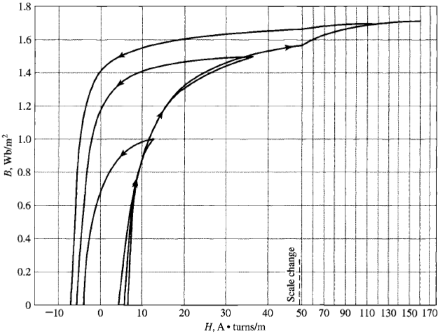
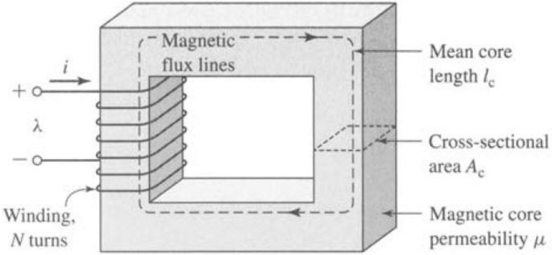


Figure 6: B-H loops of M-5 grain-oriented electrical steel 0.012 in thick (image taken from reference[6])

- Magnetic hysteresis- Remanant field in the material after external magnetic field is brought to zero
- B-H relation - non-linear and multi-valued
- B-H curve determined empirically (experiment on test samples)
- Knowledge of B-H curve (or hysteresis loop) while solving numerical problems

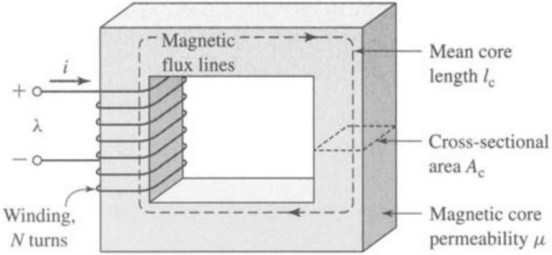
[6] 'Electric Machinery' by Fitzgerald, Kinsley and Umans

AC Excitation



- Let $\varphi(t) = \phi_{max}\sin\omega t = A_c B_{max}\sin\omega t$
  ϕ_{max} - amplitude of core flux ψ in webers
  B_{max} - amplitude of flux density in teslas
  ω - angular frequency $= 2\pi f$
  f - frequency in Hz
- Voltage induced in N turn winding, $e(t) = \omega N\phi_{max} cos(\omega t) = E_{max}\cos\omega t$
- $E_{max} = \omega N\phi_{max} = 2\pi f N A_c B_{max}$

AC Excitation



- $E_{rms} = \sqrt{2}\pi f N A_c B_{max}$
- $I_{\varphi,rms} = \frac{l_c H_{c,rms}}{N}$
- RMS volt-amperes required to excite the core to a specified flux density

$$E_{rms} I_{\varphi,rms} = \sqrt{2}\pi f B_{max} H_{rms} A_c l_c \qquad (35)$$

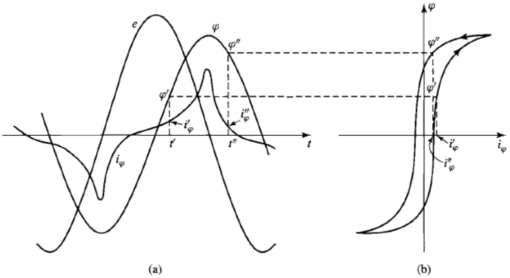


Figure 7: (a) Voltage, flux and exciting current; (b) corresponding hysteresis loop (Image taken from reference[7])

- Peaky exciting current- flattening of hysteresis loop
- Same magnitude of excitation current while rising and falling - different values of magnetic field

[7] 'Electric Machinery' by Fitzgerald, Kinsley and Umans

Losses due to Time Varying Fluxes

Two types of loss - Ohmic and hysteresis

- Ohmic loss - Induced currents in core material (eddy currents)
- Eddy currents circulate in the core material
- These oppose changes in flux density in the material
- Reduction in eddy currents- laminated core

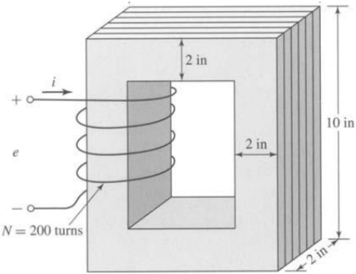


Figure 8: Laminated steel core with winding (Image taken from reference[8])

[8]'Electric Machinery' by Fitzgerald, Kinsley and Umans

- Hysteresis loss W

$$W = A_c l_c \oint H_c dB_c \qquad (36)$$

- Each cycle- net input energy to the material
- Energy required to move around the magnetic dipoles
- Hysteresis loss proportional to area of hysteresis loop (shaded area) and volume of material
- Hysteresis loss proportional to frequency of excitation

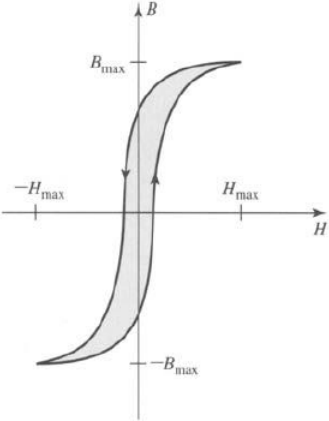


Figure 9: Hysteresis loop (Image taken from reference[9])

[9] 'Electric Machinery' by Fitzgerald, Kinsley and Umans

EE111

Lecture Notes

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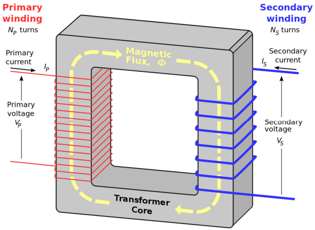
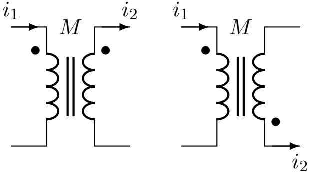
Contents

Mutual Inductance

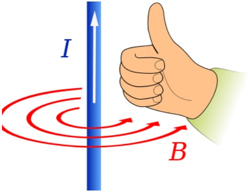
- When two circuits are magnetically connected
- Voltage induced in the second circuit is linked to time varying current in the first circuit by parameter called mutual inductance
- Coupling is generally indicated with dot markings. The dot mark basically indicates the polarity of the induced voltage and depends on how the coils are physically wound.
- Mutually Coupled coils have self and mutual inductances



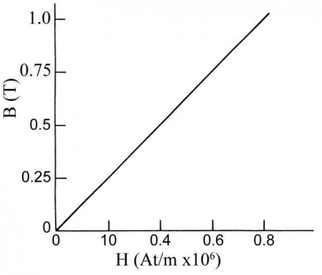


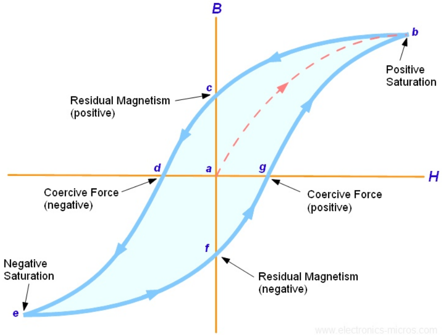
Mutual Inductance

- Consider N turn coil
- Flux lines are designated as ϕ which depends on current (i)
- Direction of flux depends on the direction of current and is obtained by Flemings Right Hand Rule
- Flux Linking Coil is designated as $\lambda = N\phi(\text{webers})$
- $\phi = \mathbf{P}Ni$. $\mathbf{P}$ is permeance of material which is magnetic property of the material. For non-magnetic material ϕvs.i is linear and for ferromagnetic materialsϕvs.i is nonlinear
- $\lambda = \mathbf{P}N^2 i \rightarrow v = \frac{d\lambda}{dt} = \mathbf{P}N^2\frac{di}{dt}$
- $L = \mathbf{P}N^2 \rightarrow$inductance
- We can show that $M = k\sqrt{L_1 L_2}$



Mutual Inductance





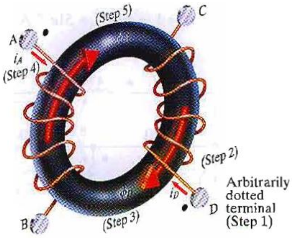
Linear (left) and Non linear (right) B-H or $\phi - i$ curves

$$
\begin{aligned}
\phi_1 &= \phi_{11} + \phi_{12} \\
\mathbf{P}_1 N_1^2 i &= \mathbf{P}_{11} N_1^2 i + \mathbf{P}_{12} N_2^2 i \\
v &= N_1^2 \left(\mathbf{P}_{11} + \mathbf{P}_{12}\right) \frac{di}{dt} \\
\frac{d\lambda_2}{dt} &= N_1 N_2 \mathbf{P}_{12} \frac{di}{dt} \\
L &= N_1^2 \left(\mathbf{P}_{11} + \mathbf{P}_{12}\right) \\
M &= N_1 N_2 \mathbf{P}_{12}
\end{aligned}
$$

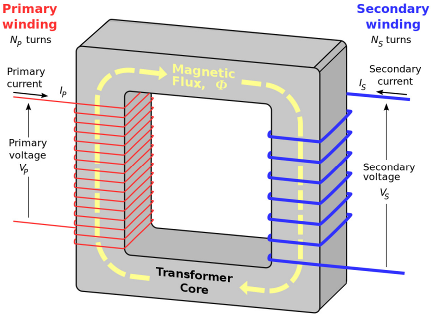
$$
\begin{aligned}
\mathbf{P}_{21} &= \mathbf{P}_{12} \\
L_1 L_2 &= N_1^2 N_2^2 \mathbf{P}_1 \mathbf{P}_2 \\
L_1 L_2 &= \left(N_1 N_2 \mathbf{P}_{12}\right)^2 \\
&\quad \left(1 + \frac{\mathbf{P}_{11}}{\mathbf{P}_{12}}\right)\left(1 + \frac{\mathbf{P}_{22}}{\mathbf{P}_{12}}\right) \\
L_1 L_2 &= \frac{M^2}{k^2} \\
M &= k\sqrt{L_1 L_2} \qquad (1) \\
0 \leq\, & k \leq 1
\end{aligned}
$$

Use this process instead of the detailed discussion done in the class.

- Arbitrarily select a terminal of one of the coils and mark it with a dot.
- Define current entering into the terminal and find direction of the flux using right hand rule.
- Arbitrarily pick one terminal of the second coil and define current entering in the coil.
- Find the direction of the flux produced by current in the second coil.
- Compare the directions of the two fluxes. If the fluxes have the same reference direction, place a dot on the terminal of the second coil where the test current enters. If the fluxes have different reference direction place a dot on the terminal of the second coil where the test current leaves.

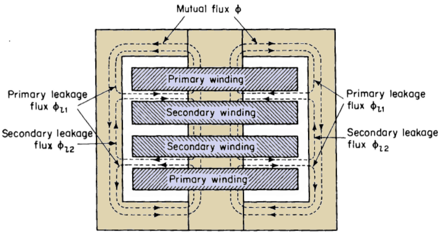


- The main components of a transformer are the windings and a core. It basically consists of mutually coupled coils and the coupling occurs through the core.
- There are two general types of transformer construction – Shell type and Core type.
- They differ in the manner in which the windings are wound around the core.



Shell type Transformers

- The core limbs surround the windings.
- The windings are divided and wound over the central limb in an interleaved manner.
- The flux in the central limb divides equally and returns through the outer limbs.
- They are used in low voltage low power applications.



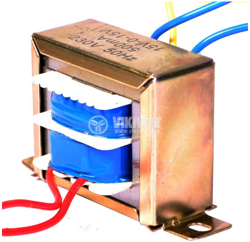


Figure: Shell type Transformer

Construction of Transformers

Core type Transformers

- The windings suround the core. They are are divided into half and wound in a concentric manner.
- The LV coil is placed adjacent to the steel core, and the HV winding is placed outside it.
- Lesser insulation and iron requirements.
- They are used for high voltage high power applications.

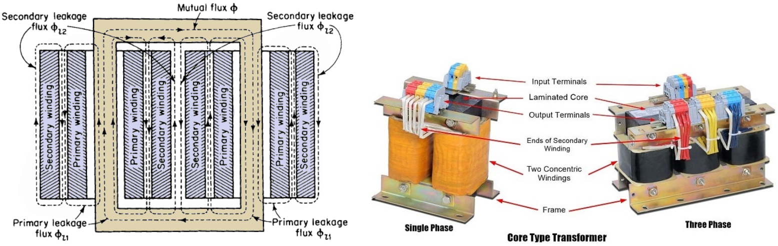


Figure: Core type Transformer

Construction of Transformers

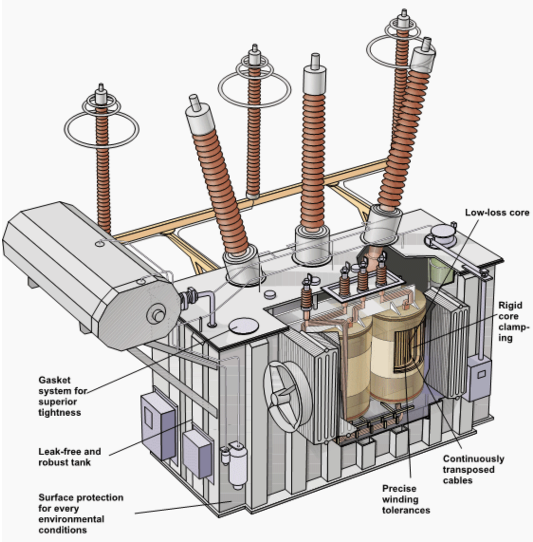
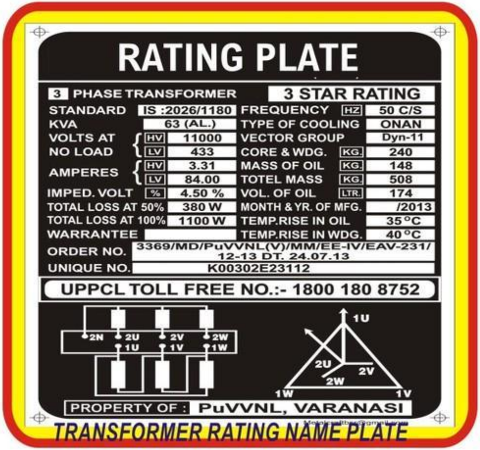


Figure: Internal view of a transformer



Transformer windings (l) and name plate of a transformer (r)

Construction of Transformers

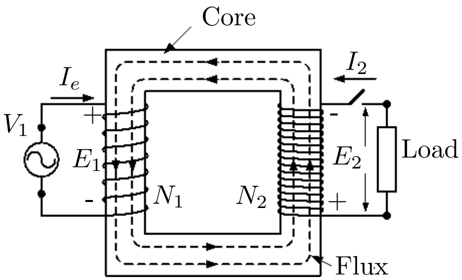
Size Comparison : A worker next to a high voltage power transformer

Principle of Transformer Action

The schematic of the transformer is shown in Figure.

For analysis, the transformer is considered as ideal, meaning –

- Winding resistances are negligible
- All the magnetic flux is confined to the core
- There are no losses in the machine.



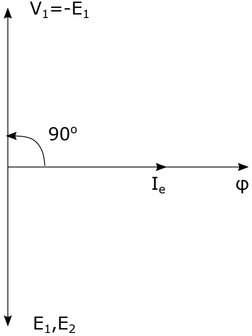


Figure: Ideal Transformer (left) and Phasor Diagram (right)

Primary Winding

- Initially, we assume that the secondary side is open.
- Sinusoidal V_1 is applied and I_e flows in the primary side.
- Since V_1 and I_e are sinusoidal, flux ϕ is also sinusoidal.
- $\phi = \phi_{max} sin(\omega t)$ is set up in the primary side coil.
- Due to alternating ϕ, emf induced in the primary coil will have a polarity such as to oppose the cause, I_e as per Lenz's law.

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \omega \phi_{max} cos(\omega t) = E_{1max} sin(\omega t - \frac{\pi}{2})$$

, where $E_{1max} = 2\pi \mathrm{f} N_1 \phi_{max}$.

- The primary winding acts as load for the source, V_1, and so the polarity of E_1 will be as shown in Figure.
- RMS value of primary winding emf $E_1 = \sqrt{2}\pi f N_1 \phi_{max}$.

Secondary Winding

• The alternating flux also induces a voltage on secondary side.

$$e_2 = -N_2\frac{d\phi}{dt} = -N_2\omega\phi_{max}cos(\omega t) = E_{2max}sin(\omega t - \frac{\pi}{2})$$

where $E_{2max} = 2\pi \mathrm{f} N_2\phi_{max}$.

• Now, if the secondary side is connected to a load, current flows in the secondary winding.

• Current direction must be such that it opposes the core flux.

• Thus, the current will flow from bottom to top through load.

• The secondary winding acts as a source for the load. Hence, the polarity of E_2 will be as shown in Figure.

• RMS value of secondary winding emf $E_2 = \sqrt{2}\pi f N_2\phi_{max}$.

• The winding may be wound in such a way that '+' terminal goes to the top.

Effect of Secondary Side Current

- When load current flows in the secondary side, a mmf will be established in the secondary winding.
- It tends to oppose (reduce) main flux (by Lenz's Law).
- If flux reduces, E_1 will reduce. However, if primary side resistance is neglected, $|E_1| = |V_1|$ (source voltage), and V_1 will not change.
- Hence any change in E_1 is countered by increasing the primary current (say $I_1^{'}$), such that

$$\text{Primary MMF } N_1 I_1^{'} = \text{Secondary MMF } N_2 I_2$$

Turns Ratio and Leakage Flux

$$E_1 = \sqrt{2}\pi f N_1 \phi_{max} \qquad E_2 = \sqrt{2}\pi f N_2 \phi_{max}$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \sqrt{2}\pi f \phi_{max} \therefore \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

Turns Ratio

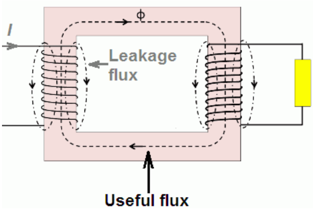
The quantity $\frac{N_1}{N_2}$ is termed as turns ratio.
Hence, the primary and secondary voltages are related by the turns ratio.

Leakage Flux

In iron core transformers, some flux leaks into the non-magnetic material surrounding the core, such as air, insulation, etc.
This flux is wasted and it decreases the amount of useful flux.
It is desirable to reduce this leakage flux by appropiate construction and winding placement.

Leakage Inductance

- Leakage flux in effect represents a reduction in actual voltage producing the main flux.
- This is modelled as an inductive voltage drop. It is inductive since the voltage drop across it produces the leakage flux.
- Thus, this inductance is termed leakage inductance.



Magnetizing Inductance

- It is used to model the main flux producing current.
- E_1 lags the flux producing current I_e by $90°$. Also, $E_1 = -V_1$.
- Hence, V_1 leads I_e by $90°$.
- Thus, I_e, in effect, flows through an equivalent reactance X_m, such that $I_e = \frac{V_1}{X_m}$.
- This reactance X_m is termed as the magnetizing inductance.

Equivalent Circuit

The equivalent circuit of any machine or device is useful to understand its performance and analyse its behaviour.

- The equivalent circuit of a practical transformer deviates from that of an ideal one in many ways.
- Starting from the circuit of an ideal transformer, actual equivalent circuit can be arrived at in a stepwise manner.

Developing transformer equivalent circuit

- Winding resistances, which were previously neglected, are now considered for both primary and secondary sides.
- They lead to voltage drops in both the sides.
- Voltage drops across the leakage reactances in both primary and secondary sides are also included.

Equivalent Circuit

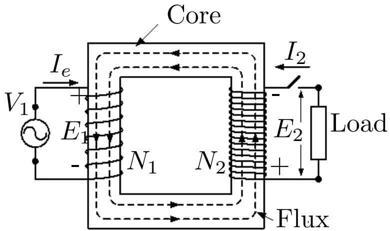


Figure: Idealised transformer circuit

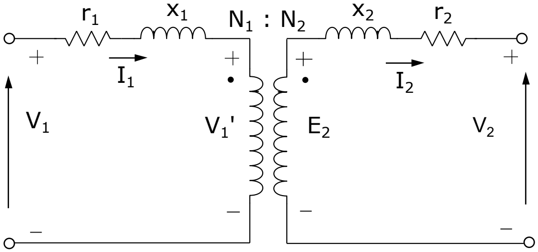


Figure: Equivalent circuit with Exciting Current neglected

Equivalent Circuit

- The primary current can be divided into two parts.
  - Load component $I_1{}'$ which counteracts secondary mmf.
  - No load or excitation current I_e.
  - Excitation current again consists of
    —Core loss component I_c, representing core losses.
    —Magnetizing component I_m.
- Modelling these primary current components gives the exact equivalent circuit.

Modelling Core loss component I_c

- Core loss occurs in the transformer core and causes heating. It depends on applied voltage.
- It can be modelled as a resistive loss.
- Hence, R_c in parallel with V_1 represents core loss given by
$$P_c = \frac{{V_1}^2}{R_c}.$$

Equivalent Circuit

Modelling magnetizing component I_m

- This is the component of I_e (and not I_e itself as previously shown for convenience) that produces the main flux.
- I_m flows through magnetizing reactance X_m placed in parallel to applied voltage.

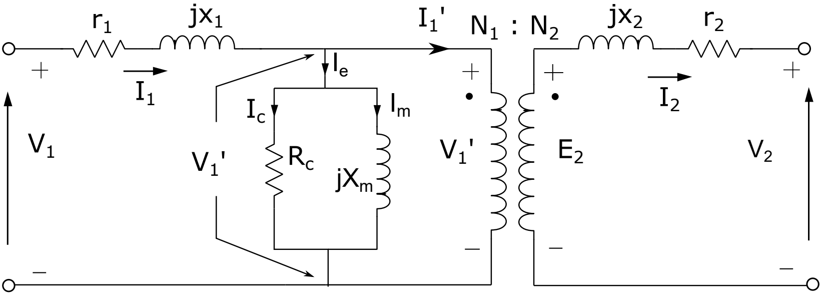


Figure: Exact Equivalent circuit

Equivalent Circuit

Transferring quantities across sides

- Quantities (such as resistance, current, etc.) can be transferred across sides.
- Example – secondary drop $I_2 r_2$ can be transferred by

$$\text{Secondary resistive drop transferred to primary} = (I_2 r_2)\frac{N_1}{N_2}$$
$$= (I_1 \frac{N_1}{N_2} r_2)\frac{N_1}{N_2} = I_1 {r_2}', \text{ where } {r_2}' = r_2 \frac{{N_1}^2}{{N_2}^2}$$

- Thus secondary resistive drop is transferred to primary by placing a resistance ${r_2}'$.
- Here, ${r_2}'$ is called secondary resistance referred to primary.
- Therefore, effective primary resistance $r_{e1} = r_1 + {r_2}'$.
- Similarly, primary to secondary transformation can be done.
- The leakage reactance can also be transferred in the same way.

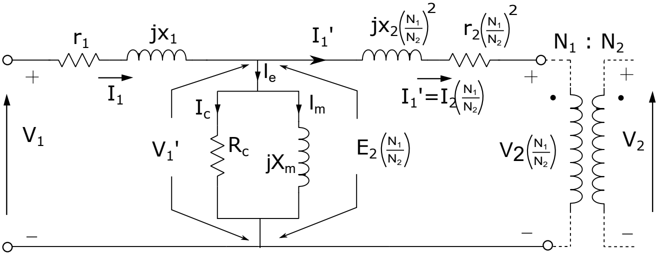


Figure: Equivalent circuit referred to Primary

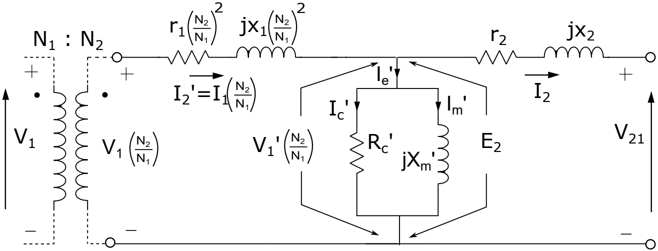


Figure: Equivalent circuit referred to Secondary

Phasor Diagram

The phasor diagram of the equivalent circuit referred to primary can be drawn in stepwise manner.
The magnetizing current I_m is taken as reference.
There are 2 general cases : output current I_2 $(= I_1^{'})$ either **lags** or **leads** output voltage $V_2^{'}$ depending upon load conditions.

I_2 lags $V_2^{'}$

- Secondary induced emf E_2 leads I_m by 90° ($\because I_m$ lags $V_1^{'}$, and $E_2 = V_1^{'}$).
- Voltage drops $I_2 r_2^{'}$ and $jI_2 x_2^{'}$ added to $V_2^{'}$ gives E_2.
- $I_c = \frac{V_1^{'}}{R_c}$ will be in phase with $V_1^{'}$.
- Phasor sum of I_m and I_c gives I_e.
- Phasor sum of I_e and I_2 gives I_1.
- Finally, adding the drops $I_1 r_1$ and $jI_1 x_1$ to $V_1^{'}$ gives V_1.

I_2 leads $V_2{}'$

Similar process can be followed when I_2 leads $V_2{}'$.

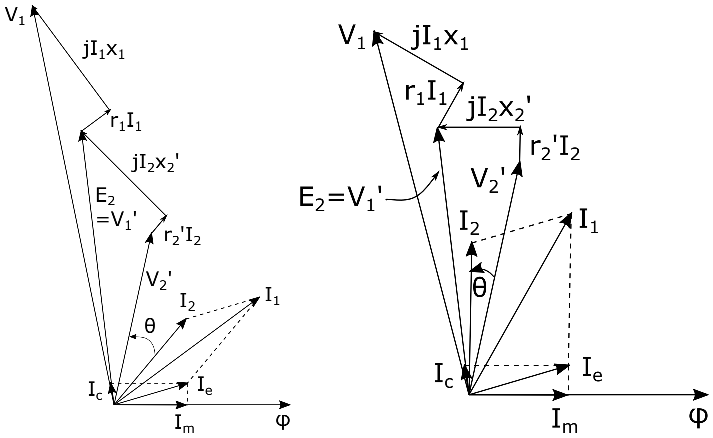


Figure: Phasor Diagrams (left) lagging I_2 (right) leading I_2

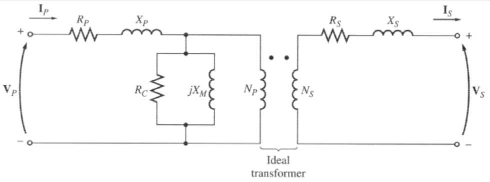
Main parameters of the equivalent circuit

- R_{01} as referred to primary (or secondary R_{02})
- The equivalent leakage reactance X_{01} as referred to primary (or secondary X_{02})
- Magnetising susceptance B_0 (or reactance X_0)
- core loss conductance G_0 (or resistance R_o)

To be determined by 2 tests

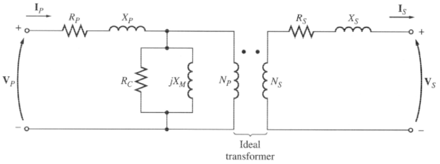
The above constants can be easily determined by two tests

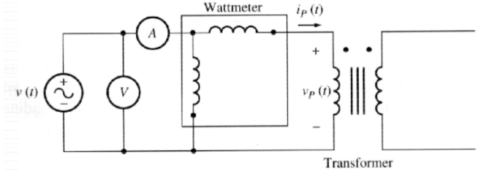
- Open circuit tests (O.C. test/No Load test)
- Short Circuit tests (S.C. test/Impedance test)

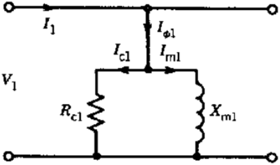


Circuit Parameters : Open-Circuit Test

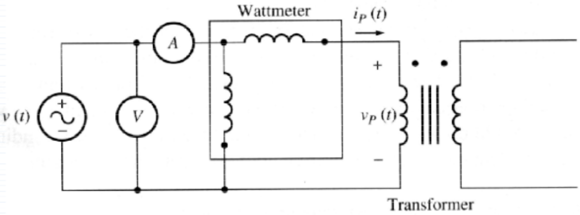
- Transformer's secondary winding is open-circuited.
- Primary winding is connected to a full-rated line voltage. All the input current must be flowing through the excitation branch of the transformer.
- The series elements R_p and X_p are too small in comparison to R_C and X_M to cause a significant voltage drop, so essentially all the input voltage is dropped across the excitation branch.
- Input voltage, input current, and input power to the transformer are measured.

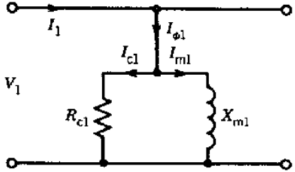






Circuit Parameters : Open-Circuit Test





The magnitude of excitation admittance

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$

The open-circuit power factor (we can calculate power facter angle accordingly)

$$PF = cos(\theta) = \frac{P_{OC}}{V_{OC} I_{OC}}$$

The power factor is always lagging for a transformer, so the current will lag the voltage by the angle q. Therefore, the admittance Y_E is:

$$Y_E = \frac{1}{R_C} = j\frac{1}{X_M} = \frac{I_{OC}}{V_{OC}} \angle - cos^{-1}(PF)$$

Therefore, it is possible to determine values of R_C and X_M in the open-circuit test.

Short-Circuit Test

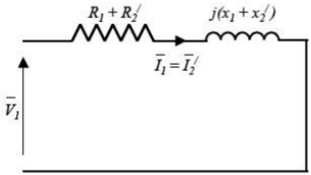
Fairly low input voltage is applied to the primary side of the transformer. This voltage is adjusted until the current in the secondary winding equals to its rated value.
The input voltage, current, and power are again measured.

Since the input voltage is low, the current flowing through the excitation branch is negligible; therefore, all the voltage drop in the transformer is due to the series elements in the circuit. The magnitude of the series impedance referred to the primary side of the transformer is:

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

The power factor of the current is given by:

$$PF = cos(\theta) = \frac{P_{SC}}{V_{SC}I_{SC}}$$



Therefore :

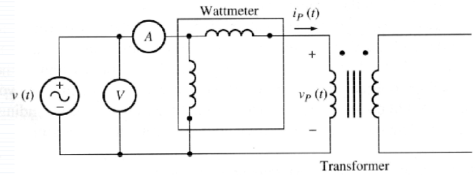
$$Z_{SE} = \frac{V_{SC}\angle 0^o}{I_{SC}\angle -\theta^o} = \frac{V_{SC}}{I_{SC}}\angle\theta^o$$

Since the serial impedance Z_{SE} is equal to:

$$Z_{SE} = R_{eq} + jX_{eq}$$

$$Z_{SE} = (R_P + a^2R_S) + j(X_P + a^2X_S)$$

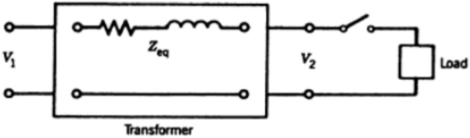
It is possible to determine the total series impedance referred to the primary side of the transformer. However, there is no easy way to split the series impedance into primary and secondary components.
The same tests can be performed on the secondary side of the transformer.
The results will yield the equivalent circuit impedances referred to the secondary side of the transformer.



$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

$$PF = cos(\theta) = \frac{P_{SC}}{V_{SC}I_{SC}}$$

At no-load, $V_{2|NL} = \dfrac{V_1}{a}$
After closing the switch,
$V_{2|L} = V_{2|NL} \pm \Delta V_2$



Definition

Because a real transformer has series impedance within it, the output voltage of a transformer varies with the load even if the input voltage remains constant. To compare transformers in this respect, the quantity called a full-load voltage regulation (VR) is defined.

The voltage regulation of a transformer is the change in the magnitude of the secondary terminal voltage from no-load to full-load.

$$VR = \frac{V_{s,nl} - V_{s,\beta}}{V_{s,\beta}}.100\% = \frac{V_p/a - V_{s,\beta}}{V_{s,\beta}}.100\%$$

where, $V_{s,nl}$ and $V_{s,fl}$ are the secondary no load and full load voltages. Note that VR of an ideal transformer is zero.

Transformer Inrush Current

The transformer inrush current or inrush of magnetizing current is the maximum instantaneous current drawn by the primary of the transformer when their secondary is open circuit. During the inrush current, the maximum value attained by the flux is over twice the normal flux.

The steady-state magnetizing current for a transformer is very low, but the momentary current when it is first energized can be quite high.

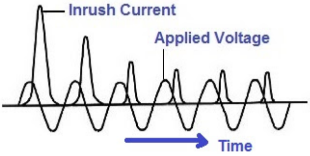
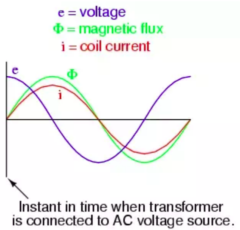
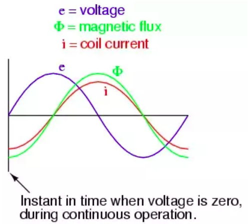


Figure: Transformer Inrush Current

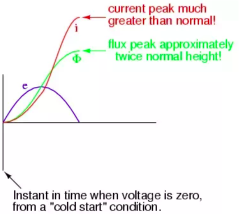
Inrush Current



When connected when voltage is at its positive peak, magnetic flux of rapidly increasing value must be generated. The result is that winding current increases rapidly, but actually no more rapidly than under normal conditions.



When connected at voltage equal to zero, both flux and winding current are at its negative peak. As voltage builds to its peak, flux and current build to maximum rate of change and on upwards till voltage descends to zero.



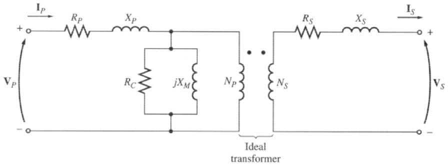
When a transformer is taken off-line, a certain amount of residual flux remains in the core. When voltage is reapplied, the flux introduced builds upon that already existing in the core. In order to maintain the level, the transformer can draw current well in excess of the transformers rated full-load current (Inrush Current).

Transformer Efficiency

The efficiency of a transformer is defined as,

$$\eta = \frac{P_{out}}{P_{in}}.100\% = \frac{P_{out}}{P_{out} + P_{loss}}.100\%$$

Note : The same equation describes the efficiency of motors and generators.



Considering the transformer equivalent circuit, we notice 3 types of losses:

- Copper losses (I^2R) - are accounted for by the series resistance.
- Hysteresis losses - are accounted for by the resister R_c.
- Eddy current losses - are accounted for by the resistor R_c.

Since the output power is, $P_{OUT} = V_s I_s cos(\theta_s)$

The transformer efficiency is, $\eta = \dfrac{V_s I_s cos(\theta)}{P_{Cu} + P_{core} + V_s I_s cos(\theta)}.100\%$

EE 111

Lecture Notes: Induction Machine

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Contents

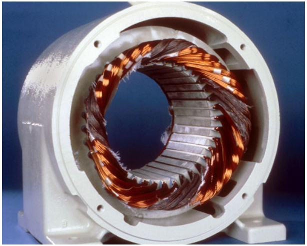
Introduction

- Induction Machine was invented by *Nikola Tesla* in 1888
- Induction Machine is an AC electro-mechanical energy conversion device
- Induction Motors are the most common and frequently encounterd machine in industry
- Available in a wide range of power ratings (from fractional kW to few MW)
- Simple design, rugged construction, less maintenance, reliable operation

Construction

Induction machine has two main parts

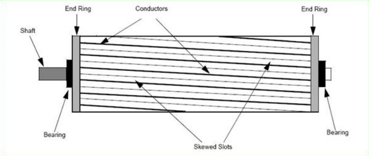
- Stator
  - Stationary Part
  - Windings held in laminated steel core, enclosed and supported by cast iron/steel frame

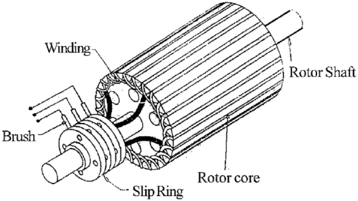


- Rotor
  - Rotating Part
  - Composed of stacked laminations to form slots for housing rotor windings

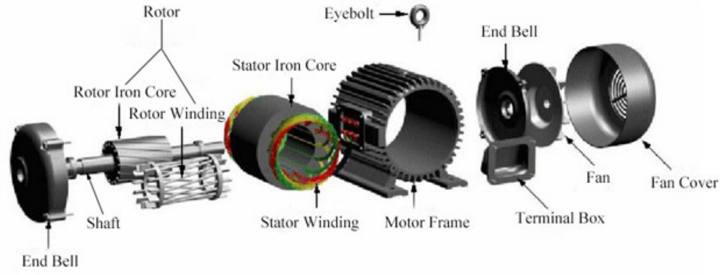
Based on rotor construction IM is classified into

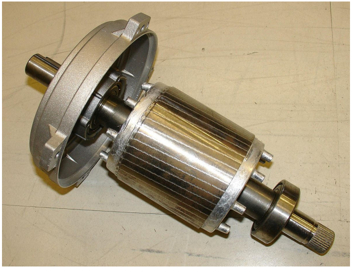
- Squirrel Cage
  - Conducting bars kept in the rotor slots and shorted at both ends by endrings
- Wound Rotor
  - 3 phase windings similar to that of stator are wound in the rotor slots
  - Winding ends are connected to 3 slip rings and brushes on the rotor shaft

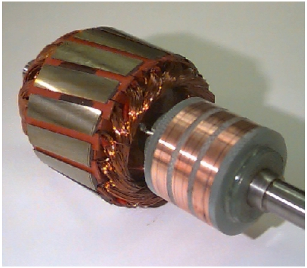




Construction Contd..







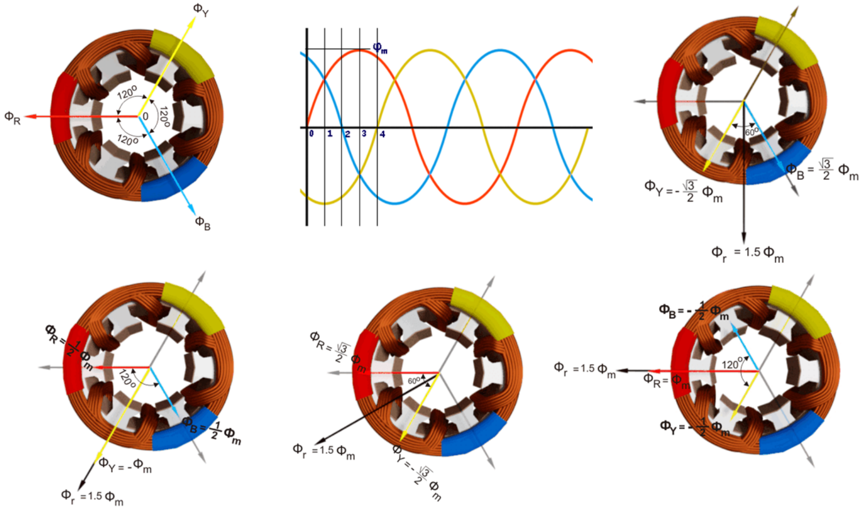
Rotating Magnetic Field

- When 3 phase balanced winding is connected to a 3 phase voltage supply
- 3 phase current will flow in the windings, which will induce 3 phase flux
- These flux will rotate at a speed called a Synchronous Speed, N_s creating a Rotating Magnetic Field

$$N_s = \frac{120f}{P} \tag{1}$$

where f = supply frequency
P = number of poles

Rotating Magnetic Field



Rotating Magnetic Field

The magnetic flux produced by the current in each phase can be represented by the equations given below

$$\phi_R = \phi_m sin(\omega t) \tag{2}$$

$$\phi_Y = \phi_m sin(\omega t - 120^\circ) \tag{3}$$

$$\phi_B = \phi_m sin(\omega t - 240^\circ) \tag{4}$$

At any instant of time the magnitude of resultant flux will be $\phi_r = 1.5\phi_m$ e.g. at $\omega t = 0$

$$\phi_R = 0 \tag{5}$$

$$\phi_Y = -\frac{\sqrt{3}}{2}\angle{-120^\circ} \tag{6}$$

$$\phi_B = \frac{\sqrt{3}}{2}\angle{-240^\circ} \tag{7}$$

So resultant flux vector will be

$$\phi_r = \phi_R + \phi_Y + \phi_B = 1.5\phi_m\angle 90^\circ$$

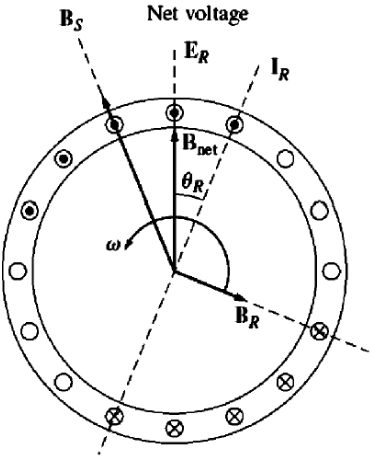
Torque Generation

- When A rotating magnetic field (RMF) B_s is produced when the stator winding is given 3 phase supply
- This RMF cuts the rotor windings and produces an induced voltage in the rotor windings according to ***Faraday's law of electromagnetic induction***
- As the rotor windings are short-circuited rotor current will flow producing a magnetic field B_r
- Due to the interaction between this two magnetic fields rotor experiences a torque which is given by

$$\tau_{ind} = kB_r \times B_s \tag{8}$$

- This torque will try to rotate the rotor in the direction of RMF

Torque Generation



Due to the torque experienced by the rotor it will rotate at speed N_r lesser than the synchronous speed.

Concept of Slip

- Both the rotor and stator magnetic fields B_r and B_s rotates together at synchronous speed N_s
- But the rotor rotates at a slower speed.
- Voltage induced in the rotor windings depends on the relative speed between rotor and magnetic fields
- This relative speed is termed as slip speed and is defined by the equation given below

$$slip\ speed\ (n_{slip}) = N_s - N_r \tag{9}$$

- Slip speed expresssed on a perunit or percentage basis is call slip

$$slip = s = \frac{N_s - N_r}{N_s} \tag{10}$$

Concept of Slip Contd..

- The rotor can be expressed in terms of slip and synchronous speed.

$$rotor\ speed\ N_r \ = \ N_s\,(1-s) \qquad (11)$$

- The working principle of induction motor is similar to that of transformer. So it is sometimes called as rotating transformer
- Unlike transformer the rotor (secondary) frequency is not same stator (stator) frequency.
- The frequency of induced current or voltage in the rotor depends on the relative speed between rotor windings and magnetic fields and is given by

$$rotor\ frequency\ f_r \ = \ \frac{PN}{120} \qquad (12)$$

where relative speed, $N = N_s - N_r = sN_s$
So now equation 12 becomes

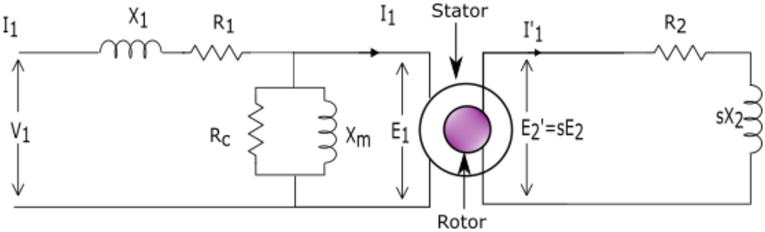
$$rotor\ frequency\ f_r = \frac{sPN_s}{120} \tag{13}$$

But we know that $\dfrac{PN_s}{120} = f$
So equation 13 can be rewritten as

$$rotor\ frequency\ f_r = sf \tag{14}$$

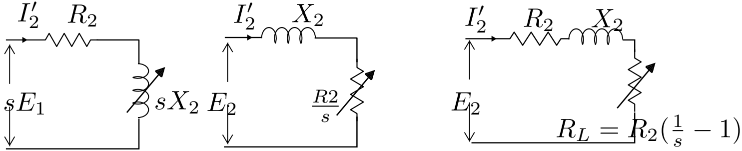
i.e. Rotor Current Frequency = Fractional Slip x Supply Frequency

- The equivalent circuit of induction motor is similar to that of transformer.
- Only difference is the variable frequency on secondary (rotor) side of induction motor equivalent circuit.



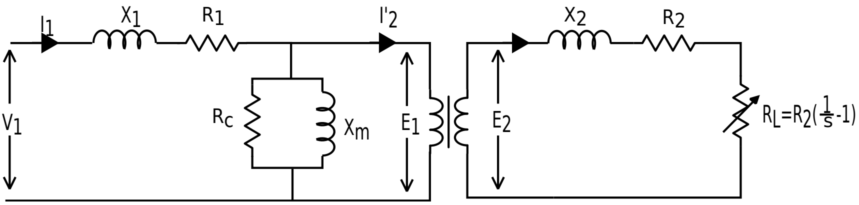
Equivalent Circuit and Phasor Diagram

- All of the rotor effects due to varying rotor speed can be considered to be caused by a varying impedance supplied with power from a constant voltage source.
- All the slip effects in rotor equivalent circuit concentrated in resistor.

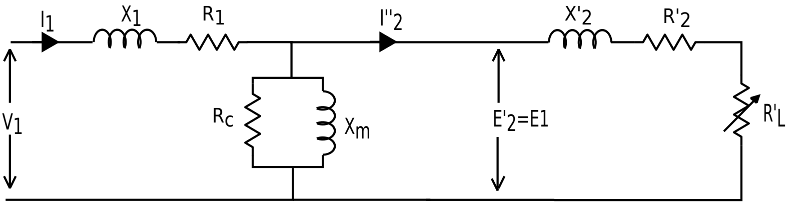


- Finally total resistance divided into parts representing rotor copper loss and mechanical power developed.
- $R_L = R_2(\frac{1}{s} - 1)$ represents the mechanical power developed.

Equivalent Circuit and Phasor Diagram

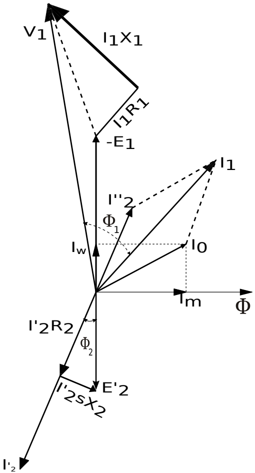


- The final and complete equivalent circuit of induction motor is as shown below.



Equivalent Circuit and Phasor Diagram

- The phasor diagram of induction motor based on its equivalent circuit is given below.



Torque Vs. Slip/Speed

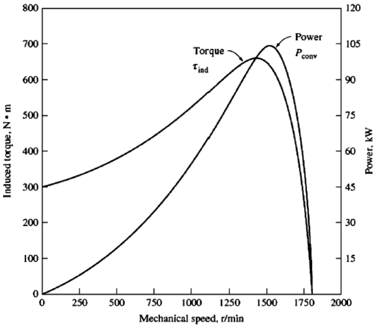
- Power input to stator is $P_{in} = 3V_1 I_1 cos\phi$
- After subtracting stator Cu loss and core loss, airgap power or rotor input is $P_{AG} = P_{in} - P_{SCL} - P_{core}$
- But $P_{AG} = \dfrac{3(I_2'')^2 R_2'}{s}$
- Rotor Cu loss $P_{RCL} = 3(I_2'')^2 R_2'$
- After rotor Cu loss remaining power converted to mechanical form $P_{conv} = P_{AG} - P_{RCL}$
$$= \frac{3(I_2'')^2 R_2'(1-s)}{s}$$
- $P_{RCL} = sP_{AG}$
- $P_{conv} = (1-s)P_{AG}$
- If friction and windage loss and stray losses are there then final output power is $P_{out} = P_{conv} - P_{F\&W} - P_{misc}$

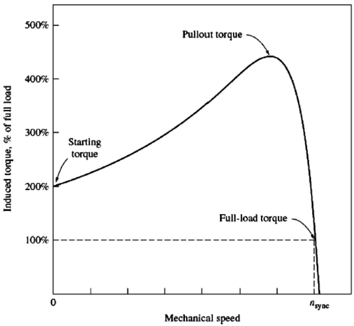
Torque Vs. Slip/Speed

- The mechanical power is converted into torque. Developed gross torque $\tau_g = \dfrac{P_{conv}}{\omega_m} = \dfrac{60P_{conv}}{2\pi N_r}$
- But $P_{conv} = \dfrac{3(I_2^{''})^2 R_2^{'}(1-s)}{s}$ and $N_r = (1-s)N_s$
- So gross torque $\tau_g = \dfrac{\dfrac{3(I_2^{''})^2 R_2^{'}}{s}}{\dfrac{2\pi N_s}{60}} = 9.55\dfrac{3(I_2^{''})^2 R_2^{'}}{sN_s}$ N - m
- After subtracting torque required to overcome friction and windage loss we get shaft torque or useful torque. So $\tau_{shaft} = \tau_g - \tau_{F\&W}$
- Under running condition the torque developed in terms of equivalent circuit parameters is $\tau_g = \dfrac{3}{2\pi N_s} \times \dfrac{s(E_2)^2 R_2}{R_2^2 + (sX_2)^2}$

Torque Vs. Slip/Speed

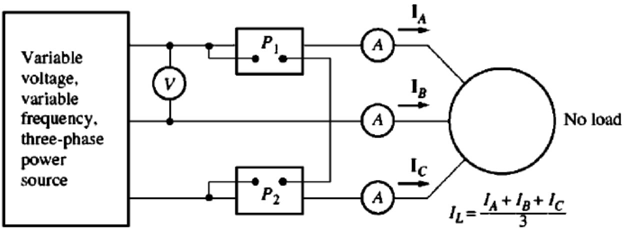
The torque-slip characteristics is given below.





No Load Test

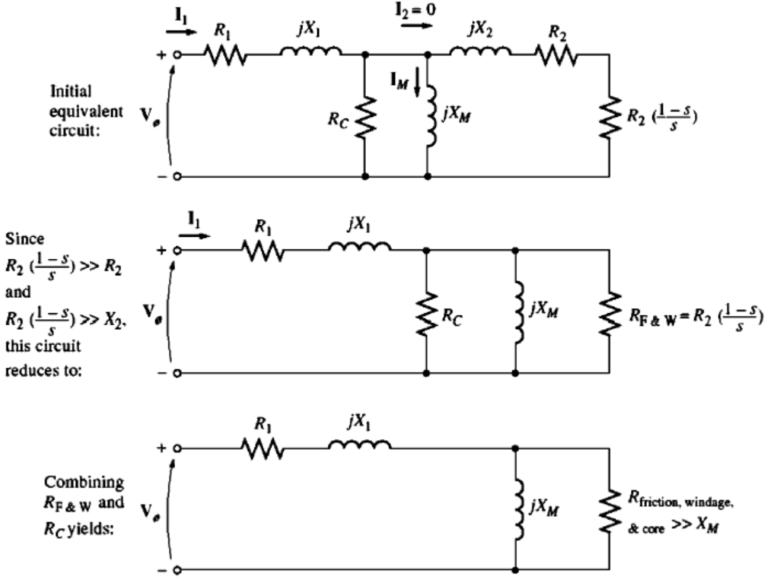
- The no load test of IM is similar to OC test of transformer.
- It gives the magnetizing branch parameters of equivalent circuit and also helps to measure the rotational losses
- rotor current and slip is very small. Rotor Cu loss can be neglected. So total input power $P_{in} = P_{SCL} + P_{rot}$
- The rotational loss $P_{rot} = P_{core} + P_{F\&W} + P_{misc}$



No Load Test

- If rotor resistance, stator resistance, magnetizing resistance (very high) can be neglected then entire supply voltage assumed to be dropped across inductiv elements.
- It gives the magnetizing branch parameters of equivalent circuit and also helps to measure the rotational losses
- $Z_{eq} = \dfrac{V_\phi}{I_{1-NL}} = X_1 + X_m$
- If all Cu losses and series impedance drop are neglected then the input power assumed to be total core loss.
- The supply appears across the magnetizing branch.
- So $I_{1-NL} = \dfrac{V_\phi}{R_c} + \dfrac{jV_\phi}{X_m}$
- Power drawn is given by $P_s = \dfrac{V_\phi^2}{R_c} \implies R_c = \dfrac{V_\phi^2}{P_s}$

No Load Test

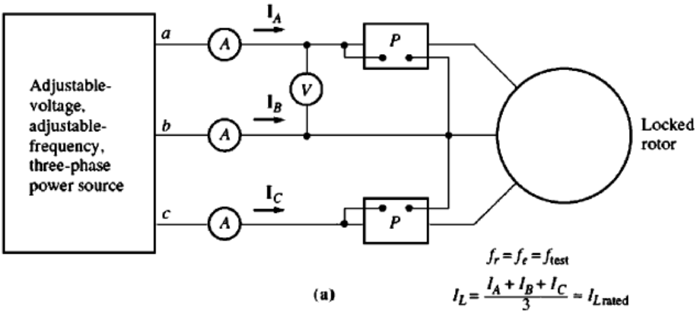


Initial equivalent circuit:
I_1
R_1
jX_1
$I_2 = 0$
jX_2
R_2
I_M
V_ϕ
R_C
jX_M
$R_2\left(\frac{1-s}{s}\right)$
Since $R_2\left(\frac{1-s}{s}\right) >> R_2$ and $R_2\left(\frac{1-s}{s}\right) >> X_2$, this circuit reduces to:
I_1
R_1
jX_1
V_ϕ
R_C
jX_M
$R_{F\,\&\,W} = R_2\left(\frac{1-s}{s}\right)$
Combining $R_{F\,\&\,W}$ and R_C yields:
R_1
jX_1
V_ϕ
jX_M
$R_{\text{friction, windage, \& core}} >> X_M$

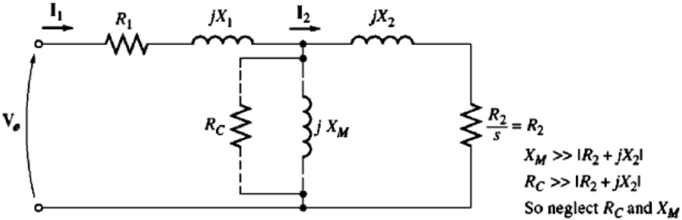
Blocked Rotor Test

- The blocked rotor test of IM is similar to SC test of transformer and gives the circuit parameters of IM
- The rotor is blocked so that it can't rotate, while a small voltage is applied only to flow rated current.
- Rotor is not moving. So slip $s = 1$. The magnetizing branch can be assumed open circuited due to large impedance compared to series impedances.
- Input stator current $I_1 = \dfrac{V_\phi}{(R_1 + R_2) + j(X_1 + X_2)}$
- Total input power $P_s = |I_1|^2(R_1 + R_2)$
- This test gives only series combination values not individual stator and rotor parameters.

Blocked Rotor Test



(a)



(b)

Speed Control of Induction Machine

- The speed of an IM can be changed by two ways
  - Changing the synchronous speed
  - Changing the slip of machine
- The synchronous speed is given by $N_s = \frac{120f}{P}$
- So it can be changed by 2 ways
  - Changing the frequency
  - Changing the poles of the machine
- The slip control can be achieved by rotor resistance control, terminal voltage control etc.
- Also in solid state IM drives both input frequency and voltage can be controlled by pulse width modulation to vary the speed.
- Speed Control can also be achieved by cascaded control, in which two machine are connected to a common shaft.
- The slip ring output of one machine is given to the stator of other machine